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Tensor product based control of the Single Pendulum Gantry process with stable neural network based friction compensation

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***Abstract*—Fast and accurate positioning and swing minimization of the containers and other loads in crane manipulation are demanding and in the same time conflicting tasks. For accurate positioning, the main problem is nonlinear friction effect, especially in the low speed region. In this paper authors propose position controller realized as hybrid controller. It consists of the tensor product based nonlinear feedback controller with additional friction self-learning neural compensator. The experimental results show that friction compensator is able to remove position error in steady state.**

***Index Terms*—Single Pendulum Gantry, Neural Network, Friction Compensation, RBFnetwork, on-line network learning.**

I. Introduction

Translational gantry cranes are widely used for the heavy loads transfer in modern industrial systems. The problem faced in load transfer is a negative influence of the crane acceleration required for the motion. Any change of the reference position causes an undesirable load swing, having negative consequences on the system control and safety performances.

In order to achieve acceptable system performances for a fast load positioning (i.e. minimal load transfer time), the swing of the suspended load should be controlled as well. This conflicting control demands can be solved with state feedback controller, designed according to linear quadratic optimum criteria, [4]. This design technique is imposed as a logical solution and it is used by several authors for solving similar control tasks.

Although the load swing problem is generally nonlinear most of the solutions are based on the linearized mathematical model. Typical control approaches are adaptive (gain-scheduling logic with optimal controllers used by Corriga, Giua and Usai in [4]), optimal (Wang and Surgenor in [15]) or robust (G. Bartolini et al. in [2]), applied on the similar types of the electromechanical systems.

Due to crane system complexity and the fact that linearised mathematical model only partially represents the real system, some authors used fuzzy controller, [10], [8], [14]. Controller based on fuzzy logic can partially solve an undesirable effects caused by the system nonlinearities, [14].

Recently, nonlinear control approach based on tensor product model representation (TP) of the process is proposed [1], [13] and successfully applied to control of Single Pendulum Gantry process [7]. The TP model represents the linear parameter varying state-space models by the parameter varying combination of Linear Time Invariant (LTI) models.

However, neither of proposed techniques provide solution for the main problem of accurate positioning - friction. Due to its highly nonlinear characteristics complex nonlinear behavior like limit cycle may occur if the controller includes the integral action, [6]. In order to reduce or eliminate the impact of friction, a neural network based compensator is proposed as additional feedforward loop to TP model based feedback controller. However, in real application friction characteristics is only partially known or even completely unknown the neural network compensator parameters need to be updated in a on-line manner. In order to ensure stability of both control and network parameters updating law they have to derived using Lyapunov stability analysis.

The rest of the paper is organized as follows. In section II mathematical model of the single pendulum gantry is presented. In section III TP based controller controller design procedure is given while in section IV neural network based friction compensator is presented. Experimental results are given in section V.

II. Mathematical Model of the Single Pendulum Gantry

The single pendulum gantry mounted on the linear cart is presented in the Fig.1, [5]. When facing the cart, a positive direction of the cart motion is to the right and a positive sense of the pendulum rotation is defined as counter clockwise. Also, the zero angle, corresponds to a suspended pendulum vertical rest down position. Single pendulum gantry can be represented as a system with one input u (motor voltage), and two outputs: α (pendulum angle) and x_c (cart position). Mathematical equations of the system motion can be derived via Lagrange equations, by defining total potential and kinetic energy of the system as a functions of generalized coordinates: cart position x_c and pendulum swing angle α. The result is the nonlinear model represented by equations (1) and (2).

The parameters of the single pendulum gantry linear model are given in Table I.

$$\ddot{x}_c = \frac{-(I_p + M_p l_p^2) B_{eq} \cdot \dot{x}_c + (M_p^2 l_p^3 + l_p M_p I_p)\sin(\alpha)\cdot\dot{\alpha}^2 + M_p l_p \cos(\alpha) B_p \cdot \dot{\alpha}}{(M_c + M_p) I_p + M_c M_p l_p^2 + M_p^2 l_p^2 \sin^2(\alpha(t))} + \\ + \frac{M_p^2 l_p^2 g \cos(\alpha)\sin(\alpha) - (I_p + M_p l_p^2)(\frac{\eta_g K_g^2 \eta_m K_t K_m \cdot \dot{x}_c}{R_m r_{mp}^2} + (I_p + M_p l_p^2)\frac{\eta_g K_g \eta_m K_t}{R_m r_{mp}} U_m)}{(M_c + M_p) I_p + M_c M_p l_p^2 + M_p^2 l_p^2 \sin^2(\alpha)} \tag{1}$$

$$\ddot{\alpha} = \frac{-(M_c + M_p) B_p \cdot \dot{\alpha} - M_p^2 l_p^2 \sin(\alpha)\cos(\alpha)\cdot\dot{\alpha}^2 + M_p l_p \cos(\alpha) B_{eq} \cdot \dot{x}_c}{(M_c + M_p) I_p + M_c M_p l_p^2 + M_p^2 l_p^2 \cdot \sin^2(\alpha)} + \\ + \frac{-(M_c + M_p) M_p g l_p \sin(\alpha) + M_p l_p \cos(\alpha)\frac{\eta_g K_g^2 \eta_m K_t K_m \cdot \dot{x}_c}{R_m r_{mp}^2} - M_p l_p \cos(\alpha)\frac{\eta_g K_g \eta_m K_t}{R_m r_{mp}} U_m)}{(M_c + M_p) I_p + M_c M_p l_p^2 + M_p^2 l_p^2 \cdot \sin^2(\alpha)} \tag{2}$$

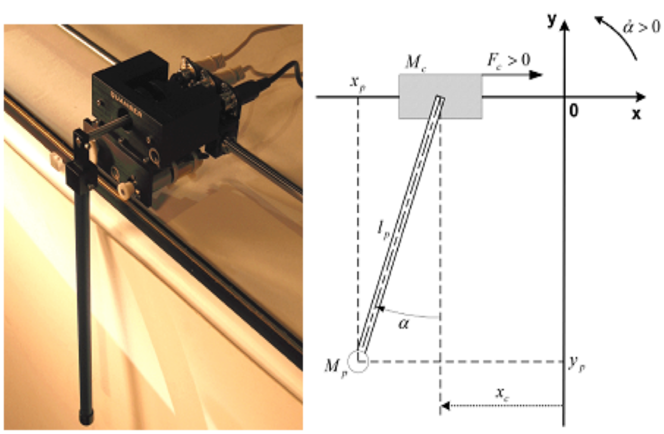

Fig. 1. Single pendulum gantry (SPG) electromechanical model as experimental model of gantry crane

III. Tensor Product Transformation Based Controller Design

Tensor Product (TP) transformation is recently proposed procedure of transforming a wide class of nonlinear systems, represented as a Linear Parameter Varying (LPV) models, into parameter varying combination of Linear Time Invariant (LTI) models. Linear Parameter Varying (LPV) system can be written in the following form:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \mathbf{S}\big(\mathbf{p}(t)\big) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix} \tag{3}$$

where $\mathbf{u}$(t) input vector , $\mathbf{y}$(t) system output vector, and $\mathbf{x}$(t) state vector. Matrix $\mathbf{S}\big(\mathbf{p}(t)\big)$ which contains varying parameters, is defined as:

$$\mathbf{S}\big(\mathbf{p}(t)\big) = \begin{bmatrix} \mathbf{A}\big(\mathbf{p}(t)\big) & \mathbf{B}\big(\mathbf{p}(t)\big) \\ \mathbf{C}\big(\mathbf{p}(t)\big) & \mathbf{D}\big(\mathbf{p}(t)\big) \end{bmatrix} \tag{4}$$

where $\mathbf{p}(t) \in \mathbf{\Omega}$ time varying N-dimensional parameter vector that is the element of the closed hypercube $\mathbf{\Omega} = [p_{1,min}, p_{1,max}] \times [p_{2,min}, p_{2,max}] \times \ldots \times [p_{N,min}, p_{N,max}] \subset \Re^N$. $\mathbf{p}$(t) also may contain system states (elements of the matrix $\mathbf{x}$) and thus (3) can be viewed as a special class of nonlinear dynamic systems in state space.

The goal of TP transformation is to transform the system (3) into the following form:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \mathbf{S}(\mathbf{p}(t)) \otimes_{n=1}^{N} w_n\big(\mathbf{p}_n(t)\big) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}, \tag{5}$$

that can be also written as:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \sum_{n=1}^{N} w_n\big(\mathbf{p}_n(t)\big)\mathbf{S}_n\big(\mathbf{p}(t)\big) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix} \tag{6}$$

where the vector $\mathbf{w}_n(p_n)$, $n = 1...N$ contains weighting coefficients of the membership functions $w_{n,i_n}(p_n)$, $i_n = 1...I_n$. The parameter $I_n < \infty$ denotes the number of the membership functions used in n-th dimension of $\mathbf{\Omega}$. At the same time membership functions satisfy $w_{n,i_n} \in [0,1]$, $\forall n$, p_n : $\sum_{i_n=1}^{i_N} w_{n,i_n} = 1$.

TABLE I
Parameters of the Single Pendulum Gantry System

Parameters	Description
$B_{eq} = 5.4[Nms/rad]$	Equivalent viscous damping coefficient as seen at the motor pinion
$B_p = 0.0024[Nm/s]$	viscous damping coefficient as seen at the pendulum axis
$\eta_g = 1$	Planetary gearbox efficiency
$\eta_m = 1$	Motor efficiency
$g = 9.81[m/s^2]$	Gravitational constant of earth
$J_p = 0.0078838[kgm^2]$	Pendulum moment of inertia
$J_m = 3.9e-7[kgm^2]$	Rotor moment of inertia
$K_g = 3.71$	Planetary gearbox gear ratio
$K_m = 0.0076776$	Back electro-motive force (EMF) constant
$K_t = 0.007683$	Motor torque constant
$l_p = 0.3302[m]$	Pendulum length from pivot to center of gravity
$M_c = 1.0731[kg]$	Lumped mass of the cart system, including the rotor inertia
$M_p = 0.23[kg]$	Pendulum mass
$R_m = 2.6[\Omega]$	Motor armature resistance
$r_{mp} = 0.00635[m]$	Motor pinion radius

For the system (5) the controller has the same polytopic form as TP model. Therefore, the control signal is given by:

$$\mathbf{u}(t) = -(\mathcal{K}(\mathbf{p}(t)) \otimes_{n=1}^{N} w_n(p_n))\mathbf{x}(t) \quad (7)$$

where the LTI feedback gains $K_{i_1,i_2 \dots i_N}$, are stored in tensor $\mathcal{K}$.

Based on system description (5) feedback controller can be designed using various procedures. In this paper we adopted procedure based on Linear Matrix Inequalities (LMI) where different constraints can be incorporated into design process. Beside closed loop stability, constraints on control and output signal is used as follows:

1) **Asymptotic stability**: TP model (5) with control law (7) is asymptotically stable if there exist $\mathbf{X} > 0$ and $\mathbf{M}_j$ such that the following inequality holds:

$$-\mathbf{X}\mathbf{A}_j^T - \mathbf{A}_j\mathbf{X} + \mathbf{M}_j^T\mathbf{B}_j^T + \mathbf{B}_j\mathbf{M}_j > 0 \quad (8)$$

for all j, i

$$\begin{aligned} &-\mathbf{X}\mathbf{A}_j^T - \mathbf{A}_j\mathbf{X} - \mathbf{X}\mathbf{A}_s^T - \mathbf{A}_s\mathbf{X} + \mathbf{M}_s^T\mathbf{B}_j^T + \\ &+ \mathbf{B}_j\mathbf{M}_s + \mathbf{M}_j^T\mathbf{B}_s^T + \mathbf{B}_s\mathbf{M}_j \geq 0. \end{aligned} \quad (9)$$

Feedback controller gains are calculated from matrices $\mathbf{X}$ i $\mathbf{M}_j$ as:

$$\mathbf{K}_j = \mathbf{M}_j \cdot \mathbf{X}^{-1} \quad (10)$$

2) **Constraint on control signal**: We assume that $\|\mathbf{x}(0)\| \leq \varphi$, where $\mathbf{x}(0)$ is generally unknown, but upper bounded by μ. Constaint $\|\mathbf{u}(t)\|_2 \leq \mu$ holds for all $t \geq 0$ if the following LMI's holds:

$$\begin{aligned} \varphi^2\mathbf{I} &\leq \mathbf{X} \\ \begin{bmatrix} \mathbf{X} & \mathbf{M}_i^T \\ \mathbf{M}_i & \mu^2\mathbf{I} \end{bmatrix} &\geq 0 \end{aligned} \quad (11)$$

3) **Constraint on output signal**: We assume that $\|\mathbf{x}(0)\| \leq \varphi$, where $\mathbf{x}(0)$ is generally unknown, but upper bounded by φ. Constraint $\|\mathbf{y}(t)\|_2 \leq \lambda$ holds for all $t \geq 0$ if the following LMI's are satisfied:

$$\begin{aligned} \varphi^2\mathbf{I} &\leq \mathbf{X} \\ \begin{bmatrix} \mathbf{X} & \mathbf{X}\mathbf{C}_i^T \\ \mathbf{C}_i\mathbf{X} & \lambda^2\mathbf{I} \end{bmatrix} &\geq 0 \end{aligned} \quad (12)$$

A. TP transformation of the Single Pendulum Gantry process

Based on equations (1) and (2) Single Pendulum Gantry process can be described in form of LPV model as given with the following equation:

$$\begin{bmatrix} \dot{x}_C \\ \ddot{x}_C \\ \dot{\alpha} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_1/a_x & a_2/a_x & a_3/a_x \\ 0 & 0 & 0 & 1 \\ 0 & a_4/a_x & a_5/a_x & a_6/a_x \end{bmatrix} \cdot \begin{bmatrix} x_C \\ \dot{x}_C \\ \alpha \\ \dot{\alpha} \end{bmatrix} + + \begin{bmatrix} 0 \\ b_1/a_x \\ 0 \\ b_2/a_x \end{bmatrix} \cdot U_m \quad (13)$$

where:

$$a_1 = -(I_p + M_p l_p^2)\left(B_{eq} + \frac{\eta_g K_g^2 \eta_m K_t K_m}{R_m r_{mp}^2}\right)$$

$$a_2 = \frac{M_p^2 l_p^2 g \cos(\alpha)\sin(\alpha)}{\alpha}$$

$$a_3 = (M_p^2 l_p^3 + l_p M_p I_p)\sin(\alpha) \cdot \dot{\alpha} + M_p B_p l_p \cos(\alpha)$$

$$a_4 = M_p l_p \cos(\alpha)\left(B_{eq} + \frac{\eta_g K_g^2 \eta_m K_t K_m}{R_m r_{mp}^2}\right)$$

$$a_5 = \frac{-(M_c + M_p) M_p g l_p \sin(\alpha)}{\alpha}$$

$$a_6 = -(M_c + M_p)B_p - M_p^2 l_p^2 \sin(\alpha)\cos(\alpha) \cdot \dot{\alpha}$$

$$a_x = (M_c + M_p)I_p + M_c M_p l_p^2 + M_p^2 l_p^2 \sin^2(\alpha)$$

$$b_1 = (I_p + M_p l_p^2) \cdot \frac{\eta_g K_g \eta_m K_t}{R_m r_{mp}}$$

$$b_2 = -M_p l_p \cos(\alpha) \cdot \frac{\eta_g K_g \eta_m K_t}{R_m r_{mp}}$$

Applying the TP transform to the LPV system (13) leads to convex combination of the LTI models (5) where matrices $\mathbf{A}_{ij}$ and $\mathbf{B}_{ij}$ are given by equation (14), while matrices $\mathbf{C}$ and $\mathbf{D}$ are constant.

Feedback controller which satisfies the conditions (8)-(12) is obtained using the Yalmip[1]/SeDuMi[2] optimization software and given by:

$$\begin{aligned} \mathbf{K}_{11} &= [\ 41,5267 \quad 25,5667 \quad -55,4898 \quad 6,3188\ ] \\ \mathbf{K}_{21} &= [\ 34,1430 \quad 20,4908 \quad -51,1911 \quad 4,5319\ ] \\ \mathbf{K}_{31} &= [\ 37,0720 \quad 22,5380 \quad -53,0596 \quad 5,2122\ ] \\ \mathbf{K}_{41} &= [\ 34,0835 \quad 20,4521 \quad -51,1550 \quad 4,5016\ ] \\ \mathbf{K}_{12} &= [\ 41,5266 \quad 25,5666 \quad -55,4899 \quad 6,3189\ ] \\ \mathbf{K}_{22} &= [\ 34,0846 \quad 20,4463 \quad -51,1305 \quad 4,5065\ ] \\ \mathbf{K}_{32} &= [\ 37,0719 \quad 22,5379 \quad -53,0595 \quad 5,2122\ ] \\ \mathbf{K}_{42} &= [\ 34,1393 \quad 20,4949 \quad -51,2144 \quad 4,5261\ ] \end{aligned}$$

IV. NETWORK NETWORK BASED FRICTION COMPENSATOR

In order to compensate nonlinear friction effect an artificial neural network is employed. Neural networks commonly consist of big number of highly interconnected neurons organized in two or more layers [9]. These interconnections give them universal approximation capability, i.e. a neural network can approximate arbitrary continuous nonlinear function with desired accuracy [3],[12] if consisted of at least two layers. Generally, a two-layer neural network with linear output layer can be described by the following equation:

$$\mathbf{y}_{NN} = \mathbf{W}_2 \cdot \sigma(\mathbf{W}_1, \mathbf{u}), \quad (15)$$

[1]http://control.ee.ethz.ch/research/software.en.html
[2]http://sedumi.ie.lehigh.edu/

$$\mathbf{A}_{11} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -12,4369 & 2,1005 & 0,0064 \\ 0 & 0 & 0 & 1 \\ 0 & 35,3734 & -30,0862 & -0,0896 \end{bmatrix} \quad \mathbf{B}_{11} = \begin{bmatrix} 0 \\ 1,6335 \\ 0 \\ -4,6462 \end{bmatrix}$$
$$\mathbf{A}_{21} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -11,2698 & 1,2506 & 0,0274 \\ 0 & 0 & 0 & 1 \\ 0 & 22,9577 & -24,2707 & -0,1313 \end{bmatrix} \quad \mathbf{B}_{21} = \begin{bmatrix} 0 \\ 1,4803 \\ 0 \\ -3,0154 \end{bmatrix}$$
$$\mathbf{A}_{31} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -11,8140 & 1,6363 & 0,0052 \\ 0 & 0 & 0 & 1 \\ 0 & 28,4820 & -26,8846 & -0,0854 \end{bmatrix} \quad \mathbf{B}_{31} = \begin{bmatrix} 0 \\ 1,5517 \\ 0 \\ -3,7410 \end{bmatrix}$$
$$\mathbf{A}_{41} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -11,2906 & 1,2657 & -0,0185 \\ 0 & 0 & 0 & 1 \\ 0 & 23,1794 & -24,3745 & -0,0324 \end{bmatrix} \quad \mathbf{B}_{41} = \begin{bmatrix} 0 \\ 1,483 \\ 0 \\ -3,0445 \end{bmatrix} \tag{14}$$
$$\mathbf{A}_{12} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -12,4369 & 2,1005 & 0,0066 \\ 0 & 0 & 0 & 1 \\ 0 & 35,3734 & -30,0862 & -0,0900 \end{bmatrix} \quad \mathbf{B}_{12} = \begin{bmatrix} 0 \\ 1,6335 \\ 0 \\ -4,6462 \end{bmatrix}$$
$$\mathbf{A}_{22} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -11,2698 & 1,2506 & 0,019 \\ 0 & 0 & 0 & 1 \\ 0 & 22,9577 & -24,2707 & -0,0314 \end{bmatrix} \quad \mathbf{B}_{22} = \begin{bmatrix} 0 \\ 1,4803 \\ 0 \\ -3,0154 \end{bmatrix}$$
$$\mathbf{A}_{32} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -11,8140 & 1,6363 & 0,0052 \\ 0 & 0 & 0 & 1 \\ 0 & 28,4820 & -26,8846 & -0,0852 \end{bmatrix} \quad \mathbf{B}_{32} = \begin{bmatrix} 0 \\ 1,5517 \\ 0 \\ -3,7410 \end{bmatrix}$$
$$\mathbf{A}_{42} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -11,2906 & 1,2657 & -0,027 \\ 0 & 0 & 0 & 1 \\ 0 & 23,1794 & -24,3745 & -0,1306 \end{bmatrix} \quad \mathbf{B}_{42} = \begin{bmatrix} 0 \\ 1,483 \\ 0 \\ -3,0445 \end{bmatrix}$$

where $\mathbf{y}_{NN}$ is neural network output, $\mathbf{W}_1$, $\mathbf{W}_2$ are parameter matrices of hidden and output network layer, respectively. Various neural network structures are obtained by choosing different kernel functions. In this paper Radial Basis Function is used as a network kernel function and such the network is usually called RBF neural network. The parameters of the hidden network layer $\mathbf{W}_1$ are set offline while the output layer weight matrix is to be adapted on-line. Taking into account universal approximation capability and assuming that friction characteristic can be described by the Stribeck model [11], following approximation can be written:

$$f(\dot{x}_C) = \mathbf{W}_2^* \sigma(\mathbf{W}_1 \dot{x}_C) + \varepsilon, \tag{16}$$

where $f(\dot{x}_C)$ is the friction force, $\mathbf{W}_2^*$ is optimal (but unknown) output parameter matrix. Thus, neural network compensator can be described by:

$$y_{NN} = \hat{\mathbf{W}}_2 \sigma(\mathbf{W}_1 \dot{x}_c), \tag{17}$$

where the matrix $\mathbf{W}_2$ is updated using the on-line learning procedure. Additionally, it is assumed that the network approximation error is quadratically bounded as:

$$\varepsilon^T \mathbf{P}_\varepsilon \varepsilon < \bar{\varepsilon} \tag{18}$$

The neural network based compensator design procedure is summarized in the following theorem.

Theorem 1. *Let the system is described by Eq.* (5) *and feedback controller satisfies conditions* (8)-(12)*. Then neural network parameters adaptation law given by:*

$$\dot{\tilde{\mathbf{W}}}_2^T = \Big(\sum_{i=1}^{N} h_i(x) \mathbf{B}_i^T\Big) \mathbf{P} \mathbf{x} \sigma^T(\mathbf{W}_1 \dot{x}_C) \mathbf{P}_W^{-1} \tag{19}$$

ensures that system states are stable and converge to the hyperball of radius $\sqrt{\bar{\epsilon}/\lambda_{\min}(\mathbf{Q})}$.

Proof: The closed loop dynamic of the system with neural network based friction compensator can described by:

$$\begin{aligned} \dot{\mathbf{x}} &= \sum_{i=1}^{N} \sum_{j=1}^{N} w_i(x) w_j(x) (\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j) \mathbf{x} + \\ &+ \Big(\sum_{i=1}^{N} w_i(x) \mathbf{B}_i\Big) \tilde{\mathbf{W}}_2 \sigma(\mathbf{W}_1 \dot{x}_C) + \Big(\sum_{i=1}^{N} w_i(x) \mathbf{B}_i\Big) \epsilon \end{aligned} \tag{20}$$

Let the Lyapunov function candidate is chosen as:

$$V(x) = \mathbf{x}^T \mathbf{P} \mathbf{x} + tr\{\tilde{\mathbf{W}}_2^T \mathbf{P}_W \tilde{\mathbf{W}}_2\} \tag{21}$$

Lyapunov function time derivative is given by:

$$\dot{V}(x) = \dot{\mathbf{x}}^T \mathbf{P} \mathbf{x} + \mathbf{x}^T \mathbf{P} \dot{\mathbf{x}} + 2tr\{\dot{\tilde{\mathbf{W}}}_2^T \mathbf{P}_W \tilde{\mathbf{W}}_2\} \tag{22}$$

Combining equations (20) and (22) leads to the following expression:

$$\dot{V}(x) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i(x)w_j(x)\mathbf{x}^T\mathbf{A}_{C,ij}^T\mathbf{P}\mathbf{x}+ \\ + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i(x)w_j(x)\mathbf{x}^T\mathbf{P}\mathbf{A}_{C,ij}\mathbf{x}+ \\ + 2\mathbf{x}^T\mathbf{P}\Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\Big)\tilde{\mathbf{W}}_2\sigma(\mathbf{W}_1\dot{x}_C) \\ + 2\mathbf{x}^T\mathbf{P}\Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\Big)\epsilon - 2tr\{\tilde{\mathbf{W}}_2^T\mathbf{P}_W\dot{\hat{\mathbf{W}}}_2\}, \tag{23}$$

which can be also written as:

$$\dot{V}(x) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i(x)w_j(x)\mathbf{x}^T(\mathbf{A}_{C,ij}^T\mathbf{P}+\mathbf{P}\mathbf{A}_{C,ij})\mathbf{x}+ \\ + 2\mathbf{x}^T\mathbf{P}\Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\Big)\epsilon \\ + 2tr\Big\{\tilde{\mathbf{W}}_2^T\Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i^T\Big)\mathbf{P}\mathbf{x}\sigma^T(\mathbf{W}_1\dot{x}_C) - \tilde{\mathbf{W}}_2^T\mathbf{P}_W\dot{\hat{\mathbf{W}}}_2\Big\} \tag{24}$$

With the neural network parameters adaptation law chosen as:

$$\dot{\hat{\mathbf{W}}}_2^T = \Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i^T\Big)\mathbf{P}\mathbf{x}\sigma^T(\mathbf{W}_1\dot{x}_C)\mathbf{P}_W^{-1} \tag{25}$$

time derivative of the Lyapunov function become:

$$\dot{V}(x) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i(x)w_j(x)\mathbf{x}^T(\mathbf{A}_{C,ij}^T\mathbf{P}+ \\ + \mathbf{P}\mathbf{A}_{C,ij})\mathbf{x} + 2\mathbf{x}^T\mathbf{P}\Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\Big)\epsilon \tag{26}$$

Due to symmetry of the matrix $\mathbf{P}$ and using the following matrix inequality:

$$\mathbf{X}^T\mathbf{Y} + (\mathbf{X}^T\mathbf{Y})^T \leq \mathbf{X}^T\mathbf{\Lambda}^{-1}\mathbf{X} + \mathbf{Y}^T\mathbf{\Lambda}\mathbf{Y}, \tag{27}$$

that is satified for every $\mathbf{X},\mathbf{Y} \in \Re^{n\times k}$ and positive definite symmetric matrix $0 < \mathbf{\Lambda} = \mathbf{\Lambda}^T \in \Re^{n\times n}$, term $2\mathbf{x}^T\mathbf{P}\Big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\Big)\epsilon$ in equation (26) can be written as:

$$\mathbf{x}^T\mathbf{P}\big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\big)\epsilon + \Big(\mathbf{x}^T\mathbf{P}\big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\big)\epsilon\Big)^T \leq \\ \leq \mathbf{x}^T\mathbf{\Lambda}^{-1}\mathbf{x} + \epsilon^T\big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i^T\big)^T\mathbf{P}^T\mathbf{\Lambda}\mathbf{P}\big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\big)\epsilon \tag{28}$$

where ϵ is assumed to be quadratically bounded as:

$$\epsilon^T\big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i^T\big)^T\mathbf{P}^T\mathbf{\Lambda}\mathbf{P}\big(\sum_{i=1}^{N} w_i(x)\mathbf{B}_i\big)\epsilon = \\ \epsilon^T\mathbf{P}_\epsilon\epsilon \leq \bar{\epsilon} \tag{29}$$

Taking the previous equation into account time derivative of the Lyapunov function can be written as:

$$\dot{V}(x) \leq -\mathbf{x}^T\mathbf{Q}\mathbf{x} + \bar{\epsilon} \tag{30}$$

where:

$$Q = -\sum_{i=1}^{N}\sum_{j=1}^{N} w_i(x)w_j(x)(\mathbf{A}_{C,ij}^T\mathbf{P}+\mathbf{P}\mathbf{A}_{C,ij}) - \Lambda^{-1}. \tag{31}$$

With matrix P already obtained from TP feedback controller design such that satisfies:

$$\mathbf{A}_{C,ii}^T\mathbf{P}+\mathbf{P}\mathbf{A}_{C,ii} < 0,\ i = 1,...,N \tag{32}$$

and $\forall j < i$ except $\forall \mathbf{p}(t) : w_i(\mathbf{p}(t))w_j(\mathbf{p}(t)) = 0$:

$$\left(\frac{\mathbf{A}_{C,ij}+\mathbf{A}_{C,ji}}{2}\right)^T\mathbf{P} + \left(\frac{\mathbf{A}_{C,ij}+\mathbf{A}_{C,ji}}{2}\right)\mathbf{P} \leq 0, \tag{33}$$

it is possible to choose $\mathbf{\Lambda}$ such that $\mathbf{Q} > 0$. In that case following assessment is valid:

$$\dot{V} \leq -\lambda_{min}(\mathbf{Q})\cdot\|\mathbf{x}\|^2 + \lambda_{max}(\mathbf{P}_\epsilon)\cdot\|\epsilon\|^2 \tag{34}$$

from which it can be concluded that time derivative of the Lyapunov function $\dot{V}$ is negative whenever $\lambda_{min}(\mathbf{Q})\cdot\|\mathbf{x}\|^2 > \lambda_{max}(\mathbf{P}_\epsilon)\cdot\|\epsilon\|^2$. Thus the system states will surely converge to the hyperball of radius $\sqrt{\bar{\epsilon}/\lambda_{\min}(\mathbf{Q})}$.

■

V. Experimental results

Experimental verification of the proposed algorithm has been made on laboratory Single Pendulum Gantry process using two different reference signals:

1) filtered step reference signal,
2) sinusoidal reference signal.

Proposed algorithm has been compared to TP based controller without friction compensator. Experimental results for both reference signals are in Figs 2-5. From Figs 2 and 3 it can be seen that steady state position error is significantly reduced (from 4.5 cm to 5mm) using the proposed control/compensation scheme. However, in order to overcome the static friction force compensation algorithm produced more active control signal in steady state (Fig. 4). Good performance of the compensation algorithm is also apparent in the case of sinusoidal reference signal (Fig. 5)

VI. Conclusion

In this paper TP based controller with neural network based friction compensator for Single Pendulum Gantry process is proposed. Controller/compensator design procedure consists of two steps: (i) TP based feedback controller design neglecting the friction effect and (ii) neural network compensator design using the results from step (i). The main advantage of the proposed procedure is that no a-priory knowledge on friction characteristic is needed since the neural network parameters are adapted in an on-line manner. Stability of the the overall control/learning system is guaranteed using Lyapunov stability theory.

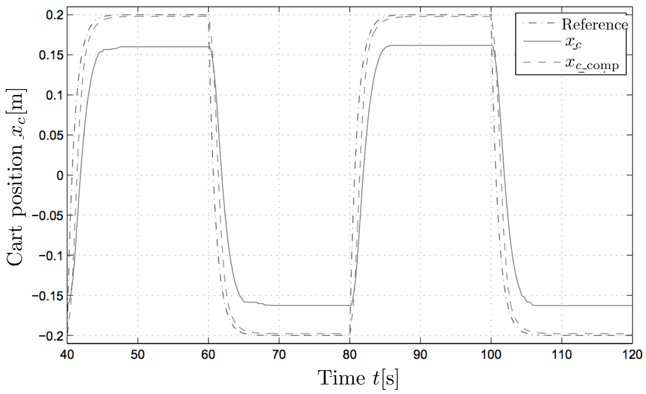

Fig. 2. Cart position with step reference signal

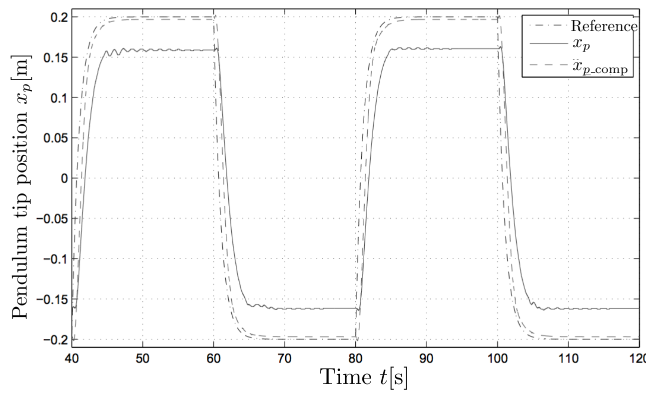

Fig. 3. Pendulum tip position with step reference signal

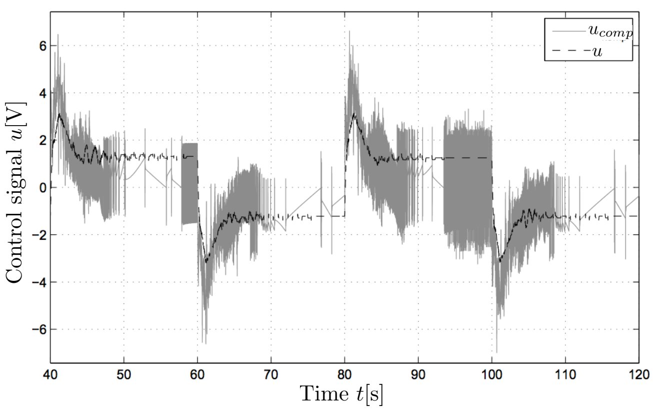

Fig. 4. Control effort for step reference signal

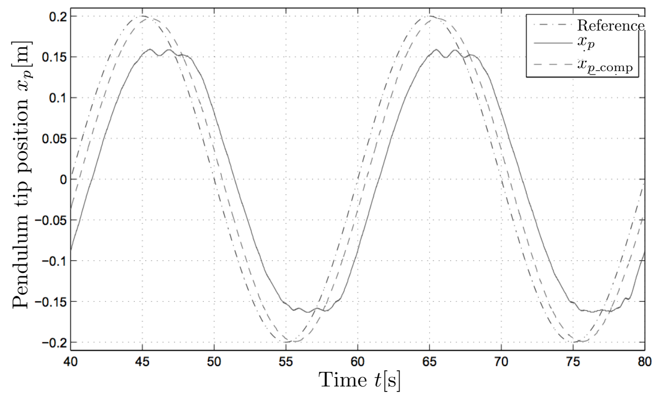

Fig. 5. Pendulum tip position for sinusoidal reference signal

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