Non-linear image noise filtering algorithm based on SVD block processing

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Abstract

A non-linear algorithm for image noise filtering, based on singular value decomposition (SVD) is presented in this paper. Changes in singular values and singular vectors caused by additive white Gaussian noise are described. The non-linear filtering algorithm is based on eliminating that changes in singular values and singular vectors. Noise variance knowledge is not required. Value proportional to noise variance is estimated in the first phase of the algorithm, using least singular values. Once computed SVD of image blocks is then used for filtering operations. Processing of image in smaller blocks makes SVD-based procedure computationally feasible. Experimental results on various images have demonstrated the validity of the approach.

1 Introduction

Singular value decomposition (SVD) is optimal unitary transform for a given image, in the sense that the energy packed in a given number of transformation coefficients is maximized. Although applicable in many image restoration applications, SVD is not often used as a domain of transformation because of a large number of computations required for calculating singular values and singular vectors of large image matrices [3]. Applications of SVD in image processing include image coding, linear space invariant and linear space variant pseudoinverse filtering, image enhancement [1], separation of 2-D filtering operations into 1-D filtering operations, generation of small convolution kernels [5], etc.

Photographic camera, CCD, tube photodetectors and all other image acquisition devices have inherent noise sources due to particular image formation process. Signal to noise ratio of such devices is one of the parameters that describe its quality. Filtering of noise is important also because noise present in blurred image is the main limiting factor for successful image deblurring. There are many various noise filtering algorithms [3]. Some of the well-known representatives are mean filter, median filter, Wiener noise smoother, and reduced update Kalman filter (RUKF). Among the filters that operate in Discrete Fourier Transform (DFT) domain, particularly interesting and efficient is Short Space Spectral Subtraction filter [4]. For each image block the value proportional to noise variance is subtracted from the current block spectrum, while keeping the phase unchanged. In this work, a new method for non-linear noise filtering in the SVD domain is proposed.

2 Image SVD and influence of additive white Gaussian noise

Discrete image is often presented as a $I \times J$ matrix of finite precision numbers. Adding the noise to the non-corrupted image $\mathbf{F}$ will produce random perturbation on every matrix element. Noised image $\mathbf{G}$ is of the same size as the original image:

$$\mathbf{G} = \mathbf{F} + \mathbf{N}, \tag{1}$$

where $\mathbf{N}$ is a random $I \times J$ noise field. Added noise will degrade original information contained in $\mathbf{F}$. Degradation is characterized with the type and the amount of the noise. In the further text we will assume that noise is Gaussian and white.

Noised image is divided into nonoverlapping square blocks of size $b \times b$. For simplicity, let us suppose that $I = Kb$ and $J = Lb$. Then each image block has SVD representation

$$\mathbf{G}_{kl} = \mathbf{U}_{kl}\,\mathbf{S}_{kl}\,\mathbf{V}_{kl}^{\tau} \quad , \ (k = 1,2,\ldots,K \ , \ l = 1,2,\ldots,L)\,, \tag{2}$$

where $\mathbf{U}_{kl}$ is $b \times b$ unitary matrix of left singular vectors, $\mathbf{S}_{kl}$ is $b \times b$ diagonal matrix of singular values, and $\mathbf{V}_{kl}$ is $b \times b$ unitary matrix of right singular vectors [2]. Equation (2) can be interpreted as an outer product expansion, a sum of base images,

$$\mathbf{G}_{kl} = \sum_{r=1}^{R} s_{klr}\mathbf{u}_{klr}\mathbf{v}_{klr}^{\tau} \quad , \tag{3}$$

where R is the rank of $\mathbf{G}_{kl}$, s_{klr} are singular values, $\mathbf{u}_{klr}$ are left singular vectors, and $\mathbf{v}_{klr}$ are right singular vectors. Singular values and corresponding singular vectors contain complete information about the image block. Image degradation, that includes

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blurring and noising, is reflected on changes in singular values and singular vectors. This is true for SVD representation of the whole $I \times J$ image, as well for SVD representation of the individual $b \times b$ image blocks.

One of characteristic values that describes square matrix is its rank. Let the mean rank of the image be the mean of the ranks of all image blocks that constitute the image:

$$\bar{r}_F = \frac{1}{KL} \sum_{k=1}^{K} \sum_{l=1}^{L} rank(\mathbf{F}_{kl}) \ . \tag{4}$$

It has been observed experimentally that natural world images have the mean rank $\bar{r}_F$ very close to b. For images with S/N ratio lower than 20 dB, the mean rank $\bar{r}_G$ of the noised image is always equal to the full rank. We can express the previous statements using the following inequality:

$$\bar{r}_F \le \bar{r}_G \le b \tag{5}$$

Undistorted image is a discrete field of highly correlated elements. Adding the noise causes appearance of false, spotty details. For their representation the base images that contain spots are required. These base images correspond to higher singular vectors and values. This guides us to an intuitive conclusion that the smaller singular values will be increased, what agrees with (5). We approached the problem experimentally. Observed changes in singular values and singular vectors that are common to all tested images manifested certain regularity.

a) The typical increase of singular values (averaged over all image blocks and normalized to one) for a class of real-world scenes corrupted by Gaussian white noise, S/N ratio=15 dB, is depicted in Figure 1. Shape of function depends on image and noise statistics, and S/N ratio value. The depicted function will be more or less skewed for different images and noise types, and typically skewed to the left for lower S/N ratios.

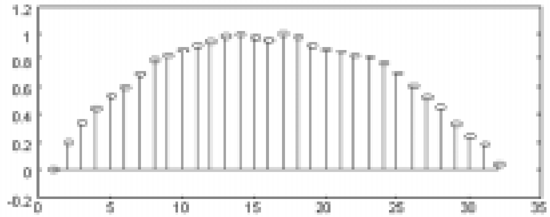

Figure 1: The normalized difference between singular values of noised and original test image, S/N ratio=15 dB, averaged for 256x256 image partitioned in blocks of size b=32.

b) Influence of additive white Gaussian noise on singular vectors was analyzed using DFT spectrum. For 256×256 test image DFT spectrum of every left singular vector was calculated for original and noised image. Noised/original singular vector DFT spectrum ratio is presented as a column on a Figure 2. White corresponds to higher values.

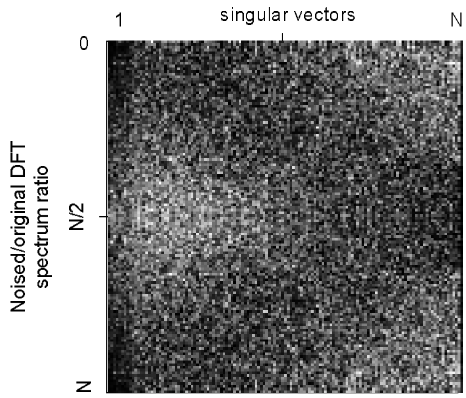

Figure 2: Noised/original left singular vector DFT spectrum ratio , N=256, S/N =10 dB.

Singular vectors that correspond to bigger singular values have significant increase in higher frequencies, while singular vectors that correspond to smaller singular vectors have significant increase in lower frequencies. This behavior is evident for both left and right singular vectors, and doesn’t depend significantly on the image block size and noise variance. Relative change is greater for lower S/N ratios.

3 Block SVD filtering algorithm

The proposed algorithm is based on eliminating changes in singular values and singular vectors, changes that resulted from additive white Gaussian noise in the image. Singular values that correspond to higher r values in equation (3) are very small for undistorted image, if not zero. The additive noise increases their values, proportional to noise variance. Value that is proportional to noise variance is estimated using few last singular values [1]. Noised image is divided into square blocks of size $b \times b$. For each block the singular value decomposition is performed. In the consequent step, the average sum of the last t singular values is calculated over all image blocks:

$$n_s = \frac{1}{KL} \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{r=b-t+1}^{b} s_{klr} \ . \tag{6}$$

Calculated SVD of image blocks will now be used for filtering. The first step of the filtering procedure is decreasing of singular values s_{klr} for every noised image block :

$$\hat{s}_{klr} = s_{klr} - p_1 \cdot n_s \cdot w(r) \ , \tag{7}$$

where $\hat{s}_{klr}$ is reconstructed original singular value, p_1 is image dependent parameter, and $w(r)$ is a weighting function

$$w(r) = 1 - \left(\frac{r-1}{(b/2)} \right)^2 \quad , \ (r = 1,2,...,b) \ , \tag{8}$$

what is a parabolic approximation of the function depicted in Figure 1.

The second step is filtering of singular vectors. Incautious changes in singular vectors can produce big changes in images. Filtering of singular vectors is based on the characteristic increase in DFT spectrum components of singular vectors, depicted in Figure 2. Influence of singular vectors to image noise is proportional to corresponding singular value. This is the reason why we perform DFT filtering on the first *b/2* pairs of singular vectors only (r=1,2, … *b*/2). Equations (9a)-(9c) describe filtering procedure for left singular vectors:

$$\mathbf{u}_{klrF} = DFT(\mathbf{u}_{klr}) \ , \tag{9a}$$

$$\hat{\mathbf{u}}_{klrF}(n) = p_2 \, \mathbf{u}_{klrF}(n) \ , \ (n = b/2 - d + 1, ... b/2 + d) \tag{9b}$$

$$\hat{\mathbf{u}}_{klr} = IDFT(\hat{\mathbf{u}}_{klrF}) \ , \tag{9c}$$

where $\hat{\mathbf{u}}_{klr}$ is filtered left singular vector. The singular vectors are DFT transformed, and the part of DFT that corresponds to *d*+1 higher frequencies is multiplied with factor $p_2 \leq 1$. The filtering operation is performed for both, left and right singular vectors.

When filtering operation is performed on singular values and singular vectors, the filtered image block is calculated:

$$\hat{\mathbf{F}}_{kl} = \hat{\mathbf{U}}_{kl} \, \hat{\mathbf{S}}_{kl} \, \hat{\mathbf{V}}_{kl} \quad (k = 1,2,...,K \ , \ l = 1,2,...,L) \ , \tag{10}$$

where $\hat{\mathbf{U}}_{kl}$ is matrix of filtered left singular vectors, $\hat{\mathbf{S}}_{kl}$ is diagonal matrix of restored singular values, and $\hat{\mathbf{V}}_{kl}$ is matrix of filtered right singular vectors. The complete filtered image $\hat{\mathbf{F}}$ is reconstructed using all filtered blocks $\hat{\mathbf{F}}_{kl}$.

4 Results and discussion

The proposed algorithm was tested for different types of images and a wide range of S/N ratios. Original images were noised with white Gaussian noise, and then restored using block SVD filtering algorithm. As a measure of quality of proposed algorithm we used an S/N ratio improvement :

$$\eta_{nr} = 10 \log_{10} \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} (F(i,j) - G(i,j))^2}{\sum_{i=1}^{I} \sum_{j=1}^{J} (F(i,j) - \hat{F}(i,j))^2} \tag{11}$$

Best restoration results were obtained for blocks of size *b*=32. The parameters p_1 and p_2 are trade-off between noise smoothing and smearing of the edges. Greater values for p_1 and smaller for p_2 cause smearing of the edges. The value of parameter p_1 is slightly determined by the type of the image, although selection p_1=3 performed well for different types of images. The optimal value of p_2 parameter depends on the noise variance and should range 0.5-1. For images with S/N ratio better than 20 dB p_2 should be equal to one. The original 256×256 Lena image is noised with white Gaussian noise to produce degraded image with S/N ratio equal to 10 dB. The restoration results obtained using block SVD filtering algorithm are presented in Figure 3 and Figure 4. Filtering was performed with *b*=32, *d*=7, *t*=3, p_1=3 and p_2=0.70. The obtained S/N ratio improvement is 3.6 dB. Use of overlapping blocks increases numerical complexity almost four times, for additional S/N ratio improvement of 0.36 dB.

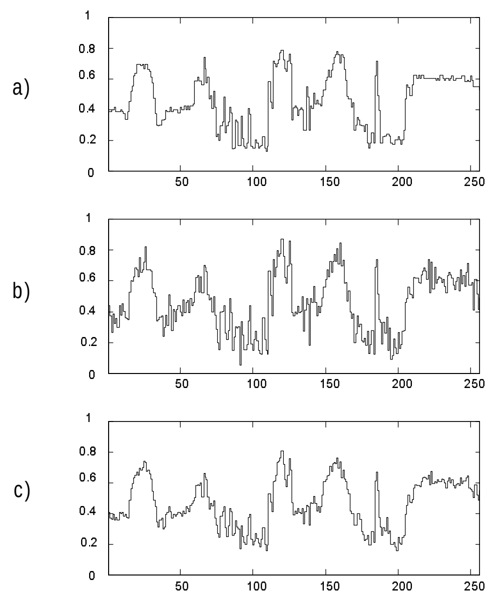

Figure 3. Gray levels of the 128. image row, a) original image, b) degraded image, S/N ratio 10 dB, c) restored image, S/N ratio improvement 3.6 dB.

a)

b)

c)

Figure 4. a) original image, b) degraded image, S/N ratio 10 dB, c) restored image, S/N improvement 3.6 dB.

5 Conclusion

A non-linear algorithm for noise filtering based on SVD filtering of image blocks is presented. Filtering of noise is performed through eliminating the changes in singular values and singular vectors that resulted from additive white Gaussian noise in the image. Noise variance knowledge is not required, because value that is proportional to the noise variance is estimated in the first step of the algorithm. Restoration results are comparable with other methods. SVD computations are reduced by use of block processing. Proposed method can be applied to colored and image dependent noise with similar restoration results. Future work will include research on the noise smoothing using SVD block filtering as a preprocessing step in a more complex procedure for image restoration.

6 References

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