SVD Block Processing for Non-linear Image Noise Filtering

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Abstract : *A new algorithm for noise filtering, based on non-linear processing of image in blocks, using singular value decomposition (SVD) is presented in this paper. Noise filtering is performed in the domain of singular values and singular vectors. A priori noise variance knowledge is not required, because a singular value-based noise variance estimation is performed in the first phase of the procedure. The non-linear filtering is based on eliminating changes in singular values and singular vectors caused by additive Gaussian white noise. Processing of image in smaller blocks makes SVD-based procedure computationally feasible. Experimental results have demonstrated the validity of the approach.*

Keywords: image processing, image enhancement, noise filtering, non-linear filtering

1. Introduction

Singular value decomposition (SVD) has been successfully applied to many image restoration problems. Usual applications include linear space invariant and linear space variant pseudoinverse filtering, image enhancement (Andrews and Patterson, 1976), separation of 2-D filtering operations into 1-D filtering operations, generation of small convolution kernels (Pratt, 1991) etc. Among all unitary transformations, SVD is optimal for given image in the sense that the energy packed in a given number of transformation coefficients is maximized. Although applicable in many image restoration applications, SVD is severely limited because of a large number of computations required for calculating singular values and singular vectors of large image matrices (Jain, 1989).

Sources of noise in images are various and are connected with particular image formation process (Andrews and Hunt, 1977). In case of a photographic film it is granularity of silver deposed on the film. CCD and tube photodetectors have their inherent noise sources. Filtering of noise is particularly important because noise present in blurred image is the main limiting factor for successful image deblurring. Noise filtering algorithms are numerous and diverse (Jain, 1989). Basic division is to algorithms that operate in space domain and to algorithms that operate in transform domain. Some of the well-known representatives are mean filter, median filter, Wiener noise smoother, and reduced update Kalman filter (RUKF). Among the filters that operate in Fourier domain, particularly interesting and efficient is short space spectral subtraction filter (Lim, 1980). For each image block, the value that corresponds to noise variance is subtracted from the current block spectrum, keeping the phase unchanged. Another example of variance subtraction is COSAR filter (Jain, 1989), where subtraction is more complex and is performed in the cosine transform domain.

In this work, a new method for non-linear image noise filtering in the SVD domain is proposed. The paper is organized as follows. The application of SVD analysis to image enhancement is discussed in Section 2. The proposed algorithm is presented in Section 3. Results and discussion are provided in Section 4, followed by the conclusion in Section 5.

2. Image SVD and influence of additive noise

Let the original, non-corrupted image **F** be represented as a *KxL* matrix. Adding the noise to the original **F** image will produce the noised image **G** of the same size

$$\mathbf{G} = \mathbf{F} + \mathbf{N} \qquad (1)$$

where **N** is a random *KxL* noise field. Added noise will degrade original information contained in **F**.

The proposed algorithm is based on block processing, so we will further proceed in that fashion. Noised image is divided into square blocks of size *b*x*b*. For simplicity, let us suppose that *K=kb* and *L=lb*. Then each image block has SVD representation

$$\mathbf{G}_{ij} = \mathbf{U}_{ij}\,\mathbf{S}_{ij}\,\mathbf{V}_{ij}^{T} \quad , \quad (i = 1,2,\ldots,k \;\; , \;\; j = 1,2,\ldots,l) \qquad (2)$$

where $\mathbf{U}_{ij}$ is *b*x*b* unitary matrix of left singular vectors, $\mathbf{S}_{ij}$ is *b*x*b* diagonal matrix of singular values, and $\mathbf{V}_{ij}$ is *bxb* unitary matrix of right singular vectors (Daniel and Noble, 1988). Singular values and corresponding singular vectors contain complete information about the image block. Equation (2) can be interpreted as an outer product expansion, a sum of base images

$$\mathbf{G}_{ij} = \sum_{r=1}^{R} s_{ijr}\,\mathbf{u}_{ijr}\,\mathbf{v}_{ijr}^{\tau} \qquad (3)$$

where *R* is the rank of $\mathbf{G}_{ij}$, s_{ijr} are singular values, $\mathbf{u}_{ijr}$ are left singular vectors, and $\mathbf{v}_{ijr}$ are right singular vectors. Image degradation, that includes blurring and noising, is reflected in changes in singular values and singular vectors. This is true for SVD representation of the whole *KxL* image, as well as for SVD representation of the individual *bxb* image blocks.

If we look at *b*x*b* image block as a square matrix, one characteristic value that describes that matrix is its rank. Let the mean rank of the image be the mean of the ranks of all blocks that constitute the image:

$$\bar{r}_F = \frac{1}{k\,l} \sum_{i=1}^{k} \sum_{j=1}^{l} rank(\mathbf{F}_{ij}) \qquad (4)$$

It has been observed experimentally, that natural-world images have the mean rank $\bar{r}_F$ very close to *b*. Some procedures (for example: interpolation, image coding) can cause a mean rank loss. This is caused by the fact that most of image energy is grouped in the higher singular values, and procedures which neglect the smallest singular values can be detected in this way. Undistorted images that contain text or block-diagrams, graphs etc. have mean rank that is much less than *b*. It has been observed that for images with S/N ratio lower than 20 dB, the mean rank $\bar{r}_G$ of a noised image is always equal to the full rank. We can express the previous statements using the following inequality:

$$\bar{r}_F \le \bar{r}_G \le b \qquad (5)$$

Image noise manifests itself as an increased “spatial activity”. To represent this spatial activity, the base images that contain spots are required. These base images correspond to higher r indexes in equation (4). This guides us to a conclusion that the smaller singular values will be increased. It has been observed that added noise manifests itself in SVD domain in this manner :

a) singular values are nonuniformly increased (although some may decrease) by amount that depends on the image and the noise statistics. The typical increase of the singular values (averaged over all blocks and normalized to one) for a class of real-world scenes corrupted by Gaussian white noise is depicted in Fig. 1.

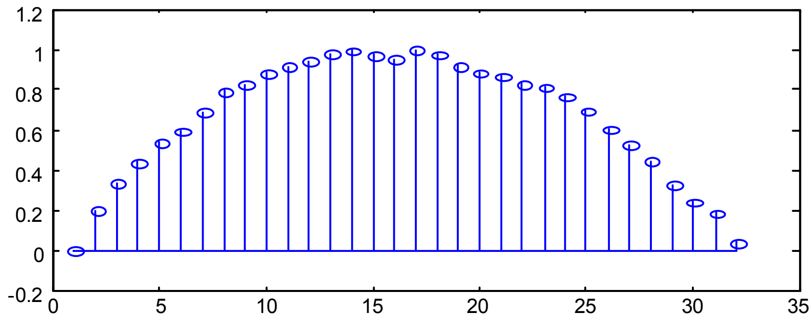

Figure 1: *The normalized difference between singular values of noised and original image, averaged for 256x256 image partitioned in blocks of size b=32.*

We can see that the medium values are increased by largest amount, although smallest singular values have the largest relative change. The depicted function will be more or less skewed for different images and noise types.

b) singular vectors are noised. Singular vectors that correspond to smaller singular values are much more perturbed. It is hard, if not impossible to analytically describe influence of noise on singular vectors. But, if we look at discrete Fourier transform of singular vectors, a common effect is that higher frequencies, that correspond to added noise, are increased.

Degradation from noising of singular vectors is much bigger than that caused by increased singular values. Amount of noise in each of them is proportional to the variance of the additive noise. We have already mentioned that singular values that correspond to higher i values in equation (3) are very small, if not zero. The additive noise increases their values, proportional to noise variance. A consequence of this fact is a well-known procedure for noise variance estimation based on the last few singular values (Daniel and Noble, 1988).

3. Block SVD filtering algorithm

The proposed algorithm has two basic steps: first, the noise variance is estimated, and then filtering is performed on singular values and vectors. Noised image is divided into square blocks of size *bxb*. For each block the singular value decomposition is performed. In the consequent step, the average sum of the last *t* singular values is calculated over all image blocks:

$$n_s = \frac{1}{k\,l} \sum_{i=1}^{k} \sum_{j=1}^{l} \sum_{r=b-t+1}^{b} s_{ijr} \qquad (6)$$

This is not a true value of noise variance, but a value that is proportional to it.

Previously calculated SVD of image blocks will now be used for filtering. First step in filtering is decreasing of noised singular value s_{ijr} for every image block :

$$\hat{s}_{ijr} = s_{ijr} - p_1 \cdot n_s \cdot w(r) \qquad (7)$$

where $\hat{s}_{ijr}$ is filtered singular value, p_1 is image dependent parameter and $w(r)$ is a weighting function that determines percentage of estimated noise variance to be subtracted from noised singular value s_{ijr}. The weighting function used in this work is chosen to be:

$$w(r) = 1 - \left(1 - \frac{r-1}{(b/2)}\right)^2 \quad , \quad (r = 1, 2, \ldots, b) \qquad (8)$$

what is a parabolic interpolation of function depicted in Fig. 1.

$p_2 \le 1$

In the second step a filtering of singular vectors is performed. Incautious changes in singular vectors can produce catastrophic changes in images. This is the reason why the filtering operations are limited to slight filtering of noise in singular vectors. Singular vectors are DFT transformed, and then part of Fourier transform that corresponds to higher frequencies is multiplied by a factor p_2 . The filtering operation is performed for both, left and right singular vectors.

After the filtering operation is applied to noised image block, the filtered image block $\hat{\mathbf{F}}_{ij}$ is calculated :

$$\hat{\mathbf{F}}_{ij} = \hat{\mathbf{U}}_{ij} \hat{\mathbf{S}}_{ij} \hat{\mathbf{V}}_{ij}^T \quad , \quad (i = 1,2,\ldots,k\,, j = 1,2,\ldots,l) \qquad (9)$$

where $\hat{\mathbf{U}}_{ij}$ is *b*x*b* matrix of filtered left singular vectors, $\hat{\mathbf{S}}_{ij}$ is *b*x*b* diagonal matrix of restored singular values, and $\hat{\mathbf{V}}_{ij}$ is *bxb* matrix of filtered right singular vectors. The complete filtered image $\hat{\mathbf{F}}$ is reconstructed using all filtered blocks $\hat{\mathbf{F}}_{ij}$.

4. Results and discussion

The proposed method was tested on various types of images. Blocks of size b=32 demonstrated best results. The number of smallest singular values, t, was equal to 3. The parameters p_1 and p_2 are a trade-off between noise smoothing and smearing of the edges. Algorithm does not require knowledge of noise variance, and in that sense we can say that it is blind noise smoothing algorithm. However, some *a priori* knowledge is needed. Parameter p_1 is determined mainly by the type of image. Values for p_1 between 1.5-5 were used in this work. Images with more details are sensitive to blurring, so the value of parameter p_1 should be smaller than in the case of images with less details. Value p_1=3 performed well for different image types. The optimal value of p_2 parameter depends on the noise variance and it should range from 0.5-1. For images with S/N ratio higher than 20 dB, p_2 should be equal to one. For images with less noise the filtering of singular vectors only blurs the edges. The method is not very sensitive to exact choice of parameters p_1 and p_2.

Restoration results for 256x256 Lena image are presented in Fig. 2. The original image was noised with white Gaussian noise to produce image with S/N ratio equal to 10 dB. Filtering of noise was performed using block SVD algorithm with block size 32x32, p_1=3, p_2=0.75. S/N ratio improvement was calculated as:

$$\eta_{snr} = 10 \log_{10} \frac{\sum_{i,j=1}^{N} (F(i,j) - G(i,j))^2}{\sum_{i,j=1}^{N} (F(i,j) - \hat{F}(i,j))^2} \qquad (10)$$

In this experiment the S/N ratio is improved by 3.3 dB. This value could be increased, but the price would be increased blurring of the edges. Without filtering of singular vectors, S/N ratio improvement is equal to 2.3 dB. Similar results in improvement proposed filter demonstrated for colored noise and image dependent noise.

a) b)

c)

Figure 2: *a) original image, b) noised image, S/N ratio 10 dB, c) restored image with S/N ratio improvement 3.3 dB , b=32, p1=3, p2=0.75.*

5. Conclusion

A new algorithm for non-linear image noise filtering based on SVD processing of image blocks is presented. Noise filtering is performed in SVD domain by filtering of singular vectors and singular values. The obtained filtering results are comparable with other methods such as nonadaptive Wiener noise smoother for Gaussian white noise. The presented filter could be adapted to other types of noise by slight modifications. Numerical complexity of the SVD computation is reduced by use of image blocks.

The proposed noise smoothing algorithm for image restoration is under investigation. Future work will include research on the noise smoothing using SVD block filtering as a preprocessing step in a more complex procedure for image restoration.

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