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Conductivity in a disordered one-dimensional system of interacting fermions

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Dynamical conductivity in a disordered one-dimensional model of interacting fermions is studied numerically at high temperatures and in the weak-interaction regime in order to find a signature of many-body localization and vanishing dc transport coefficients. On the contrary, we find in the regime of moderately strong local disorder that the dc conductivity σ_0 scales linearly with the interaction strength while being exponentially dependent on the disorder. According to the behavior of the charge stiffness evaluated at the fixed number of particles, the absence of the many-body localization seems related to an increase in the effective localization length with the interaction.

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The interplay of correlations and disorder in fermionic systems is one of the challenging open questions in the solid-state physics. The phenomenon of Anderson localization of single-electron eigenstates[1] is by now well understood in systems of noninteracting (NI) fermions. In particular, in one-dimensional (1D) systems all states become localized[2] for arbitrary small disorder[3] and hence there is no dc linear transport response at any temperature $T \geq 0$. However, it had been long ago realized[4] that correlations among electrons as introduced via Coulomb electron-electron repulsion could qualitatively change transport properties of the system.

So far, firm results and conclusions have been reached for the $T=0$ ground state of 1D tight-binding fermionic system with a diagonal Anderson disorder. In particular, it has been shown by the density-matrix renormalization-group (DMRG) numerical studies[5,6] that in spite of correlations the many-body (MB) states remain localized, preventing the dc transport. The $T>0$ behavior appears to be much harder to deal with[7,8] and, at present, the existence of MB localization beyond the ground state is controversial.[9]

Since a finite-temperature phase transition[10,11] between the MB insulator at $T<T^*$ and a conductor at $T>T^*$ implies a qualitative change in character of MB states across the eigenspectrum[11] it is as relevant and highly nontrivial to study systems at high $T\to\infty$.[12] In this context, recent studies of energy-level statistics,[12] the effective hopping in the configuration space[13] and the decay of correlation functions[14] indicate a possible MB localization at very large disorder strength W.[13] The conclusions from the scaling analysis of the conductivity of such models appear similar,[15] as well as the time evolution and the entanglement of wave functions is concerned.[16]

On the other hand, recent direct numerical evaluation of the $T>0$ transport coefficients in disordered anisotropic XXZ model[17] (model being equivalent in 1D to a tight-binding fermionic system with nearest-neighbor interaction) does not show any indication of a crossover to a MB localization at low T or at larger W. This questions the conductor-insulator phase diagram and the relation to above mentioned studies.

Our aim is to extend previous numerical study[17] of transport properties of the 1D disordered system, modeled by the t-V model of spinless fermions, in order to explore the phase diagram at high T with respect to the dc conductivity σ_0. In contrast to most previous works, in which the interaction strength $\Delta = V/(2t)$ has been mainly kept fixed and possible MB localization has been considered at large disorder values W, we start with a disordered system of NI electrons ($\Delta=0$), characterized by the vanishing dc transport at all T, i.e., $\sigma_0=0$. By increasing gradually, at fixed W, the repulsive interaction $\Delta>0$ we monitor a possible conductor-insulator transition in σ_0. Dealing with a finite-size system, instead of a singular behavior we expect that the conductor-insulator transition should manifest itself as a crossover in σ_0 vs Δ. This crossover can be then used as a signature of a qualitative (gradual or abrupt) change in MB states with respect to the dc transport in the thermodynamic limit $T\to\infty$.

As the prototype model for the interplay of correlations and disorder we study the disordered 1D t-V model. The Hamiltonian represents a tight-binding band of spinless fermions on a chain, the repulsion occurs between nearest neighbors while the disorder is in site energies,

$$H = -t\sum_i (c_{i+1}^\dagger c_i + \text{H.c.}) + V\sum_i n_{i+1}n_i + \sum_i \epsilon_i n_i. \qquad (1)$$

By choosing site energies randomly in the interval $-W<\epsilon_i<W$, we obtain in the NI limit $V=0$ the Anderson-localization model. In order to avoid the interaction-induced Mott-type insulator at $\Delta = V/(2t) > 1$, we restrict our study to the regime $\Delta<1$ [note that for $\Delta=1$ the model (1) can be mapped on the isotropic Heisenberg model in a random field]. We assume the chain with periodic boundary conditions and L sites. Furthermore, $t=1$ is used as the unit of energy.

To probe transport response we evaluate the dynamical conductivity $\sigma(\omega)$,

$$\sigma(\omega) = \frac{1-e^{-\omega/T}}{\omega L}\text{Re}\int_0^\infty dt e^{i\omega t}\langle j(t)j\rangle \qquad (2)$$

with the current operator

$$j = it\sum_i (c_{i+1}^\dagger c_i - \text{H.c.}). \tag{3}$$

We adopt the view that possible conductor-insulator transition needs to be a manifestation of the character of MB quantum states[12,13,16] (hence not directly related to other thermodynamic quantities). Therefore one may as well restrict the study to the regime $T\to\infty$, $\beta\to 0$, where, unlike for low temperatures when only the bottom of the spectrum is important, all MB states contribute with equal weight to $\sigma(\omega)$, including those in the middle of the spectrum which apparently have the lowest chance to be localized. In this limit, the relevant (and nontrivial) quantity is $\tilde{\sigma}(\omega)=T\sigma(\omega)$.

$\tilde{\sigma}(\omega)$ is calculated by employing the microcanonical-Lanczos method (MCLM),[17,18] particularly suited for dynamical quantities at elevated T. In the following we present results for systems with $L=16-24$ sites and for generic cases of half filling and quarter filling $n=N_e/L=1/2,1/4$, respectively. A sampling over $N_r\sim 100$ random ϵ_i configurations is made to obtain the relevant average response.

Finite-size effects should not affect significantly our analysis in both, the energy and space domains. Concerning the space domain, we focus on disorder parameters W for which the NI-electron localization length ξ_0 is much shorter than the size of the system, $\xi_0 \ll L$. One may use the estimate (for $V=0$) $\xi_0\sim 28.5/W^2$,[5] which for $W=2-4$ gives $\xi_0=7-1.8\ll L$. Moreover, the $T=0$ DMRG calculations show that the interaction reduces the localization length ξ compared with the NI case, whereby ξ exhibits the power-law behavior in W with an interaction-dependent exponent.[5]

The energy resolution of our spectra is much smaller than the average level spacing associated with the largest system size $L=24$ studied. Approximate eigenfunctions corresponding to $T\to\infty$ limit are converged in $M_1\sim 2000$ Lanczos steps, providing the energy resolution of $\delta E\sim 0.004$ (for $L=24$). In the next step, $M_2\sim 4000$ Lanczos iterations are used to evaluate $\tilde{\sigma}(\omega)$, leading to an estimation of the final frequency resolution $\delta\omega\sim 0.005$. Since for the largest $L=24$ the studied sector contains $N_{st}\sim 2\times 10^6$ MB states, the average level spacing $\Delta E\sim 10^{-5}\ll\delta\omega$, so the discreteness of the exact eigenspectrum due to finite-system size plays practically no role in our results.

In Fig. 1 we present typical high-T spectra for $\tilde{\sigma}(\omega)$, showing different $W=1.5,2,3,4$ for fixed $\Delta=0.5$. Since we deal with a substantial disorder, $\tilde{\sigma}(\omega)$ are essentially different from the weak-scattering Drude-type form. Curves in Fig. 1 reveal maxima at $\omega_m>0$. This is well pronounced in the inset of Fig. 1, where ω_m is plotted vs W. $\omega_m=0$ only for a weak disorder $W<1$, when $\sigma(\omega)$ is closer to Drude form. While the optical sum rule is for $T\to\infty$ independent of W and Δ,[17]

$$\int_0^\infty \tilde{\sigma}(\omega)d\omega = \frac{\pi}{2L}\langle j^2\rangle = \pi t^2 n(1-n), \tag{4}$$

the dc value $\tilde{\sigma}_0=\tilde{\sigma}(0)$, being the central quantity studied further on, shows a pronounced variation with W.

In Figs. 2(a) and 2(b) the emphasis is given to the low-ω window, which is relevant for the extraction of the dc value $\tilde{\sigma}_0$. $W=2$ spectra for the $n=1/2$ case are smoothed

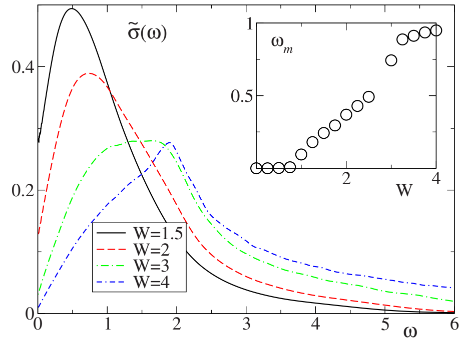

FIG. 1. (Color online) Dynamical high-T conductivity $\tilde{\sigma}(\omega)$ for $\Delta=0.5$ and different disorders $W=1.5,2,3,4$ evaluated for a half-filled system $n=1/2$ and $L=24$ sites. Inset: the position ω_m of maxima of $\tilde{\sigma}(\omega)$ as a function of W.

with an ω-dependent damping $\eta=\eta_0+(\eta_\infty-\eta_0)\tanh^2(\omega/\omega_0)$, $\eta_0=0.002\lesssim\delta\omega$, $\eta_\infty=0.02$, and $\omega_0=0.2$. Such a damping, used hereafter, preserves the sensitivity for lowest $\omega\to 0$ frequencies while smoothing configuration-dependent fluctuations (most pronounced at $\Delta\to 0$) at higher ω.

As argued above for $\xi_0<L$ cases, Fig. 2(a) confirms the absence of any evident L dependence (at least for $\Delta=0.5$). One can make an additional observation that, unlike for $\Delta=0.5$, the fluctuations of $\tilde{\sigma}(\omega)$ even at low frequencies remain substantial for the NI case $\Delta=0$. One finds that these fluctuations diminish with the increase in the sampling N_r over random-disorder configurations, indicating that the repulsive interaction $\Delta>0$ suppresses the sensitivity to the particular disorder configuration. In addition, $\Delta=0.5$ case in Fig. 2(a) reveals a remarkable linearity, $\tilde{\sigma}(\omega)\sim\tilde{\sigma}_0+\alpha|\omega|$, being apparently generic[17] for all $\Delta>0$.

By varying $\Delta=0-0.5$, the role of the interaction on the

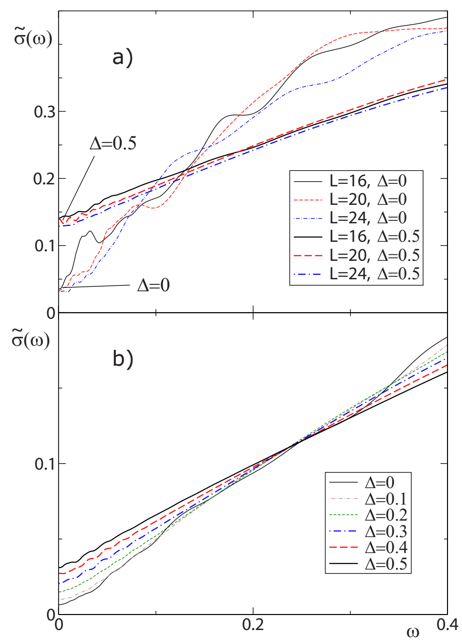

FIG. 2. (Color online) Low-ω part of $\tilde{\sigma}(\omega)$ for $n=1/2$: (a) $\Delta=0,0.5$, disorder $W=2$, and different $L=16,20,24$ and (b) $W=3$, $L=24$, and different $\Delta=0-0.5$.

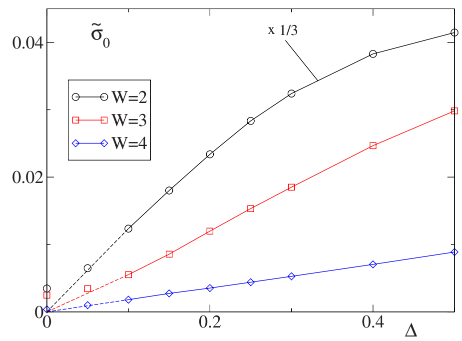

FIG. 3. (Color online) dc conductivity $\tilde{\sigma}_0$ vs Δ for half filling $n=1/2$ and different $W=2,3,4$. Dashed lines for $\Delta<0.1$ are interpolations to the theoretically known value $\tilde{\sigma}_0(\Delta=0)=0$.

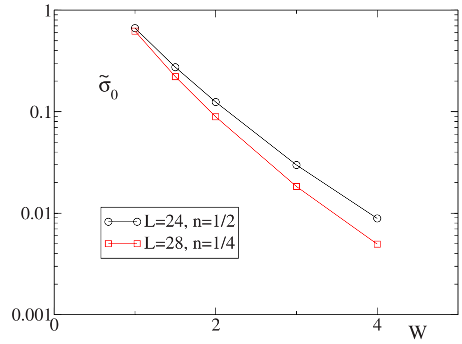

FIG. 4. (Color online) $\tilde{\sigma}_0$ vs W for $\Delta=0.5$ and fillings $n=1/2,1/4$.

MB localization is investigated in Fig. 2(b). Results are given for fixed disorder $W=3$, $L=24$, and $n=1/2$. Again, the remarkable linearity $\tilde{\sigma}(\omega)$ at low ω may be observed being very reproducible for $\Delta>0.1$ in spite of very small dc values $\tilde{\sigma}_0$ involved. Even more important, there is no signature of a presumable qualitative change in $\tilde{\sigma}(\omega)$ (at least for $\Delta>0.1$), which would point to the crossover for finite Δ from the MB localization, present at $\Delta=0$, to a conducting regime $\tilde{\sigma}_0>0$.

Based on the same numerical analysis of $\tilde{\sigma}(\omega)$ used for Figs. 2(a) and 2(b), in Fig. 3 we show the extracted dc values $\tilde{\sigma}_0$ vs Δ. To suppress the effects of the configuration-fluctuating component of $\tilde{\sigma}(\omega)$ as much as possible, $\tilde{\sigma}_0$ is evaluated from a linear fit of $\tilde{\sigma}(\omega)$ in the frequency interval $\omega<0.1$. $\tilde{\sigma}_0$ calculated in this way are given by symbols for $W=2,3,4$, respectively. For $\Delta<0.1$, dashed lines are used in Fig. 3 to interpolate $\tilde{\sigma}_0$ to the theoretical value $\tilde{\sigma}_0=0$ of the NI $\Delta\to 0$ limit.

Results in Fig. 3 are central to this work. In spite of small values of $\tilde{\sigma}_0$, in particular, for the $W=4$ case, the extracted $\tilde{\sigma}_0$ show very consistent behavior. Namely, it is quite evident that in the interval $W=2-4$ we do not find any signature of possible crossover in the behavior $\tilde{\sigma}_0$ vs Δ, which could be interpreted as the onset of the MB localization for $\Delta<\Delta_c(W)$. In fact, the simplest dependence $\tilde{\sigma}_0\propto\Delta$ seems to represent well our results in the investigated regime $2\leq W\leq 4$.

Since the possible MB localization at $\Delta>0$ should be more plausible at lower doping, where the condition $\xi<1/n$ is stronger, we investigate the quarter-filling case $n=1/4$ as well. Results for $\tilde{\sigma}_0$ vs Δ are, however, even quantitatively similar to the $n=1/2$ case, although, as expected, somewhat smaller values of $\tilde{\sigma}_0$ are obtained.

As presented in Fig. 4, it is instructive to follow the dependence of the dc value $\tilde{\sigma}_0$ as function of the disorder W. We investigate the $\Delta=0.5$ case for two fillings, $n=1/2,1/4$, and the data for $W=1,1.5$ are included. It is evident from Fig. 4 that the dependence is exponential, i.e., $\tilde{\sigma}_0\sim a\exp(-bW)$, with $b\approx 1.7,2$ for $n=1/2,1/4$, respectively.

Figure 4 gives also a clear limitation to our numerical approach in the regime $W\gg 1$. Since the low-ω slope α of $\tilde{\sigma}(\omega)$ shows weaker dependence on W ($\alpha\propto 1/W$ from Fig. 1), a reliable evaluation of $\tilde{\sigma}_0$ requires a very high resolution $\delta\omega\ll 1$. The latter is determined in our MCLM method by M_1,M_2 Lanczos steps but in the final stage also by the MB level density and the average MB level spacing $\Delta E\propto 1/N_{st}\propto\exp(-\zeta L)$. The reason for this is that macroscopic results for the transport become plausibly relevant (for $\Delta>0$ case) only if $\delta\omega\gg\Delta E$.

The above results show that a weak repulsive interaction in 1D disordered tight-binding systems is capable of destroying the phenomena of MB localization. At the present stage of investigations, we do not have a clear analytical or phenomenological explanation for this property. However, we have so far discussed cases by keeping the fermion density n fixed. On the other hand, one can investigate the behavior of the MB states by fixing the number of fermions N_e while changing the system size L. Several such studies have been reported,[19] suggesting that the two-particle localization length ξ in the presence of interaction becomes enhanced in comparison with the single-particle case. Because the context investigated has been rather different, we study here this effect from the quantity directly relevant to the coherent charge transport, i.e., from the charge stiffness D. For $T>0$, D is defined by[20]

$$D=\frac{\beta}{2LZ}\sum_n e^{-\beta E_n}|\langle n|j|n\rangle|^2. \tag{5}$$

As before, we are focused on the high-T regime $\beta\to 0$, when all the levels are probed, $Z=N_{st}$. In the absence of disorder and for finite $\Delta>0$, $\tilde{d}=2LTD/N_e$ remains finite because of the integrability of the model.[20] With disorder switched on and N_e fixed, one expects an exponential suppression of $\tilde{d}$ with the increase in the system size L, $\tilde{d}\propto\exp(-L/\xi)$. Furthermore, for the particular case of NI fermions ξ should be independent of N_e, i.e., $\xi=\xi_0$, with ξ_0 denoting the single-fermion localization length.

Since for the localized phase (e.g., for $V>0$ and $T=0$) the stiffness $\tilde{d}$ is distributed for different realizations of the disorder according to the log-normal distribution,[5] we present in Fig. 5 the average $\bar{d}=\exp(\langle\ln\tilde{d}\rangle)$ vs L, with $N_e=2$, $\Delta=1$, and various $W=0-4$. $\tilde{d}$ is obtained by the full diagonalization (for $N_e=2$, $N_{st}\propto L^2$). It is evident from the figure that, for larger disorders, $W\geq 2$, $\bar{d}$ decays exponentially with L, which is consistent with the MB localization in the $n\to 0$ limit. On the other hand, it is remarkable that the interaction

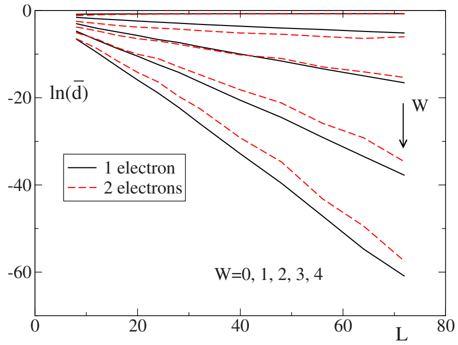

FIG. 5. (Color online) Logarithm of the rescaled charge stiffness $\bar{d}$ vs L for $N_e=1,2$, $\Delta=1$, and various $W=0-4$.

$\Delta=1$ increases $\bar{d}$ at large L. It seems from Fig. 5 that a crossover between the $\xi<\xi_0$ and $\xi>\xi_0$ behaviors occurs for $W\approx 2$ and $L^*\approx 40$. For strong disorder $W=3,4$, the crossover appears already at $L^*\sim 10$. These results suggest that the increase in ξ due to interaction may be an argument for the elimination of the MB localization at high T for finite fermion densities $n=1/4,1/2$ discussed in this work.

In conclusion, our numerical results reveal a steady and uniform increase in $\tilde{\sigma}_0$ with the repulsive interaction $\Delta>0$ at high T and at fixed disorder W, which does not support a possible MB localization at $\Delta>0$ in the considered regime. In order to put our results in broader context, let us comment the relation to other works. Authors advocating a finite-T insulator-conductor transition[11] give an estimate for the transition temperature $T^*\propto 1/(\mathcal{N}\xi\lambda \ln \lambda)$, where $\mathcal{N}$ is the (single-particle) density of states and λ a characteristic matrix element for the electron-hole pair creation. For high $T\gg 1$ and weak interaction $\Delta<1$ one may translate the estimate[12] to a critical $\Delta^*\propto\lambda^*\propto 1/(\mathcal{N}\xi_0)$. Hence, for larger $W>2$ we are in the regime where at least some crossover should be observed for $\Delta\sim\Delta^*>0$. Still, we do not observe any clear sign of the latter, at least it is not evident enough.

On the other hand, some recent numerical studies using different criteria, seem to point to the possible conductor-insulator transition and the MB localization at high T at much larger W, essentially within the same model with typically fixed interaction $\Delta\sim 1$.[13–15] Translating the definitions of disorder W, their estimates for the onset of localization would be $W>W^*\sim 6-10$, consistent with the observed qualitative change in the level statistics.[12] It should be observed that such cases correspond to extreme disorder, which would require within our (or an analogous) approach the observation (see Fig. 4) of $\tilde{\sigma}_0<10^{-4}$. The corresponding resolution $\delta\omega<10^{-4}$ and large MB density of states leading to $\Delta E\propto LW/N_{st}<\delta\omega$ may be in principle obtained, e.g., by the full diagonalization for large enough L. However, the latter is not reachable by up-to-date numerical methods. Hence, we cannot exclude such a scenario for the onset of the MB localization at $W>W^*$ but on the other hand, such extreme disorder would also put limits to its theoretical as well as experimental verification and relevance.

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