



Mladen Pavičić: Grafičko računanje kvantno-mehaničkih nelinearnih jednadžbi

Projekt: Kvantno računanje: paralelnost i vizualizacija

(082-0982562-3160); voditelj: M. Pavičić

Program: Distrada i znanstvena vizualizacija

(0982562); voditelj: Karolj Skala

e-Science day, IRB 17.12.2009

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Pozadina



Gatesova ideja: Riješiti probleme sirovom silom.

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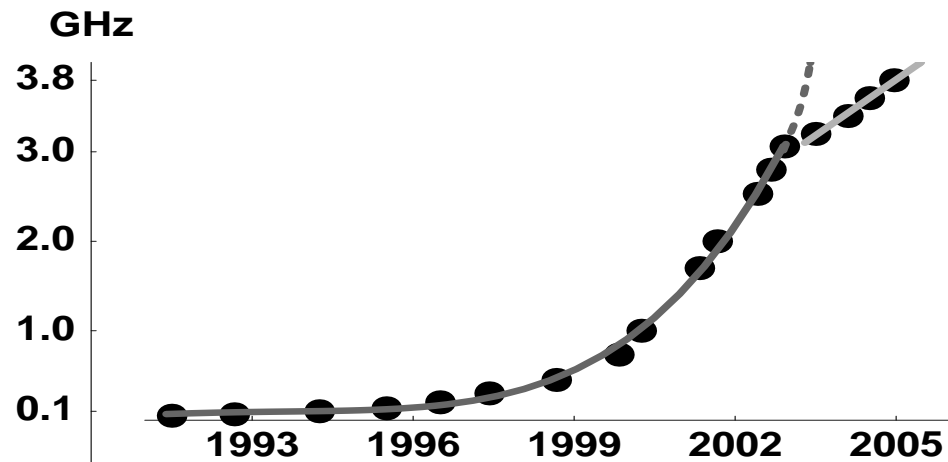
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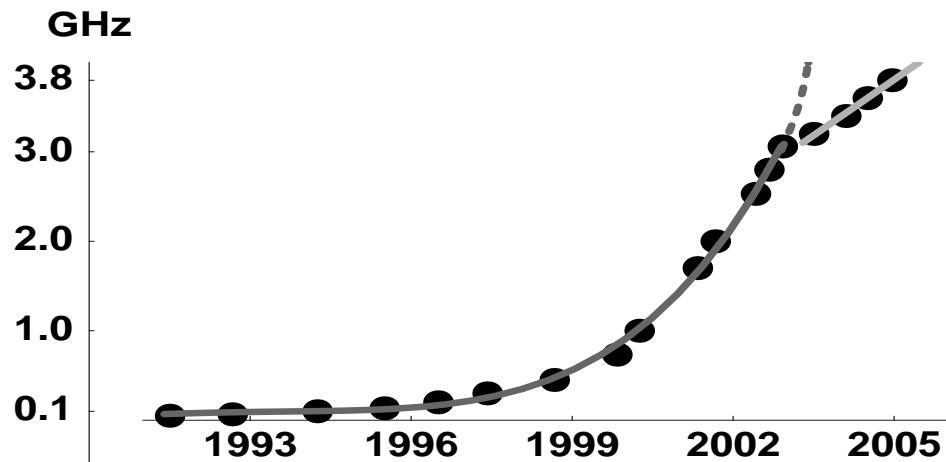


Pozadina



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Intel CPU introductions: 486-50 MHz Jun 1991, DX2-66 Aug 92, P(entium)-100 Mar 94, P-133 Jun 95, P-200 Jun 96, PII-300 May 97, PII-450 Aug 98, PIII-733 Oct 99, PIII-1.0 GHz Mar 00, P4-1.7 Apr 01, P4-2.0 Aug 01, P4-2.53 May 02, P4-2.8 Aug 02, P4-3.0 Apr 03, P4-3.4 Apr 04, P4-3.6 Jun 04, P4-3.8 Nov 04.

Problem



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Izračunavanje orijentacija uređaja za pripremu i mjerenje kvantnih stanja traži masivno računanje pomoću clustera i grida. Rezultati su premnogobrojni → primjenjuje se 2D, 3D i 4D grafička vizualizacija. Priprema stanja koja su isključivo kvantna: Kochen-Specker-ova stanja.



MULTIPARTY ENTANGLEMENT IN GRAPH STATES

TABLE II. The number of vertices $|V|$ and edges $|E|$, Schmidt measure E_S , rank index (see Sec. V) RI_3 and RI_2 (for splits with 2 or 3 vertices in the smaller partition), number of nonisomorphic but LU-equivalent graphs [LU class], and the 2-colorable property 2-col for the graph classes in Figs. 4 and 5.

No.	[LU class]	$ V $	$ E $	E_S	RI_3	RI_2	2-col
1	1	2	1	1			yes
2	2	3	2	1			yes
3	2	4	3	1		(0,3)	yes
4	4	4	3	2		(2,1)	yes
5	2	4	4	1		(0,10)	yes
6	6	5	4	2		(6,4)	yes
7	10	5	4	2		(8,2)	yes
8	3	5	5	$2 < 3$		(10,0)	no
9	2	6	5	1	(0,0,10)	(0,15)	yes
10	6	6	5	2	(0,6,4)	(8,7)	yes
11	4	6	5	2	(0,9,1)	(8,7)	yes
12	16	6	5	2	(0,9,1)	(11,4)	yes
13	10	6	5	3	(4,4,2)	(12,3)	yes
14	25	6	5	3	(4,5,1)	(13,2)	yes
15	5	6	6	2	(0,10,0)	(12,3)	yes
16	5	6	6	3	(4,6,0)	(12,3)	yes
17	21	6	6	3	(4,6,0)	(14,1)	yes
18	16	6	6	3	(6,4,0)	(15,0)	yes
19	2	6	6	3	(10,0,0)	(15,0)	yes

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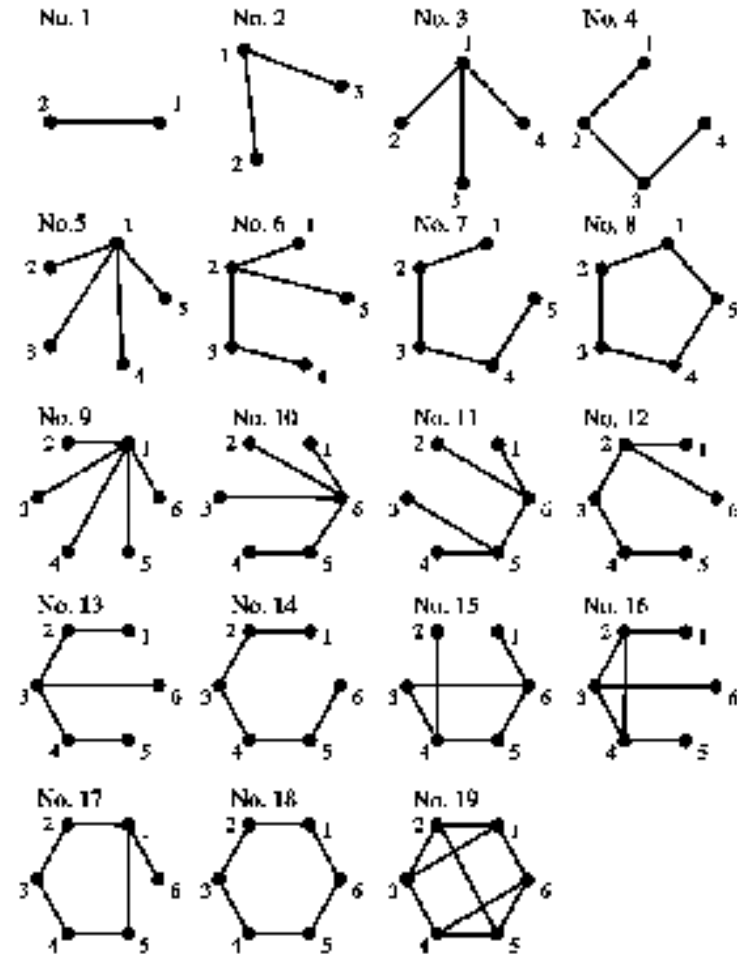


FIG. 4. List of connected graphs with up to six vertices that are not equivalent under LU transformations and graph isomorphisms.

Multiparty entanglement in graph states



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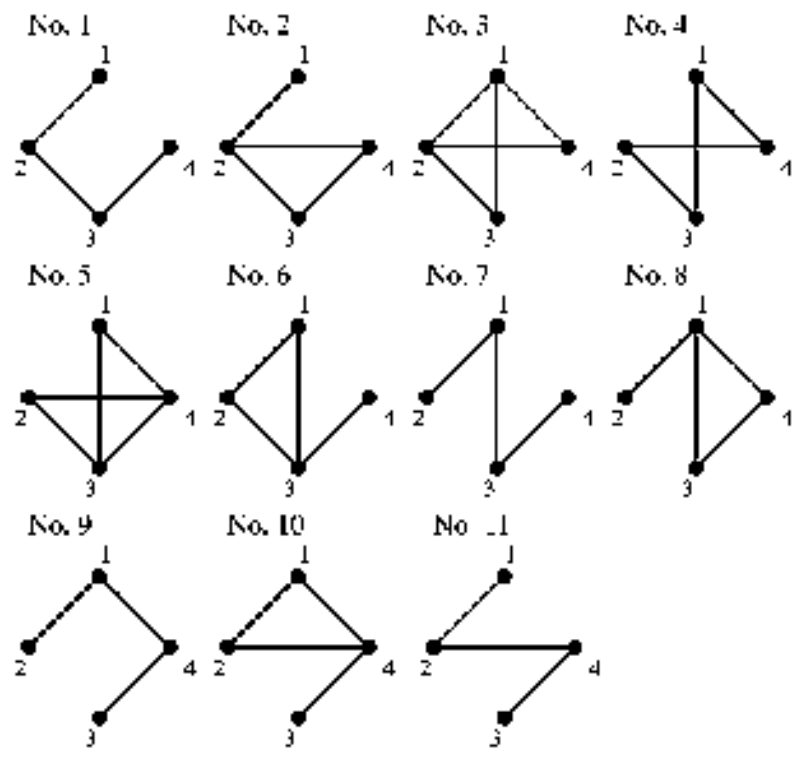
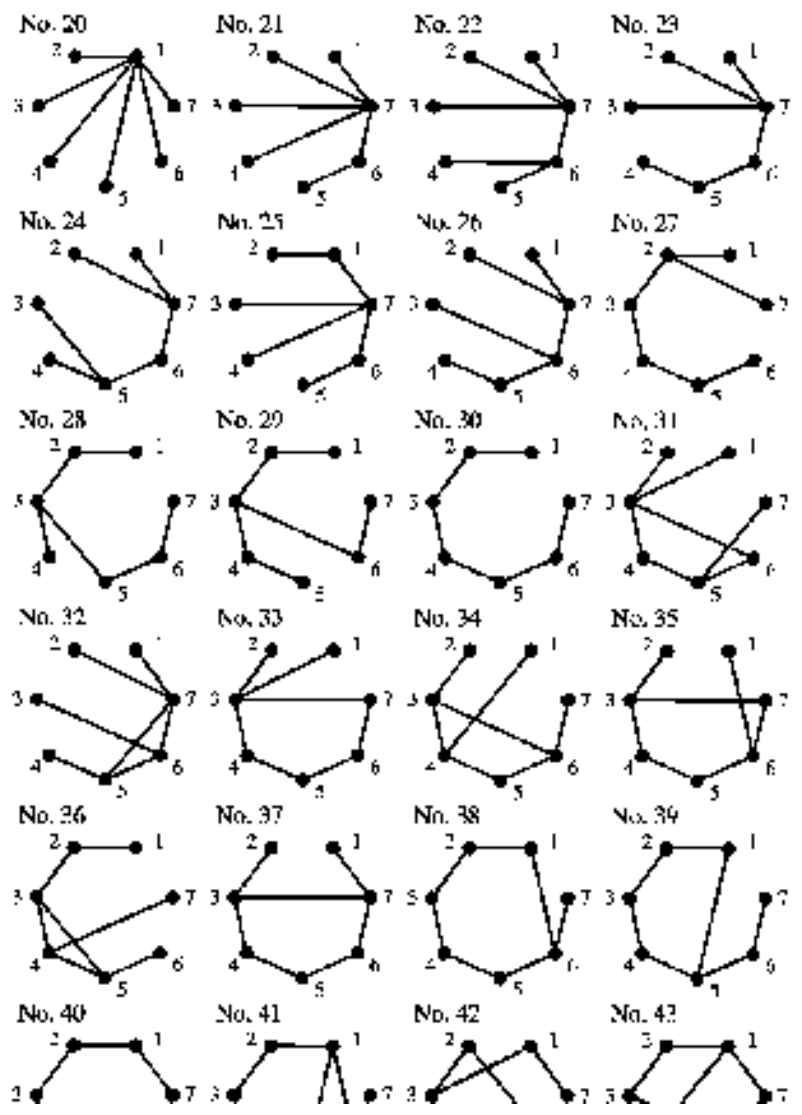


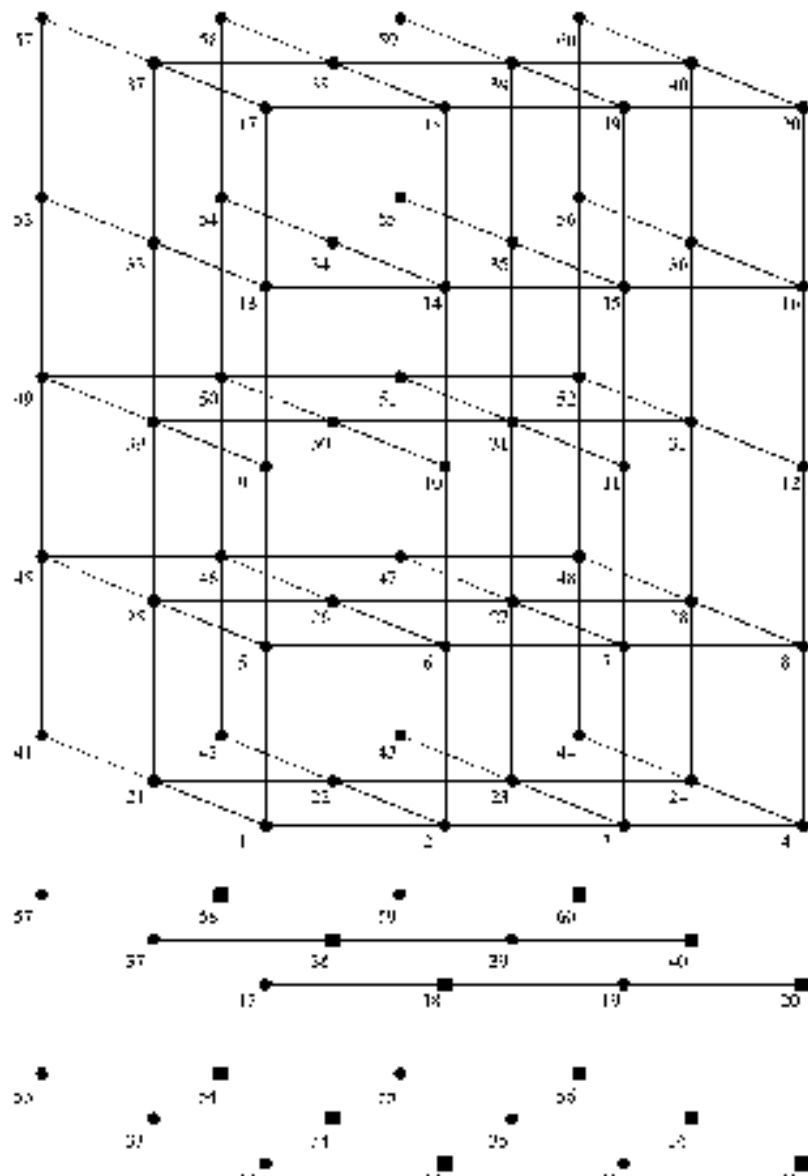
FIG. 6. An example for an successive application of the LU rule, which exhibits the whole equivalence class associated with graph No. 1. The rule is successively applied to that vertex of the predecessor, which is written above the arrows of the following diagram:

$$\begin{array}{ccccccccccc} & & 3 & & 2 & & 3 & & 1 & & 3 & & 1 \\ \text{No. 1} & \rightarrow & \text{No. 2} & \rightarrow & \text{No. 3} & \rightarrow & \text{No. 4} & \rightarrow & \text{No. 5} & \rightarrow & \text{No. 6} & \rightarrow & \text{No. 7} \\ & & 3 & & 4 & & 1 & & 2 & & & & \\ \rightarrow & \text{No. 8} & \rightarrow & \text{No. 9} & \rightarrow & \text{No. 10} & \rightarrow & \text{No. 11} \end{array}$$



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inductive proof may be of interest also for other graph classes. ■

Example 3: The Schmidt measure of an entangled ring with an even number $|V|$ of vertices is given by $|V|/2$.

Proof. This is a 2-colorable graph, which gives on the one hand the upper bound of $|V|/2$ for the Schmidt measure. On the other hand, by choosing the partitions $A=\{1,2\}$ and $B=\{3,4\}$ on the first four vertices, which are increased (for $|V|>4$) alternately by the rest of the vertices, yielding the partitioning with

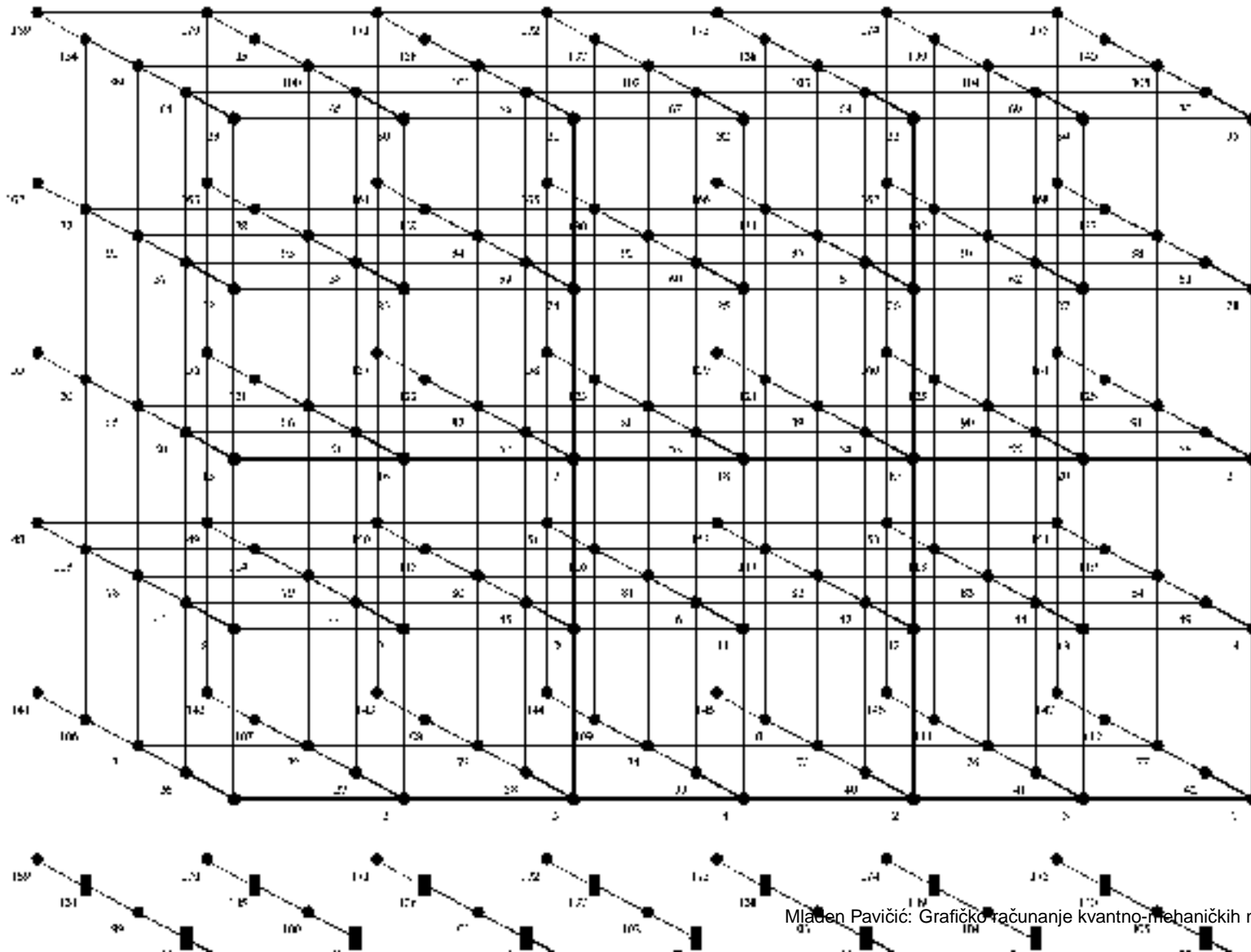
$$A = \{1, 2, 5, 7, \dots, 2k + 5, \dots, |V| - 1\} \quad (102)$$

$$B = \{3, 4, 6, 8, \dots, 2k + 6, \dots, |V|\}, \quad (103)$$

one obtains a bipartitioning (A, B) , which has maximal Schmidt rank $E_S^{(A,B)} = |V|/2$ according to Proposition 4 (see Fig. 14). ■

Example 4: All connected graphs up to seven vertices.

We have computed the lower and upper bounds to the Schmidt measure, the Pauli persistency, and the maximal partial rank, for the nonequivalent graphs in Figs. 4 and 5. They are listed in Table II, where we have also included the *rank index*. By the rank index, we simply compressed the information contained in the Schmidt rank list with respect to all bipartite splittings, counting how many times a certain rank occurs in splittings with either two or three vertices in the smaller partition. For example, the rank index $RI_3 = (20, 12, 3)$ of graph number 29 means that the rank 3 occurs 20 times in all possible 34 splits, the rank 2 twelve times, and the rank 1 only three times. (Note that here we





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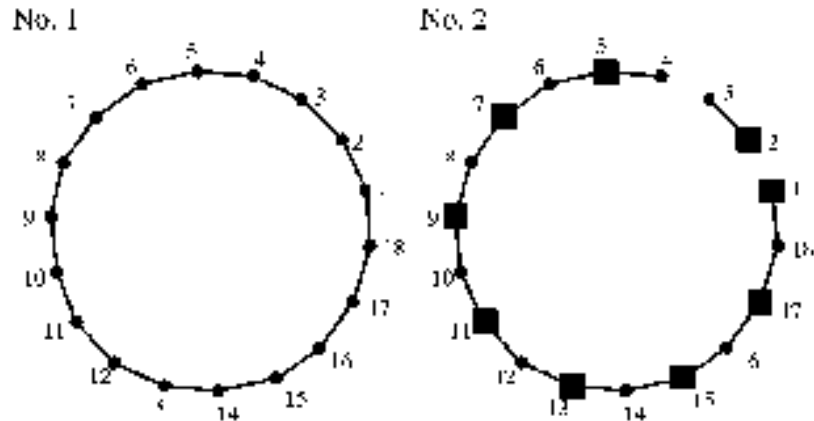
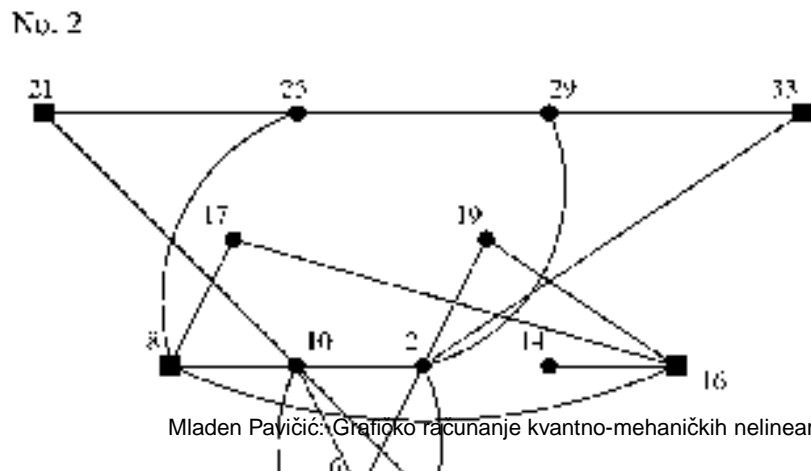
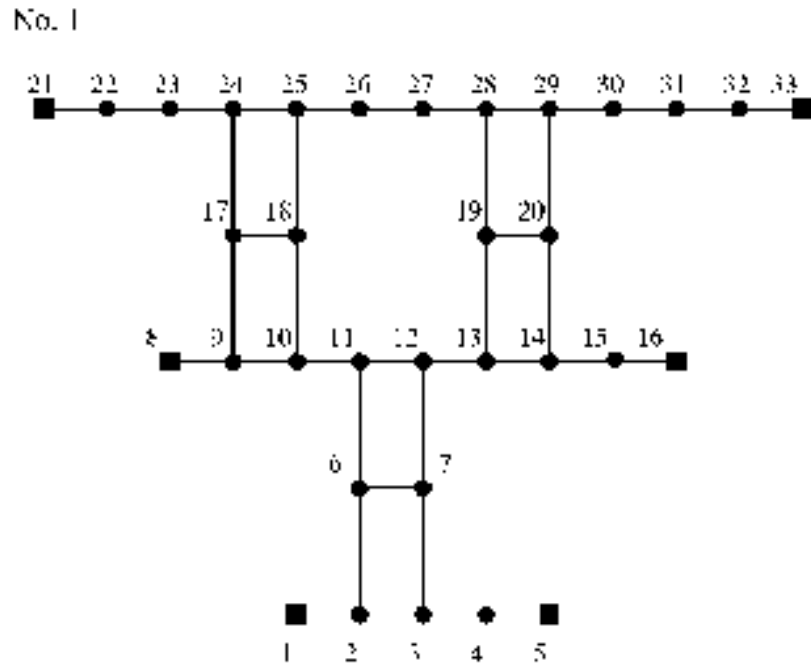


FIG. 14. Graph No. 1 is an entangled ring on 18 vertices. Graph No. 2 represents the resulting graph between the partitions A , whose vertices are depicted by boxes, and the partition B , whose vertices are depicted by discs.

ties, which makes it “difficult” to disentangle the state by few measurements. From this one can understand why the gap between the lower and upper bound occurs in such cases. As discussed in Sec. III B of all graph codes with less than seven vertices only these two are candidates for strongly error detecting graph codes introduced in Ref. [7].

Example 5. Concatenated [7,1,3]-CSS code.

The graph G depicted in Fig. 15 represents an encoding procedure for the concatenated $[7,1,3]$ -CSS code. The corresponding graph state has Schmidt measure 28. For encoding, the qubit at the vertex \circ can be in an arbitrary state. With the rest of the vertices (initially prepared in the state corresponding to $|x, +\rangle$), it is then entangled by the 2-qubit unitary $U^{(a,b)}$ introduced in Eq. (10). Encoding the state at vertex \circ



Orthogonal Spins



We have to measure spins in 3, 4, 5, ... dimensions.

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Orthogonal Spins



We have to measure spins in 3, 4, 5, ... dimensions. Of course in a Hilbert space. It corresponds to outgoing ports in our 3-dim lab. Vectors are orthogonal \Rightarrow nonlinear equations

$$\mathbf{a}_B \cdot \mathbf{a}_C = a_{B1}a_{C1} + a_{B2}a_{C2} + a_{B3}a_{C3} + a_{B4}a_{C4} = 0,$$

$$\mathbf{a}_B \cdot \mathbf{a}_D = a_{B1}a_{D1} + a_{B2}a_{D2} + a_{B3}a_{D3} + a_{B4}a_{D4} = 0,$$

$$\mathbf{a}_B \cdot \mathbf{a}_E = a_{B1}a_{E1} + a_{B2}a_{E2} + a_{B3}a_{E3} + a_{B4}a_{E4} = 0,$$

$$\mathbf{a}_C \cdot \mathbf{a}_D = a_{C1}a_{D1} + a_{C2}a_{D2} + a_{C3}a_{D3} + a_{C4}a_{D4} = 0,$$

$$\mathbf{a}_C \cdot \mathbf{a}_E = a_{C1}a_{E1} + a_{C2}a_{E2} + a_{C3}a_{E3} + a_{C4}a_{E4} = 0,$$

$$\mathbf{a}_D \cdot \mathbf{a}_E = a_{D1}a_{E1} + a_{D2}a_{E2} + a_{D3}a_{E3} + a_{D4}a_{E4} = 0.$$

Mission Impossible



To solve these equations for all possible combinations for at least 18 vectors (no solutions below 18) we would need a million ages of the universe on all today's processors on the Globe working in parallel.

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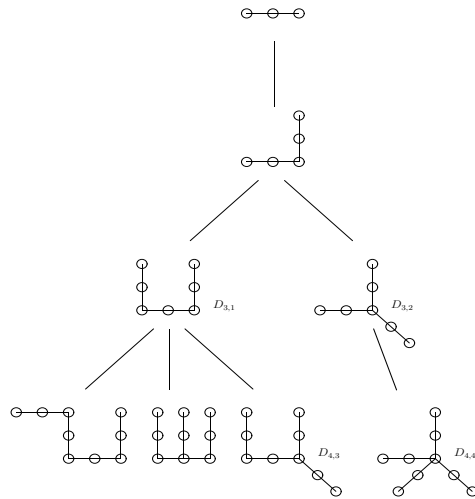


Figure 1: Generation tree for MMP diagrams with loops of size 5 for 3-dim vectors.



Use MMP hyper-graphs instead of equations.

Exponential \Rightarrow Polynomial



We first “translate” nonlinear equations into linear hypergraphs.

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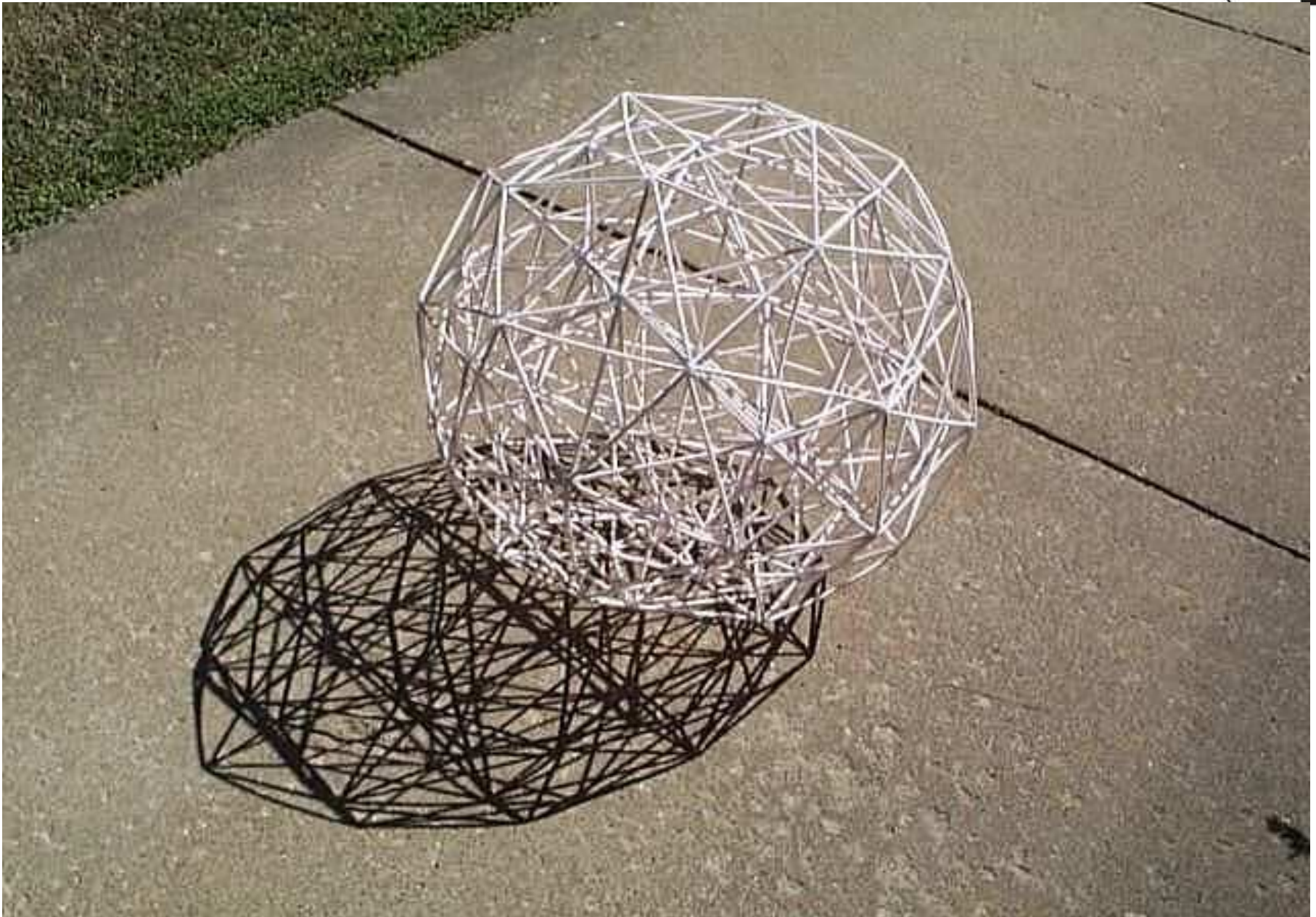
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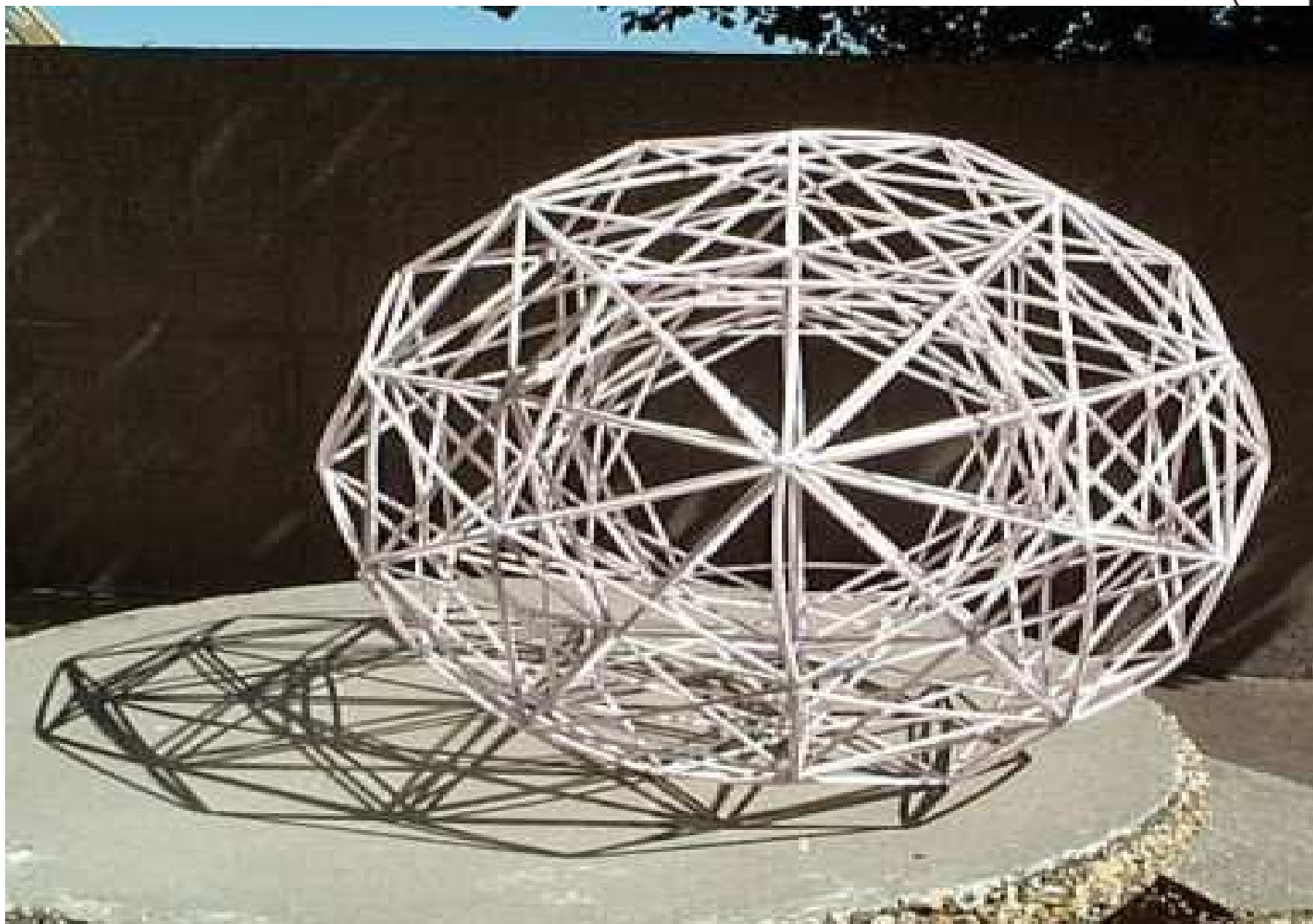
In the end we translate a rather “small” number (millions) of hypergraphs back into equations and solve them by means of interval analysis method. Its programs are SPC as well.

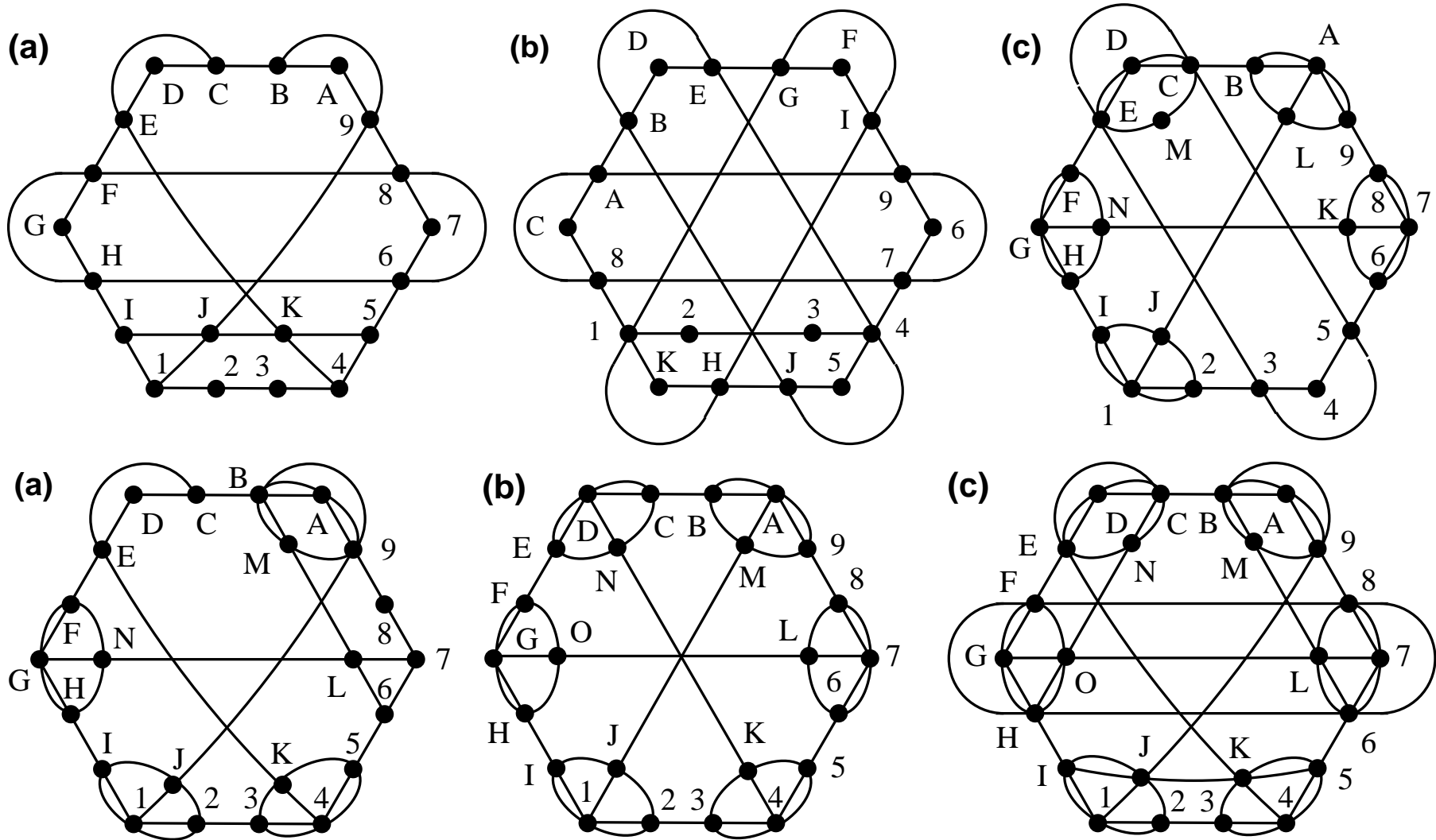
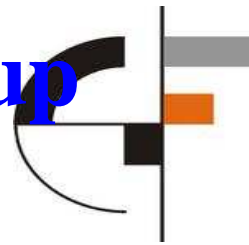
David A. Richter

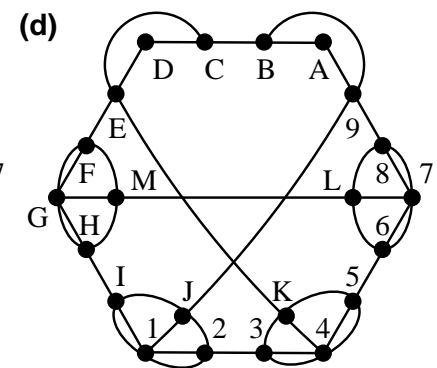
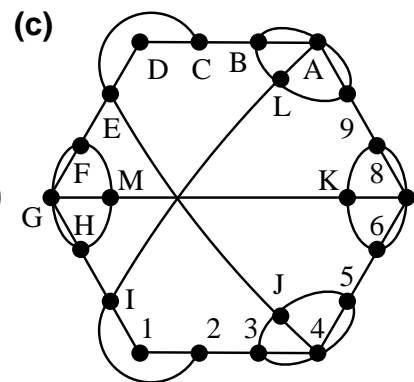
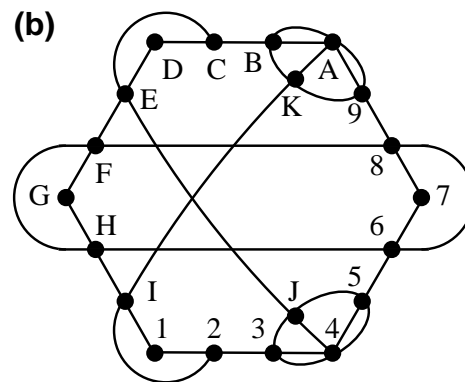
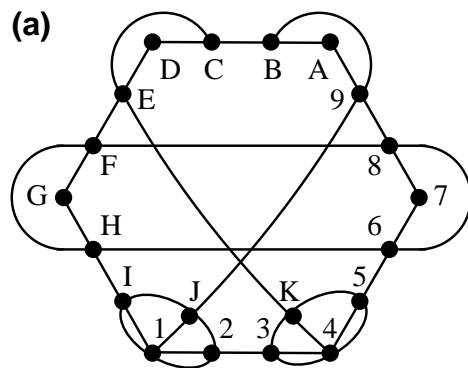
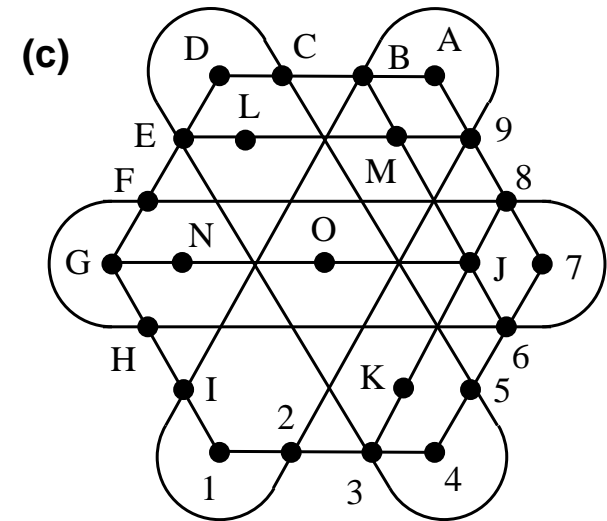
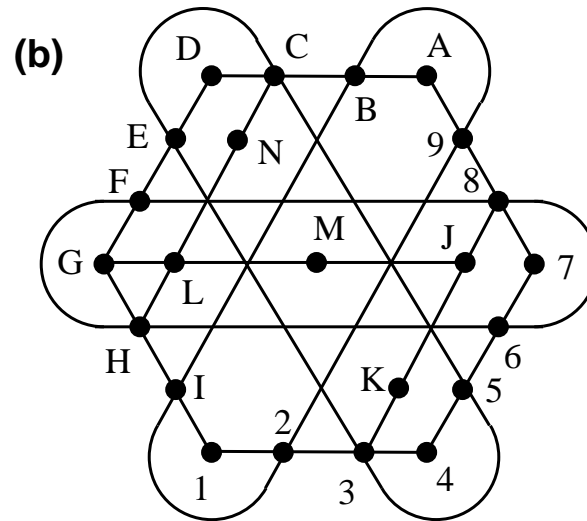
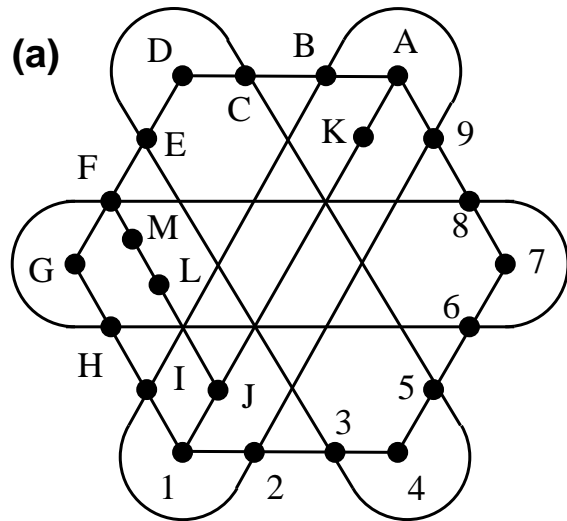












Solutions



\	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<i>total</i>
18	1																1
19		1															1
20		1	4+	1													7
21			2	11	4	1											18
22			1	9	36	23	12	3	1								85
23				2	19	76	79	58	27	11	3	1					276
24				1	6	39	137	187	188	136	83	40+	1	18	6	2	845
<i>total</i>	1	2	8	24	65	139	228	248	216	147	86	42	18	6	2	1	1233

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Boxed in red are the solutions previously found by humans.

Solutions

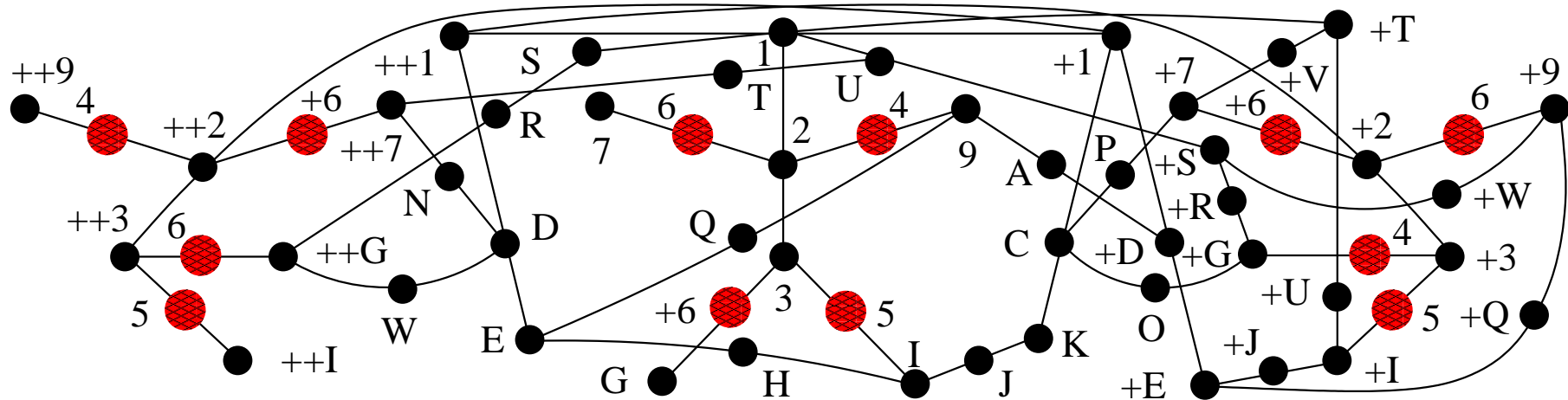
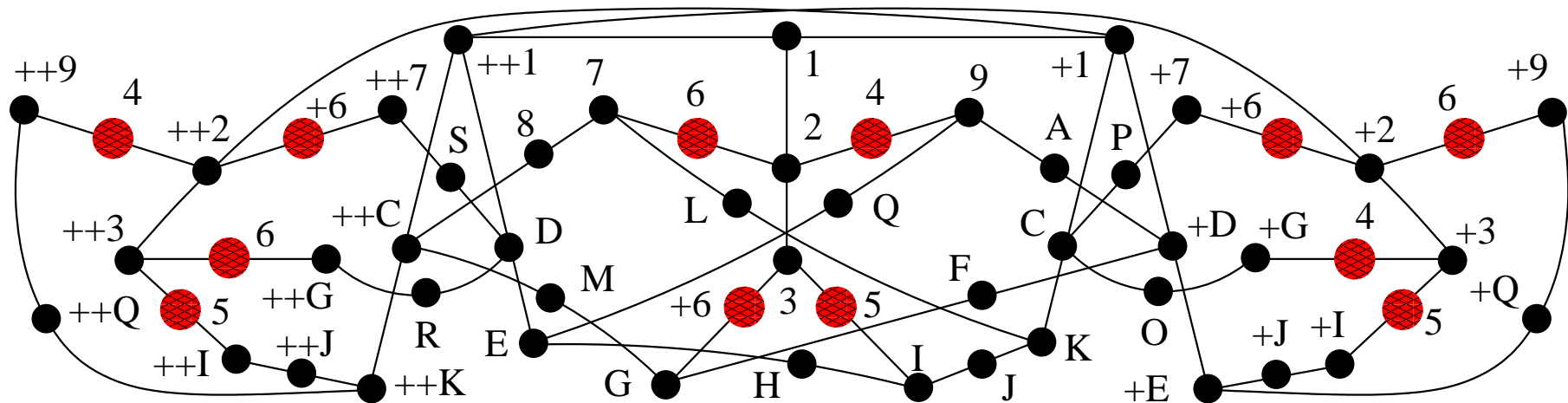


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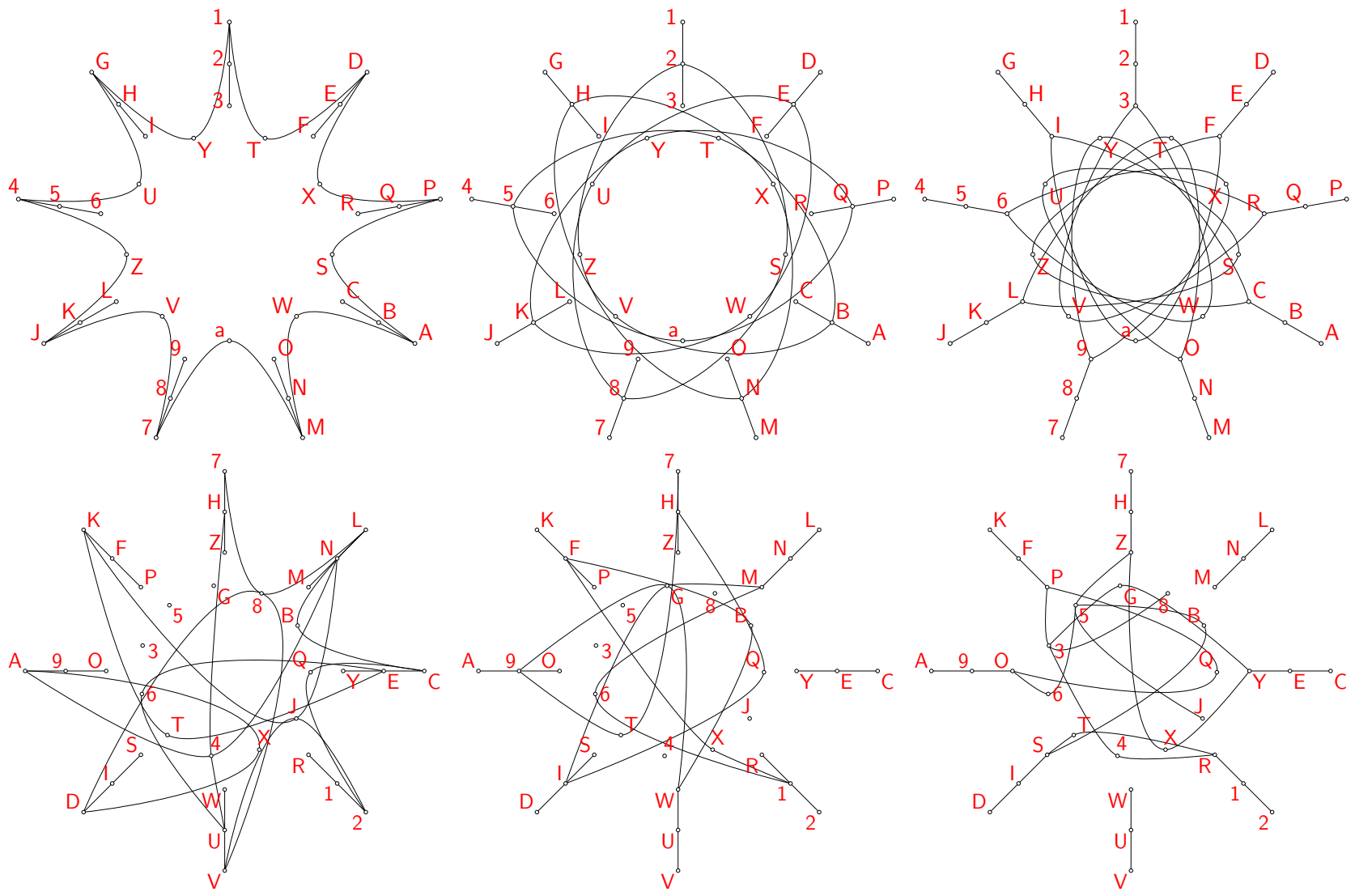
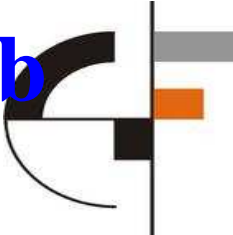
Boxed in red are the solutions previously found by humans.

It would take over 150 years to generate all the solutions on a single PC





Krešimir Fresl, Građevinski fakultet, Zagreb



A i Danko Bosanac, IRB ...



je na projektu, ali u njegovim drugim teorijskim dijelovima, a ne na konkretnim clusterskim i grid računanjima.



CRONGI Grid Report for Mon, 26 Oct 2009 03:11:52 +0100

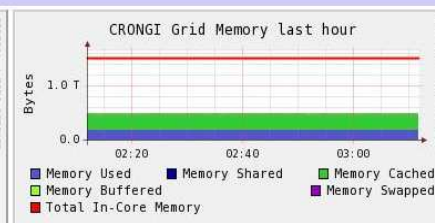
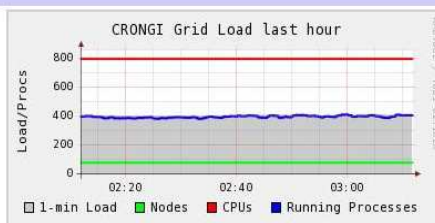
[Get Fresh Data](#)Last Sorted CRONGI Grid >

CRONGI Grid (5 sources) (tree view)

CPUs Total: 796
Hosts up: 77
Hosts down: 0

Avg Load (15, 5, 1m):
50%, 51%, 51%

Localtime:
2009-10-26 03:11

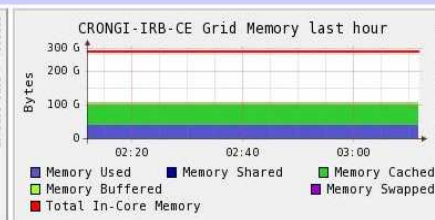
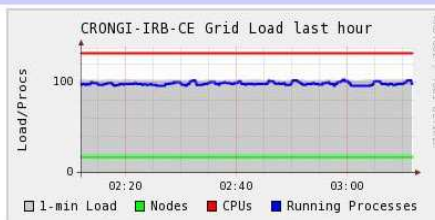


CRONGI-IRB-CE Grid (tree view)

CPUs Total: 132
Hosts up: 17
Hosts down: 0

Avg Load (15, 5, 1m):
77%, 77%, 77%

Localtime:
2009-10-26 03:11

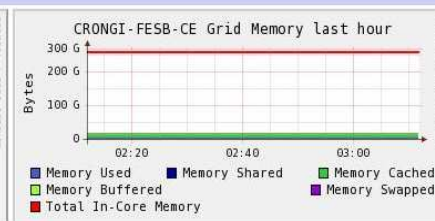
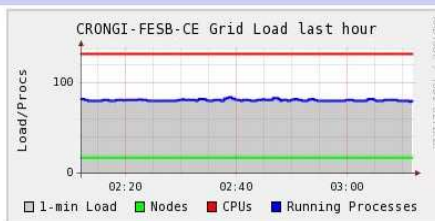


CRONGI-FESB-CE Grid (tree view)

CPUs Total: 132
Hosts up: 17
Hosts down: 0

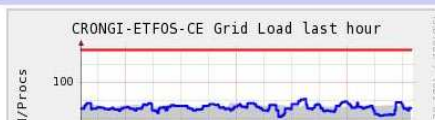
Avg Load (15, 5, 1m):
61%, 61%, 61%

Localtime:
2009-10-26 03:11



CRONGI-ETFOS-CE Grid (tree view)

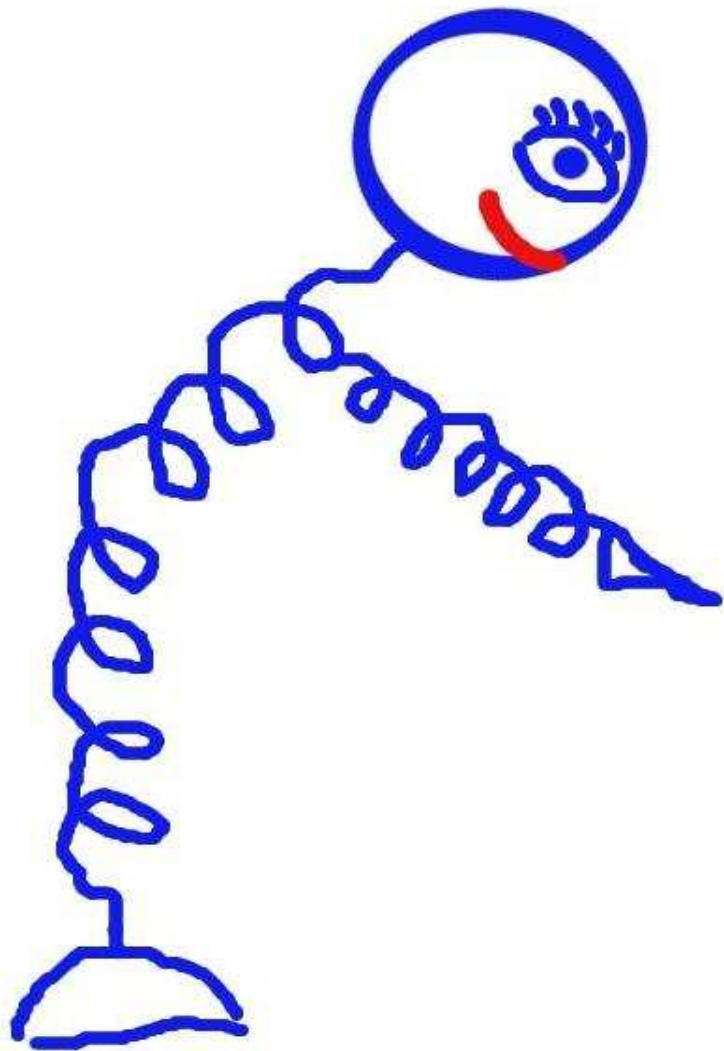
CPUs Total: 136
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PARALELIZAM & VIZUALIZACIJA ...



... ili kako paralelno vizualizirano grid računanje gradi kvantnu—inherentno paralelnu—simulaciju.



Hvala!