GPDs from DVCS at LO and beyond

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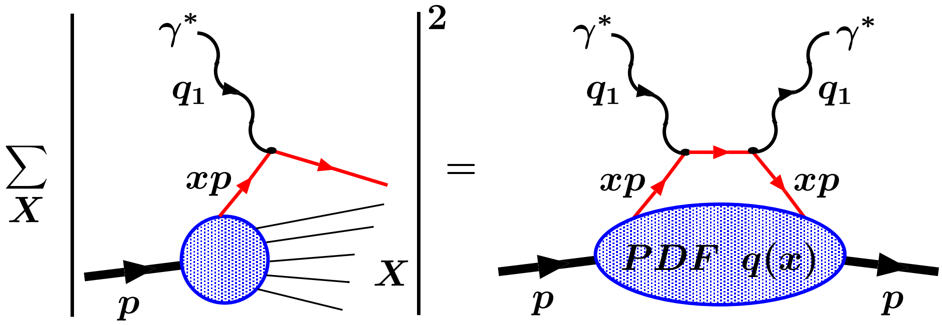
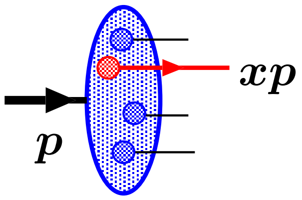
Diffractive and electromagnetic processes at the LHC
Trento, 4–10 Jan 2010

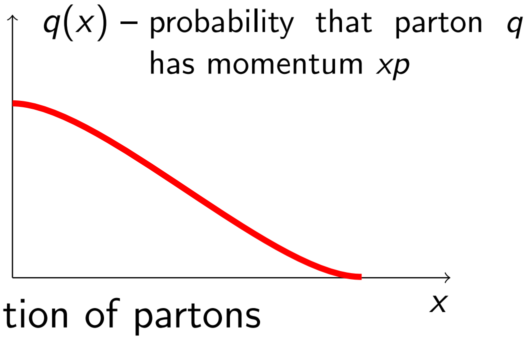
Outline

Parton distribution functions

- Deeply inelastic scattering, $-q_1^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \text{const}$

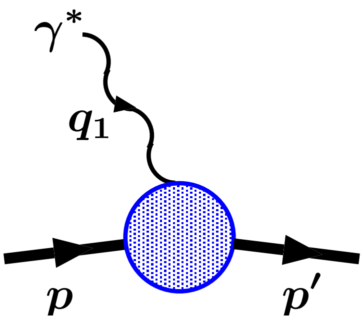






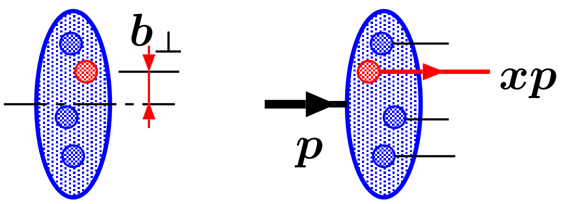
- no information on spatial distribution of partons

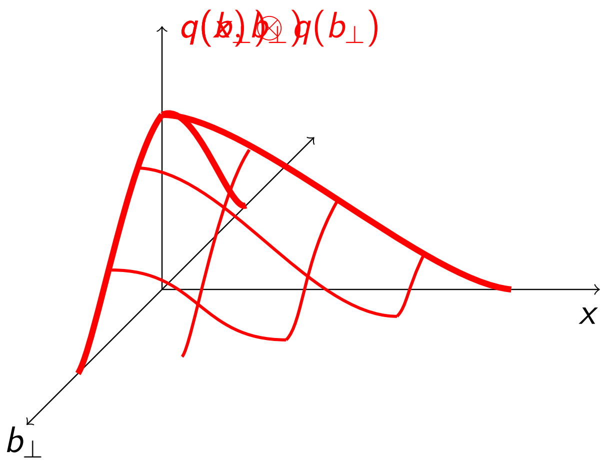
Electromagnetic form factors



- Dirac and Pauli form factors:

$$q(b_\perp) \sim \int dq_1 \, e^{iq_1 \cdot b_\perp} F_{1,2}(t = q_1^2)$$

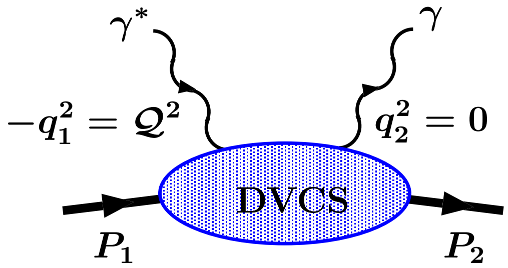




- "skewless" GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp \, e^{i\Delta \cdot b_\perp} q(x, b_\perp)$

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]



$$P = P_1 + P_2$$
$$q = (q_1 + q_2)/2$$

generalized Bjorken limit:

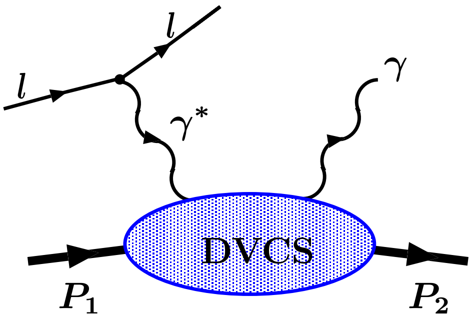
$$-q^2 (\overset{\text{DVCS}}{\simeq} \mathcal{Q}^2/2) \to \infty \qquad \vartheta = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \approx \frac{\eta}{\xi} \overset{\text{DVCS}}{=} 1$$

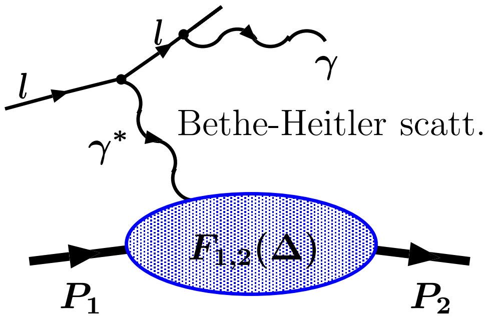
$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const } (\textit{as } x_B) \qquad t = (P_2 - P_1)^2$$

- cross-section can be expressed in terms of Compton form factors (CFFs): $\mathcal{H}(\xi, t, \mathcal{Q}^2), \mathcal{E}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2), \ldots$

Deeply virtual Compton scattering

- Measured in leptoproduction of a real photon:





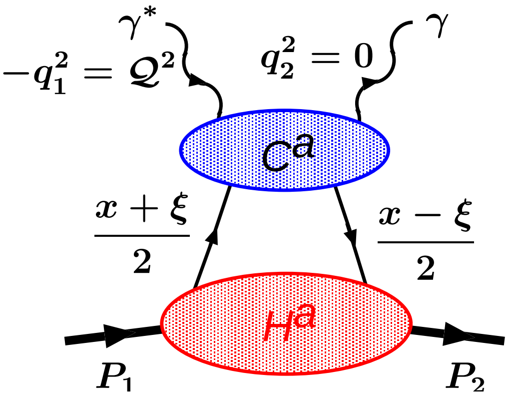
- There is a background process but it can be used to our advantage:

$$\sigma \propto |\mathcal{T}_{\text{DVCS}}|^2 + |\mathcal{T}_{\text{BH}}|^2 + \mathcal{T}^*_{\text{DVCS}}\mathcal{T}_{\text{BH}} + \mathcal{T}_{\text{DVCS}}\mathcal{T}^*_{\text{BH}}$$

- Using $\mathcal{T}_{\text{BH}}$ as a referent "source" enables measurement of the phase of $\mathcal{T}_{\text{DVCS}}$

Factorization of DVCS $\longrightarrow$ GPDs

[Collins and Freund '99]



- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \; H^a(x, \eta=\xi, t, \mu^2)$$

a=NS,S(Σ,G)

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)

Properties of GPDs

- Forward limit ($\Delta \to 0$): $\Rightarrow$ GPD $\to$ PDF

$$F^q(x,0,0) = H^q(x,0,0) = \theta(x) q(x) - \theta(-x) \bar{q}(-x)$$

Sum rules:

$$\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x,\eta,\Delta^2) \\ E^q(x,\eta,\Delta^2) \end{array} \right. = \left\{ \begin{array}{l} F_1(\Delta^2) \\ F_2(\Delta^2) \end{array} \right.$$

- Possibility of solution of proton spin problem

$$\frac{1}{2} \int_{-1}^{1} dx\, x \Big[H^q(x,\eta,\Delta^2) + E^q(x,\eta,\Delta^2) \Big] = J^q(\Delta^2)$$

[Ji '96]

DVCS at LO and beyond

based on:

- [DVCS using Mellin-Barnes representation, going to higher-orders and fitting GPDs]
  D. Müller,
  Next-to-next-to leading order corrections to deeply virtual Compton scattering: The Non-singlet case, [hep-ph/0510109]

  K. Kumerički, D. Müller, K. Passek-K., A. Schäfer,
  Deeply virtual Compton scattering beyond next-to-leading order: the flavor singlet case, [hep-ph/0605237]

  K. Kumerički, D. Müller, K. Passek-K.,
  Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond, [hep-ph/0703179]

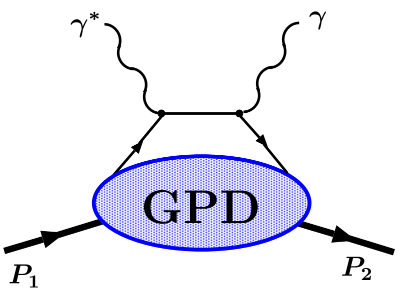
- [Getting the right information from LO]
  K. Kumerički, D. Müller, K. Passek-K.,
  Sum rules and dualities for generalized parton distributions: Is there a holographic principle?, [arXiv:0805.0152 [hep-ph]]

  K. Kumerički, D. Müller,
  *D*eeply virtual Compton scattering at small x(B) and the access to the GPD H, [arXiv:0904.0458 [hep-ph]]

$$ {}^{a}\mathcal{H}(\xi,t,\mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x,\xi,\mathcal{Q}^2/\mu^2,\alpha_s(\mu)) \; H^a(x,\eta=\xi,t,\mu^2) $$

a=NS,S(Σ,G)

- C^a (hard-scattering amplitude i.e. Wilson coefficient):
  - **LO, NLO** (1st order in α_s)
    [Ji et al, Belitsky et al, Mankiewicz et al, '97]
    $\Rightarrow$ need **NNLO** to stabilize perturbation series and investigate convergence
- H^a (GPD):
  - Complete deconvolution is impossible, so to extract GPDs from the experiment we need to **model their functional dependence**.
  - **Evolution** known to NLO order and not trivial to implement.



$+ \dots$

- factorization formula for singlet DVCS CFFs:

$$^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x\, \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}(x, \xi, t, \mu^2)$$

- ...in terms of conformal moments

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$= 2\sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, t, \mu^2)$$

$$H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots)$$

...

H_j^a even polynomials in η with maximal power η^{j+1}

- series summed using Mellin-Barnes integral over complex j:

$$= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right)\right] \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, t, \mu^2)$$

Advantages of conformal moments and Mellin-Barnes representation

- **NNLO corrections** accessible by making use of conformal OPE and known NNLO DIS results
- enables simpler inclusion of **evolution** effects
- possible efficient and stable numerical treatment $\Rightarrow$ stable and fast **computer code** for evolution and fitting
- powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
- opens the door for interesting **modelling of GPDs**
- moments are equal to matrix elements of local operators and are thus directly accessible on the lattice

Modelling conformal moments

- η-dependence inspired by SO(3) partial wave decomposition of $\gamma^*\gamma \to pp$ scattering (similar to "dual" parametrization [Polyakov, Shuvaev '02])

$$\mathbf{H}_j(\eta, t) = \underbrace{\begin{pmatrix} N'_\Sigma F_\Sigma(t)\,\mathrm{B}\big(1+j-\alpha_\Sigma(0), 8\big) \\ N'_G F_G(t)\,\mathrm{B}\big(1+j-\alpha_G(0), 6\big) \end{pmatrix}}_{\text{Leading partial wave}} + \begin{pmatrix} s_\Sigma \\ s_G \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}$$

- Leading wave (we have used in [hep-ph/0703179]):
  - Regge-inspired ansatz
  - for $t = 0$ corresponds to x-space PDFs of the form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\,(1-x)^7 \,; \qquad G(x) = N'_G\, x^{-\alpha_G(0)}\,(1-x)^5$$

- at NLO data can be fitted with leading wave only
- but at LO we need η-dependence!
  $\longrightarrow$ included in the new LO analysis

Sum rules and GPDs from LO

[Teryaev '05; Kumerički, Müller and Passek-K. '07, '08; Diehl and Ivanov '07;]

- LO perturbative prediction

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mu^2 = \mathcal{Q}^2)$$

$$\Downarrow$$

$$\frac{1}{\pi}\Im\mathrm{m}\mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} H(x, \xi = x, t, \mathcal{Q}^2) - H(-x, \xi = x, t, \mathcal{Q}^2)$$

- dispersion relation

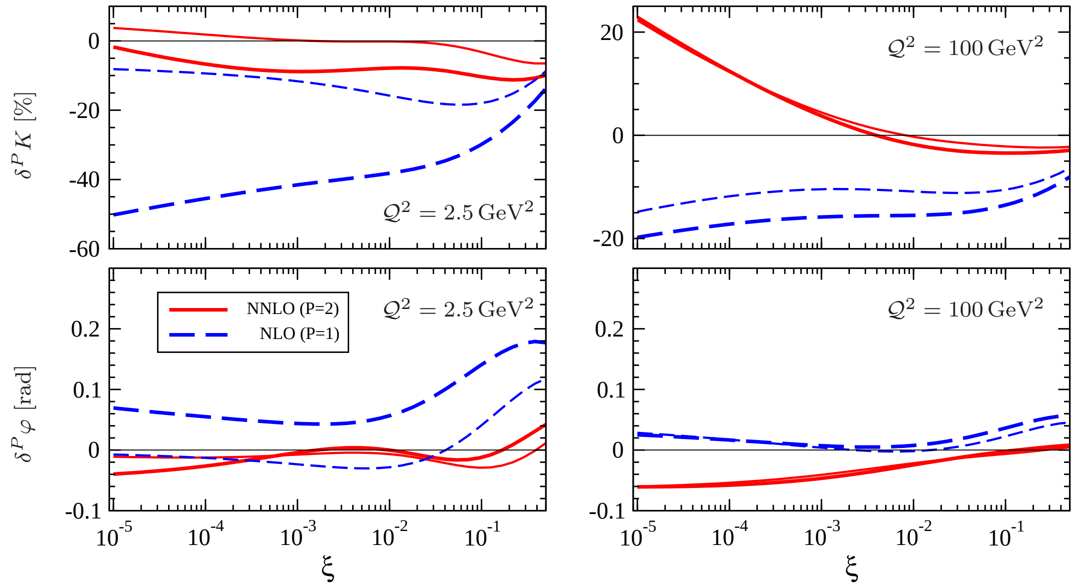
$\Rightarrow$ various sum rules ...

The goal is to reveal from DVCS data the GPDs at their cross-over trajectory $\eta (= \xi) = x$ and to obtain a generic understanding of the skewness effect.

$\longrightarrow$ model dependent extraction of H at the cross-over trajectory also for large ξ (JLab data) [Kumerički and Müller '09]

NLO and NNLO corrections

for generic parameters

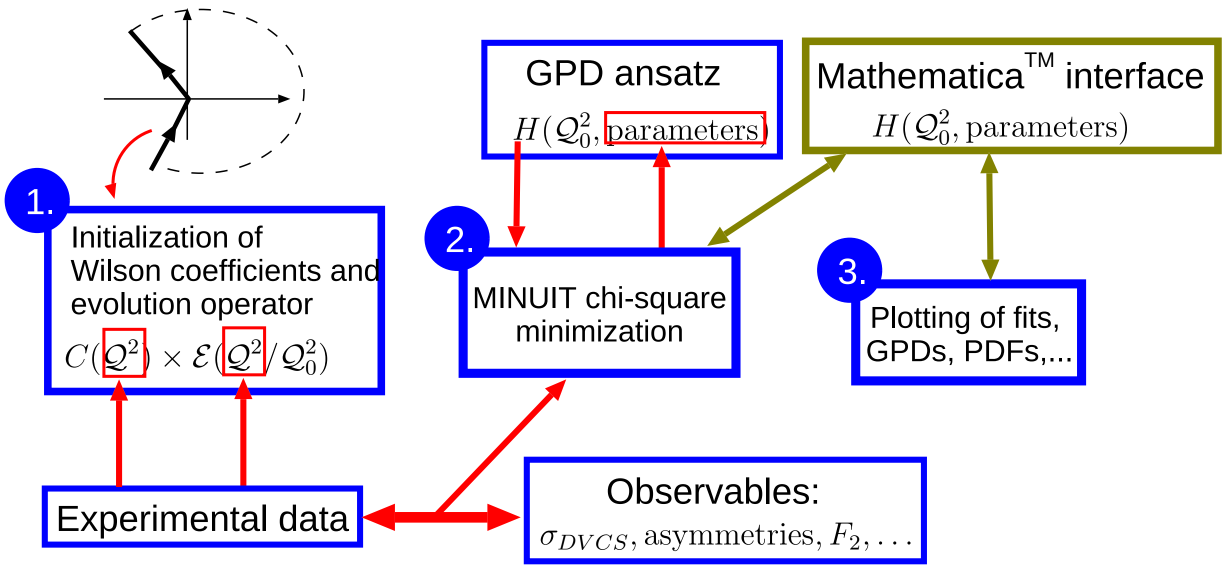


Thick lines: "hard" gluon $N_G = 0.4$ $\alpha_G(0)=\alpha_\Sigma(0) + 0.05$

Thin lines: "soft" gluon $N_G = 0.3$ $\alpha_G(0)=\alpha_\Sigma(0) - 0.02$

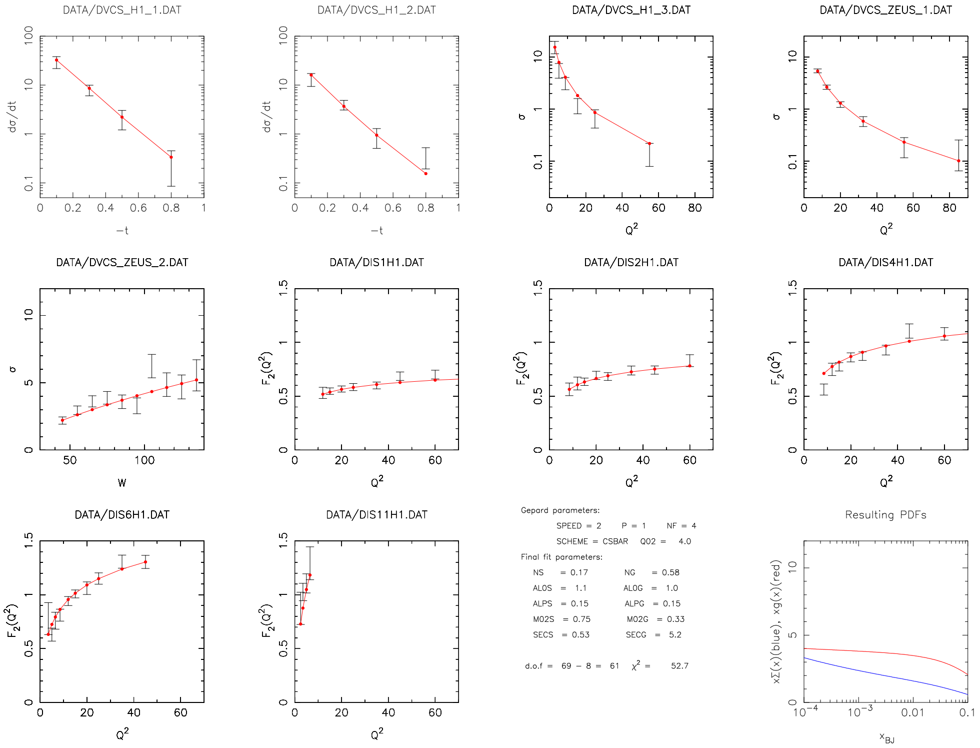
$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

Fast fitting routine (GeParD)

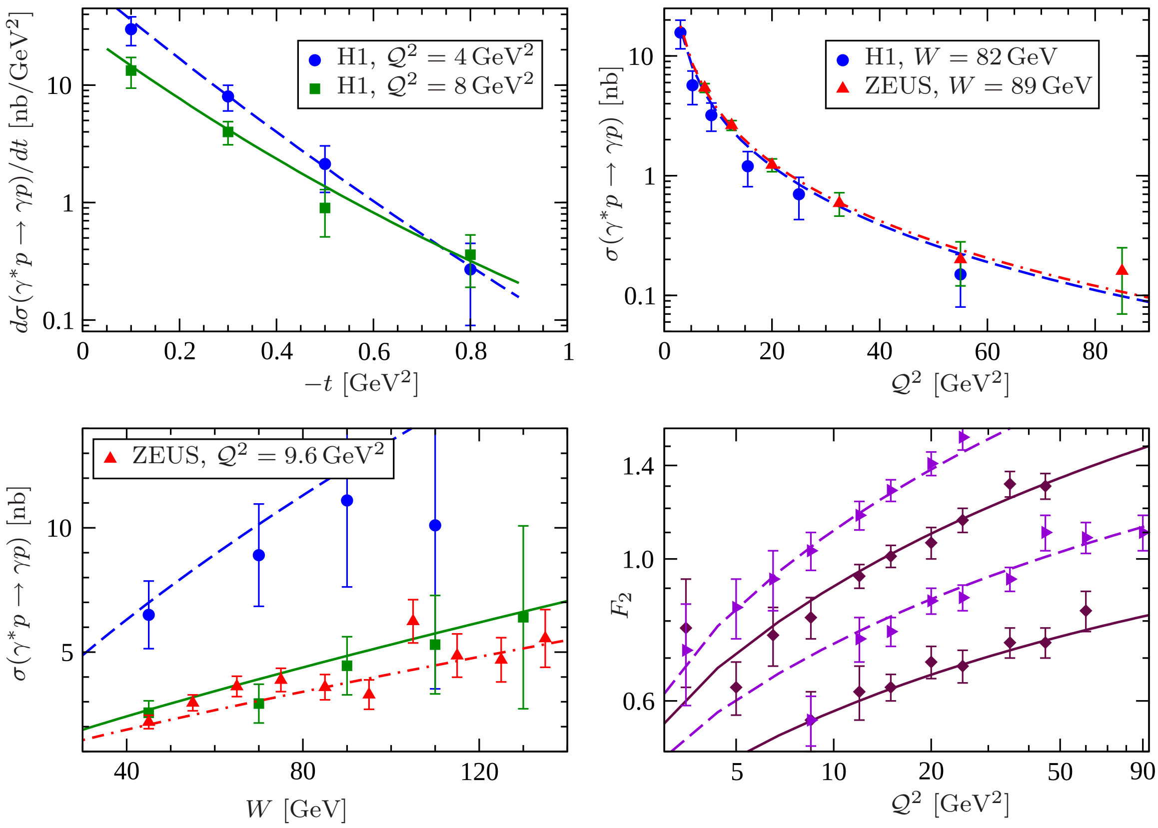


- $\int \mathrm{d}j\ C_j(Q^2) \times \mathcal{E}_j(Q^2, Q_0^2) \times H_j(Q_0^2) \Rightarrow$ Observable
- Check by comparison to QCD-PEGASUS [Vogt '04] and evolution of Les Houches benchmark PDFs

Fits (GeParD output)

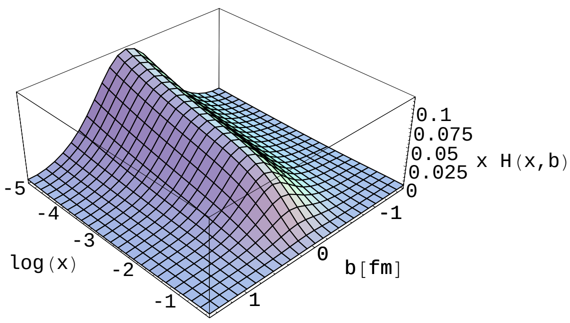


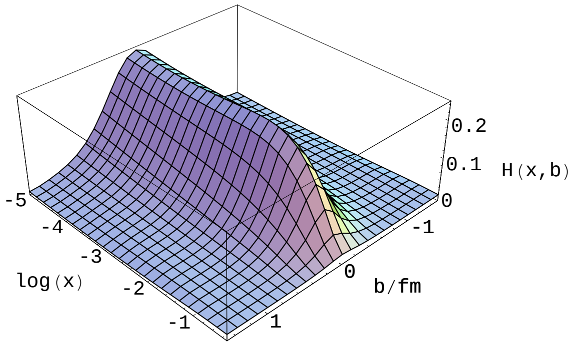
NNLO fit to HERA DVCS+DIS data

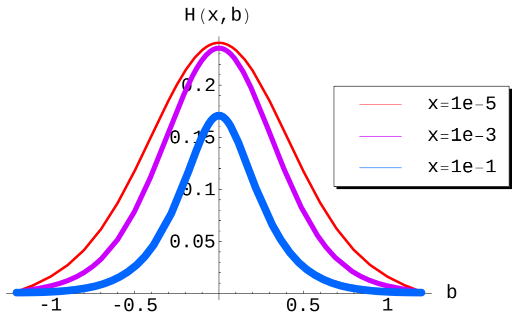


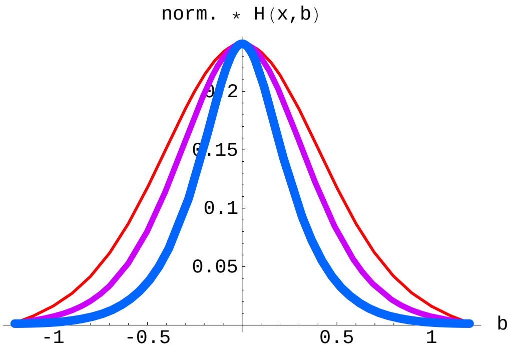
Three-dimensional image of a proton

Quarks: Gluons:

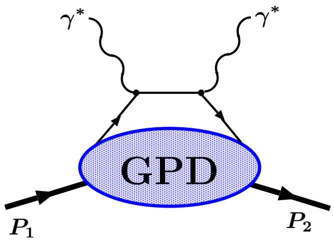








Complementary processes

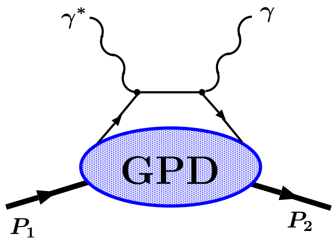


(general) DVCS

$\gamma^* p \to \gamma^* p$

$(ep \to epl^+ l^-)$

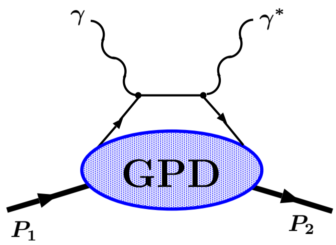
[Belitsky, Müller '02,'03; Guidal, Vanderhaeghen '02]



spacelike DVCS

$\gamma^* p \to \gamma p$

$(ep \to ep\gamma)$

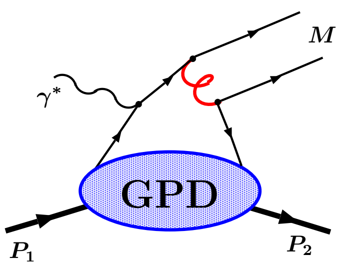


timelike DVCS

$\gamma p \to \gamma^* p$

$(\gamma p \to p l^+ l^-)$

[Berger, Diehl, Pire '01; Pire et al '08, Afanasev et al '09]



deeply virtual electroproduction of mesons (DVEM)

more difficult, but access to flavours

$\gamma^* p \to M p$

NLO: [Belitsky and Müller '01, Ivanov et al '04]

Timelike DVCS

$\gamma(q_1)p(P_1) \to \gamma^*(q_2)p(P_2)$, $q^2 = q_2^2 = \mathcal{Q}^2 \to \infty$

- experimentaly accesible from exclusive photoproduction of lepton pairs ($\gamma p \to l^+ l^- p$)
- Bethe-Heitler amplitude much more important $\to$ always bigger then DVCS
- offers relatively simple access to the real part of the Compton amplitude via the angular distribution of the lepton pair
- preliminary analysis was performed at LO [Berger, Diehl, Pire '01]
- in timelike regime important contributions and effects possible at higher-orders
- To do:
  - modification of [Belitsky and Müller '03] formulas for $pp \to ppl^+l^-$ process relevant for, say, ALICE
  -applying the formalism and higher-order expressions developed for spacelike DVCS to the timelike case
  -analysis of the experimental feasibilities (see, e.g., [Pire et al '08])

Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVEM . . . different processes offer different insight and should provide more complete picture.
- Spacelike DVCS analyzed up to NNLO:
  - Using conformal moments of GPDs has several advantages, including
    - elegant approach to NLO and NNLO corrections
    - providing convenient framework for GPD modelling
  - NLO corrections can be sizable; NNLO corrections are small,supporting perturbative framework of DVCS; scale dependence is not so conclusive: large NNLO effects for $\xi \lesssim 10^{-3}$
  - Fits to available DVCS and DIS data also work well and give access to transversal distribution of partons; in order to get good LO fits, one needs more sophisticated GPD modelling.
- The analysis of DVEM and timelike DVCS along the same lines is under way.

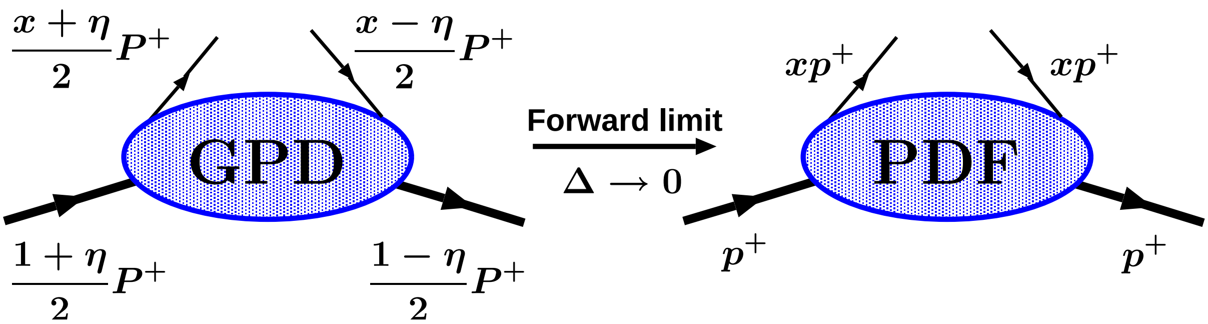
The End

Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,t=\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$



$$P = P_1 + P_2\ ; \qquad \Delta = P_2 - P_1\ ; \qquad \eta = -\frac{\Delta^+}{P^+} \quad \text{(skewedness)}$$

Properties of GPDs

- Decomposing into helicity conserving and non-conserving part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

- Forward limit ($\Delta \to 0$): $\Rightarrow$ GPD $\to$ PDF

$$F^q(x,0,0) = H^q(x,0,0) = \theta(x)q(x) - \theta(-x)\bar{q}(-x)$$

Sum rules:

$$\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x,\eta,\Delta^2) \\ E^q(x,\eta,\Delta^2) \end{array} \right. = \left\{ \begin{array}{l} F_1(\Delta^2) \\ F_2(\Delta^2) \end{array} \right.$$

- Possibility of solution of proton spin problem

$$\frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\eta,\Delta^2) + E^q(x,\eta,\Delta^2)\right] = J^q(\Delta^2) \qquad \text{[Ji '96]}$$

OPE

- DVCS belongs to a class of two-photon processes (DIS, DVCS, two-photon production of hadronic states ...) calculable by means of OPE

$$T_{\mu\nu}(q,P,\Delta) = \frac{i}{e^2}\int d^4x\, e^{ix\cdot q}\langle P_2,S_2|Tj_\mu(x/2)j_\nu(-x/2)|P_1,S_1\rangle$$
$$\to C_j\, O_j$$

$$\downarrow$$

$$\left.\begin{array}{l}\text{generalized Bjorken kinematics}\\ \text{conformal symmetry}\end{array}\right\} \to \text{unified description}$$

Conformal OPE (COPE)

- COPE prediction for general kinematics reads

$$C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^* = fixed)$$
$$= c_j(\alpha_s^*)\, {}_2F_1\!\left(\left.\begin{array}{c}(2+2j+\gamma_j(\alpha_s^*))/4,\,(4+2j+\gamma_j(\alpha_s^*))/4\\ (5+2j+\gamma_j(\alpha_s^*))/2\end{array}\right|\frac{\eta^2}{\xi^2}\right)\left(\frac{\mu^2}{Q^2}\right)^{\gamma_j(\alpha_s^*)}$$

- $\eta = 0$: DIS

$$\lim_{\eta\to 0} C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^*) = c_j^{DIS}(\alpha_s^*)|_{\beta=0}\left(\frac{\mu^2}{Q^2}\right)^{\gamma_j(\alpha_s^*)/2}$$

- $\eta = \xi$: DVCS
- $\eta = 1$: photon-to-pion transition form factor

Breaking of conformal symmetry

- massless QCD is conformally symmetric at the tree level
- conformal symmetry broken at the loop level (renormalization introduces mass scale)
  - running of the coupling constant $\Rightarrow \beta \neq 0$
  - renormalization of the composite operators $\Rightarrow$ non-diagonal anomalous dimensions $\gamma_{jk} = \delta_{jk}\gamma_j + \gamma_{jk}^{\mathrm{ND}}$

$$\mu \frac{d}{d\mu} O_j(\dots,\mu^2) = -\sum_{k=0}^{j} \gamma_{jk}(\alpha_s(\mu))\, \eta^{j-k}\, O_k\,(\dots,\mu^2)\,,$$

$$\mu \frac{d}{d\mu} C_j(\dots, Q^2/\mu^2, \alpha_s(\mu))] = \sum_{k=j}^{\infty} C_k(\dots, Q^2/\mu^2, \alpha_s(\mu))\gamma_{kj}(\alpha_s(\mu)) \left(\frac{\eta}{\xi}\right)^{k-j}$$

Conformal scheme

- non-diagonal terms of anomalous dimensions ($\overline{\mathrm{MS}}$ scheme) can be removed by finite renormalization, i.e, specific choice of factorization scheme $\rightarrow$ conformal subtraction ($\overline{\mathrm{CS}}$) scheme:

$$C^{\overline{\mathrm{MS}}}\, O^{\overline{\mathrm{MS}}} = C^{\overline{\mathrm{MS}}}\, B\, B^{-1}\, O^{\overline{\mathrm{MS}}} = C^{\overline{\mathrm{CS}}} O^{\overline{\mathrm{CS}}}$$

$$\gamma_{jk}^{\overline{\mathrm{CS}}} = \delta_{jk}\gamma_k + \frac{\beta}{g}\Delta_{jk}$$

- however, there is ambiguity in $\overline{\mathrm{MS}} \rightarrow$ CS rotation matrix:

$$B = B^{(\beta=0)} + \frac{\beta}{g}\delta B$$

and by judicious choice of δB one can "push" mixing to NNLO ($\overline{\mathrm{CS}}$ scheme, [Melić et al. '02]) $\rightarrow \Delta_{jk}$ — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected

NNLO DVCS

- Finally

$$C_j^{\overline{\rm CS},\rm DVCS}(Q^2/\mu^2,\alpha_s(\mu)) = C_k(1,\alpha_s(Q))\,\mathcal{P}\exp\left\{\int_Q^\mu \frac{d\mu'}{\mu'}[\gamma_j(\alpha_s(\mu'))\delta_{kj}\right\}$$

with

$$C_j(1,\alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2}\Gamma\left(\frac{5}{2}+j+\gamma_j(\alpha_s)/2\right)}{\Gamma(3/2)\Gamma\left(3+j+\gamma_j(\alpha_s)/2\right)}\; c_j^{\overline{\rm MS},\rm DIS}(\alpha_s)$$

- we take
  $c_j^{\overline{\rm MS},\rm DIS}(\alpha_s)$ from [Zijlstra, v. Neerven '92,'94, v. Neerven and Vogt '00]
  γ_j from [Vogt, Moch and Vermaseren '04]

Conformal algebra

- Conformal group restricted to light-cone $\sim O(2,1)$

$$L_+ = -iP_+ \qquad L_- = \frac{i}{2}K_- \qquad L_0 = \frac{i}{2}(D+M_{-+})$$

$$[L_0, L_\mp] = \mp L_\mp \qquad [L_-, L_+] = -2L_0$$

Casimir: $L^2 = L_0^2 - L_0 + L_- L_+$

conf.spin j: $[L^2, \mathbb{O}_{n,n+k}] = j(j-1)\mathbb{O}_{n,k}$

(D — dilatations, K_- — special conformal transformation (SCT))

Operator Product Expansion

$$J_{\rm em}(x) J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{1}{x^2}\right)^2 x_-^{n+k+1} C_{n,k} O_{n,k}$$

$k = 0$: $\quad O_{n,0} \equiv \quad \bar\psi \gamma^+ (i \overleftrightarrow{D}_+)^n \psi \qquad i\partial_+ \stackrel{\rm M.E.}{\rightarrow} -\Delta_+$

$\overleftrightarrow{D}_+ \equiv \overrightarrow{D}_+ - \overleftarrow{D}_+$

- $C_{n,0}$ and γ_n of $O_{n,0}$ are well known from DIS up to NNLO.
- But $C_{n,k}$ and $\gamma_{n,k}$ are not so well known.
- $\gamma_{n,k} \neq 0 \;\Rightarrow$ operators $O_{n,k}$ mix under evolution.
- Choosing operator basis in which $\gamma_{n,k}$ is diagonal would help. But how to diagonalize unknown matrix?!
- (At least) to LO answer is: use conformal operators.

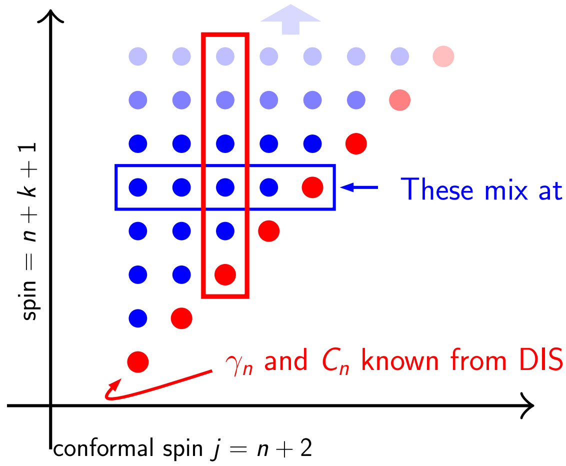
Conformal operators

$$\mathbb{O}_{n,n+k} = (i\partial^+)^{n+k}\, \bar\psi \gamma^+ C_n^{3/2}\left(\frac{\overleftrightarrow{D}^+}{\partial^+}\right) \psi$$

- they have well-defined conformal spin $j = n+2$
- massless QCD is conformally symmetric at the tree level $\Rightarrow$ conformal spin is conserved
- mixing of operators with different n is forbidden by conformal symmetry, while mixing of those with different $n+k$ is forbidden by Lorentz symmetry $\Rightarrow \mathbb{O}_{n,n+k}$ don't mix at LO
- conformal symmetry broken at the loop level (renormalization introduces mass scale, dimensional transmutation) $\Rightarrow$
  - running of the coupling constant $\partial g / \partial \ln \mu \equiv \beta \neq 0$
  - anomalous dimensions of operators $\gamma_{jk} = \delta_{jk}\gamma_j + \gamma_{jk}^{\rm ND}$

$\Rightarrow \mathbb{O}_{n,n+k}$ start to mix at NLO

Conformal Towers



- Diagonalize in artificial $\beta = 0$ theory by changing scheme

$$\mathbb{O}^{\text{CS}} = B^{-1}\mathbb{O}^{\overline{\text{MS}}} \qquad \text{so that} \qquad \gamma_{jk}^{\text{CS}} = \delta_{jk}\gamma_k$$

- $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0} \quad \Rightarrow$ summing complete tower

$\beta \neq 0$ (I)

- In full QCD $\beta \neq 0$ and NLO diagonalization is spoiled:

$$\gamma_{jk}^{\text{CS}} = \delta_{jk}\gamma_k + \frac{\beta}{g}\Delta_{jk}$$

- However, there is also ambiguity in $\overline{\text{MS}} \to \text{CS}$ rotation matrix:

$$B = B^{(\beta=0)} + \frac{\beta}{g}\delta B$$

- By judicious choice of δB one can "push" mixing to NNLO ($\overline{\text{CS}}$ scheme, [Melić, Müller and Passek '02]).
- But how to calculate rotation matrix B? This is problem equivalent to calculation of $\gamma_{j,k}$.

$\beta \neq 0$ (II)

- The $B^{(\beta=0)}$ is constrained by conformal Ward identities ...

$$B_{jk}^{(\beta=0)\mathrm{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi}\theta(j>k)\frac{\gamma_{jk}^{\mathrm{SCT,\ LO}}}{a_{jk}}$$

(a_{jk} — known matrix)
[Müller '93]

SCT $\equiv$ special conformal transformation

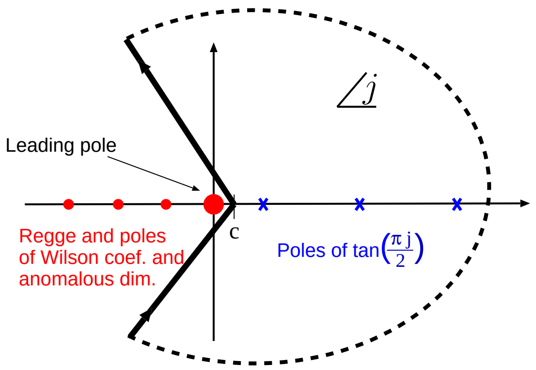
- ...and, as a consequence

$$^{\overline{\mathrm{MS}}}\gamma_{jk}^{\mathrm{ND},(1)} = \frac{\left[\gamma^{\mathrm{SCT},\ (0)} - \beta_0 \frac{b}{g}, \gamma^{(0)}\right]_{jk}}{a_{jk}}$$

- Final result:
n-loop DIS (diagonal) result + $(n-1)$-loop SCT anomaly =
n-loop non-diagonal prediction

Mellin-Barnes representation of CFFs

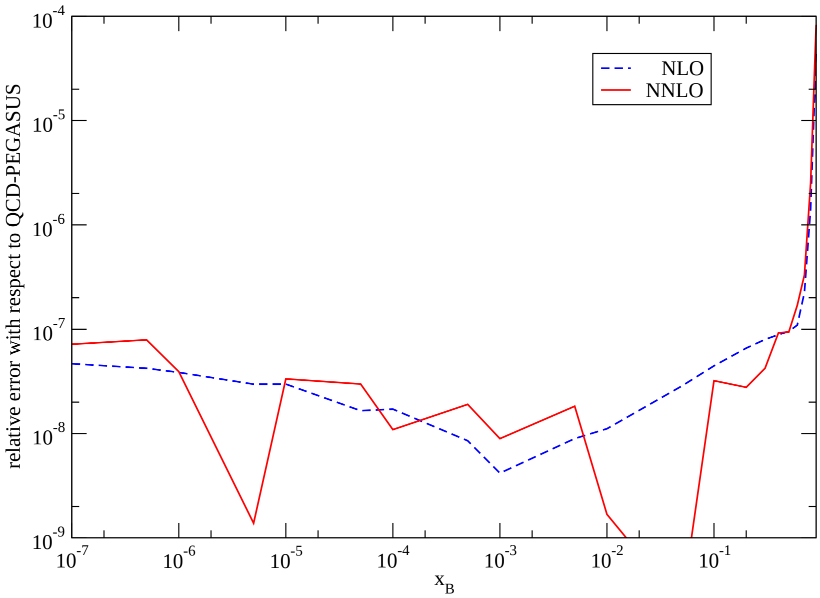
$$^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right)\right] \xi^{-j-1}\mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\,\mathbf{H}_j(\xi, t, \mu^2)$$



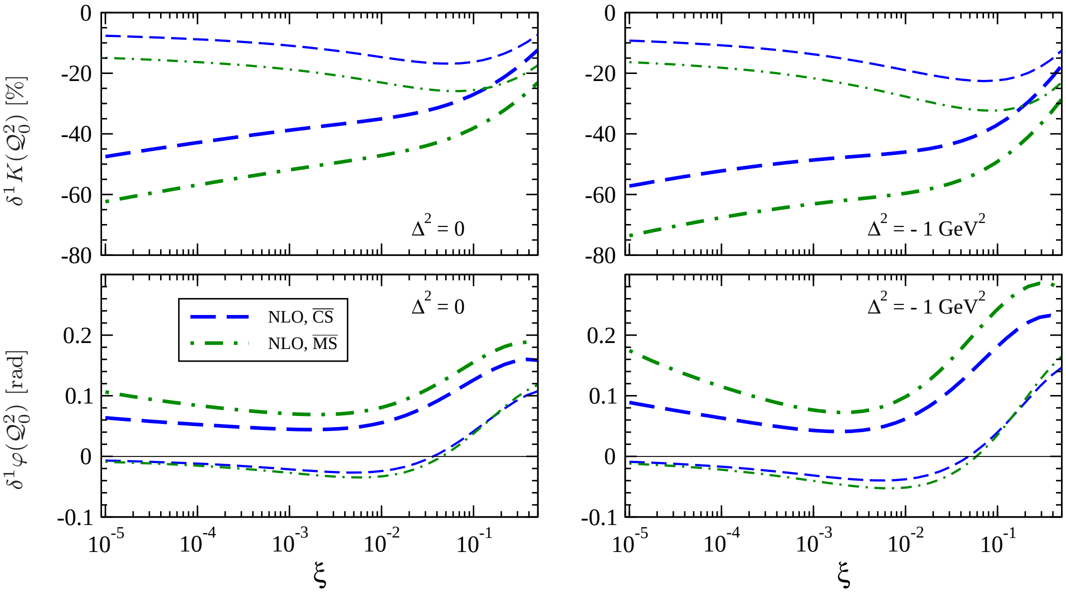
- We have used previously developed formalism to
  1. investigate size of NNLO corrections to non-singlet [D. Müller, Phys.Lett. **B634** (2006), hep-ph/0510109] and singlet Compton form factors in $\overline{\mathrm{CS}}$ scheme [K. Kumerički, D. Müller, K. P-K., A. Schäfer, Phys.Lett. **B648** (2007), hep-ph/0605237]
  2. compare the $\overline{\mathrm{CS}}$ NLO predictions to complete $\overline{\mathrm{MS}}$ NLO predictions (non-diagonal evolution included) and analyze the latter [K. Kumerički, D. Müller, K. P-K., Nucl. Phys. **B794** (2008), hep-ph/0703179]
  3. perform fits (in both schemes) to DVCS (and DIS) data and extract information about GPDs [K. Kumerički, D. Müller, K. P-K., Nucl. Phys. **B794** (2008), hep-ph/0703179]
  4. analyze the new HERA data to LO (investigating η-dependent GPD model) [K. Kumerički and D. Müller, arXiv:0904.0458 [hep-ph]]

Check

- Check by comparison to QCD-PEGASUS [Vogt '04]
- evolution of Les Houches benchmark PDFs:



NLO corrections



Thick lines: "hard" gluon $N_G = 0.4$ $\alpha_G(0) = \alpha_\Sigma(0) + 0.05$

Thin lines: "soft" gluon $N_G = 0.3$ $\alpha_G(0) = \alpha_\Sigma(0) - 0.02$

$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

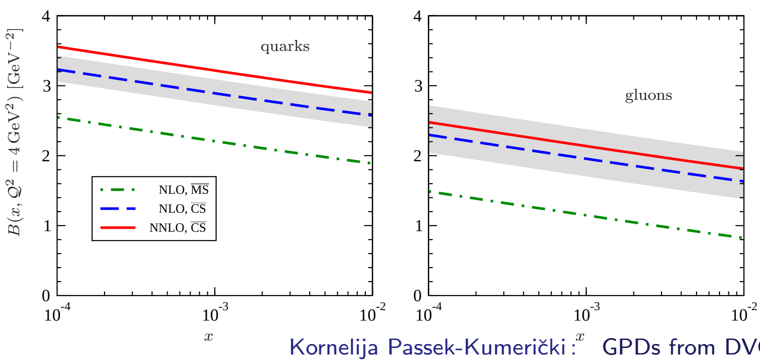
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b [Burkardt '00, '02]

$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2)\,,$$

- Average transversal distance :

$$\langle \vec{b}^2 \rangle(x, \mathcal{Q}^2) = \frac{\int d\vec{b}\, \vec{b}^2 H(x, \vec{b}, \mathcal{Q}^2)}{\int d\vec{b}\, H(x, \vec{b}, \mathcal{Q}^2)} = 4B(x, \mathcal{Q}^2)\,,$$



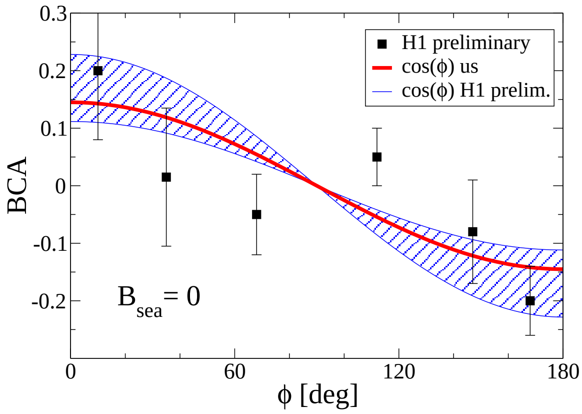
(at $\mathcal{Q}^2 = 4\,\mathrm{GeV}^2$)

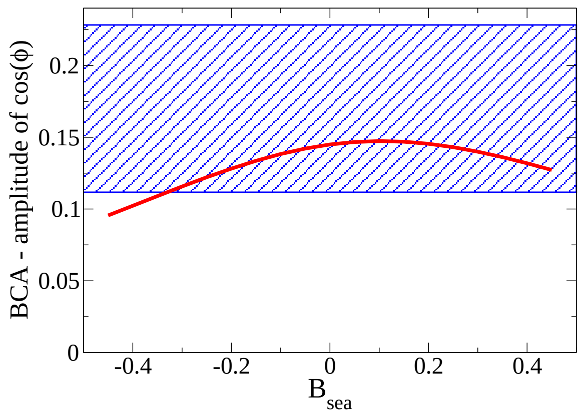
$\langle \vec{b}^2 \rangle_{\mathrm{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04}\,\mathrm{fm}^2$

Beam charge asymmetry

$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{\mathcal{A}_{\rm Interference}}{|\mathcal{A}_{\rm DVCS}|^2 + |\mathcal{A}_{\rm BH}|^2} \overset{\rm LO}{\propto} F_1 \mathcal{H} + \frac{|t|}{4M^2} F_2 \mathcal{E}$$

- Model $E_{\rm sea}$ as $\kappa_{\rm sea} H_{\rm sea}$ and take $B_{\rm sea} \equiv \int dx\, x E_{\rm sea}$ as parameter

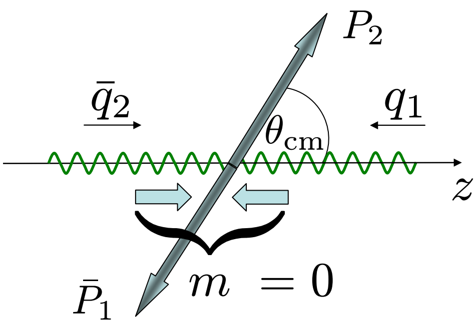




- H1 data enable exclusion only of very negative $B_{\rm sea}$

Modelling conformal moments of GPDs

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$



When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

- ...and dependence on $\theta_{\rm cm}$ in t-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

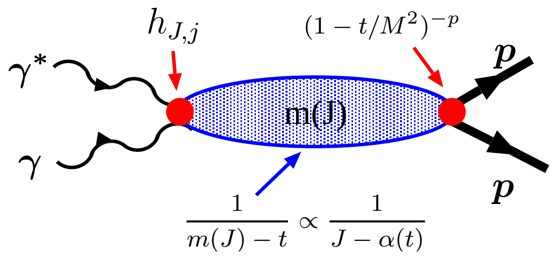
- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$, as well as trivial crossing properties of Wilson coefficients C_j, leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

- and t-channel partial waves are modelled as:

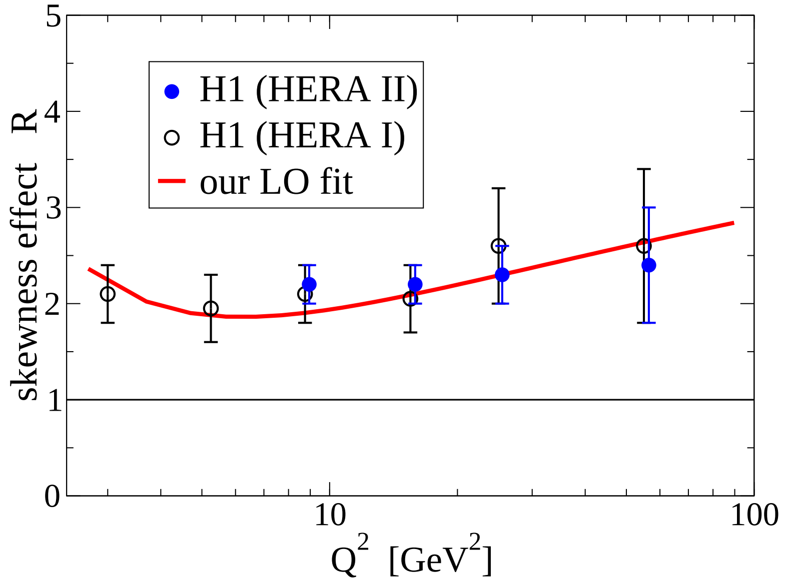


$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p} \eta^{j+1-J} d^J_{0,\nu}$$

- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

Skewness effect (I) — R

$$R \equiv \left.\frac{\mathcal{I}m\, A_{\rm DVCS}}{\mathcal{I}m\, A_{\rm DIS}}\right|_{t=0} \stackrel{LO}{=} \frac{H(\xi,\xi)}{H(2\xi,0)}$$ [Shuvaev et al. '99]

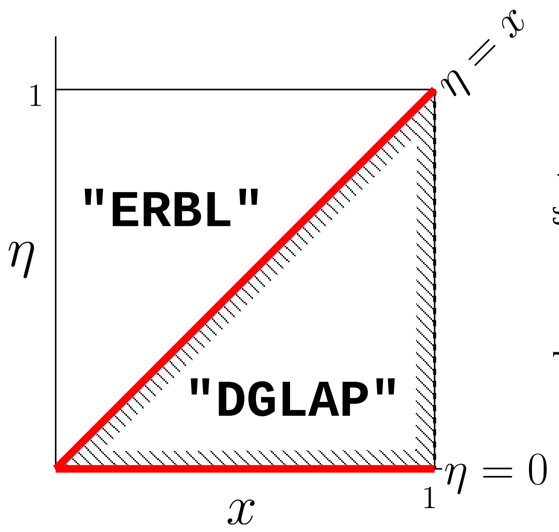


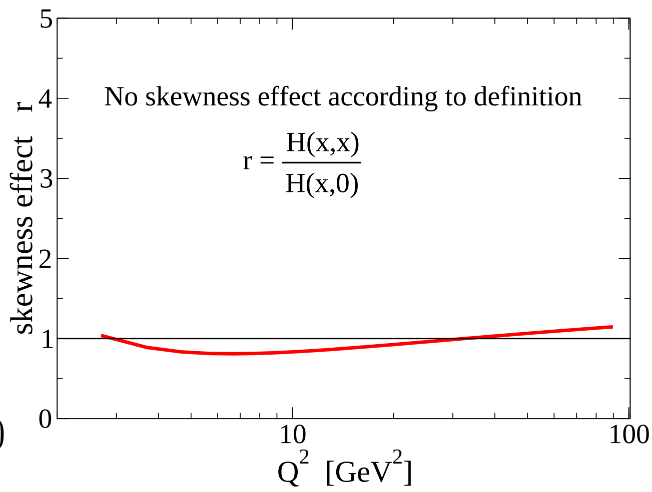
- Significant skewness effect?

Skewness effect (II) — r

- Skewness effect is naturally defined by ratio of GPDs $H(x, \eta)$ at two physically relevant trajectories: $\eta = x$ and $\eta = 0$

$$r = \frac{H(x,x)}{H(x,0)} \stackrel{LO}{\approx} \frac{1}{2^{\alpha}} R \qquad \text{for} \quad q(x \to 0) \sim x^{-\alpha} \qquad \alpha \approx 1$$





App: GPDs App: Conf. approach App: Results App: Mod. GPDs App: SR and GPDs at LO App: Experimental issues

Skewness effect (III)

- To get the correct normalization and t-dependence, one has to compensate "natural" DVCS-to-DIS enhancement factor [Shuvaev et al. '99]

$$\left.\frac{2^{j+2}\Gamma(j+5/2)}{\sqrt{\pi}\,\Gamma(j+3)}\right|_{j=\alpha-1\approx 0.2} \approx 1.5$$

  - at NLO radiative corrections take care of that
  - at LO resummed subleading partial waves have to give negative contribution:

$$\mathbf{H}_j(\eta,t) = \begin{pmatrix} N'_\Sigma F_\Sigma(t)\,\mathrm{B}\big(1+j-\alpha_\Sigma(0),8\big) \\ N'_G F_G(t)\,\mathrm{B}\big(1+j-\alpha_G(0),6\big) \end{pmatrix} + \underbrace{\begin{pmatrix} s_\Sigma \\ s_G \end{pmatrix}\begin{pmatrix}\text{subleading partial waves, } \eta\text{-dependence!}\end{pmatrix}}_{<0}$$

negative "intrinsic skewness"

GPDs and sum rules

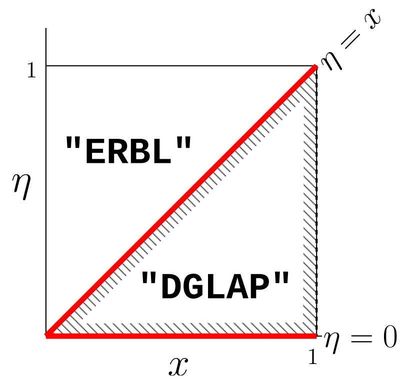
[K. Kumerički, D. Müller, K. Passek-K., arXiv:0805.0152 [hep-ph]],
[Teryaev '05; Diehl and Ivanov '07]

- LO perturbative prediction

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mu^2 = \mathcal{Q}^2)$$

$$\Downarrow$$

$$\frac{1}{\pi} \Im\mathrm{m} \mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} H(x, \xi = x, t, \mathcal{Q}^2) - H(-x, \xi = x, t, \mathcal{Q}^2)$$



- LO perturbative prediction

$$\boxed{\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} \int_{0}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H^{-}(x, \xi, t, \mu^2 = \mathcal{Q}^2)}$$

$$\Downarrow$$

$$\frac{1}{\pi} \Im\mathrm{m} \mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} H^{-}(x, \xi = x, t, \mathcal{Q}^2)$$

$$\Re\mathrm{e} \mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} PV \int_{0}^{1} dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) H^{-}(x, \xi, t, \mu^2 = \mathcal{Q}^2)$$

- dispersion relation

$$\boxed{\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int_{0}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) \frac{\Im\mathrm{m} \mathcal{H}(x, t, \mathcal{Q}^2)}{\pi} + \mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)}$$

$$\Re\mathrm{e} \mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} PV \int_{0}^{1} dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) H^{-}(x, x, t, \mathcal{Q}^2) + \mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)$$

- $\Rightarrow$ various sum rules

One strategy for extracting GPD information to LO accuracy:

- model the t-dependency of the GPD at $\eta = 0$
- parameterize the skewness function S defined in

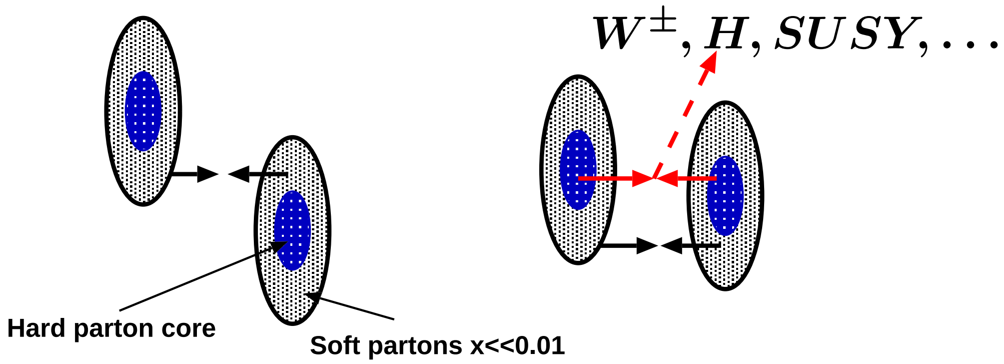
$$H^{-}(x,x,t,\mathcal{Q}^2) = \left[1 + S(x,t,\mathcal{Q}^2|H^{-})\right] H^{-}(x,\eta=0,t,\mathcal{Q}^2)\,.$$

- fit parameters to measured observables (CFFs)

The goal of this LO analysis and of this concept is to reveal GPDs at their cross-over trajectory $\eta = x$ from DVCS data and to obtain a generic understanding of the skewness effect.

More work in this direction needed...

Relevance of GPDs for collider physics



- heavy particle production $\Rightarrow$ collision is more central $\Rightarrow$ larger probability for multiple parton collisions
- [Frankfurt, Strikman and Weiss '04]
- No influence on total inclusive cross sections, but event structure is sensitive to transversal parton distributions.
- Relevant for LHC?