(2.32)

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COMPUTATIONAL UNCERTAINTIES OF SHIP HULL GIRDER RELIABILITY MODELLING

Summary

The aim of this paper is to investigate the computational uncertainties by applying different methods in reliability analysis of ship hull girder. The International Maritime Organization (IMO) proposals for hull girder reliability assessment are undertaken for probabilistic modelling of ship ultimate strength under combined longitudinal still water and wave bending moments both for hogging and sagging conditions. The example in the paper considers a typical service profile of a chemical tanker in as-built state and in corroded hull condition. The three methods of reliability analysis are compared in the paper: the First Order Second Moment (FOSM), the Advanced First Order Reliability Method (FORM) and the Monte Carlo Simulation (MCS). The conclusions resume the results of comparison of applied methods and discuss the appropriateness of numerical reliability methods on digital computers in ship hull girder safety assessments.

Key words: *hull girder, reliability, uncertainty, FOSM, FORM, MCS, chemical tanker.*

RAČUNSKE NEIZVJESNOSTI MODELIRANJA POUZDANOSTI BRODSKOG TRUPA

Sažetak

Cilj ovog rada je ispitati računske neizvjesnosti primjenom različitih postupaka analize pouzdanosti brodskog trupa. Za modeliranje granične čvrstoće brodskog trupa pod zajedničkim djelovanjem uzdužnih momenata savijanja na mirnoj vodi i na valovima, kako u progibu tako i u pregibu, preuzete se preporuke Međunarodne organizacije za pomorstvo (IMO) za ocjenu pouzdanosti. Primjer u članku razmatra tipičnu službu tankera za kemikalije za tek izgrađeni brod i za trup izložen hrđanju. U članku su uspoređena tri postupka analize pouzdanosti: Metoda prvog reda sa statističkim momentima drugog reda (FOSM), Napredna metoda prvog reda (AFORM) i Monte Carlo simulacija (MCS). U zaključcima se rezimiraju podaci usporedbi primijenjenih numeričkih postupaka za proračun pouzdanosti na elektroničkim računalima te se raspravljaju odgovarajući postupci za ocjenu sigurnosti brodskog trupa.

Ključne riječi: *brodski trup, pouzdanost, neizvjesnost, FOSM, FORM, MCS, tanker za kemikalije*

1. Introduction

In addition to a number of serious environmental, material and workmanship uncertainties with respect to ship hull structural reliability, the problem of idealization and modelling uncertainties also involve computational uncertainties due to usually complex and often iterative mathematical operations. Pure analytical methods are seldom appropriate for practical reliability calculations. Among a number of numerical methods for practical engineering computations on digital computers on disposal for shipbuilders are the semi-probabilistic and probabilistic methods of various levels of complexity on one hand and a number of crude and accelerated Monte-Carlo simulation methods on the other. None of these numerical methods can assure exact results of complex engineering reliability calculations. Therefore this note presents the numerical results of ship hull girder ultimate strength reliability calculations 1-22 by applying various numerical methods 23-33 on a recently built tanker in order to identify the differences in their efficiency and accuracy. In spite of the intense calculation efforts on digital computers the paper could not guaranty the exact results of the hull girder ultimate longitudinal strength reliability calculation.

2. Structural safety assessments

The structural safety problems are rationally considered in terms of a finite set of basic variables $\mathbf{X}=(X_1,X_2,\cdots,X_n)$ describing loads, material properties and geometry. Depending on the character of the basic variables the structural safety can be viewed as deterministic, probabilistic or as mixed problem.

The deterministic basic variables are defined solely by their nominal values and possibly by their tolerances when available. The stochastic basic variables are defined depending on the available statistical data usually by their mean values and dispersion (range, variance, standard deviation, coefficient of variation), correlations or by complete statistical distributions when available.

Many structural problems in engineering can be represented by structural capability (C) or resistance (R) also denoted sometimes as strength (S) and of demands (D) or loads (L) depending on the whole set or on the subset of basic variables **X**.

The commonly recognized safety measures normally employ the basic variables to relate the structural capabilities and demands and can be presented by safety margin *m*, safety factors *f* and usage factors *i* as shown:

$$m(\mathbf{X})=C(\mathbf{X})-D(\mathbf{X}),\ \ f(\mathbf{X})=\frac{C(\mathbf{X})}{D(\mathbf{X})}\ \text{and}\ i(\mathbf{X})=\frac{D(\mathbf{X})}{C(\mathbf{X})}.$$

Other formulations of safety measures can be used for some specific problems.

The limit state function *g(***X***)* of the basic variables **X** that divides the design space of basic variables into fail set **F** and safe set **S** can be formulated in terms of safety measures depending on structural properties as shown:

$$g(\mathbf{X})\begin{cases}C<D & \text{for}\,\mathbf{X}\in\mathbf{F}\\ C=D & \text{for}\,\mathbf{X}\,\text{on limt state}\,.\\ C>D & \text{for}\,\mathbf{X}\in\mathbf{S}\end{cases}$$

The limit state functions in general can be linear, nonlinear, continuous or not, implicit or explicit, derivable or not and in all this cases appropriate calculations should be applied.

Nominal safety measures apply to deterministic definition of basic variables also using tolerances where available, as follows:

$$m = (C - t_C) - (D + t_D), \quad f = \frac{C - t_C}{D + t_D} \text{ and } i = \frac{D + t_D}{C - t_c}.$$

3. Structural reliability assessments

The structural reliability assessment reconsiders the system safety accounting for data dispersion by employing available statistical data on several levels. The firstly applied and the simplest approximation in the reliability assessment calculations that accounts for the data uncertainties undertakes the well known theorem of the linear combination of normally distributed random variables. For the normally distributed random variables Xi given by their mean value X and standard deviation, the standard unit variable is given by normal transformation $x_i = (X_i - \bar{X}_i) / \sigma_{X_i}$.

The central safety measures denoted as level zero reliability assessment are based on mean values of variables that define the mean capability $\bar{C}$ and mean demand $\bar{D}$ as shown:

$$M = \bar{C} - \bar{D} \qquad f = \frac{\bar{C}}{\bar{D}} \qquad i = \frac{\bar{D}}{\bar{C}}.$$

The direct consequence of this approximation is the firstly published Cornell's safety index (level I method) defined by means and variances of capabilities and demands as follows:

$$\beta_C = \frac{\bar{C} - \bar{D}}{\sqrt{\sigma_C^2 + \sigma_D^2}} \tag{1}$$

For the linear limit state function $g(\mathbf{X}) = -a_o + \sum_{i=1}^{n} a_i x_i < 0$ and for the whole set of basic variables the general term for the safety index is denoted as the Second Moment (SM) method, which is not using other distributional properties except statistical moments up to second order, can be put down as follows:

$$\beta_{SM} = \frac{-a_o + \sum_i a_i \cdot \bar{x}_i}{\sqrt{\sum_i (a_i \cdot \sigma_{xi})^2}} \tag{2}$$

For the non-linear limit state function and for the whole set of basic variables the general term for the safety index is denoted as the First Order Second Moment (FOSM) method since it applies the first order Taylor series expansion (level II method, distribution-free method) as shown:

$$\beta_{FOSM} = \frac{-a_o + \sum_i \left(\frac{\partial g}{\partial x_i} \right)_{\bar{x}_i} (\bar{x}_i - x_i)}{\sqrt{\sum_i \left(\frac{\partial g}{\partial x} \right)^2_{X^*} \sigma^2_{X_i}}} \tag{3}$$

If the linearization takes place in the mean value the method is denoted as the Mean Value First Order Second Moment (MVFOSM). However, the results for non linear limit state functions depend on the linearization point as well as on the formulation of the limit state function.

The ultimate goal of the structural reliability analysis is the assessment of the failure probability generally defined as the integral over the failure set **F** defined by $g(\mathbf{X}) \geq 0$ as follows:

$$P_F = P(g(\mathbf{X}) \leq 0) = \iiint\limits_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} \tag{4}$$

The simplest approximation of the failure probability based on FOSM is obtained as the inverse value of the standard normal function for formerly calculated safety indices as follows:

$$P_F = \Phi^{-1}(-\beta) \tag{5}$$

For FOSM the probability approximation implies that all the random variables are normal and the limit state function is either originally linear or lately linearized up to the first order. The numerical computation is under specified conditions fast and accurate but the results can diverge significantly from the correct results.

The aim of the next level of approximation is to account for original statistical distributions of random variables.

Normal tail approximation offers the approximation of true statistical distributions with normal distribution. Such an approximation allows the extension of FOSM methods to more general problems of different statistical distributions of basic variables.

More accurate of the normal tail approximation is the direct transformation of random variables that can be implemented in computer codes.

The aim of the next approach is the improvement of the FORM methods with respect to invariance of the results for different linearization points and formulations of the limit state functions. Advanced First Order Reliability Method (AFORM) (level III) applies the first order linearization of the limit state functions in the design point instead in the mean value in combination of the direct transformation of original random variables into normal random variables of either correlated or non-correlated types using Rosenblatt or Nataf transformations. Such a transformation provides the opportunity for integration in order to obtain the failure probability.

The Second Order Reliability Method (SORM) (level III) applies the second order linearization of the limit state functions and the direct transformation of original random variables into normal random variables of either correlated or non-correlated types. Such a transformation provides the opportunity for integration of nonlinear failure functions in order to obtain the failure probability.

Monte Carlo Simulation (MCS) is the only method so far that provide the possibility to integrate the failure probability for optional definition of the limit state functions and for arbitrary statistical distributions of basic variables. However, the method provides assessment of the results within accuracy intervals depending on the number of samples. Normally, the crude Monte Carlo simulation (CMCS) simulation requires enormous number of samplings. Significant improvements are attainable by importance sampling simulation (ISMCS) methods when the sampling takes place in the area where most of the probability contents is placed that is normally around and in the vicinity of the design point. A number of other different accelerated MCS are developed for reliability assessments in structural problems 23-33 .

4. Reliability formulation of hull ultimate strength

The limit-state equation with respect to hull-girder ultimate failure under vertical bending moments, reads:

$$\hat{\chi}_u M_u - \hat{M}_{sw} - \psi \hat{\chi}_w \hat{\chi}_{nl} \hat{M}_w < 0 \tag{6}$$

where:

M_u - deterministic ultimate hull-girder bending moment;
$\hat{M}_{sw}$ - random variable extreme vertical still-water bending moment;
$\hat{M}_w$ - random variable extreme vertical wave bending moment;
ψ - load combination factor between still water loads and wave loads;
$\hat{\chi}_u, \hat{\chi}_w, \hat{\chi}_{nl}$ - random variables representing modelling uncertainty of ultimate strength, linear wave load and non-linearity of wave load.

The reliability analysis 1-22 according to limit-state equation (6) is performed separately for two independent failure modes – sagging and hogging. The hull-girder reliability in each of the two failure modes is calculated for three elementary loading conditions – full load condition (FL), ship in ballast (BL) and partial loading condition (PL). For rational reliability assessment, the percentage of time that a ship spends in each of these loading conditions has to be estimated. The operational profile for chemical tankers differs from the profile of oil tankers since oil tankers are rarely sailing extended voyages in ballast condition, which is result of global spreading of production and consumption of chemicals 2 , Table 1.

Table 1 Operational profile adopted for chemical tanker 2

Tablica 1. Radni profil prihvaćen za tanker za prijevoz kemikalija

Load cond.	Harbour	Full load	Ballast load	Partial Load
Percentage of spent time	15%	35%	15%	35%
Voyage duration (days)	---------	23.5	23.5	2.0

5. Example ship

The ship analyzed in the present study is an existing, a relatively small chemical carrier, with corrugated both centreline and transverse bulkheads fully satisfying the contemporary rules for design and construction of steel ships including IACS UR S11 2 . The particulars of the chemical tanker are presented in Table 2.

Table 2 Main characteristics of chemical tanker

Tablica 2. Glavne izmjere tankera za prijevoz kemikalija

Length between perpendiculars *Lpp*		120 m
Moulded breadth	*B*	17 m
Moulded depth	*D*	9 m
Scantling draught	*T*	7 m
Deadweight	*DWT*	7900 dwt

The longitudinal elements of the ship hull amidships is shown in Figure 1 2 , where some of typical features of chemical carriers may be noticed. In the first place, all structural members are removed from the cargo tank boundaries. This is achieved by placing deck longitudinals and deck girders on the topside of deck plating. Accordingly, web frames are also placed on the top of the deck plating in way of the cargo area. Deck, bottom and inner bottom panels are typical longitudinally framed structures, commonly used on chemical tankers. Transversal framing system is adopted for side- and inner-shell structures.

Cargo hold area is entirely built of mild steel and covered by special type of epoxy coating 2 . It is to be mentioned that cargo tanks of similar ships are alternatively often constructed of corrosion-resistant stainless steel. The ship might carry wide range of chemicals specified in the International Code for the Construction and Equipment of Ships Carrying Dangerous Chemicals in Bulk (IBC code), with densities between 0.7 and 1.5 t/m^3.

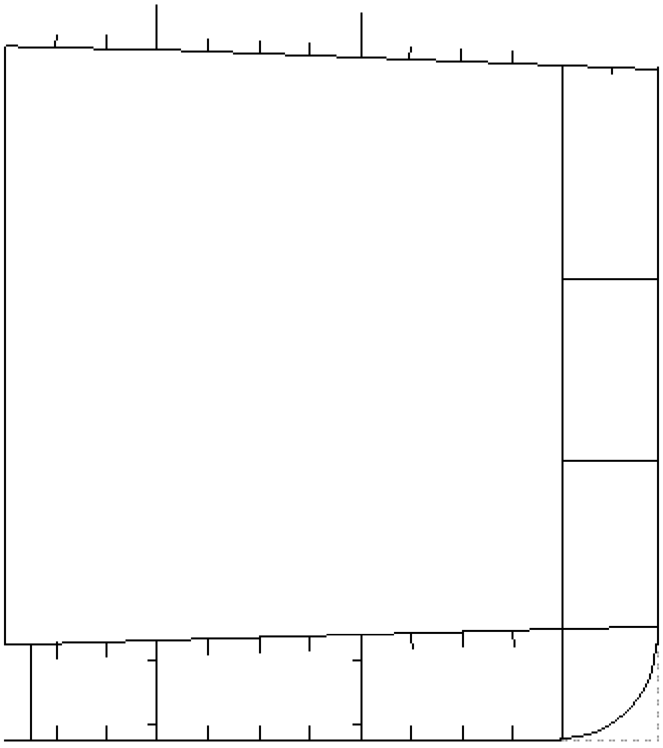

Fig. 1 Longitudinal elements of the ship hull amidships

Slika 1. Uzdužni elementi brodskog trupa u sredini

The annual safety indices were calculated for as-built state of the ship and for corroded state according to the corrosion deduction thickness from CSR, by the COMREL program using the FORM method 2 .

Safety indices are calculated for three different loading conditions (full load, ballast and partial load) in hogging failure mode. Analysis is performed independently for oil tanker operational scenarios presented in Tables 1. The summary of the stochastic model adopted is shown in Tables 3 where the notation adopted in equation (1) is used.

Table 3 Summary of stochastic model adopted

Tablica 3. Pregled prihvaćenih podataka stihastičkog modela

Variable	Distribution	mean		COV
$\hat{M}_{sw}$ (MNm)	Gumbel	FL	102.6	0.22
		PL	184.7	0.13
		BL	219.1	0.25
$\hat{M}_{w}$ (MNm)	Gumbel	FL	299.0	0.09
		PL	273.1	0.09
		BL	255.9	0.09
M_u (MNm)	Deterministic	As-built	813	
		Corroded	663	
$\hat{\chi}_w$	Gaussian	0.9		0.15
$\hat{\chi}_{nl}$	Gaussian	0.95		0.15
$\hat{\chi}_u$	Log-normal	1.14		0.13
ψ	Deterministic	FL	0.92	
		PL	0.91	
		BL	0.80	

In addition to the reported results 2 the study performs five other checking calculations (Table 4a and Table 4b):

FORM
FOSM
Crude Monte Carlo (CMCS) using Fortran programming facilities
Importance Sampling Monte Carlo (ISMCS) using Fortran
Crude Monte Carlo using MathCad (CMCS-MC)

Table 4a Summary of results for FORM, FOSM, CMCS, ISS reliability calculations

Tablica 4a. Pregled rezultata proračuna FORM, FOSM, CMCS i ISS postupcima

As-built					
		FL	BL	PL	p_f
FORM(COMREL)		4.75	3.84	4.5	3.82
	P_f	1.02E-06	6.15E-05	3.40E-06	6.59E-05
FORM(RELI)		4.75	3.85	4.49	3.84
	P_f	9.93E-07	5.70E-05	3.55E-06	6.15E-05
FOSM(RELI)		4.4	3.85	4.01	3.74
	P_f	5.36E-06	5.65E-05	3.05E-05	9.24E-05
CMCS		3.52	3.84	4.41	3.51
	P_f	2.14E-04	6.34E-06	5.38E-06	2.26E-04
IMCS		4.73	3.84	4.43	3.82
	P_f	1.13E-06	6.20E-05	4.85E-06	6.80E-05
CMCS-MC		4.80(+1.034/-0.91)	3.96(+0.15/-0.14)	4.47(+0.61/+0.63)	3.93
	P_f	7.93E-07	3.75E-05	3.91E-06	4.22E-05

Table 4b. Summary of results for FORM, FOSM, CMCS, ISS reliability calculations

Tablica 4b. Pregled rezultata proračuna FORM, FOSM, CMCS i ISS postupcima

Corroded					
		FL	BL	PL	p_f
FORM(COMREL)		3.84	3.02	3.46	2.95
	P_f	6.15E-05	1.26E-03	2.70E-04	1.60E-03
FORM(RELI)		3.84	3.03	3.46	2.96
	P_f	5.93E-05	1.22E-03	2.69E-04	1.55E-03
FOSM(RELI)		3.65	3.03	3.19	2.87
	P_f	1.27E-04	1.20E-03	7.04E-04	2.03E-03
CMCS			2.97	3.38	2.85
	P_f		1.49E-03	3.57E-04	2.16E-03
ISMCS		3.83	3.05	3.42	2.96
	P_f	6.50E-05	1.17E-03	3.23E-04	1.56E-03
CMCS-MC		3.90(+0.10,-0.11)	2.84(+0.02/-0.02)	3.67(+0.07/-0.08)	2.82
	P_f	4.81E-05	2.26E-03	1.21E-04	2.43E-03

6. Conclusion

The exact solution of complex structural reliability problems can hardly be attained. Numerical methods provide approximations based on first order or second moment linearization of the limit state functions. The iterative procedure applies finite accuracy of stopping criteria. The Monte Carlo methods provide results within confidence intervals depending on the number of samples. The problems of accuracy became even greater in systemic reliability analysis with more independent or dependent failure modes. Therefore the structural reliability analysis has to cope with reasonably set goals of accuracy at least in order to provide relative ordering of probabilistic safety measures within classes of problems instead of the absolute values of safety measures. Accuracy is a prevailing goal of numerical calculations. However, practical problems in engineering require also computational efficiency. Therefore the two criteria: accuracy and efficiency are considered for selection of the appropriate calculation procedure often depending on the type and size of the problem.

The FORM results calculated by Fortran program RELI are in good agreement with reference values produced by computer program COMREL. The small differences might be the consequence of different tolerances in stopping criteria of the Rackwitz-Fiesler iterative algorithm. The computation is fast and reliable in most of the cases of well-defined mathematical models. The FORM procedure is appropriate to multiple failure mode engineering problems and deals with correlated basic variables and failure modes. Normally it is applied as a computer code due to its iterative character.

The results of FOSM calculation by Fortran program RELI showed surprisingly close results to reference values. The deviations from the FORM results are increasing with increase of safety indices what is the consequence in increasing differences in tails of applied statistical distributions other then assumed normal distributions as it is adopted in FOSM. Some improvements are possible by using he normal tail approximation procedure. The FOSM procedure is simple enough, fast and reliable but it is not useful for multiple failure problems although it can handle correlated basic variables. The procedure can be easily performed even by using standard spreadsheet facilities of digital computers.

The MCS method provided the results that can be considered as close to accurate within the calculated confidence intervals when efficient random number generators are employed. However, the test in this study showed that the CMCS might be somehow problematic when very small probabilities (high safety indices) are considered since the number of samples is very high, about 10 times of the reciprocal value of probability for only 0,3 confidence interval. The CMCS in the examples was performed for more than 10^6 samples.

The Importance Sampling Simulation approach gave good agreement with reference values. This approach is very sensitive to sampling parameter selection and requires tuning of calculation parameters such as the translocation of IS function as close as possible to design point, type of IS function (original, normal) and the mean and variance of IS function. The ISMCS requires significantly smaller number of samples of about 10^3 trials.

The Crude Monte Carlo simulations performed within the MathCad programming environment provide reliable results after a enormously high number of samples of about 10^8 for high safety indices within relatively wide confidence intervals for small failure probabilities.

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