Near-Deterministic Discrimination of All Bell States with Linear Optics

Mladen Pavičić*
Chair of Physics, Faculty of Civil Engineering, University of Zagreb, Zagreb 10001, Croatia & Institute for Theoretical Atomic, Molecular and Optical Physics at Physics Department at Harvard University and Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, USA
(Dated: July 18, 2011)

For a reliable implementation of quantum teleportation, a near-deterministic (close to 100%) discrimination of all four Bell states of entangled qubits is required. One can carry it out with linear optical elements only if conditional dynamics are allowed. Here we present a setup in which we repeatedly disentangle and re-entangle photons in three of four states, so as to separate photons in one of them, conditioned on keeping the other two at bay. The efficiency of a realistic implementation of our setup with current technology is over 90% for an ideal source of photons on demand.

PACS numbers: 03.67.Hk, 42.50.p, 03.67.a, 03.65.Bz

Predicted future implementations of quantum computers [1], quantum repeaters [2], and quantum error correction [3] devices often rely on quantum teleportation and swapping [4, 5],[6, Sec. 8.2] and superdense coding [7]. They are all based on the discrimination of the Bell states, i.e., a linearly independent basis of a 4-dim Hilbert space of entangled qubits. [6, Secs. 4.1.2, 8.2]

The qubits used for the above protocols are usually photons. Therefore, a method for discriminating Bell states that is technologically the easiest to implement uses only linear optical devices, such as beam splitters, wave plates, and single photon detectors, and only one degree of freedom, usually polarisation. This is opposed to two or more degrees of freedom the use of which is called *hyperentanglement*.

In 1999, Vaidman's [8] and Lütkenhaus' [9] groups put forward a *no-go proof* which states that one can achieve 100% efficiency of discriminating Bell states, i.e., discriminate all four of them, only "in a limit." If we exclude a conditional dynamics, then only 50% efficiency is possible, i.e., only two Bell states can be unambiguously discriminated. [10] By "conditional dynamics," "we mean that we monitor one selected mode while keeping the other modes in a waiting loop. Then we can perform some linear operation on the remaining modes." [9]

The no-go proofs allow nonlinear conditional setups, nonlinear interactions, and hyperentanglements, though. Therefore several proposals that include the evolution of a subsystem conditioned on the state of another subsystem have been put forward [11], and several experiments with nonlinear interactions [12] and hyperentanglements [13, 14] have been carried out. Knill, Laflamme, and Milburn have given a proof of principle that one can carry out a near-deterministic protocol with linear optics, based on conditional sign flips. These can be implemented by means of nonlinear sign gates that make use of ancilla photons. [15] They also exploit feedback from detectors.

Here we show that the above no-go proofs also allow linear optical setups in which a near-deterministic separation of three Bell states from each other is carried by means of their conditional separation. Our approach does not make use of ancillas, does not require nonlinear spin gates, and does not exploit feedback. It is therefore not only simpler and feasible with the current technology, but it also provides us with a new method of re-entangling photons and of separating the obtained entanglements.

In the standard all-optical implementation of teleportation, we want to transfer an unknown state

$$|\psi\rangle_1 = \alpha|H\rangle_1 + \beta|V\rangle_1 \tag{1}$$

of photon 1 to photon 3 from an independently produced pair of photons {2,3} (H and V represent horizontal and vertical polarisations of photon 1, respectively.) The pair {2,3} is in one of the four Bell states

$$|\Psi^\pm\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2|V\rangle_3 \pm |V\rangle_2 H\rangle_3),$$
$$|\Phi^\pm\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2|H\rangle_3 \pm |V\rangle_2 V\rangle_3). \tag{2}$$

Let it be in a $|\Psi^+\rangle_{23}$ state. Bob combines photons 1and 2

$$|\psi\rangle_1 \otimes |\Psi^+\rangle_{23} = \frac{1}{2}\left[|\Psi^+\rangle_{12}|\psi\rangle_3 + |\Psi^-\rangle_{12}(\alpha|H\rangle_3 - \beta|V\rangle_3)\right.$$
$$\left.+|\Phi^+\rangle_{12}(\beta|H\rangle_3 + \alpha|V\rangle_3) + |\Phi^-\rangle_{12}(\beta|H\rangle_3 - \alpha|V\rangle_3)\right]. \tag{3}$$

Then he determines which Bell state photons 1 and 2 were in and communicates his result to Alice via a classical channel. She makes use of half wave plates (HWPs) to recover the unknown state $|\psi\rangle$. [6, 12]

Let us first sketch how Bob's device works. Photons 1 and 2 reach a beam splitter (BS) from its opposite sides. If in state $|\Psi^-\rangle$, they will emerge from the opposite sides of the BS (they *split*) and states can be identified just by the reactions of detectors at those opposite sides. If in any of the other three states, they will both emerge from the same side of the BS (they *bunch* together). Then they are directed to concatenated Mach-Zehnder (MZ) interferometers. From the first MZ, 3/4 of photons in state $|\Psi^+\rangle$ will split and 1/4 will bunch together. Photons in states $|\Phi^\pm\rangle$ will exit as they entered—bunched

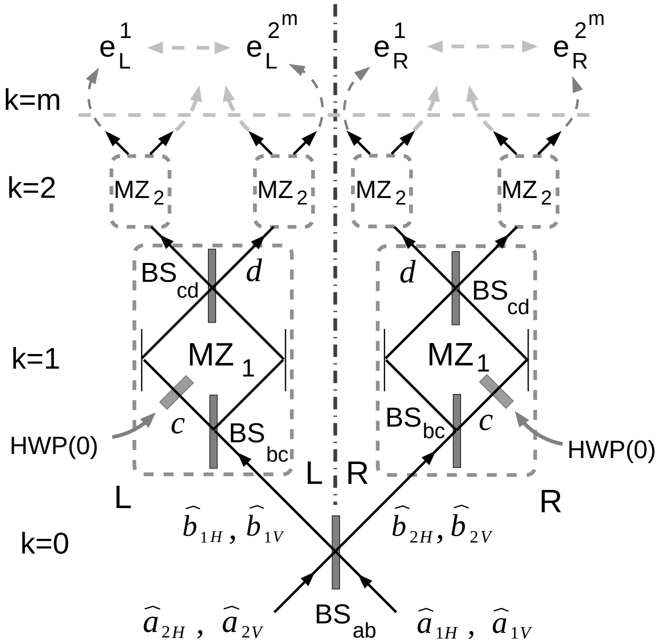

FIG. 1. Schematic of →100% discrimination of all four Bell states. In MZ_1 3/4 of $|\Psi^+\rangle$ pairs split; 1/4 emerge bunched together from either side of BS_{cd}. States $|\Phi^\pm\rangle$ bunch together. States are sent for further separation to MZ_k, $k = 2, \ldots, m$ (which are the same as MZ_1) and then to 2^m $e_{L,R}$ ports.

together. At the next MZs, 15/16 of the former photons will split again, at the next ones 63/64, and so on. This ideally amounts to a separation of 75%, 94%, 98%, etc. The latter photons will stay bunched together after each stage and by letting them through a HWP and a polarizing beam splitter (PBS), we separate $|\Phi^+\rangle$ from $|\Phi^-\rangle$. The details are as follows.

At a BS input-output creation-operator relations have the following form (see Fig. 1)

$$\hat{b}^\dagger_{1H} = (\hat{a}^\dagger_{1H} + \hat{a}^\dagger_{2H})/\sqrt{2}, \quad \hat{b}^\dagger_{2H} = (\hat{a}^\dagger_{1H} - \hat{a}^\dagger_{2H})/\sqrt{2},$$
$$\hat{b}^\dagger_{1V} = (\hat{a}^\dagger_{1V} + \hat{a}^\dagger_{2V})/\sqrt{2}, \quad \hat{b}^\dagger_{2V} = (\hat{a}^\dagger_{1V} - \hat{a}^\dagger_{2V})/\sqrt{2}. \quad (4)$$

Creation operators generate the photon states when applied to the Fock vacuum state $|0\rangle$. For instance, $|H\rangle_{a1} = \hat{a}^\dagger_{1H}|0\rangle$ is an input and $|V\rangle_{b1} = \hat{b}^\dagger_{1V}|0\rangle$ is an output state at a BS. Here, as well as in the indices below, a (b) refers to photons coming to (emerging from) BS_{ab}. Index 1 (2), here and below, refers to photons arriving at BS_{ab} from the right (left) side (see Fig. 1) and emerging from its left (right) side.

When photons in Bell states arrive at BS_{ab}, then $|\Psi^-\rangle_{a12} \to |\Psi^-\rangle_{b12}$, while $|\Psi^+\rangle_{a12}, |\Phi^\pm\rangle_{a12}$ emerge from one side of BS_{ab} and stick together; e.g., as $|\Psi^+\rangle_{b11}$, $|\Phi^-\rangle_{b22}$, and $|\Phi^+\rangle_{b11}$. [1, 6, 16] That means that photons in state $|\Psi^-\rangle_{b12}$ split and all the other three Bell states bunch together.

Next, we shall follow the levels b, c, d shown in Fig. 1. Photon pairs are sent to BS_{bc} and then to BS_{cd}, which together with a HWP(0) form MZ_1. Photons emerging from MZ_1s go to MZ_2s, which are replica of MZ_1s.

To calculate what happens with $|\Psi^+\rangle$ at BS_{bc}, we use Eq. (4). We form the inverse transformation to obtain the outcoming states. The incoming state disentangles at BS_{bc} [b (c) refers to photons before (after) BS_{bc}]:

$$\hat{b}^\dagger_{1V}\hat{b}^\dagger_{1H} = (\hat{c}^\dagger_{1H}\hat{c}^\dagger_{2V} + \hat{c}^\dagger_{1V}\hat{c}^\dagger_{2H} + \hat{c}^\dagger_{1H}\hat{c}^\dagger_{1V} + \hat{c}^\dagger_{2H}\hat{c}^\dagger_{2V})/2. \quad (5)$$

The HWP(0)s change the signs of any "V" terms that pass through it (not all do, though), and the first two terms in Eq. (5) yield $\frac{1}{2}|\Psi^-\rangle_{c-12}$. The last two terms yield

$$\text{either} \quad \hat{c}^\dagger_{1H}\hat{c}^\dagger_{1V} - \hat{c}^\dagger_{2H}\hat{c}^\dagger_{2V} = \hat{d}^\dagger_{1H}\hat{d}^\dagger_{2V} + \hat{d}^\dagger_{1V}\hat{d}^\dagger_{2H}, \quad (6)$$
$$\text{or} \quad \hat{c}^\dagger_{1H}\hat{c}^\dagger_{1V} + \hat{c}^\dagger_{2H}\hat{c}^\dagger_{2V} = \hat{d}^\dagger_{1H}\hat{d}^\dagger_{1V} - \hat{d}^\dagger_{2H}\hat{d}^\dagger_{2V}, \quad \text{i.e.,} \quad (7)$$
$$\text{either} \quad \frac{1}{2}|\Psi^+\rangle_{d-12}, \quad \text{or} \quad \frac{1}{2}(|H\rangle_{d1}|V\rangle_{d1} - |V\rangle_{d2}|H\rangle_{d2}), \quad (8)$$

depending on whether the photons pass through HWP(0) or not, respectively. [Index $c-12$ ($d-12$) means that photons split at BS_{bc} (BS_{cd}) and $c1$ ($c2$) that they bunch together at the left (right) exit of BS_{bc}.] The photons bunch together and take the arm with HWP(0) with 50% probability. Thus at the first level, $k = 1$, the former photons and half of the latter photons split (to both $c1$ and $c2$ arms). Taken together, the photons split with 75% probability and bunch together with 25% probability.

The split photons emerge from BS_{cd} in either the state $|\Psi^-\rangle_{d-12}/2$ or in $|\Psi^+\rangle_{d-12}/2$ one. Which one of them the photons will be in is irrelevant, since for our final measurement it is only important that they emerge from the opposite sides of BS_{cd}. The bunched photons arrive at BS_{de} in MZ_2 and disentangle as in Eq. (5). Again, only 25% of them reappear bunched together from MZ_2. The probability of detecting correlated clicks for photons emerging from MZ_2 is therefore 93.75%, at $k = 3$ it is 98.44%, and at $k = m$ it is $100 \cdot (1 - 2^{-2m})$%.

To calculate what happens with $|\Phi^\pm\rangle$ at BS_{bc}, we again make use of Eq. (4):

$$\hat{b}^\dagger_{1H}\hat{b}^\dagger_{1H} \pm \hat{b}^\dagger_{1V}\hat{b}^\dagger_{1V} = \hat{c}^\dagger_{1H}\hat{c}^\dagger_{2H} \pm \hat{c}^\dagger_{1V}\hat{c}^\dagger_{2V}$$
$$+\frac{1}{2}(\hat{c}^\dagger_{1H}\hat{c}^\dagger_{1H} \pm \hat{c}^\dagger_{1V}\hat{c}^\dagger_{1V}) + \frac{1}{2}(\hat{c}^\dagger_{2H}\hat{c}^\dagger_{2H} \pm \hat{c}^\dagger_{2V}\hat{c}^\dagger_{2V}). \quad (9)$$

and obtain the states [which the HWP(0)s will not change]:

$$|HH\rangle_{c12} \pm |VV\rangle_{c12} + \frac{1}{2}|2H\rangle_{c11} \pm \frac{1}{2}|2V\rangle_{c11} + \frac{1}{2}|2H\rangle_{c22}$$
$$\pm\frac{1}{2}|2V\rangle_{c22} = |\Phi^\pm\rangle_{c-12} + \frac{1}{2}|\Phi^\pm\rangle_{c-11} + \frac{1}{2}|\Phi^\pm\rangle_{c-22} \quad (10)$$

in which photons either split (to the $c1$ and $c2$ arms; indices $c12$ and $c-12$) or bunch together (to either $c1$ or $c2$; indices $c11$, $c22$, $c-11$, $c-22$). (Cf. [6, Eq. (4.29)].)

At BS_{cd}, the terms in brackets in Eq. (9) transform to

$$\frac{1}{2}(\hat{d}^\dagger_{1H}\hat{d}^\dagger_{1H} \pm \hat{d}^\dagger_{1V}\hat{d}^\dagger_{1V}) + (\hat{d}^\dagger_{1H}\hat{d}^\dagger_{2H} \pm \hat{d}^\dagger_{1V}\hat{d}^\dagger_{2V})$$
$$+\frac{1}{2}(\hat{d}^\dagger_{2H}\hat{d}^\dagger_{2H} \pm \hat{d}^\dagger_{2V}\hat{d}^\dagger_{2V}) - (\hat{d}^\dagger_{1H}\hat{d}^\dagger_{2H} \pm \hat{d}^\dagger_{1V}\hat{d}^\dagger_{2V}), \quad (11)$$

The last two terms in each row cancel each other, and we obtain the states $(|\Phi^{\pm}\rangle_{d-11} + \frac{1}{2}|\Phi^{\pm}\rangle_{d-22})/2$, which have the same functional form as $|\Phi^{\pm}\rangle_{c-11}$ and $|\Phi^{\pm}\rangle_{c-22}$ in Eq. (10). This recombination of photons at a beam splitter within an MZ interferometer is a Bell version of the NOON state. [6, Sec. 5.3, Eq. (5.44), $P_2 = 0$].

States $|\Phi^{\pm}\rangle_{c-12}$ from (10) transform to $(|\Phi^{\pm}\rangle_{d-11} - \frac{1}{2}|\Phi^{\pm}\rangle_{d-22})/2$. The sign "−" here is of no importance since we will not recombine left and right outputs.

Thus the states $|\Phi^{\pm}\rangle_{d-11}$ and $|\Phi^{\pm}\rangle_{d-22}$ are the only states in which photons, that originally were in states $|\Phi^{\pm}\rangle_{a-12}$, will appear. When we send the photons from MZ_k to MZ_{k+1} they will always enter and exit MZs in the same state. In effect, we keep them at bay with respect to the $|\Psi^+\rangle$ mode whose photons are being more and more separated. In a real experiment this "keeping at bay" is counterfactual because we will never have two Bell states in the system at the same time. However, in our calculation, we conditionally take care of all modes simultaneously, because all photons in whatever mode pass through the same elements of the setup.

When the photons exit the last MZ_m, they are directed to the device shown in Fig. 2, where they first pass through a half-wave plate oriented at 22.5° [HWP($\frac{\pi}{8}$)], which rotates the polarisation direction for 45° in the path of photons emerging from BS (unnormalised).

$$\mathrm{HWP}(\frac{\pi}{8})|\Phi^-\rangle = (|H\rangle + |V\rangle)(|H\rangle + |V\rangle)$$
$$-(|H\rangle - |V\rangle)(|H\rangle - |V\rangle) = |HV\rangle + |VH\rangle = |\Psi^+\rangle. \quad (12)$$

Hence, photons in state $|\Phi^-\rangle$ split at the PBS behind the plate. The HWP does not affect photons in state $|\Phi^+\rangle$.

Detectors with photon number resolution can recognize the $|\Phi^+\rangle$ state in one step, and they should be used for future applications. But their efficiency is currently under 90%, [17, 18] so, for the best testing of our results we propose single photon detectors (arranged as in Fig. 2), whose current efficiencies are > 98%, [19] However, the first postselection experiments can also be carried with today's standard detectors (efficiency around 60%). They would provide us with detection patterns that indicate projections onto any of the four Bell states, and they can be carried out with as few as five detectors. Of course, the overall efficiency would then drop significantly, but for a postselection verification, the overall efficiency is not relevant. To see how this works, let us consider a detector on the L(eft) side in Fig. 1. Its "click" together with a "click" of the one on the R(ight) side detect $|\Psi^-\rangle$. The latter detector and a 3rd one at another $\mathrm{e_R}$ detect $|\Psi^+\rangle$. The 3rd and a 4th one on the L and R sides of PBS in Fig. 2, respectively, detect $|\Phi^-\rangle$. Finally, the 4th and a 5th one on the R side of PBS detect $|\Phi^+\rangle$.

In Fig. 2 photons enter BSs from one side only.

$$\hat{f}^{\dagger}_{1H}\hat{f}^{\dagger}_{1H} + \hat{f}^{\dagger}_{1V}\hat{f}^{\dagger}_{1V} = \hat{g}^{\dagger}_{1H}\hat{g}^{\dagger}_{2H} + \hat{g}^{\dagger}_{1V}\hat{g}^{\dagger}_{2V}$$
$$+\frac{1}{2}(\hat{g}^{\dagger}_{1H}\hat{g}^{\dagger}_{1H} + \hat{g}^{\dagger}_{1V}\hat{g}^{\dagger}_{1V}) + \frac{1}{2}(\hat{d}^{\dagger}_{2H}\hat{d}^{\dagger}_{2H} + \hat{d}^{\dagger}_{2V}\hat{d}^{\dagger}_{2V}). \quad (13)$$

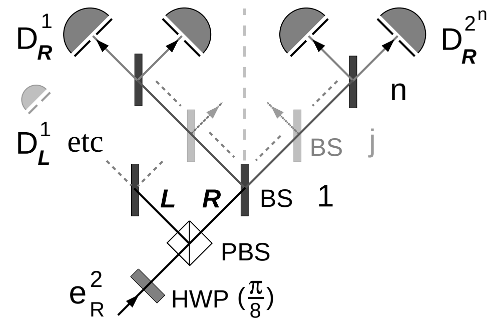

FIG. 2. Concatenated polarisation-preserving beam splitter device for splitting $|\Phi^+\rangle$ photons emerging from ports shown in Fig. 1. ($|\Phi^-\rangle$ splits at PBS.) The figure above is made for port $\mathrm{e_R^2}$. Devices for other ports are the same. The indicated left (L) branch of the device is the same as the right one (R).

So, 50% of photons emerge from the opposite sides of a BS. [20, 21] Photons go through n BSs as shown in Fig. 2. The procedure requires $2^n - 1$ beam splitters and 2^n detectors or just 5 detectors in a postselection mode. The probability of discriminating $|\Phi^+\rangle$ by detecting photons coming out from the nth row of BSs in coincidence is $1 - 2^{-n}$. For example for $n = 6$ the probability is 98.4%.

Taken together, the coincidence clicks shown in Table I correspond to a deterministic discrimination of all four Bell states in the $\to$ 100% limit, i.e., for $m, n \to \infty$.

	"clicks" traced from ports (see Fig. 1)		
$\lvert\Psi^-\rangle$	e_L^i	AND	e_R^j
$\lvert\Psi^+\rangle$	e_L^j	AND	e_L^l
$\lvert\Psi^+\rangle$	e_R^j	AND	e_R^l
	"clicks" (traced from either any $\mathrm{e_L}$ or any $\mathrm{e_R}$) of		
$\lvert\Phi^-\rangle$	D_L^o	AND	D_R^p
$\lvert\Phi^+\rangle$	D_L^p	AND	D_L^q
$\lvert\Phi^+\rangle$	D_R^p	AND	D_R^q
$i, j, l = 1, \ldots, 2^m$; $j \neq l$; $o, p, q = 1, \ldots, 2^n$; $p \neq q$			

TABLE I. Ideal discrimination of all four Bell states for $n, m \to \infty$. For $m = 3$ and $n = 6$ the efficiency is 95.3%.

A realistic experiment is feasible with the current technology. Path-length differences for MZs should be set to zero, and frequencies of both photons should be as equal as possible. [22] When one changes path-length difference of an MZ [after subtraction of accidental detections (e.g., 1 or 3 "clicks")], the visibility of the fourth-order interference fringes is ∼1 [23], so an efficient concatenation of MZs should be feasible if demanding. Wave packet calculation can be made according to calculations carried out in [6, Ch. 5] & [24].

As for the losses in the system, they are minimal. Metallic BSs can be gold-coated with losses as low as 1%. Alternately, we can utilise dielectric BSs (with the losses of 0.1%; they were used, e.g., in [13]). Our equations and Table I should then be recalculated by substituting Eqs. (4.2) and (4.3) from [6, Sec. (4.1)] for Eqs. (4). Therefore we estimate that an efficiency of 98% per MZ is achievable

We obtain the maximal number of MZs that we can efficiently concatenate, and with it the maximal efficiency of separating $|\Psi^+\rangle$ photons that we can achieve, as follows. For MZ_1 we have $0.75 \cdot 0.98 \approx 0.74$, for MZ_2 $0.94 \cdot 0.98^2 \approx 0.9$, for MZ_3 $0.984 \cdot 0.98^3 \approx 0.94$, and for MZ_4 $0.996 \cdot 0.98^4 \approx 0.92$. Hence, with 98% efficiency for each MZ, we reach the maximal overall efficiency of 94% with three MZs. Only with MZ efficiency $\to 100\%$ can we achieve $|\Psi^+\rangle$ separation and Bell state discrimination $\to 100\%$. The 94% efficiency mentioned above is what we should obtain in a postselection experiment even with standard detectors. If we included the losses at detectors, we would still obtain an overall realistic efficiency $> 90\%$ (with the best achievable detectors).

A source of *event ready* photons, i.e., *photons on demand* is currently the least efficient element of the setup. Four photon [25] and six photon down conversion schemes for obtaining such a source have been proposed. The latter scheme has recently been realized with an efficiency of up to 77% but with the success probability of order 10^{-6}. [26] So, substituting fiber couplers for beam splitters would be an efficient option in spite of higher losses.

A comparison of a possible realistic realisation of the present proposal with the already obtained experimental results can be made with respect to two main applications: teleportation and superdense coding.

"The use of hyperentanglement of photons, unfortunately, does not offer advantages for teleportation ... having only 50% probability of success." [27] Nonlinear setups, on the other hand, cannot offer efficiency over 50% [12] with the current technology. Thus, our setup is apparently by far the most efficient one for generating near-deterministic teleportation.

Superdense coding—sending up to two bits of information by manipulating just one qubit—recently achieved a postselection channel capacity of $1.63 > \log_2 3 \approx 1.585$. [14] In our setup, Alice's postselection suffers from $|\Psi^+\rangle \leftrightarrow |\Phi^+\rangle$ ambiguities: for $m = 3$, 1.6% of $|\Psi^+\rangle$ photons stay bunched together, and $\mathrm{HWP}(\frac{\pi}{8})$ converts their $|\Psi^+\rangle$ state into $|\Phi^-\rangle$ one. So, at the PBS after the HWP, both photons will either go through or be reflected and will be indistinguishable from $|\Phi^+\rangle$. The efficiencies of detecting the other two states, $|\Psi^-\rangle$ and $|\Phi^-\rangle$, are 100%. Thus we can unambiguously transfer $2 + 2 \cdot 0.98 \approx 3.97$ messages via one qubit, i.e., our channel capacity is $\log_2 3.97 \approx 1.99$. Note that in a realistic postselection measurement scheme, even $n = 1$ suffices in Fig. 2, and all clicks that correspond to any losses of photons (one or none) at BSs are discarded. Hence, the inefficiencies of devices that generate photons on demand are also eliminated.

Taken together, we achieve an overall efficiency of detecting pre- as well as postselected photons in Bell states that convincingly beat all the previous schemes.

Acknowledgments. Supported by the Ministry of Sci. Edu & Sport of Croatia (project 082-0982562-3160) and by a US Nat. Sci. Found. grant to the Institute for Theoretical Atomic, Molecular and Optical Physics at Harvard University and Smithsonian Astrophysical Observatory. I thank my host Hossein Sadeghpour for his hospitality.

* mpavicic@grad.hr

[1] M. Pavičić, *Quantum Computation and Quantum Communication* (Springer, New York, 2005).

[2] W. Dür, H. Briegel, J. I. Cirac, and P. Zoller, *Phys. Rev. A* **59**, 169 (1999).

[3] A. R. Calderbank and P. W. Shor, *Phys. Rev. A* **54**, 1098 (1996).

[4] C. H. Bennett, et al., *Phys. Rev. Lett.* **70**, 1895 (1993).

[5] M.Pavičić and J.Summhammer, *Phys.Rev.Lett.* **73**, 3191 (1994). D. Bouwmeester, et al. *Nature* **390**, 575 (1997).

[6] Z.-Y. J. Ou, *Multi-Photon Quantum Interference* (Springer, New York, 2007).

[7] C. H. Bennett and S. J. Wiesner, *Phys. Rev. Lett.* **69**, 2881 (1992).

[8] L. Vaidman and N. Yoran, *Phys. Rev. A* **59**, 116 (1999).

[9] N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, *Phys. Rev. A* **59**, 3295 (1999).

[10] J. Calsamiglia and N. Lütkenhaus, *App. Phys. B* **72**, 67 (2001).

[11] M. G. A. Paris, et al., *Phys. Lett. A* **273**, 153 (2000).

[12] Y.-H. Kim, S. P. Kulik, and Y. Shih, *Phys. Rev. Lett.* **86**, 1370 (2001).

[13] C. Schuck, G. Huber, C. Kurtsiefer, and H. Weinfurter, *Phys. Rev. Lett.* **96**, 190501 (2006).

[14] J. T. Barreiro, T.-C. Wei, and P. G. Kwiat, *Nature Phys.* **4**, 282 (2008).

[15] E. Knill, R. Laflamme, and G. J. Milburn, *Nature* **409**, 46 (2001). T.C.Ralph, A. G. White, W. J. Munro, and G. J. Milburn, *Phys. Rev. A* **65**, 012314 (2003).

[16] M. Pavičić, *Phys. Rev. A* **50**, 3486 (1994).

[17] D. Rosenberg, A. E. Lita, A. J. Miller, and S. W. Nam, *Phys. Rev. A* **71**, 061803 (2005).

[18] R. H. Hadfield, *Nature Photonics* **3**, 697 (2009).

[19] A. E. Lita, A. J. Miller, and S. W. Nam, *Optics Express* **16**, 3032 (2008). A. Lita, et al., in *SPIE symposium; Photon Counting IV, April 5-9, 2010,* (Orlando, FL, 2010).

[20] R. A. Campos, B. E. A. Saleh, and M. C. Teich, *Phys. Rev. A* **40**, 1371 (1989).

[21] R. Lange, J. Brendel, E. Mohler, and W. Martienssen, *Europhys. Lett.* **5**, 619 (1988).

[22] J. G. Rarity, et al., *Phys. Rev. Lett.* **65**, 1348 (1990).

[23] A. Yoshizawa and H. Tsuchida, *Appl. Phys. Lett.* **82**, 2457 (2004).

[24] R. A. Campos, B. E. A. Saleh, and M. C. Teich, *Phys. Rev. A* **42**, 4127 (1990).

[25] M. Pavičić, *Opt. Commun.* **142**, 308 (1997).

[26] S. Barz, G. Cronenberg, A. Zeilinger, and P. Walther, *Nature Photonics* **4**, 553 (2010).

[27] T. Wei, J. T. Barreiro, and P. G. Kwiat, *Phys. Rev. A* **75**, 060305 (2007).