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DYNAMIC MODELS IN IMPERATIVE LOGIC

Imperatives in Action: Changing Minds and Norms

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1 WHY STUDY IMPERATIVES?

The theory of imperatives is philosophically relevant since in building it — some of the long standing problems need to be addressed, and presumably some new ones are waiting to be discovered. The relevance of the theory of imperatives for philosophical research is remarkable, but usually recognized only within the field of practical philosophy.

> Imperatives lie at the heart of both practical and moral reasoning. . . [22]

Unlike the quote above, the emphasis can be put on problems of theoretical philosophy. Proper understanding of imperatives is likely to raise doubts about some of our deeply entrenched and tacit presumptions. In philosophy of language it is the presumption that declaratives provide the paradigm for sentence form; in philosophy of science it is the belief that theory construction is independent from the language practice, in logic it is the conviction that logical meaning relations are constituted out of logical terminology, in ontology it is the view that language use is free from ontological commitments. The list is not exhaustive; it includes only those presumptions that this paper concerns.

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1.1 Philosophy of language: "declarative fallacy"

Belnap has defined 'declarative fallacy' as a tendency of reducing logical phenomena to those obtaining between declarative sentences. The following quote vividly introduces the notion.

> … in our culture when a logician, or nearly any trained philosopher, says 'sentence,' what is meant is a declarative sentence a sentence capable of having, as they say, a truth-value, or maybe truth-conditions, a sentence that can be used to 'say' something, a sentence expressing a proposition, a sentence that can play a role in inference as either premiss or conclusion, a sentence that might occur in someone's (say Quine's) 'canonical language.' This is what is to be rejected. This is the Declarative Fallacy. Instead, one should recognize that from the beginning there are not only declarative sentences, but, at least, both interrogatives and imperatives. The grammarians are right and those teachers of elementary logic that seem to have miseducated most of us are wrong: give all sentences equal time, and do not take declaratives as a paradigm for what can happen between full stops. [6, p. 1]

Neglecting of non-declaratives in philosophical analysis is a prime example of declarative fallacy. But even when other sentence moods are taken into consideration, the avoidance of the said fallacy is not guaranteed. According to Belnap, the purported existence of common element in all types of sentences represents a variant of declarative fallacy. His approach will be employed here in the analysis of the quote below. The quote relies on the idea of two component structure of speech acts, which of itself does not represent a declarative fallacy, but does so if it is assumed that there is a common propositional or semantic content for any type of speech act. Such an additional assumption is presupposed by Green's term 'common element' [23].

> In chemical parlance, a radical is a group of atoms normally incapable of independent existence, whereas a functional group is the grouping of those atoms in a compound that is responsible for certain of the compound's properties. Analogously, a proposition is itself communicatively inert; for instance, merely expressing the proposition that snow is white is not to make a move in a "language game". Rather, such moves are only made by putting forth a proposition with an illocutionary force such as assertion, conjecture, command, etc. The chemical analogy gains further plausibility from the fact that just as a chemist might isolate radicals held in common among various compounds, the student of language may isolate a common element held among 'Is the door shut?', 'Shut the door!', and 'The door is shut'. This common element is the proposition that the door is shut, queried in the first sentence, commanded to be made true in the second, and asserted in the third. [23, p. 435]

If we assume that syntactically identical radical parts (shown in Table 1) of non-declarative sentences (1)–(3) are also semantically identical, and that the radical part of (1) can serve as a paradigm for the other two, then we have committed Belnap's "declarative fallacy."

Where does the difference between syntactically identical parts lie? Several possible interpretations of purportedly the same radical come to mind. One possibility is:

1. In 'It is the case that *the door is shut*' the radical describes a generic state of affairs.
2. In 'Let it be the case that *the door is shut*' the radical can be understood as a description of an action which is to be performed in the "external" world.
3. In 'Is it the case that *the door is shut*' the radical talks about an action in the "internal world," i.e. the act of making sure the interrogator knows whether the door is shut.

One can oppose the last two interpretations by pointing out that there is also a possibility of state-of-affairs interpretation. In particular the expression 'Let it be the case that . . . ' seems to offer such an option. But this expression is closer in form to an optative (a *fiat*, an expression of a wish) than to an imperative. The imperatival form 'See to it that the . . . ' clearly reveals its agentive content. Thus, one is faced with alternative interpretations of imperatives: on the one side, there is the propositional content interpretation suggested by declarative paradigm, and, on the other, the agentive content interpretation suggested by non-declarative paradigm. The interpretational preference ought to be justified. As Donald Davidson [17, p. 140] has put it: "much of the interest in logical form comes from an interest in logical geography." In other words, it is at the level of meaning relations between sentences, and not at the level of an isolated sentence, where the justification of our interpretational preferences should be sought for. When meaning relations are not respected, the communication breakdown occurs, like the one depicted in Example 1.1.1 where the speaker refuses the sentence to which he/she is committed and the hearer entitled.

Example 1.1.1. Speaker: Is the door shut? Recipient: So you want to know whether the door is shut. Speaker: No, I don't. Recipient (*confused*): Huh?

Table 1: Common element assumption.

	Modal element Illocutionary force (indicator)	Sentence radical Semantic content
(1)	It is the case that	the door is shut.
(2)	Let it be the case that	the door is shut!
(3)	Is it the case that	the door is shut?

Non-propositional interpretation of sentence radicals figures prominently in Belnap's (et al.) stit-theory [8]. According to stit-theory the canonical form [α *stit* : Q] of agentives consists of three parts: agent term (α), verb phrase (see to it that; *stit* in acronym notation), and declarative complement (Q). Imperative content thesis is one of the central claims in stit-theory:

> Regardless of its force on an occasion of use, the content of every imperative is agentive. [8, p. 10]

In 1965. Åqvist had put forward the thesis regarding the content of interrogatives:

> In general, the idea is to equate questions with the kind of *epistemic imperatives* (or optatives, perhaps), whose primary function, or use, consists in their serving as means of widening the questioner's knowledge, of increasing the amount of information in his possession. [3, p. 6]

Belnap's imperative content thesis and Åqvist's interrogative content thesis, taken together, imply that interrogatives have agentive content. The difference between imperatives and interrogatives, with respect to their content, lies in the type of action that is being asked for. In the case of interrogatives the agentive is of a peculiar kind: it is an "epistemic action" of widening the questioner's knowledge about the issue raised.

There is an asymmetry here; according to these approaches "propositional content," typically found in declaratives, cannot play the role of sentence radical in imperatives and interrogatives. Since the agentive radical may occur in indicative mood too, it appears that the imperative rather than the declarative provides a paradigmatic sentence form. Therefore, the imperatives ought to be studied not only for the negative reason of avoiding the declarative fallacy.

1.2 Philosophy of human sciences and their methodological autonomy

The last decades of the 19th century saw the start of discussion on the nature of human sciences. One of the key insights of the philosophy of human sciences is that besides having their own method (understanding) and object (action), they also enjoy a linguistic autonomy consisting in use of specific vocabulary and logical syntax.

Wilhelm Dilthey (1833–1911) delineated human sciences and natural sciences as epistemologically distinct categories of science. Dilthey understood the human sciences as having a practical component, and therefore imperatives do belong to their language.

> The human sciences, ... contain three distinct classes of assertions. One class describes reality given in perception. These assertions comprise the historical component of knowledge. The second class explicates the uniform behavior of partial contents of this reality,

> which are separated out by abstraction. These assertions form the theoretical component of the human sciences. The last class expresses value judgments and prescribes rules. These assertions contain the practical component of the human sciences. The human sciences consist of these three classes of statements: facts, theorems, value judgments and rules. [18, p. 78]

Wilhelm Windelband (1848–1915) coined the adequate names for the two categories of empirical sciences: idiographic sciences and nomothetic sciences.

> In their quest for knowledge of reality, the empirical sciences either seek the general in the form of the law of nature or the particular in the form of the historically defined structure. On the one hand, they are concerned with the form which invariably remains constant. On the other hand, they are concerned with the unique, immanently defined content of the real event. The former disciplines are nomological sciences. The latter disciplines are sciences of process or sciences of the event. The nomological sciences are concerned with what is invariably the case. The sciences of process are concerned with what was once the case. If I may be permitted to introduce some new technical terms, scientific thought is nomothetic in the former case and idiographic in the latter case. [52, p. 175]

Donald Davidson (1917–2003) pointed out the differences of the languages employed in human and natural sciences both in terms of their vocabulary and logic: the language of the former creates an "intensional context" which cannot occur in the language of the latter. The respective vocabularies alongside with their transformational syntaxes are termed 'mental' (or "vocabulary of thought and action") and 'physical vocabulary.'

> The nomological irreducibility of the psychological means, if I am right, that the social sciences cannot be expected to develop in ways exactly parallel to the physical sciences, nor can we expect ever to be able to explain and predict human behavior with the kind of precision that is possible in principle for physical phenomena. This does not mean there are any events that are in themselves undetermined or unpredictable; it is only events as described in the vocabulary of thought and action that resist incorporation into a closed deterministic system. These same events, described in appropriate physical terms, are as amenable to prediction and explanation as any. [17, p. 230]

Georg Henrik von Wright (1916–2003) has put forward the thesis that practical syllogism grounds methodological autonomy of sciences of man.

> Practical reasoning is of great importance to the explanation and understanding of action. ... the practical syllogism provides the sciences of man with something long missing from their methodology: an explanation model in its own right which is a definite alternative to the subsumption-theoretic covering law model. Broadly speaking, what the subsumption-theoretic model is to causal explanation and explanation in the natural sciences, the practical syllogism is to teleological explanation and explanation in history and the social sciences. [50, p. 27]

Interweaving the matching threads from the quotes above we arrive at the thesis that human sciences, if conceived as idiographic, use distinctive and irreducible language (mental-vocabulary language) in forming a peculiar type of theoretical constructions (providing the 'practical inferences' whose conclusion describes the act being understood or interpreted).

1.2.1 What is practical syllogism?

Aristotle has discovered practical inference as different in kind from the theoretical inference (for Aristotle's account of practical inference see e.g. *Nicomachean ethics* 1112b, 1147b; *Metaphysics* 1032b, *De Motu Animalium* 701a). The conclusions of these two categories of inference give answer to different questions: practical inference provides an answer for what-to-do question, while theoretical inference answers to what-is-the-case question. Although being already outlined in Aristotle's works, the structure of reasoning that leads to action or decides upon its normative value or provides the understanding of the Self and the Other has remained theoretically unclear in spite of its utmost importance in human life. The diagnosis given half a century ago by Elizabeth Anscombe is still valid: practical inference is a logical form of invaluable significance but its character remains unknown.

> 'Practical reasoning,' or 'practical syllogism,' which means the same thing, is one of Aristotle's best discoveries. But its true character has been obscured. [2, pp. 57–58]

1.2.2 An exemplar of practical inference

Practical syllogism has drawn considerable attention in philosophy from the 1960s onwards, and from the 1990s in artificial intelligence (so called BDI model of rational agency). There is no consensus on the exact form of practical syllogism and its logical validity. Two philosophical accounts will be presented and briefly discussed.

In 1969 James D. Wallace gave the following account of practical syllogism:

> (1) S genuinely wants p to be the case for its own sake.
> (2) Only if S does X will p be the case.
> (1) and (2) constitute *prima facie* grounds for
> (C) S should do X.
> That is, if one agrees that (1) and (2) are true, and if one grants that it is in S's power to do X and to make p the case, then one cannot deny (C) without committing oneself to the existence of a case either for the claim that S should refrain from doing X or for the claim that S should refrain from making p the case. If S can do X and can attain p, then if (1) and (2) are true and it is also true that
> (α) There are no grounds for claiming that S should refrain from doing X or from making p the case,
> then there are logically sufficient grounds for (C). Where it is assumed that S can do

> these things, the assertion of (1), (2), and (α), together with the denial of (C), would be unintelligible. [51, pp. 443–444]

It should be noted that Wallace pointed out the peculiar nature of logical relation between premises and conclusion in practical syllogism. On the one hand, the conclusion is only partially justified in the prima-facie way, and it can be defeated by additional premises. On the other hand, if there are no relevant reasons against some of the premises or the conclusion, then the conclusion holds in the classical or Tarskian way. The first property can be termed as ‘non-monotonicity,’ while the second does not seem to have received so far a fuller theoretical explication. The kindred property of “premise completeness” will be discussed below (see 1.4.6). Although Wallace’s formalization shows recognition of a subtle nature of logical relations in practical inference, an objection must be raised: the formalization does not make a clear distinction between “internal reasons” (i.e. propositional attitudes, intentional states, mental states of practical reasoner) and “external reasons” (i.e. reasons ascribed by an interpreter). The sentence schemata (2), (C), and (α) should be reformulated in terms of mental states of S in order to display the logical form of practical inference as a nexus of mentality.

A significant part of Georg Henrik von Wright’s philosophical opus was devoted to the problem practical inference. His views on the issue underwent some subtle modifications over the years (compare e.g. [48] and [50]), but the formalizations offered show that practical inference is construed as an internal relation between intentional states.

$$\begin{array}{l} A \text{ intends to bring about } p. \\ A \text{ considers that he cannot bring about } p \text{ unless he does } a. \\ \hline \text{Therefore } A \text{ sets himself to do } a. \end{array}$$

> A schema of this kind is sometimes called a practical inference (or syllogism). I shall use this name for it here, without pretending that it is historically adequate, and consciously ignoring the fact that there are many different schemas which may be grouped under the same heading. [50, p. 96]

If we approach the analysis of the exemplar schema in question form the standpoint of modal logic, the number of modal operators needed becomes high.

Example 1.2.1.

$$\begin{array}{l} A\ \overset{\mathit{modal}\,1.a}{\text{intends to}}\ \overset{\mathit{modal}\,2.a}{\text{bring about}}\ p. \\ A\ \overset{\mathit{modal}\,3}{\text{considers that}}\ \text{he}\ \overset{\mathit{modal}\,4}{\text{cannot}}\ \overset{\mathit{modal}\,2.a}{\text{bring about}}\ p\ \text{unless}\ \overset{\mathit{modal}\,2.b}{\text{he does}}\ a. \\ \hline \text{Therefore } A\ \overset{\mathit{modal}\,2.c}{\text{sets himself to do}}\ a. \end{array}$$

In Von Wright's schema there are at least four expressions that invoke modal logic treatment. First, the modalities of intentionality are required to capture the logical form of the schema:

1. bouletic modality $[\mathrm{I}_A]$ for (1.a) 'A intends to...';
2. praxeological modality $[\mathrm{Do}_A]$ for (2.a) 'A brings it about that...,' (2.b) 'A does so that...,' and 'A sets himself to do ...';
3. doxastic modality $[\mathrm{B}_A]$ for (3) 'A considers that ...'.

Second, an additional modality is required:

4. alethic modality $\Diamond$ (e.g. 'it is possible that ...') or perhaps ability modality (see e.g. [12]) is needed for (4) 'can.'

$$\begin{array}{ll} [\mathrm{I}_A] & p \\ [\mathrm{B}_A] & (\Diamond p \rightarrow [\mathrm{Do}_A]\ a) \\ \hline [\mathrm{Do}_A] & a \end{array}$$

Notice that if all four modal operators are erased, then we get *modus ponendo ponens*.

Practical inference is usually understood as an exemplar form of teleological explanation: agent A's action a, $[\mathrm{Do}_A]a$, is teleologically explained in terms of agent's intention, $[\mathrm{I}_A]p$, whose content is the goal p, and agent's belief, $[\mathrm{B}_A](\Diamond p \rightarrow [\mathrm{Do}_A]\ a)$, that agent's doing a is necessary for the realization of the intended goal p.

Practical inference belongs to the realm of intentionality. But the logic of intentional states is not clear even for single modalities, let alone their combinations. In this respect, one can repeat Anscombe's words: the true character of the logic of intentionality is still obscure.

1.2.3 Imperatives and human sciences

What theory of imperatives has to do with methodology of sciences of man? Is there any connection between intentional states and sentence moods? I will try to provide the evidence for the first and argue for the affirmative answer as to the second question.

There is an important similarity between the types of intentional states and the categories of sentence moods regarding their "direction of fit" (Table 2). This similarity does not appear to be accidental. Dynamic semantics gives us a way of thinking that might reveal the source of connection between intentional states and sentence moods.[1]

[1] The beginnings of dynamic approach in philosophical logic can be traced back to two papers of David Lewis from 1979: [31] and [30]. Now there is a number of semantical theories that can and have been classified under the heading 'dynamic semantics' (e.g. discourse representation theory of Hans Kamp). We will discuss an offspring of the family of dynamic semantical systems that have been developed over the last

Table 2: Directions of fit.

From ... to world	From world to ...
mind-to–world fit: *belief*	world-to-mind fit: *desire*, *intention*
word-to-world fit: *declarative*	world-to-word fit: *imperative*

Let us take a look at an early formulation of the theory that shows the tendency towards the weakening of the semantic/pragmatic distinction:

> ... the meaning of a sentence does not lie in its truth conditions, but rather in the way it changes (the representation of) the information of the interpreter. The utterance of a sentence brings us from a certain state of information to another one. The meaning of a sentence lies in the way it brings about such a transition. [25, p. 43]

By the equation 'meaning = change-potential' pragmatics and semantics are blended whilst speech acts rather than sentences become the objects of logical analysis.

Table 3: A sketch of typical changes for declaratives and imperatives

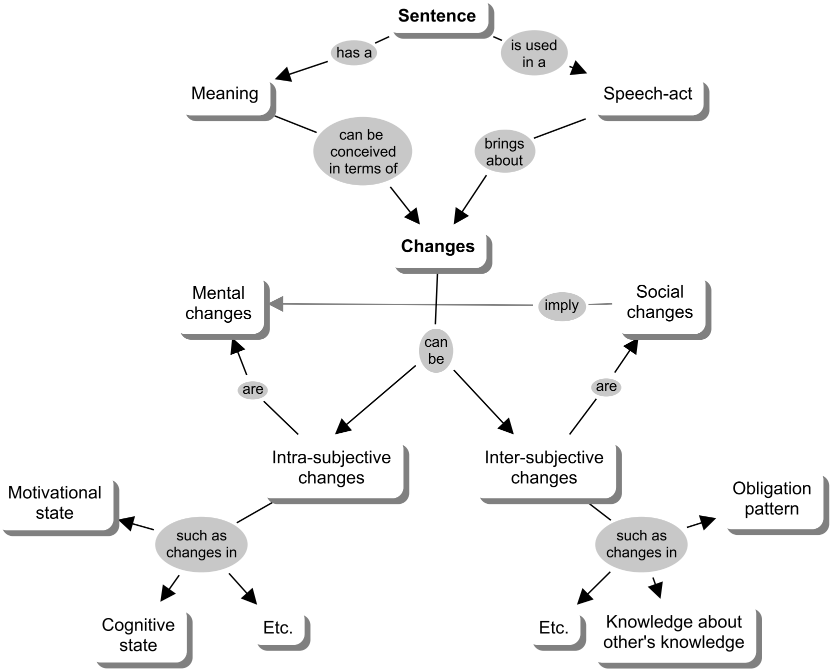
	Mental changes	Social changes
Declaratives	cognitive	group knowledge
Imperatives	cognitive-motivational	obligation pattern

Example 1.2.2. In an ideal speech situation the speaker by uttering 'Shut the door' expresses his/her will and changes the cognitive-motivational state of the hearer so that (i) the hearer comes to believe that the door is not shut, that it is possible to shut it, that the door will not be shut unless she shuts it, and (ii) the hearer starts to want the door to be shut. In this way the hearer becomes motivated to shut the door. The "obligation pattern" between the speaker and the hearer changes too after the imperative has been uttered. The hearer is now obliged to shut the door while it becomes forbidden for the speaker to prevent the door from shutting.

It is commonly objected that this account of speech acts as the prime logical objects is too complex and based on psychology. Neither of the two counterclaims are justified. Imperatives have a multi-layered semantics and it is a theoretician's own

three decades by a number of authors associated with the Institute for Logic, Language, and Computation at University of Amsterdam: Johan van Benthem [10], Frank Veltman [46], Jeroen Groenendijk [24], Martin Stokhof , Jan van Eijck, Paul Dekker, and a large number of other researchers that have worked or studied there, or had been inspired by the approach.

Figure 1 Semantics couched in terms of pragmatics.



choice how precise the theoretical account will be. The objection on psychological foundation is flawed as well. The gap between the normative and the empirical emerges in the logic of communication in the same way as it does in the logic of thought. If in Frege's quote below the original bolded expressions are replaced by the new italicized expressions in brackets, no logic specific content will be lost, rather the scope of logic will be extended. Judge for yourself the acceptability of substitutions!

> It is not the **holding something to be true** [*performance of a speech act*] that concerns us but the laws of **truth** [*cooperation*]. We can also think of these as prescriptions for **making judgements** [*performing speech acts*]; we must comply with them in our **judgements** [*speech acts*] if we are not to **fail of the truth** [*violate the cooperativity*]. So if we call them **laws of thought** [*laws of communication*] or, better, **laws of judgement** [*laws of speech act*], we must not forget we are concerned here with laws which, like the principles of morals or the laws of the state, prescribe how we are to act, and do not, like the laws of nature, define the actual course of events. **Thinking** [*Communication*], as it actually takes place, is not always in agreement with the laws of logic any more than people's actual behavior is always in agreement with the moral law. I therefore think it better to avoid the expression '**laws of thought**' ['*laws of communication'*] altogether in

logic, because it always misleads us into thinking of **laws of thought** [*laws of communication*] as laws of nature. If that is what they were we should have to assign them to **psychology** [*sociology*]. [19, p. 246–247]

1.2.4 Sentences projected on a psychological screen

The basic idea of dynamic semantics is that a sentence φ acts upon interlocutor's mental (i.e. intentional) state σ, and changes it into state σ'; φ is "projected" onto σ'. In the typical case φ is projected onto $\sigma' = \sigma[\varphi]$ in such a way that it is "accepted" there (σ' is its fixed point): $\sigma[\varphi] = (\sigma[\varphi])[\varphi]$. The effects of sentence φ can also be expressed in dynamic-logic style as $[\varphi]\psi$ in the sense 'always after φ has been uttered ψ holds' (see e.g. [54]).

In the formal sense intentional states can be modeled using the standard structures, first order and possible worlds structures as the building blocks. The changes induced by utterances are sometimes called Tarskian and Kripkean variations. After the sentences have been projected, the resulting models can be characterized using an appropriate logic of intentionality (e.g. doxastic) or some other logic (e.g. deontic logic).

It seems that it is undisputable that imperatives as used in commands, orders or requests change both the hearer's intentional state and the obligation pattern between the speaker and the hearer. So, at least three modal logics will be needed to describe the impact of imperatives:

- doxastic logic, i.e. logic of belief;
- bouletic logic, i.e. logic of intentional states having "world-to-mind" direction of fit, e.g. logic of desire;
- deontic logic, i.e. logic of obligations.

The remaining numerous uses of imperatives (e.g. advice, suggestion, permission, wish, threat,...), described by linguists, unsurprisingly show that there are more uses than there are sentence moods. In all of the "non-canonical" uses of imperatives it seems that "pragmatic/semantic field" is exploited only partially.[2]

The logic of intentionality is **the** logic of idiographic human sciences if their language uses "mental vocabulary." Their "methodological autonomy" consists in use of "practical inference" whose nature still remains obscure. The working hypothesis of this paper is that the incomplete cartography of logical geography of intentionality can be improved by employing the imperative logic. The connection between logics will be established if we show that one of them, e.g. logic of desire, can be embedded into another, e.g. into logic of imperatives.[3] For that purpose we have to show how

[2] Portner [35] makes a distinction between deontic, bouletic, and teleological readings of imperatives.

[3] Theorem 1.3.12 establishes the connection between Cross's logic of desire and $L_!$ imperative logic.

to translate the sentences from the language of logic of desire into the language of imperative logic in such a way that sentences translated in the language of target logic somehow reflect, perhaps within a restricted semantic space, the “logic geography” of the source logic. Formally, this can be done if it is proved that the source logic is a “sublogic” (in the sense of García-Matos and Väänänen [20] definition) of the target logic, or a “corridor” (in the sense of Mossakowski, Diaconescu and Tarlecki [34]definition) from the source logic to the target, or for the special case when the source logic has classical negation, if there is a “corridor” with parsimonious projection and a “translational constant” (see Theorem 1.2.1 and Figure 2).

Theorem 1.2.1 (Žarnić [56])**.** *Let logic* $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ *be a logic with strong negation. Then for any logic* $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$ *it holds that if there are: a sentence* $\kappa \in \Phi_2$*, a parsimonious function* $\pi^* : Mod(\{\kappa\}, \Sigma_2) \to \Sigma_1$*, and a function* $\tau : \Phi_1 \to \Phi_2$ *such that*

$$\pi^*(\sigma_2) \models_1 \varphi_1 \textit{ iff } \sigma_2 \models_2 \tau(\varphi_1)$$

for any $\varphi_1 \in \Phi_1$ *and* $\sigma_2 \in Mod(\{\kappa\}, \Sigma_2)$*, then* τ *is a semantic relations preserving translation, i. e.*

$$\Gamma_1 \models_1 \varphi_1 \Leftrightarrow \tau(\Gamma_1) \models_2^* \tau(\varphi_1)$$

where $\models_2^* \subseteq Mod(\{\kappa\}, \Sigma_2) \times \Phi_2$ *and* $\tau(\Gamma_1) = \{\tau(\varphi_1) \mid \varphi_1 \in \Gamma_1\}$.

Remark. The proof of Theorem 1.2.1 is given here in section 1.3.2.

Figure 2 The existence of the translation-projection pair $\langle \tau, \pi^* \rangle$ shows that the logical geography of the logic of desire $L_D = \langle \mathcal{L}_D, \Sigma_D, \models_D \rangle$ can be represented in the imperative logic $L_! = \langle \mathcal{L}_!, \Sigma_!, \models_! \rangle$.

$$\begin{tikzcd}
\mathcal{L}_D \arrow[rr, "\tau"] \arrow[d, dashed, no head] & & \mathcal{L}_! \arrow[d, dashed, no head] \\
\models_D \arrow[d, dashed, no head] & & \models_! \arrow[d, dashed, no head] \\
\Sigma_D & & \Sigma_! \arrow[ll, "\pi^*"']
\end{tikzcd}$$

Imperatives and practical inference To understand a language encompasses comprehension of the meaning relations obtaining between its sentences, getting a grasp of its logical geography. To understand the logic of the language of intentionality means to comprehend the workings of natural language. A conjecture worth considering is that an adequate theory of imperatives requires examination of its logical connections with the logic of intentionality *et vice versa*. Furthermore, Davidson’s thesis that the action belongs to the realm of intentionality in combination with

Belnap's thesis that "the content of every imperative is an agentive" reveals the crucial importance of the interconnection between logic of action and imperative logic. Therefore, if the methodological autonomy of human sciences is based upon "practical inference" whose logic cannot be studied in isolation, then studying imperatives is highly relevant for the philosophical foundation of human sciences.

1.3 Appendix: embedding logic of desire into imperative logic

1.3.1 Connecting logics through a narrow corridor

Semantically characterized logic can be defined as a triple consisting of a language, a set of structures (interpretations), and a satisfaction relation.

Definition 1.3.1. Logic L is a triple $\langle \Phi, \Sigma, \models \rangle$.

Definition 1.3.2. Satisfaction relation $\models$ is a binary relation between structures Σ and formulas Φ:

$$\models \subseteq \Sigma \times \Phi.$$

Definition 1.3.3. The set $Mod(\Gamma, \Sigma)$ of models (the intension) of a set Γ of formulas within a set Σ of structures with respect to satisfaction relation $\models$ is the set of structures satisfying each formula in the set:

$$Mod(\Gamma, \Sigma) = \{\sigma \in \Sigma \mid \forall \varphi (\varphi \in \Gamma \to \sigma \models \varphi)\}.$$

Remark. The precise notation would require explicit mention of the satisfaction relation under consideration. For example, for $L_a = \langle \Phi_a, \Sigma_a, \models_a \rangle$, $\Gamma_a \subseteq \Phi_a$ and $\Sigma \subseteq \Sigma_a$ we should write $Mod(\Gamma_a, \Sigma, \models_a)$. From the context it will be obvious which satisfaction relation is being used, so shorthand notation $Mod(\Gamma_a, \Sigma)$ will be used instead.

Definition 1.3.4. Set Γ is satisfiable in Σ iff $Mod(\Gamma, \Sigma) \neq \emptyset$.

Definition 1.3.5. Consequence relation $\models \subseteq \wp\Phi \times \Phi$ for a logic $\langle \Phi, \Sigma, \models \rangle$ is the relation

$$\Gamma \models \varphi \text{ iff } Mod(\Gamma, \Sigma) \subseteq Mod(\{\varphi\}, \Sigma)$$

Notation. Following the convention the symbol $\models$ will be used as a duplicate symbol denoting both satisfaction and consequence relation.

Remark. The consequence relation defined in this way is a Tarskian consequence relation. Its provable properties include: 1. reflexivity, $\{\varphi\} \models \varphi$; 2. monotony, if $\Gamma \subseteq \Gamma'$ and $\Gamma \models \varphi$, then $\Gamma' \models \varphi$; 3. transitivity, if for all $\psi \in \Delta$, $\Gamma \models \psi$ and $\Delta \models \varphi$, then $\Gamma \models \varphi$.

Sublogic In the literature the sublogic relation is defined in different ways. Let us first examine Definition 1.3.6 given in [20, p. 21]!

Definition 1.3.6 (GMV). An abstract logic $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ is a sublogic of another abstract logic $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$, in symbols

$$L_1 \leq L_2,$$

if there are (i) a sentence $\kappa \in \Phi_2$, and functions (ii) $\pi : \Sigma_2 \longrightarrow \Sigma_1$ and (iii) $\tau : \Phi_1 \longrightarrow \Phi_2$ such that

1. $\forall \sigma_1 (\sigma_1 \in \Sigma_1 \rightarrow \exists \sigma_2 (\sigma_2 \in \Sigma_2 \land \pi(\sigma_2) = \sigma_1 \land \sigma_2 \models_2 \kappa))$ and

2. $\forall \varphi_1 \forall \sigma_2 ((\varphi_1 \in \Phi_1 \land \sigma_2 \in \Sigma_2) \rightarrow (\sigma_2 \models_2 \kappa \rightarrow (\sigma_2 \models_2 \tau(\varphi_1) \leftrightarrow \pi(\sigma_2) \models_1 \varphi_1)))$.

Superlogic L_2 inherits some logical properties of its sublogic L_1, like compactness and decidability [20, p. 21–22]. However, we are interested here in the question whether a superlogic can, metaphorically speaking, contain a map of its sublogic thus representing the logical geography of the latter. More formally, we are interested whether there is a translation τ from source to target logic that will preserve *sequitur*, $\models_1$, and *non sequitur* relations, $\not\models_1 = \wp\Phi_1 \times \Phi_1 - \models_1$ of source logic so that $\Gamma_1 \models_1 \varphi_1 \Leftrightarrow \tau(\Gamma_1) \models_2 \tau(\varphi_1)$ holds for any Γ_1 and φ_1, where $\tau(\Gamma)$ is a shorthand notation for $\{\tau(\varphi) \mid \varphi \in \Gamma\}$. Sublogic relation does not guarantee the existence of image of source logic geography in a target logic, i.e. for some logics L_1 and L_2 such that $L_1 \leq L_2$ it may hold that $\Gamma_1 \models_1 \varphi_1$ and $\tau(\Gamma_1) \not\models_2 \tau(\varphi_1)$.

Example 1.3.1. According to the definition it may well be the case that $\Gamma_1 \models_1 \varphi_1$ and still for some $\sigma_2 \in \Sigma_2 - Mod(\kappa, \Sigma_2)$ it holds that $\sigma_2 \not\models_1 \varphi_1$ although $\sigma_2 \models_2 \tau(\psi_1)$ for all $\psi_1 \in \Gamma_1$.

In GMV definition the use of a “translational constant” $\kappa \in \Phi_2$ for restricting the domain of π can also be understood as addition of the formula κ to any translation. Thus the L_2-map of logical geography of L_1 could be represented by the equivalence (i) $\Gamma_1 \models_1 \varphi_1 \Leftrightarrow \tau^{\kappa}(\Gamma_1) \models_2 \tau^{\kappa}(\varphi_1)$ where superscript κ indicates that κ is added to any translation. The other way to think about the same is the way taken by García-Matos and Väänänen: restricting the consequence relation $\models_2$ to the set $Mod(\{\kappa\}, \Sigma_2)$. Denoting the restricted consequence relation by $\models_2^{\kappa}$, the equivalence that is sought for becomes (ii) $\Gamma_1 \models_1 \varphi_1 \Leftrightarrow \tau(\Gamma_1) \models_2^{\kappa} \tau(\varphi_1)$. With any of these additional conditions, either with addition of translational constant κ to any translation or with restricting consequence relation to those structures that satisfy κ, it can be easily shown that GMV sublogic relation preserves *sequitur* and *non sequitur* relations of source logic. In what follows we will use the reformulation (ii). The GMV definition does guarantee the coordination for $\models_1$ and $\models_2^{\kappa}$ where $\models_2^{*} \subseteq Mod(\{\kappa\}, \Sigma_2) \times \Phi_2$ and, therefore, the existence of a sublogic relation proves the existence of a target logic map of the logical geography of source logic.

Proposition 1.3.1. *Let* $L_1 \leq L_2$. $\Gamma_1 \models_1 \varphi_1 \Rightarrow \tau(\Gamma_1) \models_2^\kappa \tau(\varphi_1)$

Proof Assume $\Gamma_1 \models_1 \varphi_1$. Let σ_2 be any structure such that $\sigma_2 \in Mod(\tau(\Gamma_1) \cup \{\kappa\}, \Sigma_2)$. By Definition 1.3.6, it holds that $\sigma_2 \in Mod(\tau(\Gamma_1) \cup \{\kappa\}, \Sigma_2) \Leftrightarrow \pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$. Therefore, $\pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$. Since $\Gamma_1 \models_1 \varphi_1$, $\pi(\sigma_2) \in Mod(\{\varphi_1\}, \Sigma_1)$. By GMV definition, it holds that $\sigma_2 \in Mod(\tau(\varphi_1), \Sigma_2) \Leftrightarrow \pi(\sigma_2) \in Mod(\varphi_1, \Sigma_1)$. Therefore, $\sigma_2 \in Mod(\tau(\varphi_1), \Sigma_2)$. □

In order to prove the existence of an image of *non sequitur* relation we will need the fact that π is a surjective function, i.e. that the range of π equals Σ_1.

Proposition 1.3.2. *Let* $L_1 \leq L_2$. $\Gamma_1 \not\models_1 \varphi_1 \Rightarrow \tau(\Gamma_1) \not\models_2^\kappa \tau(\varphi_1)$.

Proof For contraposition, assume $\tau(\Gamma_1) \models_2^\kappa \tau(\varphi_1)$. Let σ_1 be any structure such that $\sigma_1 \in Mod(\Gamma_1, \Sigma_1)$. Since π is surjective and defined for $Mod(\{\kappa\}, \Sigma_2)$, there exists σ_2 such that $\sigma_2 \in Mod(\{\kappa\}, \Sigma_2)$ and $\pi(\sigma_2) = \sigma_1$. Let σ_2 be such a structure. By GMV definition, it holds that $\sigma_2 \in Mod(\tau(\Gamma_1), \Sigma_2) \Leftrightarrow \pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$. Given that $\sigma_1 \in Mod(\Gamma_1, \Sigma_1)$ and $\pi(\sigma_2) = \sigma_1$, we get $\sigma_2 \in Mod(\tau(\Gamma_1), \Sigma_2)$. Since $\tau(\Gamma_1) \models_2^\kappa \tau(\varphi_1)$, $\sigma_2 \in Mod(\tau(\varphi_1), \Sigma_2)$. By GMV definition, it holds that $\sigma_2 \in Mod(\tau(\varphi_1), \Sigma_2) \Leftrightarrow \pi(\sigma_2) \in Mod(\varphi_1, \Sigma_1)$. Since $\pi(\sigma_2) = \sigma_1$, $\sigma_1 \in Mod(\varphi_1, \Sigma_1)$ as required. □

Proposition 1.3.3. *Let* $L_1 \leq L_2$.

$$\Gamma_1 \models_1 \varphi_1 \Leftrightarrow \tau(\Gamma_1) \models_2^\kappa \tau(\varphi_1)$$

Proof Propositions 1.3.1 and 1.3.2. □

Corridor Mossakowski, Diaconescu and Tarlecki [34, p. 87] introduce the notion of 'corridor' (Definition 1.3.7) which resembles the notion of sublogic.

Definition 1.3.7 (MDT)**.** Corridor $\langle \tau, \pi \rangle$ is a pair of functions: (i) sentence translation function $\tau : \Phi_1 \to \Phi_2$, (ii) "model reduction function": $\pi : \Sigma_2 \to \Sigma_1$ such that

$$\sigma_2 \models_2 \tau(\varphi_1) \Leftrightarrow \pi(\sigma_2) \models_1 \varphi_1$$

for logics $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ and $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$.

Proposition 1.3.4. *If there is an MDT corridor* $\langle \tau, \pi \rangle$ *between logics, then* τ *is a translation that preserves sequitur relation.*

Remark. Since MDT corridor does not require π to be surjective, non sequitur image may fail to obtain in the target logic.

Table 4 shows which of the two relations from definitions 1.3.6 and in 1.3.7 guarantees provability of the existence of the target logic map of the source logic geography, and for each side of geography (*sequitur* and *non sequitur*) explicitly states the property that enables the proof.

1.3.2 Parsimonious projection

For the purpose of proving the existence of the target logic map of the source logic geography, the surjection condition can be weakened if the source logic has strong (classical) negation.

Definition 1.3.8. A logic $L = \langle \Phi, \Sigma, \models \rangle$ has a strong (classical) negation iff for any $\varphi \in \Phi$ there is a $\psi \in \Phi$ such that

(i) $Mod(\{\varphi\}, \Sigma) \cap Mod(\{\psi\}, \Sigma) = \emptyset$, and
(ii) $Mod(\{\varphi\}, \Sigma) \cup Mod(\{\psi\}, \Sigma) = \Sigma$.

Notation. The notation $\neg\varphi$ will be used for the classical negation of φ.

The weakened condition requires that for any set of models for a satisfiable set of sentences from the source logic there is a projection that picks at least one of them.

Definition 1.3.9. For logics $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ and $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$ a parsimonious projection π^* is a projection $\pi^* : \Sigma_2 \to \Sigma_1$ such that for any $\Gamma_1 \subseteq \Phi_1$ it holds that

$$Mod(\Gamma_1, \Sigma_1) \neq \emptyset \to \exists \sigma_2 [\sigma_2 \in \Sigma_2 \wedge \pi(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)]$$

If there is a corridor $\langle \tau, \pi^* \rangle$ between logics L_1 and L_2, and π^* is a parsimonious projection restricted to the models of translational constant κ, and if source logic L_1 has strong negation, then τ is a semantic relations preserving translation.

Theorem 1.3.5. *Let logic $L_1 = \langle \Phi_1, \Sigma_1, \models_1 \rangle$ be a logic with strong negation. Then for any logic $L_2 = \langle \Phi_2, \Sigma_2, \models_2 \rangle$ it holds that, if there are*

(i) a parsimonious function $\pi^* : Mod(\{\kappa\}, \Sigma_2) \to \Sigma_1$, and
(ii) a function $\tau : \Phi_1 \to \Phi_2$ such that $\pi^*(\sigma_2) \models_1 \varphi_1 \Leftrightarrow \sigma_2 \models_2 \tau(\varphi_1)$ for any $\varphi_1 \in \Phi_1$ and for any $\sigma_2 \in Mod(\{\kappa\}, \Sigma_2)$,

then τ is a semantic relations preserving translation, i.e. $\Gamma_1 \models_1 \varphi_1 \Leftrightarrow \tau(\Gamma_1) \models_2^\kappa \tau(\varphi_1)$

Table 4: A comparison between sublogic relation and corridor existence with respect to possibility of preservation of logical geography.

Provability of ...	*sequitur* relation?	*non sequitur* relation?
with GMV sublogic	Yes (using translation constant κ and the fact that π is total on $Mod(\kappa, \Sigma_1)$)	Yes (since π is surjective)
with MDT corridor	Yes (since π is total)	No

Proof Left to right. Assume $\Gamma_1 \models_1 \varphi_1$. Let σ_2 be any structure such that $\sigma_2 \in Mod(\tau(\Gamma_1), Mod(\{\kappa\}, \Sigma_2))$. By condition (ii) of the theorem, it holds that $\pi^*(\sigma_2) \in Mod(\Gamma_1, \Sigma_1) \Leftrightarrow \sigma_2 \in Mod(\tau(\Gamma_1), Mod(\{\kappa\}, \Sigma_2))$. Therefore, $\pi^*(\sigma_2) \in Mod(\Gamma_1, \Sigma_1)$. Since $\Gamma_1 \models_1 \varphi_1$, then $\pi^*(\sigma_2) \in Mod(\{\varphi_1\}, \Sigma_1)$. By (ii) again,

$$\pi^*(\sigma_2) \in Mod(\{\varphi_1\}, \Sigma_1) \Leftrightarrow \sigma_2 \in Mod(\tau(\varphi_1), Mod(\{\kappa\}, \Sigma_2)).$$

Therefore, $\sigma_2 \in Mod(\tau(\varphi_1), Mod(\{\kappa\}, \Sigma_2))$ as required.

Right to left. Assume $\tau(\Gamma_1) \models_2^\kappa \tau(\varphi_1)$. For *reductio ad absurdum* assume $\Gamma_1 \not\models_1 \varphi_1$. L_1 has strong negation, so there is a sentence $\neg\varphi \in \Phi_1$. From $\Gamma_1 \not\models_1 \varphi_1$ we get $Mod(\Gamma_1 \cup \{\neg\varphi\}, \Sigma_1) \neq \emptyset$. Since π^* is a parsimonious projection, then there is a $\sigma_2 \in Mod(\{\kappa\}, \Sigma_2)$ such that for some $\sigma_1 \in \Sigma_1$ it holds that $\pi^*(\sigma_2) = \sigma_1$ and $\sigma_1 \in Mod(\Gamma_1 \cup \{\neg\varphi\}, \Sigma_1)$. From the second conjunct it follows that $\sigma_1 \in Mod(\{\neg\varphi\}, \Sigma_1)$. Using condition (ii) we get $\sigma_2 \in Mod(\tau(\Gamma_1), Mod(\{\kappa\}, \Sigma_2))$ and $\sigma_2 \in Mod(\tau(\neg\varphi_1), Mod(\{\kappa\}, \Sigma_2))$. By the assumption, i.e. $\tau(\Gamma_1) \models_2 \tau(\varphi_1)$, we also get $\sigma_2 \in Mod(\tau(\varphi_1), Mod(\{\kappa\}, \Sigma_2))$. Using (ii) again we get $\sigma_1 \in Mod(\{\varphi\}, \Sigma_1)$, and for strong negation it is not possible that $Mod(\{\varphi\}, \Sigma_1) \cap Mod(\{\neg\varphi\}, \Sigma_1) \neq \emptyset$. Therefore we have arrived at the contradiction as we wanted to. □

1.3.3 Logic of desire as a sublogic of imperative logic

Cross's logic of desire First, Cross's [16] L_D logic of desire will be introduced in which four modalities are distinguished:

1. $\Delta\varphi$ for 'the agent desires (in the sense of goal belief discrepancy) that φ.' In other words, the agent desires that φ and believes that $\neg\varphi$.
2. $\nabla\varphi$ for 'the agent has a reason to (needs to) make sure that φ.' In other words, the agent desires that φ and does not know whether φ is the case.
3. $\oplus\varphi$ for 'the agent is satisfied that φ.' In other words, the agent desires that φ and believes that φ.
4. $\odot\varphi$ for 'the agent indifferently accepts that φ.' In other words, the agent is undecided about the desirability of φ whilst believes that φ.

Definition 1.3.10. Let P be a formula of the language $\mathcal{L}_P$ of classical propositional logic and let $\bigcirc \in \{\Delta, \nabla, \oplus, \odot\}$. The language $\mathcal{L}_D$ is the set of formulas φ recursively defined in Backus-Naur form as follows

$$\varphi ::= \bigcirc P \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2)$$

Remark. The language $\mathcal{L}_D$ is a language of intentionality (a language describing psychological states of an agent) and therefore no non-modal formulas occur in it. E.g. the formula $\oplus\varphi \wedge \neg\varphi$ which says that the agent is falsely satisfied can be formulated only from an objectivistic perspective in which an infallible observer compares the other's inner state $\oplus\varphi$ and the real state of affairs $\neg\varphi$. Nevertheless, the formulas of $\mathcal{L}_P$ will be considered in the definition of semantics of $\mathcal{L}_D$ for a technical purpose.

Definition 1.3.11. $V(P) \subseteq W$ for each propositional letter $P \in \mathcal{L}_P$. For compounds P and Q:

$$V(\neg P) = W - V(P)$$
$$V(P \wedge Q) = V(P) \cap V(P)$$

Definition 1.3.12. Let $\mathfrak{M}_D = \langle\langle W, R_D, R_B\rangle, V\rangle$, $P \in \mathcal{L}_P$, $\varphi \in \mathcal{L}_D$ and $\psi \in \mathcal{L}_D$.

- $\mathfrak{M}_D, w \models P$ iff $w \in V(P)$.
- $\mathfrak{M}_D, w \models \Delta P$ iff (i) for all v, if $R_D(w,v)$, then $\mathfrak{M}_D, v \models P$, and (ii) for all u, if $R_B(w,u)$, then $\mathfrak{M}_D, u \not\models P$.
- $\mathfrak{M}_{D,w} \models \nabla P$ iff (i) for all v, if $R_D(w,v)$, then $\mathfrak{M}_D, v \models P$, and (ii) there is a u such that $R_B(w,u)$ and $\mathfrak{M}_D, u \models P$, and (iii) there is a z such that $R_B(w,z)$ and $\mathfrak{M}_D, z \not\models P$.
- $\mathfrak{M}_D, w \models \oplus P$ iff (i) for all v, if $R_D(w,v)$, then $\mathfrak{M}_D, v \models P$, and (ii) for all u, if $R_B(w,u)$, then $\mathfrak{M}_D, u \models P$.
- $\mathfrak{M}_D, w \models \odot P$ iff (i) there is a v such that $R_D(w,v)$ and $\mathfrak{M}_D, v \models P$, (ii) there is a u such that: $R_D(w,u)$ and $\mathfrak{M}_D, u \not\models P$, and (iii) for all z, if $R_B(w,z)$, then $\mathfrak{M}_D, z \models P$.
- $\mathfrak{M}_D, w \models \varphi \wedge \psi$ iff $\mathfrak{M}_D, w \models \varphi$ and $\mathfrak{M}_D, w \models \psi$.
- $\mathfrak{M}_D, w \models \neg\varphi$ iff $\mathfrak{M}_D, w \not\models \varphi$.

Imperative logic $L_!$ Imperative logic $L_!$ follows the commanded-action approach. The concept of action used is a "modalized" and simplified version of G.H. von Wright's theory developed in his *Norm and Action* [47]. Von Wright semantics of action exploits three structural elements: initial-state which the agent changes or which would have changed if the agent had not been active, end-state which results from the action, counter-state which would have resulted from agent's passivity. In order to treat Von Wright's semantics in the way of modal logic the following relations will be used: the relation R_{next} for "historical possibility" representing the ways in which the world can be changed either by the course of nature or by the agent's intervention, the relation $R_!$ representing the preference for an imperative future, and the relation $R_.$ representing the information available to the agent on initial-point, i.e. the information on the state of affairs in which the agent's productive or preventive act is to commence. In $L_!$ it is assumed that 'See to it that φ' gives the general form of imperatives (notation $!^{stit}\varphi$) while the distinction between productive and preventive acts is introduced by adding an appropriate conjunct 'It is the case that $\neg\varphi$' and 'It is the case that φ,' respectively. E.g. the imperative schema 'Produce φ' will be expressed in the language $\mathcal{L}_!$ of imperative logic $L_!$ by the formula $!^{stit}\varphi \wedge \cdot\neg\varphi$. The language $\mathcal{L}_!$ also accommodates two types of suggestions: the indicative suggestion — 'It might be the case that φ' (notation $\cdot_{might}\varphi$), and the imperative suggestion — 'It might be good that φ' (notation $!_{might}\varphi$).

Definition 1.3.13. Let P be a formula of the language $\mathcal{L}_P$ of classical propositional logic and let $\bigcirc \in \{!^{stit}, \cdot, \cdot_{might}, !_{might}\}$. The language $\mathcal{L}_D$ is the set of formulas φ recursively defined in Backus-Naur form as follows

$$\varphi ::= \bigcirc P \mid \neg\varphi \mid (\varphi_1 \wedge \varphi_2).$$

Definition 1.3.14. A structure $\mathfrak{M}_! = \langle\langle W, R_!, R_\cdot, R_{next}\rangle, V\rangle$ is a model of $\mathcal{L}_!$ iff $W \neq \emptyset$, $R_! \subseteq R_{next} \subseteq W \times W$, $R_\cdot \subseteq W \times W$, and $V : \mathcal{L}_P \to \wp W$.

Remark. V is defined in the same way as in Definition 1.3.11.

Definition 1.3.15. Refinement of a relation $R_\bigcirc$ with respect to its second members by a proposition $P \in \mathcal{L}_P$ is the relation $R_\bigcirc^{*\bigcirc P}$:

$$R_\bigcirc^{*\bigcirc P} = \{\langle w, v\rangle \in R_\bigcirc \mid mem_2(R_\bigcirc) \in V(P)\}$$

where $\bigcirc \in \{!, \cdot, next\}$.

Definition 1.3.16. Eliminative shifts of a model $\mathfrak{M}_! = \langle\langle W, R_\cdot, R_\cdot, R_{next}\rangle, V\rangle$ with respect to a formula $!P$ and a formula $\cdot P$ are the models:

$$\begin{aligned} \mathfrak{M}_!^{*!P} &= \langle\langle W, R_!^{*!P}, R_\cdot, R_{next}\rangle, V\rangle \\ \mathfrak{M}_!^{*\cdot P} &= \langle\langle W, R_!, R_\cdot^{*\cdot P}, R_{next}\rangle, V\rangle \end{aligned}$$

Definition 1.3.17. Let $\mathfrak{M}_! = \langle\langle W, R_\cdot, R_\cdot, R_{next}\rangle, V\rangle$, $P \in \mathcal{L}_P$, $\varphi \in \mathcal{L}_!$ and $\psi \in \mathcal{L}_!$.

- $\mathfrak{M}_!, w \models !^{stit}P$ iff (i) for all v, if $R_!(w,v)$, then $\mathfrak{M}_!, v \models P$, (ii) there is a u such that $R_{next}(w,u)$ and $\mathfrak{M}_!, u \models P$, and (iii) there is a z such that: $R_{next}(w,z)$ and $\mathfrak{M}_!, z \not\models P$.
- $\mathfrak{M}_!, w \models \cdot P$ iff for all v, if $R_\cdot(w,v)$, then $\mathfrak{M}_!, v \models P$.
- $\mathfrak{M}_!, w \models !_{might}P$ iff $\mathfrak{M}_!^{*!P}, w \models !P$.
- $\mathfrak{M}_!, w \models \cdot_{might}P$ iff $\mathfrak{M}_!^{*\cdot P}, w \models \cdot P$.
- $\mathfrak{M}_!, w \models \neg\varphi$ iff $\mathfrak{M}_!, w \not\models \varphi$.
- $\mathfrak{M}_!, w \models \varphi \wedge \psi$ iff $\mathfrak{M}_!, w \models \varphi$ and $\mathfrak{M}_!, w \models \psi$.

Definition 1.3.18. Let $P \in \mathcal{L}_P$, $\varphi \in \mathcal{L}_D$ and $\psi \in \mathcal{L}_D$. The translation function $\tau : \mathcal{L}_D \to \mathcal{L}_!$ is defined as follows:

- $\tau(\Delta P) = \cdot\neg P \wedge !^{stit}P$

- $\tau(\nabla P) = \cdot_{might}P \wedge \cdot_{might}\neg P \wedge !^{stit}P$
- $\tau(\oplus P) = \cdot P \wedge !^{stit}P$
- $\tau(\odot P) = \cdot P \wedge !_{might}P \wedge !_{might}\neg P$
- $\tau(\neg\varphi) = \neg\tau(\varphi)$
- $\tau(\varphi \wedge \psi) = (\tau(\varphi) \wedge \tau(\psi))$

Remark. It should be noted that $\mathcal{L}_D$ and $\mathcal{L}_!$ share the same subformula propositional base given by $\mathcal{L}_P$.

1.3.4 Coordinating the two logics

In the semantics of modal logics satisfiability is defined in terms of truth at a world w in a model $\mathfrak{M}$. Coordination of satisfiability in two modal logics via a corridor prompts us to look at the point within a relational structure, so, in order to keep the metaphor of a corridor, the term 'evaluation corner' will be introduced.

Definition 1.3.19. Evaluation corner is a pair $\langle\mathfrak{M}, w\rangle$.

The projection function does not have to be total and it will be defined by restricting its domain to those models with $R_! \neq \emptyset$ and $R_{next} - R_! \neq \emptyset$, i.e. models allowing for at least one imperative future. This condition parallels on the semantical side the translational constant requirement of GMV definition. The translation constant that guarantees that $R_! \neq \emptyset$ and $R_{next} - R_! \neq \emptyset$ can be obtained by the infinitary conjunction of formulas $!_{might}A \vee !_{might}\neg A$ for each propositional letter A in the subformula propositional base of $\mathcal{L}_D$. Since infinitary conjunctions are not allowed in the language $\mathcal{L}_D$, we will relay on the option of a projection function π^* within a restricted domain.

Definition 1.3.20. Model projection function π^* for $L_!$ and L_D is a function from a proper subset of evaluation corners of $L_!$ to a set of evaluation corners of L_D such that:

- if $\mathfrak{M}_!, w \models !_{might}A$ or $\mathfrak{M}_!, w \models !_{might}\neg A$ for each propositional letter A, then

$$\pi^*(\langle\underbrace{\langle\langle W, R_!, R_\cdot, R_{next}\rangle, V\rangle}_{\mathfrak{M}_!}, w\rangle) = \langle\underbrace{\langle\langle W, R_D, R_B\rangle, V\rangle}_{\mathfrak{M}_D}, w\rangle$$

  where $R_D = R_!$ and $R_B = R_\cdot$,
- undefined otherwise.

Lemma 1.3.6. *$\mathfrak{M}_!, w \models !_{might}A$ or $\mathfrak{M}_!, w \models !_{might}\neg A$ for each propositional letter $A \in \mathcal{L}_P$ iff $\mathfrak{M}_!, w \models !_{might}P$ or $\mathfrak{M}_!, w \models !_{might}\neg P$ for any formula $P \in \mathcal{L}_P$.*

Proof Induction. □

Now it remains to prove that the pair $\langle\tau, \pi^*\rangle$ is a corridor.

Proposition 1.3.7. *The pair $\langle \tau, \pi^* \rangle$ is a corridor from L_D to $L_!$.*

Proof We use induction. In the inductive basis we will examine only one case, the case of the operator Δ, the other cases are similar. Let $\sigma_!^w = \langle \mathfrak{M}, w \rangle$ be an arbitrary evaluation corner. Assume $\pi(\sigma_!^w) \models_D \Delta P$. According to Definition 1.3.12, $\{v \mid R_D(w,v)\} \subseteq V(P)$ and $\{v \mid R_B(w,v)\} \subseteq V(\neg P)$. According to Definition 1.3.20, $R_D = R_!$ and $R_B = R_.$, and therefore the conditions (i) and (ii) from Definition 1.3.17 (first item) are satisfied for $\tau(\Delta P) = !^{stit}P$. The negative condition (iii) of Definition 1.3.17 (first item) is satisfied too since an evaluation corner $\sigma_!^w$ such $\pi(\sigma_!^w) = \sigma_D^w$ must satisfy $!_{might}P$ or $!_{might}\neg P$ (Lemma 1.3.6), which means that there is a model shift that will satisfy at least one of the disjuncts. Only the disjunct $!_{might}P$ can be satisfied, but that requires existence of $u \in \{v \mid R_{next}(w,v)\}$ such that $u \in V(\neg P)$. In that way the negative condition (iii) will be satisfied. Therefore, any $\sigma_!^w$ such that $\pi(\sigma_!^w) = \sigma_D^w$ satisfies $\tau(\Delta P)$. In the opposite direction, assume $\sigma_!^w \models_! \tau(\Delta P)$. It follows directly that the evaluation corner $\pi^*(\sigma_!^w)$, where R_{next} is dropped from the original model and other relations retained, will satisfy ΔP.

In the inductive step we will examine only the negation case. Assume inductive hypothesis $\pi(\sigma_!^w) \models_D \varphi$ iff $\sigma_!^w \models_! \tau(\varphi)$. Suppose $\pi(\sigma_!^w) \models_D \neg\varphi$. It means that $\pi(\sigma_!^w) \not\models_D \varphi$. By the inductive hypothesis $\sigma_!^w \not\models_! \tau(\varphi)$. Using semantic definition we get that $\sigma_!^w \models_! \neg\tau(\varphi)$. Using definition for translation τ we finally arrive at $\sigma_!^w \models_! \tau(\neg\varphi)$. The other direction is similar. □

Proposition 1.3.8. *If there is a corridor $\langle \tau, \pi \rangle$ between logics L_1 and L_2, then translation τ preserves satisfiability.*

Proof Easy conditional proof. □

Definition 1.3.21. For sentences $\bigcirc P \in \mathcal{L}_!$ (where $P \in \mathcal{L}_P$ and $\bigcirc \in \{!, \cdot, next\}$), $\varphi \in \mathcal{L}_!$, and $\psi \in \mathcal{L}_!$ their impact on a relation $R_\bigcirc$ is defined as follows:

(i) for relation $R_!$:

- $!^{stit}P/R_! = \{\langle w,v \rangle \in R_! \mid v \in V(P)\}$
- $\cdot P/R_! = \cdot_{might}P/R_! = !_{might}P/R_! = R_!$
- $\neg\varphi/R_! = R_! - \varphi/R_!$
- $(\varphi \wedge \psi)/R_! = \varphi/R_! \cap \psi/R_!$

(ii) for relation $R_.$:

- $\cdot P/R_. = \{\langle w,v \rangle \in R_. \mid v \in V(P)\}$
- $!^{stit}P/R_. = \cdot_{might}P/R_. = !_{might}P/R_. = R_.$
- $\neg\varphi/R_. = R_. - \varphi/R_.$

- $(\varphi \wedge \psi)/R_{.} = \varphi/R_{.} \cap \psi/R_{.}$

(iii) for relation R_{next}:

- $\varphi/R_{next} = R_{next}$

Definition 1.3.22. The Henkin-style evaluation corner

$$\sigma_{\Gamma}^{w^{\#}} = \langle\langle\langle W^{\#}, R_{!}^{\#}, R_{.}^{\#}, R_{next}^{\#}\rangle, V^{\#}\rangle, w^{\#}\rangle$$

for a set $\Gamma = \{\varphi_1, \dots, \varphi_{|\Gamma|}\}$ of sentences of $\mathcal{L}_{!}$ is the structure-point pair built in the following way:

- $W^{\#} = \wp(at(\mathcal{L}_P))$, where $at(\mathcal{L}_P)$ is the set of propositional letters in the propositional base of $\mathcal{L}_{!}$,
- $V^{\#}(A) = \{w \mid A \in w\}$ for propositional letters $A \in at(\mathcal{L}_P)$,
- $R_{!}^{\#} = \bigcap_{0 \leq i \leq |\Gamma|} R_{!}^{i}$, where $R_{!}^{0} = W^{\#} \times W^{\#}$ and $R_{!}^{i} = \varphi_i / R_{!}^{0}$ for each $\varphi_i \in \Gamma$,
- $R_{.}^{\#} = \bigcap_{0 \leq i \leq |\Gamma|} R_{.}^{i}$, where $R_{.}^{0} = W^{\#} \times W^{\#}$ and $R_{.}^{i} = \varphi_i / R_{.}^{0}$ for each $\varphi_i \in \Gamma$,
- $R_{next}^{\#} = R_{next}^{0} = W^{\#} \times W^{\#}$,
- $w^{\#} \in mem_1(R_{!}^{\#})$,

(where $|\Gamma|$ denotes the cardinality of Γ).

Proposition 1.3.9. *$\varphi \in \mathcal{L}_{!}$ is satisfiable in $L_{!}$ if and only if $\sigma_{\{\varphi\}}^{w^{\#}} \models_{!} \varphi$.*

Proof Proof by induction. In induction basis only the case of $!^{stit}P$ will be examined. Construction of $R_{!}^{0}$ guarantees that $mem_2(R_{!}^{0}) \subseteq V^{\#}(P)$. Since $R_{next}^{0} = R_{next}^{\#}$ it will contain counter-point for any P. In the opposite direction, trivially it holds that if a sentence is satisfied by the # interpretation, it is satisfied by some interpretation and, therefore, it is satisfiable.

In the inductive step only the negation case will be examined. The inductive hypothesis is: φ is satisfiable iff $\sigma_{\{\varphi\}}^{w^{\#}} \models \varphi$. In left-to-right direction let us indirectly prove the contrapositive. Suppose that $\neg\varphi$ is not satisfiable. Then $\neg\varphi$ is not true in its Henkin-style model $\sigma_{\{\neg\varphi\}}^{w^{\#}}$, i.e. $\sigma_{\{\neg\varphi\}}^{w^{\#}} \not\models \neg\varphi$. Then by Definition 1.3.17, $\sigma_{\{\neg\varphi\}}^{w^{\#}} \models \varphi$. Therefore, φ is satisfiable. From inductive hypothesis it follows that $\sigma_{\{\varphi\}}^{w^{\#}} \models \varphi$, but, as can be proved, it cannot be the case that φ is true at the same point $w^{\#}$ in two Henkin-style evaluation corners, one of which is built for φ ($\sigma_{\{\varphi\}}^{w^{\#}}$) and the other for $\neg\varphi$ ($\sigma_{\{\neg\varphi\}}^{w^{\#}}$). The opposite direction is obvious. □

Lemma 1.3.10. *For any satisfiable $\varphi \in \mathcal{L}_{!}$, the Henkin-style evaluation corner $\sigma_{\{\varphi\}}^{w^{\#}}$ for φ is in the domain of π^{*}.*

Proof Induction. In the basis case only $!^{stit}P$ will be examined. An arbitrary propositional letter A is ether logically dependent or independent on P. If dependent, the fulfilment of positive condition in $R^{\#}_{!}$ is secured by the fact that it is fulfilled for P, while the fulfillment of the positive and the negative condition for $R^{\#}_{next}$ follows from the fact that $R^{\#}_{next} = W^{\#} \times W^{\#}$. Therefore either $\sigma^{w^{\#}}_{\{!P\}} \models !_{might}A$ or $\sigma^{w^{\#}}_{\{!P\}} \models !_{might}\neg A$. If A is independent, it will not be affected by P and then there will be an A-world and an $\neg A$-world in $mem_2(R^{\#}_{!})$ as well as in $mem_2(R^{\#}_{next})$ due to maximality of a Henkin-style evaluation corner.

In the inductive step the case of negation will be examined. By inductive hypothesis the Henkin-style model for φ is in the domain of π^*, i.e. $\sigma^{w^{\#}}_{\{\varphi\}} \in domain(\pi^*)$. For reductio assume that the Henkin-style model for $\neg\varphi$ is not in the domain of π^*, i.e. $\sigma^{w^{\#}}_{\{\neg\varphi\}} \notin domain(\pi^*)$. If so, then for some letter A it holds that $\sigma^{w^{\#}}_{\{\neg\varphi\}} \not\models !_{might}A$ and $\sigma^{w^{\#}}_{\{\neg\varphi\}} \not\models !_{might}\neg A$. Then it must be the case that either (i) $\neg\varphi/R^{\#}_{next} \subseteq V^{\#}(A)$ or (ii) $\neg\varphi/R^{\#}_{next} \subseteq V^{\#}(\neg A)$. Thus by Definition 1.3.21, $\varphi/R^{\#}_{next} \subseteq V^{\#}(\neg A)$ or $\varphi/R^{\#}_{next} \subseteq V^{\#}(A)$. Neither of these can obtain according to the inductive hypothesis. □

Proposition 1.3.11. *π^* is a parsimonious projection.*

Proof Henkin-style evaluation corners are in the domain of π^*. For any satisfiable $\varphi \in \mathcal{L}_D$, $\tau(\varphi)$ is satisfiable according to 1.3.8. If so, then for some $\psi \in \mathcal{L}_!$ there is a Henkin-style corner $\sigma^{w^{\#}}_{\{\psi\}}$ such that $\sigma^{w^{\#}}_{\{\psi\}} \models \tau(\varphi)$. By proposition 1.3.7, it follows that $\pi^*(\sigma^{w^{\#}}_{\{\psi\}}) \models \varphi$ and therefore $\pi^*(\sigma^{w^{\#}}_{\{\psi\}}) \in Mod_{L_D}(\{\varphi\})$. □

Theorem 1.3.12. *There is a semantic relations preserving translation of sentences of the Cross's $L_D = \langle \mathcal{L}_D, \Sigma_D, \models_D \rangle$ logic of desire to sentences of $L_! = \langle \mathcal{L}_!, \Sigma_!, \models_! \rangle$ imperative logic.*

Proof In order to show that τ translation of $\mathcal{L}_D$ to $\mathcal{L}_!$ is conservative, i.e.

$$\Gamma \models_D \varphi \Leftrightarrow \tau(\Gamma) \models_! \tau(\varphi)$$

first we have to prove that:

(i) τ translation has consequence preserving (sequitur preserving) property, and
(ii) τ translation has non-consequence preserving (non sequitur preserving) property.

The first follows the fact that there exist a corridor from L_D to $L_!$ (Proposition 1.3.7) and Theorem 1.3.5. The second follows from Theorem 1.3.5 using the facts that the corridor's model projection function is parsimonious (Proposition 1.3.11) and that L_D has strong negation. □

1.4 Philosophy of logic: logical pluralism and the foundations of logical relations

There are implicit (empirical, pre-theoretical, pre-systematic, intuitive) notions on relations of logical consequence as is exhibited in the correct use of the adverb 'therefore' by a competent natural language speaker. The pre-theoretical notions might spring from different sources like: (h.1) understanding of logical terms (see the quote below), (h.2) recognition of the properties of a consequence relation, (h.3) combination of the two: understanding of logical terminology on the background of recognition of a type of consequence relation or on the background of recognition of a logical property (e.g. consistency). Hypothesis (h.1) describes the historically influential explication of the implicit procedural logical knowledge. This type of semantic explication Shapiro [43] describes as the one that combines the notion of logical form based on the recognition of logical terms with the notion of truth-preservation, thus founding the latter on the former.

> Let us say that a sentence Φ (in natural language) is a consequence of a set Γ of sentences in a *blended* sense if it is not possible for every member of Γ to be true and Φ false, and this impossibility holds in virtue of the meaning of the logical terms. [43, p. 663]

The consequence relation explication given by hypothesis (h.1) can be viewed as a special case of general hypothesis that the semantic relations among sentences are dependent on meaning of words occurring in them. It is the hypothesis on complex character of pre-theoretical notions, described in case (h.3) above, that will be examined here. Given the fact that the imperative logic and the logic of intentionality remain an obscure part of our pre-theoretical procedural (*logica utens*) as well as of our theoretical propositional knowledge (*logica docens*), the hypothesis (h.3) seems worth considering. Another reason for entertaining the hypothesis on complex character of pre-theoretical logical knowledge is that it can explain the non-uniform behavior of the same logical terms in different contexts, e.g. within diverse sentence moods. The fact that the connective 'or' behaves differently in declaratives and imperatives (Ross's paradox [38]) is the puzzle that has become the trademark of imperative logic.

1.4.1 Classical consequence relation

A language user implicit logical notions possess their "boundary conditions". Let us use the term 'constituive/regulative theory' for the approach to the phenomenology of the logic notions of an empirical subject in which the hypothesis (h.1) is used in the following way: the recognition of logical properties is constituted by understanding of logical terminology and regulated by general relational logical laws. It is the assumption that semantic relations have immutable properties that makes this theoretical position monistic.

An example of general relational logical laws is given in Tarski's (1928) [45] axiomatization of general properties of consequence relation.

Axiom 1. $|S| \leq \aleph_0$.
Axiom 2. If $X \subseteq S$, then $X \subseteq Cn(X) \subseteq S$.
Axiom 3. If $X \subseteq S$, then $Cn(Cn(X)) = Cn(X)$.
Axiom 4. If $X \subseteq S$, then $Cn(X) = \bigcup_{Y \subseteq X \text{ and } |Y| < \aleph_0} Cn(Y)$.
Axiom 5. There exists a sentence $x \in S$ such that $Cn(\{x\}) = S$.
[45, p. 31]

Tarski's general axioms of consequence relation, construed as the relation between sets of sentences $Cn \subseteq \wp S \times \wp S$, could be expressed in the natural language as follows: For countable languages S (Axiom 1) it holds that:

(i) consequences of sentences remain within the same language and premises are their own consequences (reflexivity; Axiom 2),
(ii) consequences of consequences of a set are already consequences of that set (transitivity; Axiom 3),
(iii) consequences of a set X do not exceed the consequences of their finite subsets Y, which are retained in their superset X consequences (compactness and monotonicity, Axiom 4),
(iv) there is at least one sentence in the language such that its consequences include all the sentences of that language (existence of *falsum*, "absurdity," "explosive sentence," "informational breakdown," etc.; Axiom 5).[4]

The pluralistic "constituive/constituive" approach to the phenomenology of empirical logical notions is governed by the hypothesis (h.3 above) that logical notions of an empirical subject result from combining the intuitions on logic operations with the intuitions on logical relations, and that these intuitions are interdependent. From the pluralistic perspective, intuitions on logical operators and intuitions on logical relations grasp the different facets of phenomena belonging to the logic *genus*. The contextual differences in the use of homonymic logical term, such as the use of the connective 'or' in declaratives and in imperatives, uncover a distinctive character that a combination of logical operators and logical relations can have. In this perspective the meaning of a logical operator is viewed as being mediated by specific relational laws. Monistic approach is not of necessity ruled out by pluralistic hypothesis, but it can rather be regarded as its restriction to particular contexts (e.g. language use in a "deductive discipline").

[4] The monotonicity condition expressed by "right to left" reading of the equation is slightly different from the one widely used in the literature for Tarski condition is restricted to finite sets: if $Y \subseteq X$ and $|Y| < \aleph_0$, then $Cn(Y) \subseteq Cn(X)$.

1.4.2 Two cases for logic pluralism

A case of explosive connective Cook's elaboration [15] of Prior's thought experiment [36] of introducing an arbitrary connective into the language, gives support to the thesis that the object of logical theoretical analysis is given by a combination of a notion of meaning of logical terminology with a notion of nature of logical relations. The prescriptions for the use of "Prior's connective" (`tonk`) can deceivably suggest that its existence would introduce the destroying explosive element into a language and that, therefore, there are some indispensable logical laws. It is a well known

Table 5: Explosiveness of '`tonk`' in the presence of a transitive consequence relation.

(1)	$\Gamma \vdash p \Rightarrow \Gamma \vdash p \ \mathtt{tonk}\ q$	`tonk`Intro
(2)	$\Gamma \vdash p \ \mathtt{tonk}\ q \Rightarrow \Gamma \vdash q$	`tonk`Elim
(3)	$\Gamma \vdash p \Rightarrow \Gamma \vdash q$	$\vdash$transitivity:1,2

fact that explosiveness of the connective `tonk` disappears in absence of transitive relation. Cook [15] has even shown that the rules for `tonk` are sound under the suitable conditions, such as these:

- valuation $v : \mathcal{L} \to \wp\{\mathrm{t}, \mathrm{f}\}$,
- consequence: $\Gamma \models q$ iff (i) $\mathrm{t} \in v(q)$ whenever $\mathrm{t} \in v(p)$ for all $p \in \Gamma$, **or** (ii) $\mathrm{f} \notin v(q)$ whenever $\mathrm{f} \notin v(p)$ for all $p \in \Gamma$,
- definition for `tonk` (where T stands for {t}, B for {t, f}, N for $\emptyset$, F for {f}):

`tonk`	T	B	N	F
T	T	B	T	B
B	T	B	T	B
N	N	F	N	F
F	N	F	N	F

The consequence relation holds for $\{p\} \models p \ \mathtt{tonk}\ q$ in virtue of truth membership preservation, and it holds for $\{p \ \mathtt{tonk}\ q\} \models q$ in virtue of falsity non-membership preservation.

> Logicians who abandon transitivity, however, will need to find some other criteria by which to reject *Tonk-Logic* as illegitimate, at least if they wish to vindicate the intuition that the 'badness' of tonk traces to some violation of general requirements on legitimate logical operators, and is not specific to particular logical systems. [15, p. 223]

Cook's result gives support to the claim that operators cannot be dealt with in isolation from the background notion of a consequence relation. More generally, the result supports the pluralistic hypothesis that pre-theoretical notions (on logical relations and properties) are complexes of interdependent intuitions dealing both with logical relations and logical terms.

A paradoxical imperative inference Let us analyze the chain of reasoning given in Example 1.4.1!

Example 1.4.1. From an imperative obligation to an universal permission.

(1) Slip the letter into the letter-box!
(2) Slip the letter into the letter-box or burn it!
(3) You must: slip the letter into the letter-box or burn it.
(4) You may: slip the letter into the letter-box or burn it.
(5) You may burn the letter.
(6) Therefore, if you ought to slip the letter into the letter box, you may burn it.

The intuitions on the acceptability of some parts in this chain of reasoning are not sharp as the analysis given in Table 6 shows. Unexpected behavior of 'or' in (2) ($\vee$Intro seems too strong) and in (4) (granting much more then $\vee$Elim would permit). Contrary to first order rules, the introduction of disjunction (2) does not seem to be adequately grounded in its apparent disjunct (1). The distribution of permission over disjunction in (4), although almost beyond doubt, radically departs from behavior of the connective 'or' in first order logic. One can easily imagine a language community in which each transition, except the last one is considered to be valid.

Table 6: Clear and unclear logical intuitions in Example 1.4.1.

TRANSITION	LOGICAL ELEMENTS	INTUITIVE ACCEPTABILITY
(1);(2)	connective 'or'	ambivalent
(2);(3)	imperative mood; deontic operator 'must'	mainly affirmative
(3);(4)	deontic operators: 'must', 'may'	affirmative
(4);(5)	deontic operator: 'may'; connective: 'or'	mainly affirmative
(1)–(5);(6)	connective: 'if ... then ...'	negative

A similar situation can be found in deontic logic. Table 7 gives a chain of sequents that correspond to deontic reading of sentences from Example 1.4.1 and their transitions.[5] The token of consequence relation (1), which is intuitively less plausible than (3), holds in normal deontic logics while (3) does not hold. At any rate, one can easily imagine a logically competent subject who endorses all the sequents from (1) to (3) and simultaneously refuses to accept their transitive closure in (4).

[5]Modal operator O stands for 'it is obligatory that . . . ' and P stands for 'it is permitted that . . . '.

[6]Scott's principle $\{(p_1 \wedge ... \wedge p_{n-1}) \rightarrow q\} \vdash (\Box p_1 \wedge ... \wedge \Box p_{n-1}) \rightarrow \Box q$ ($n \geq 1$, a theorem of classical propositional logic on the left side) characterizes normal propositional modal logic (e.g. it may replace K axiom and necessitation rule). It may be read as stating that "meaning relations" of propositional logic, i.e. meaning relations holding in virtue of meaning of truth-functional connectives, are preserved in the modal context.

Table 7: A chain of sequents in deontic logic resembling Example 1.4.1.

	SEQUENT	JUSTIFICATION
	$p \models p \vee q$	meaning of $\vee$
(1)	$\mathrm{O}p \models \mathrm{O}(p \vee q)$	Scott's principle[6]
(2)	$\mathrm{O}(p \vee q) \models \mathrm{P}(p \vee q)$	D axiom
(3)	$\mathrm{P}(p \vee q) \models \mathrm{P}q$	by "free choice permission"
(4)	$\mathrm{O}p \models \mathrm{P}q$	by $\models$ transitivity; from 1–3

1.4.3 A pluralistic conjecture

The tonk-example shows that syntactically defined logical terms have different properties given the diverse types of consequence. (Alf) Ross's paradox and free choice permission show that logical terms may change their behavior in the presence of other logical elements, sentence moods included. The odd result that if anything is obligatory, then everything is permitted (i.e. $\mathrm{O}p \Rightarrow \mathrm{P}q$) shows that one may have intuitions that confirm isolated consequence steps and still lack the intuition that confirms transitive closure of these steps. The pre-theoretical understanding of logical relations may well be holistic in character: perhaps there is no unique understanding of logical terms that is *constitutive* for the understanding of consequence relations, and perhaps there is no unique understanding of admissible consequence relations that is *regulative* for the understanding of logical terms.

Practical logic is abundant with unclear intuitions. Both on the formal and on the informal side the results and intuitions collide on the issues of existence of consequence relation for particular schemata and on the nature of consequence relation. It seems that the way out of this difficulty requires reconsideration of foundational issues in logic such as the relationship between the nature of consequence relation and the meaning of logical terms in the context created by the use of sentences in declarative and imperative moods. Let us consider one of the possible ways to conceive this relationship but within the context of the single declarative mood! Restall [37] shows by a way of example how pluralism can result from invariant "inference schemes" and changeable structural rules. The sequent example in the citation below consists of two steps, first of which employs relevantistically unacceptable "structural rule" (right weakening) while the second is justified by a logical rule (left negation rule).[7]

> In relevant logic, we do not allow weakening: that is, we do not allow the inference from $X \vdash Y$ to $X, A \vdash Y$ or to $X \vdash Y, A$, on grounds of relevance. Consider the following proof of

[7] The sequent $X \vdash Y$ may be "symmetrically" read as "asserting all of X and denying all of Y is a mistake iff $X \vdash Y$ is valid."

the relevantly invalid explosion

$$\dfrac{\dfrac{A \vdash A}{A \vdash A, B}}{A, \neg A \vdash B}$$

The relevant logician complains about the first step, not the second. A relevantist is happy to infer $A, \neg A \vdash B$ from $A \vdash A, B$, but is not happy to infer $A, \neg A \vdash B$ from $A \vdash A$.[8] The different logics considered here differ in structural rules, not in our theory of negation. So plurality is allowed in applications of inference schemes, but the schemes determining the meaning of connectives are unitary. [37, p. 442]

The weak pluralism discussed by Restall differs from strong pluralism that is endorsed in this paper (see Table 8).

Table 8: The typology of theoretical positions with respect to the relationship between structural and logical rules.

	Invariant rules for logical terminology	Variant rules for logical terminology
Invariant structural rules	Strong monism	Weak monism
Variant structural rules	Weak pluralism	Strong pluralism

1.4.4 Varieties of consequence relation

The unclear character of some meaning relations between texts and sentences in natural language presumably shows that there is an irreducible multitude of these relations. Within the context of practical discourse, the expression 'prima facie' has been used for a long time now (at least from the 15th century onwards) for a special kind of consequence or justification relation. Prima facie relation belongs to a broader class of non-monotonic consequence relations,[9] but differs from other members of the class — defeasibility of conclusion is not a matter of special character of premises (e.g. default rules). In practical logic there is a need for a more general notion of consequence relation that is both sensitive to different kinds of meaning relations, and not restricted to a particular type of semantic value (e.g. truth value) or particular relational properties (e.g. monotonicity).

In dynamic semantics the notion of consequence can be generalized in such a manner that classical consequence reveals itself to be nothing more than a special

[8] The original contains a typo in the sentence: "A relevantist is happy to infer $A, \neg A \vdash B$ from $A \vdash A, B$, but is not happy to infer $A, \neg A \vdash B$ from B. " (instead of B here $A \vdash A$ should stand). The typo is corrected here.

[9] $\Gamma \models p \nRightarrow \Gamma, \Delta \models p$

case of "meaning inclusion." It amounts to this: the use of 'therefore φ' is justified in context σ iff φ produces no change in σ, i.e. iff $\sigma[\varphi] = \sigma$. Unlike static semantics in which semantic notions, like consequence or consistency, are defined by the sentence/interpretation relation, in dynamic semantics the sentence/set-of-interpretations relation is used. In the exemplar case of so called 'update,' the semantics of an updating sentence is conceptualized as an operation on a given set of interpretations, i.e. on a context. The operation eliminates the falsifying interpretations and leaves only the verifying ones behind, creating thus a new context. In some dynamic semantical system two extreme positions can be distinguished: the empty context 0 in which all the interpretations are present, and the absurd context 1 having no interpretations left. Dynamic semantics incorporates the static one, but is not reducible to it. The advantage of dynamic perspective is that semantics of more complex speech acts and more refined text/sentence relations becomes theoretically accessible. Special types of consequence relation between text and sentence can be defined depending on whether the text order is irrelevant (test-to-test consequence) or not (update-to-test consequence), on whether the relation holds in general or is "localized" (ignorant-update-to-test consequence), and on whether some other condition is met or not.[10] The three of the above mentioned types of consequence deserve our particular attention: test-to-test consequence because it is just the classical consequence; update-to-test consequence because it does not abstract away from the order of sentences in a text; ignorant-update-to-test consequence because, as it will be argued, this type of consequence provides the explication of consequence relation in imperative logic.[11]

Definitions 1.4.1. Varieties of dynamic consequence:

Test-to-test consequence $p_0; \ldots ; p_n \models_{tt} q$ iff for all contexts σ, $\sigma[p_1] = \ldots = \sigma[p_n] = \sigma \Rightarrow \sigma[q] = \sigma$

Update-to-test consequence $p_0; \ldots ; p_n \models_{ut} q$ iff for all contexts σ, $\sigma[p_1]...[p_n] = \sigma[p_1] \ldots [p_n][q]$

Ignorant-update-to-test consequence $p_0; \ldots ; p_n \models_{0-ut} q$ iff for the empty context 0 (the context carrying no information), $0[p_1] \ldots [p_n] = 0[p_1] \ldots [p_n][q]$

1.4.5 Prima facie consequence

The older notion of meaning inclusion anticipates not only the general notion of dynamic consequence, but the notion of ignorant-update-to-test consequence as well, although in an implicit way. Let us briefly analyze two quotes. The first one comes

[10] Van Benthem has given a theoretically advanced exposition and analysis of dynamic consequence relation in his [9] book.

[11] Veltman has introduced "0-update-to-test" consequence in [46].

from Carnap and Bar-Hillel's seminal paper [13] where an explication for the notion of meaning inclusion was introduced.

> Whenever i **L**-implies j, i asserts all that is asserted by j, and possibly more. In other words, the information carried by i includes the information carried by j as a (perhaps improper) part. Using '*In*(...)' as an abbreviation for the pre-systematic concept 'the information carried by...', we can now state the requirement in the following way:
> R3-1. $In(i)$ includes $In(j)$ iff i **L**-implies j.
> By this requirement we have committed ourselves to treat information as a set or class of something. This stands in good agreement with common ways of expression,as for example, "The information supplied by this statement is more inclusive than (or is identical with, or overlaps) that supplied by the other statement." [13, p. 7]

The second quote is from a more recent work where Sagüllo terms the notion of meaning inclusion as 'information containment conception.'

> The information containment conception: P implies c if and only if the information of c is contained in the information of P. In this sense, if P implies c, then it would be redundant to assert c in a context where the propositions in P have already been asserted; i.e., no information would be added by asserting c. [39, p. 218]

The two ideas stand out in the quotes above: 'adding information' and 'information as a set or class of something.' The first one shows that sentences can do something: they can add information. The second idea indicates that semantic relations occur at the level of sets, since "information [is] a set or class of something." Putting the two ideas together, we get the thesis that sentences act on sets (of interpretations). Although it appears that there is only a single notion of information containment, that is not the case, as will be argued here. The relevant notions are:

1. Conclusion adds no information to **any** context that includes all the information contained in premises.
2. Conclusion adds no information to the context that includes **only** the information contained in premises.

The second notion corresponds to "ignorant-update-to test" consequence and a variant of it will be introduced later as *prima facie* consequence.

Adding, removing, and checking information The repertoire of speech acts is very rich. Some acts add information to the context, and these can be called 'updates.' Some remove information from the context like the acts of withdrawing or unsaying, and these can be called 'downdates.' There is also the third type of speech acts by which no information is neither added or taken away from the context. These acts can be termed 'tests.' But there are many properties that can be tested, like consistency or validity. Consistency testing is an examination whether an information can be added to a context without causing informational breakdown (i.e. without erasing all the information and thus resulting in the empty set with no interpretation left).[12] Consistency testing can be identified with acceptability testing:

$$\sigma\left[?^{consistency}\varphi\right] = \begin{cases} \sigma \text{ if } \sigma[\varphi] \neq \emptyset, \\ \emptyset \text{ otherwise.} \end{cases}$$

Local validity testing examines whether a context will be changed by adding information. Local validity testing can be identified with acceptance testing:

$$\sigma\left[?^{validity}\varphi\right] = \begin{cases} \sigma \text{ if } \sigma[\varphi] = \sigma, \\ \emptyset \text{ otherwise.} \end{cases}$$

If one thinks about semantics as something to do with the actions performed on "sets of something", then one is not obliged to treat natural language expressions 'therefore' and 'might' as metalinguistic predicates.

Example 1.4.2. Denote by $\mathcal{L}_o$ the language in which some logical constants occur. Then we need a meta language $\mathcal{L}_m$ to state that a sentence $p \in \mathcal{L}_o$ is a consequence of a set of sentences $\Gamma \subseteq \mathcal{L}_o$ since 'Γ Therefore, p' does not belong to the language $\mathcal{L}_0$. It can seem odd that by saying 'p Therefore, q' either (i) the speaker mentions sentences p and q but does not use them or (ii) the speaker simultaneously uses and mentions p and q since she is asserting p and q (by using them) as well as (mentioning them while) asserting the existence of consequence relation between 'p' and 'q'.

Table 9: Possible syntactic characterizations of expressions representing logical relations.

Metalogical predicates	*Logical operators*
Therefore$(\Gamma, p) \in \mathcal{L}_m$, i.e. $\Gamma \models p$	therefore $p \in \mathcal{L}_0$
Might$(\Gamma, p) \in \mathcal{L}_m$, i.e. $\Gamma \cup \{p\} \not\models \bot$	might $p \in \mathcal{L}_0$

[12] Relative to a context σ an update sentence φ may has one among four semantic values. A sentence is either (α) accepted and acceptable, or (β) not accepted and acceptable, or (γ) not accepted and not acceptable, or (δ) accepted and not acceptable in a context σ of available interpretations:

$$V(\varphi, \sigma) = \begin{cases} \alpha \text{ if } \sigma[\varphi] = \sigma \wedge \sigma[\varphi] \neq 1 \\ \beta \text{ if } \sigma[\varphi] \neq \sigma \wedge \sigma[\varphi] \neq 1 \\ \gamma \text{ if } \sigma[\varphi] \neq \sigma \wedge \sigma[\varphi] = 1 \\ \delta \text{ if } \sigma[\varphi] = \sigma \wedge \sigma[\varphi] = 1 \end{cases}$$

The value α can be read as of 'true in all available interpretations,' β as 'indeterminate' or 'true in some and false in other available interpretations,' γ as 'false in all available interpretations,' while δ as 'absurdity.'

There are two possible approaches to the determination of the syntactic type of expressions ‘might’ and ‘therefore’: either to classify them as logical operators or as metalogical predicates (Table 9). The logical-operator option is taken when we interpret some natural language sentences as “test functions” both for stating relative consistency and for stating local validity (context validity):

$$\text{sentence\_function(context)} = \begin{cases} \text{context} & \text{if the condition is met,} \\ \text{failure} & \text{otherwise.} \end{cases}$$

The advantages of dragging of the adverb ‘therefore’ back into the object language are that it can be treated unambiguously (instead of signifying different relations in different logics) and that there is a gain in sensitivity to local phenomena of “information containment.” The drawback is that the correct use of therefore-operator need not imply existence of consequence relation. The advantage of might-operator is that consistency assertion becomes a part of the object language. The position taken in the literature on dynamic semantics is compromised: ‘might’ is treated as a logical operator, while ‘therefore’ remains a metalogical predicate.

1.4.6 Geach’s problem

The literature on non-monotonic consequence relation often overlooks that as early as 1966, P. T. Geach had described a similar variety of consequence relation while discussing an informal pattern of practical reasoning:

> Some years ago I read a letter in a political weekly to some such effect as this. ‘I do not dispute Col. Bogey’s premises, nor the logic of his inference. But even if a conclusion is validly drawn from acceptable premises, we are not obliged to accept it if those premises are incomplete; and unfortunately there is a vital premise missing from the Colonel’s argument [. . .]’ I do not know what Col. Bogey’s original argument had been; whether this criticism of it could be apt depends on whether it was a piece of indicative or of practical reasoning. Indicative reasoning from a set of premises, if valid, could of course not be invalidated because there is a premise “missing” from the set. But a piece of practical reasoning from a set of premises can be invalidated thus: your opponent produces a fiat you have to accept, and the addition of this to the fiats you have already accepted yields a combination with which your conclusion is inconsistent. [21, p. 286]

The consequence relation described by Geach has two notable properties:

- (“locality”) conclusion holds in virtue of premises but it can be defeated by additional premises;
- (existence of the limit) if the premises are complete the conclusion cannot be defeated,

where ‘conclusion is defeated’ means ‘premises are acceptable but conclusion is not.’ By ’Geach’s problem’ I mean a problem of devising modeltheoretic notion of consequence relation that captures the pretheoretical notions of conclusion defeasibility and of “completeness of premises.”

The so-called “Tarskian consequence relation” neither can provide a model theoretical counterpart for the relation described by Geach nor for the relation referred to by the expression ‘*prima facie’* (or by closely related expression ‘*pro tanto’* in the literature on rationality and metaethics, e.g. [27]). Therefore, searching for a model-theoretic definition adequate for this variety of a consequence relation requires relying on a weaker type of semantic relation. Dynamic semantics provides a way of thinking about semantic relations in the natural language that operates at the level of generality that enables recognition of diverse types of semantic relations. Instead of using dynamic “update and upgrade” terminology, a “static” exposition of a weaker semantic relation that serves as an explication for the ‘prima facie consequence relation’ will be given. The sketch of the solution for the “Geach’s problem” as applied to imperative logic will be given in terms of a static semantical system. One of the important insights of dynamic semantics is that some semantic phenomena can be distinguished only if take into account the relations between sets of interpretations. This insight will be incorporated by introducing of an partial order between modal structures (Definition 1.4.4). The “locality” of consequence relation will explicated through notions of the “minimal structure” (Definition 1.4.7) and “prima facie consequence relation” (Definition 1.4.10). The notion of “premise completeness” will be formalized in Definition 1.4.11.

1.4.7 A simple static system

We will try to delineate the contours of Geach’s description of practical argument on the background of an imperative logic, using for that purpose a modified variant of Lemmon’s [29] syntax for change expressions and Von Wright’s action semantics [47] [49]. Imperatives are commanded actions and can be analyzed as two-part sentences combining two kinds of direction of fit:

$$(\underbrace{\overset{before}{\text{initial\_situation}}}_{word-to-world\ fit}/\underbrace{\overset{after}{\text{resulting\_situation}}}_{world-to-word\ fit})$$

Von Wright distinguishes four types of act: there are two types of productive act and two types of preventive act. If one takes imperatives as commanded actions and uses the “change expression syntax,” the four types of imperatives will be: (i) Produce $p!$ or $!(\neg p/p)$; (ii) Destroy $p!$ or $!(p/\neg p)$; (iii) Maintain $p!$ or $!(p/p)$; (iv) Suppress $p!$ or $!(\neg p/\neg p)$. The Belnap-style imperative (v) See to it that $p!$ or $!(\top/p)$, turns out to be a generalization of (i)–(iv) imperatives in which information on the initial situation is abstracted away. Disregarding the differences between the imperative variants mentioned, the truth condition for its general form $!(p/q)$ is the following: $!(p/q)$ is true iff (i) in the initial situation p is the case, (ii) q is the case in the imperative future, (iii) q is possible in the future, (iv) q is avoidable in the future.[13]

Definitions 1.4.2. Let At be a finite set of propositional letters.

- Language $\mathcal{L}_P$ is the set of formulas $\varphi ::= a \mid \top \mid \neg\varphi \mid \varphi \wedge \psi$, where $a \in At$.
- Language $\mathcal{L}_!$ is the set of formulas $\varphi ::= \cdot(p/\top) \mid !(p/q) \mid \boxdot(\top/q)$, where $p, q \in \mathcal{L}_P$.
- Language $\mathcal{L}_{!might}$ is the set of formulas $\varphi ::= p \mid \text{might } p \mid \varphi_1; \varphi_2$, where $p \in \mathcal{L}_!$.

Definitions 1.4.3. The set W^0 of worlds possible with respect to At is the set $W^0 = \wp At$.
The set Σ of informative structures is the set $\Sigma = \{\langle W, R_!, R_F\rangle \mid W \subseteq W^{\,0},\ R_! \subseteq R_F \subseteq W \times W\}$.
Ignorant structure 0 is the structure $0 = \langle W^{\,0}, W^{\,0} \times W^{\,0}, W^{\,0} \times W^{\,0}\rangle = \langle W^{\,0}, R_!^0, R_F^0\rangle$.

Definition 1.4.1. Valuation for formulas $p, q \in \mathcal{L}_P$:

- $w \models p$ iff $p \in w$ for propositional letters $p \in At$.
- $w \models \neg p$ iff $w \not\models p$.
- $w \models p \wedge q$ iff $w \models p$ and $w \models q$.

Definition 1.4.2. Truth at $w \in W$ in $\sigma = \langle W, R_!, R_F\rangle$:

- $\sigma, w \models \cdot(p/\top)$ iff $w \models p$ and $R_!(w,v)$ or $R_F(w,v)$ for some v.
- $\sigma, w \models !(p/q)$ iff (i) $w \models p$, (ii) $v \models q$ for all v such that $R_!(w,v)$, (iii) $u \models q$ for some u such that $R_F(w,u)$, and (iv) $z \not\models q$ for some z such that $R_F(w,z)$.
- $\sigma, w \models \boxdot(\top/p)$ iff $v \models p$ for all v such that $R_!(w,v)$ or $R_F(w,v)$.
- $\sigma, w \models$ might φ iff $\sigma, v \models \varphi$ for some v.
- $\sigma, w \models \varphi; \psi$ iff $\sigma, w \models \varphi$ and $\sigma, w \models \psi$.

Definition 1.4.3 (Validity in σ). φ is valid in $\sigma = \langle W, R_!, R_F\rangle$, i.e. $\sigma \models p$ iff $\sigma, w \models p$ for all $w \in W$.

Definition 1.4.4 (Substructure). $\sigma = \langle W, R_!, R_F\rangle$ is a substructure of $\sigma' = \langle W', R'_!, R'_F\rangle$, i.e. $\sigma \leq \sigma'$ iff $W \subseteq W'$ and $R_! \subseteq R'_!$ and $R_F \subseteq R'_F$.

Proposition 1.4.1. *$\leq$ is a partial order on Σ.*

[13] In the formal system $R_!$ denotes the relation between doxastically possible initial states and imperative future states whereas R_F denotes the relation between doxastically possible initial states and historically possible (from the agent's perspective) future states.

Definition 1.4.5. Intension of a propositional component $p \in \mathcal{L}_P$ is the set $[\![p]\!] = \{w \in W^0 \mid w \models p\}$.

Definition 1.4.6. $\langle W^1, R^1_!, R^1_F \rangle \Cap \langle W^2, R^2_!, R^2_F \rangle = \langle W^1 \cap W^2, R^1_! \cap R^2_!, R^1_F \cap R^2_F \rangle$

Definition 1.4.7 (Minimal structure)**.** The minimal structure $(0 \mid \varphi) \in \Sigma$ for $\varphi \in \mathcal{L}_{!might}$ is inductively defined as follows:

- $(0 \mid \cdot(p/\top)) = \langle W^0 \cap [\![p]\!], R^0_! \cap [\![p]\!] \times [\![\top]\!], R^0_F \cap [\![p]\!] \times [\![\top]\!] \rangle$
- $(0 \mid !(p/q)) = \langle W^0 \cap [\![p]\!], R^0_! \cap [\![p]\!] \times [\![q]\!], R^0_F \cap [\![p]\!] \times [\![\top]\!] \rangle$
- $(0 \mid \boxdot(\top/p)) = \langle W^0, R^0_! \cap [\![\top]\!] \times [\![p]\!], R^0_F \cap [\![\top]\!] \times [\![p]\!] \rangle$
- $(0 \mid \text{might } \varphi) = \langle W^0, R^0_!, R^0_F \rangle$
- $(0 \mid \varphi;\psi) = (0 \mid \varphi) \Cap (0 \mid \psi)$

Definition 1.4.8. σ represents solely the information contained in $\varphi \in L_{!might}$ iff $\sigma \models \varphi$ and for all σ' it holds that if $\sigma' \models \varphi$, then $\sigma' \leq \sigma$.

Proposition 1.4.2. $(0 \mid \varphi)$ *represents solely the information contained in* φ*.*

Definition 1.4.9. $(0 \mid \Gamma) = \bigcap_{\varphi \in \Gamma} (0 \mid \varphi)$

Definition 1.4.10 (Prima facie consequence)**.** $\Gamma \models_{prima\ facie} \varphi$ iff $(0 \mid \Gamma) \models \varphi$

Definition 1.4.11. Let $(0 \mid \Gamma) = \langle W, R_!, R_F \rangle$ and $(0 \mid \Gamma) \in \Sigma$. Γ is a complete set iff $|mem_1(R_!)| = 1$ and $|mem_2(R_!)| = 1$.

Remark. Prima facie consequence relation as formulated in Definition 1.4.10 is not reflexive, non-monotonic, and not transitive. An example of non-transitivity will be discussed below.

Applying the simple system Let us go back to the expanded Ross's paradox. First, the translation to $\mathcal{L}_{!might}$ will be given (in the last column of Table 10), and the presupposed premises (i) and (ii) will be included.

Table 10: Expanded Ross's paradox adapted to $\mathcal{L}_{!might}$ language and the tacit premises included.

The letter is not burned.	(i)	$\cdot(\neg B/\top)$
It is not possible that the letter is in the letter box and that it is burned.	(ii)	$\boxdot(\top/\neg L \vee \neg B)$
Put the letter into the letter box!	(iii)	$!(\neg L/L)$
Put the letter into the letter box or burn it!	(iv)	$!(\neg L \wedge \neg B/L \vee B)$
It might be good to burn the letter!	(v)	might $!(\neg B/B)$

Now the procedure for creating the largest structure (informatively minimal model) for a given set will be introduced. The structures can be thought of as being composed of two sets: $R_!$ is the set of ordered pairs (the ordered pairs hereafter in this section will be referred to as 'arrows') whose first member is an element of the set of doxastically possible initial situations while second is an element of the set imperatively possible future situations; R_F is the set of arrows, whose first member is an element of the set of initial situations and second is an element of doxastically possible future situations.[14]

The procedure is composed of the following rules successively applied for removing available arrows by writing $\times$ to the right of their first and second members:

- For $\cdot(p/\top)$ remove all the arrows starting at $\neg p$-worlds.
- For $!(p/q)$ test whether there is an R_F arrow pointing to a q world and an R_F arrow pointing to a $\neg q$ world; if so, remove all $R_!$ arrows starting in a $\neg p$ world or ending in a $\neg q$ world; otherwise, remove all arrows.
- For $\boxdot(\top/p)$ remove all arrows ending in $\neg p$-worlds.
- For might φ test whether φ would erase all arrows. If so, remove them all; otherwise, do not remove any.
- For $\varphi;\psi$ apply the rule for φ and then the rule for ψ.

The conclusion follows from the premises in the prima facie way iff it removes no arrows.

Example 1.4.3. Imperative disjunction introduction is partially vindicated (Table 11):

$$\{\underset{(i)}{\cdot(\neg B/\top)}, \underset{(ii)}{\boxdot(\top/\neg L\vee\neg B)}, \underset{(iii)}{!(\neg L/L)}\} \vDash_{prima\ facie} \underset{(iv)}{!(\neg B\wedge\neg L/B\vee L)} \tag{1}$$

Table 11: The eliminative table for (1).

Initial situation			Imperative future			Possible future		
w_1	$\{B,L\}$	$\times$ by (i)	w_1	$\{B,L\}$	$\times$ by (ii)	w_1	$\{B,L\}$	$\times$ by (ii)
w_2	$\{B\}$	$\times$ by (i)	w_2	$\{B\}$	$\times$ by (iii)	w_2	$\{B\}$	
w_3	$\{L\}$	$\times$ by (iii)	w_3	$\{L\}$		w_3	$\{L\}$	
w_4	$\emptyset$		w_4	$\emptyset$	$\times$ by (iii)	w_4	$\emptyset$	

Example 1.4.4. Free choice permission is also partially vindicated if modified, as is done here, to the licensing of suggestion (v) by the choice offering imperative (iv)

[14]The set W from the model $\langle W, R_!, R_F\rangle$ can be ignored here since the motive for its introduction is purely technical — its function was to enable definition of validity in a model.

(Table 12):

$$\{\underset{(iv)}{!(\neg B \wedge \neg L / B \vee L)}\} \vDash_{prima\ facie} \text{might } \underset{(v)}{!(\neg B/B)} \tag{2}$$

Table 12: The eliminative table for (2).

Initial situation			Imperative future			Possible future		
w_1	$\{B,L\}$	$\times$ by (iv)	w_1	$\{B,L\}$		w_1	$\{B,L\}$	
w_2	$\{B\}$	$\times$ by (iv)	w_2	$\{B\}$		w_2	$\{B\}$	
w_3	$\{L\}$	$\times$ by (iv)	w_3	$\{L\}$		w_3	$\{L\}$	
w_4	$\emptyset$		w_4	$\emptyset$	$\times$ by (iv)	w_4	$\emptyset$	

Example 1.4.5. In spite of partial vindication of imperative disjunction introduction and of permission distribution over disjuncts, the upshot of Ross paradox O$p \Rightarrow$ Pq (if anything is obligatory, the everything is permitted) is avoided (Table 13). The relation $\vDash_{prima\ facie}$ is not transitive and in this case the unwanted conclusion (v) does not follow.

$$\{\underset{(i)}{\cdot(\neg B/\top)}, \underset{(ii)}{\boxdot(\top/\neg L \vee \neg B)}, \underset{(iii)}{!(\neg L/L)}\} \nvDash_{prima\ facie} \text{might } \underset{(v)}{!(\neg B/B)} \tag{3}$$

The set $\Gamma = \cdot(\neg B/\top), \boxdot(\top/\neg L \vee \neg B), !(\neg L/L)\}$ is satisfiable but the set $\Gamma' = \Gamma \cup \{\text{might } !(\neg B/B)\}$ is not, i.e. $(0 \mid \Gamma') = \emptyset$ as Table 13 shows.

Table 13: The eliminative table for (3).

Initial situation			Imperative future			Possible future		
w_1	$\{B,L\}$	$\times$ by (i)	w_1	$\{B,L\}$	$\times$ by (ii)	w_1	$\{B,L\}$	$\times$ by (ii)
w_2	$\{B\}$	$\times$ by (i)	w_2	$\{B\}$	$\times$ by (iii)	w_2	$\{B\}$	$\times$ by (v)
w_3	$\{L\}$	$\times$ by (iii)	w_3	$\{L\}$	$\times$ by (v)	w_3	$\{L\}$	$\times$ by (v)
w_4	$\emptyset$	$\times$ by (v)	w_4	$\emptyset$	$\times$ by (iii)	w_4	$\emptyset$	$\times$ by (v)

To conclude In Ross's paradox the source of our confusion does not seem to lie in imperative disjunction introduction or in free choice permission but rather in the sequencing of logical steps in the transitive way not supported by the nature of consequence relation in imperative context.

The language practices do not support the hypothesis that understanding of meanings of logical terms is constitutive for understanding of consequence relation. The language practices do not support the hypothesis that understanding of consequence relation is regulative for understanding of meaning of logical terms. My conjecture is that understanding of logical terms and logical relations comes to us bundled together as a collection of open notions.

2 The Mainstream in Philosophical Semantics of Imperatives

There are two prominent features of the philosophical approach to imperatives:

1. use of modal logic (from static beginnings, to be discussed in 2.1 below, to recent dynamic trends), and
2. investigation of connections between imperatives and actions (to be discussed in 2.1.2 below).

Possible worlds semantics has been established as one of the main tools in philosophical analysis in the last third of the 20th century. Using the possible worlds semantics paved the way for explication of meaning of many of words which have permeated the philosophical discussion over the centuries (necessity, possibility, obligation, permission, action, knowledge, etc.). Due to modal logic, the "logical terminology" ceased to be limited to a small collection of just a few words (truth-functional connectives, simple quantifiers, and identity predicate), but started to include an open collection of words, and even the sentence moods in imperative and erotetic logic. In this way, after a short post-Fregean period of limitation to the language of mathematics and natural sciences, logic has turned back to its full scope of investigation. Given the fact that modal logic deals with the logic of language being used in philosophy as well as in human sciences, modal logic is sometimes colloquially called 'philosophical logic.'

The idea of possible worlds was envisaged by Gottfried Wilhelm Leibniz (1646–1716), but remained theoretically inert until Rudolf Carnap gave explication for possible worlds in terms of formally consistent and complete set of sentences ("state descriptions"), and Stig Kanger and Saul Kripke introduced the notion of accessibility relation that points to possible worlds which are to be considered. Since logical truth is the general truth, there are number of ways to define logical truth in possible worlds semantics: (V1) validity in the model: being true at each world; (V2) validity on the frame: being V1-valid in any model built over the frame; (V3) validity in the class of frames: being V2-valid on each frame from the class; (V4) validity in all models. In modal logic (V3) notion is used. This notion suggests, so to speak, that the semantics of modal operators (e.g. words like 'necessary,' 'obligatory,' 'known' etc.) is captured by diverse structures (their "meaning space" is given by particular structure) by means of the axioms that characterize frames, or by explicit constrains on class or frames.

2.1 Imperatives and modal logic: the beginnings

Philosophical analysis typically requires multiple modalities. In his seminal paper on modal imperative logic Brian Chellas used two binary accessibility relations, S_t for world lines that overlap up to the time point t, and R_t for relation of "imperative alternative.” Modalities ! and ¡ are standardly defined as universal (holding in all R_t alternatives) and existential (holding in some R_t alternatives).

> Now the three conditions on the relation of imperative alternativeness may be stated precisely: for each w, w', $w'' \in W$, and $t \in T$,
> (I) there is a $w' \in W$ such that $R_t(w, w')$;
> (II) if $R_t(w, w')$, then $S_t(w, w')$;
> (III) if $S_t(w, w')$, then $R_t(w, w'')$ iff $R_t(w', w'')$. [14, p. 122]

Provided that Chellas directly (i.e. within semantics) characterizes the relations, he does not need to give an axiomatic presentation of imperative logic. Chellas reads $!p$ as an optative ‘Let it be the case that p’ and interprets it as imperative obligation, while ¡ is understood as imperative permission. The symbol $\boxdot$ stands for “historical necessity”(true in all S_t relata).[15] Chellas system is purely semantical, and therefore, the theorems of it are not proved within a deductive system. Basin, Matthews and Viganò have developed a system of deduction for unimodal normal logics lying within Geach’s hierarchy (i.e. those normal modal logics whose relational theory is representable in first order language) [5]. The deductive rules are common to any universal and existential modality (e.g. $\Box$ rules are applicable to Chellas’s ! and $\boxdot$). The differences between logics are defined by a relational theory which describes frame properties. Their approach can be easily adjusted to polymodal logics [55].

The labeled deduction system for Chellas’s imperative logic Let us build a labeled deduction system for Chellas’s logic. In a labeled deduction system each formula is prefixed by a world index w. At each w we use classical rules, all of which are standard and localized to a single world, with the exception of “global negation” introduction rule (where a contradiction in an accessible world v rules out the assumption made at the world w). Rules for $\rightarrow$ and $\neg$ will be given for an illustrative purpose.[16] The rules are presented in Table 14. The relational theory is easily obtained from Chellas’ conditions (I)–(III) below, using a Skolem function f in (I):

(I) $\vdash R_t(w, f(w))$
(II) $R_t(w, v) \vdash S_t(w, v)$
(III) $S_t(w, v),\ R_t(w, u) \vdash R_t(v, u)$ and $S_t(w, v),\ R_t(v, u) \vdash R_t(w, u)$.

The labeled deduction rules for imperative (! and ¡) and historical ($\boxdot$ and $\diamonddot$) modalities are easily obtained since they are nothing but two pairs of universal and existential modalities defined over the relations R_t and S_t, respectively (see Table 15).

Example 2.1.1. Firstly, let us prove $\boxdot p \rightarrow !p$, a proposition having the Stoic flavor of desiring the unavoidable.

[15] Chellas’s system also has tense operators, but they will be disregarded here.
[16] The other non-mentioned first-order rules are similar to the ones given in [4].

Proposition 2.1.1. $\vdash_{Chellas} \boxdot p \to !p$

Proof

1			
2		$w: \boxdot p$	assumption
3	v	$R_t(w,v)$	assumption
4		$S_t(w,v)$	3/ (II)
5		$v : p$	2, 4/ $\boxdot$Elim
6		$w :!p$	2–5/ !Intro
7		$w : \boxdot p \to !p$	2–6/ $\to$Intro

□

The question arises as to whether the proposition ‘Let it be the case whatever is necessary the case’ is a theorem of logic or a thesis of a normative system, e.g. Stoic ethics?

Example 2.1.2. Ross’s paradox is easily provable.

Proposition 2.1.2. $\vdash_{Chellas} !p \to !(p \vee q)$

Table 14: The labeled deduction system: some rules for logical constants.

$\to$Intro	$\Gamma, w : p \vdash w : q \Rightarrow \Gamma \vdash w : p \to q$	
$\to$Elim	$\Gamma \vdash w : p$ and $\Gamma \vdash w : p \to q \Rightarrow \Gamma \vdash w : q$	
$\neg$Intro	$\Gamma, w : p \vdash v : \bot \Rightarrow \Gamma \vdash w : \neg p$	
$\neg$Elim	$\Gamma, w : \neg\neg p \Rightarrow \Gamma \vdash w : p$	
$\Box$Intro	$\Gamma, Rwv \vdash v : p \Rightarrow \Gamma \vdash w : \Box p$	v does not occur in Γ
$\Box$Elim	$\Gamma \vdash w : \Box p$ and $\Gamma \vdash Rwv \Rightarrow \Gamma \vdash v : p$	
$\Diamond$Intro	$\Gamma \vdash Rwv$ and $\Gamma \vdash v : p \Rightarrow \Gamma \vdash w : \Diamond p$	
$\Diamond$Elim	$\Gamma \vdash w : \Diamond p$ and $\Gamma, Rwv,\ v : p \vdash \varphi \Rightarrow \Gamma \vdash \varphi$	v does not occur in $\Gamma \cup \{\varphi\}$

PROOF

1		
2	$w : \; !p$	assumption
3	$v \mid R_t(w,v)$	assumption
4	$v : p$	2, 3/ !Elim
5	$v : \; p \vee q$	4/ $\vee$Intro
6	$w :!(p \vee q)$	3–5/ !Intro
7	$w :!p \rightarrow !(p \vee q)$	2–6/ $\rightarrow$Intro

□

Connective $\vee$ is problematic on both introduction and elimination side in imperative context. Permissions distribute over disjunctions (‘You may take an apple or a pear’ is a “free-choice permission” entailing ‘You may take an apple’ as well as ‘You may take a pear’). Usually one wants Ross’s paradox not to be provable and permission distribution to be provable. In Chellas’s system however just the opposite holds: $!p \vdash !(p \vee q)$ but $¡(p \vee q) \nvdash ¡p$.

Example 2.1.3. No obligation with respect to p implies the permission regarding $\neg p$.

Proposition 2.1.3. $\vdash_{Chellas} \neg !p \rightarrow ¡\neg p$

Table 15: The labeled deduction rules for imperative and historical modalities.

!Intro	$\Gamma, R_t wv \vdash v : p \Rightarrow \Gamma \vdash w :!p$	v does not occur in Γ
!Elim	$\Gamma \vdash w :!p$ and $\Gamma \vdash R_t wv \Rightarrow \Gamma \vdash v : p$	
¡Intro	$\Gamma \vdash R_t wv$ and $\Gamma \vdash v : p \Rightarrow \Gamma \vdash w : ¡p$	
¡Elim	$\Gamma \vdash ¡p$ and $\Gamma, R_t wv, \; v : p \vdash \varphi \Rightarrow \Gamma \vdash \varphi$	v does not occur in $\Gamma \cup \{\varphi\}$
$\boxdot$Intro	$\Gamma, S_t wv \vdash v : p \Rightarrow \Gamma \vdash w : \boxdot p$	v does not occur in Γ
$\boxdot$Elim	$\Gamma \vdash w : \boxdot p$ and $\Gamma \vdash S_t wv \Rightarrow \Gamma \vdash v : p$	
⟐Intro	$\Gamma \vdash S_t wv$ and $\Gamma \vdash v : p \Rightarrow \Gamma \vdash w :$ ⟐p	
⟐Elim	$\Gamma \vdash w :$ ⟐p and $\Gamma, S_t wv, v : p \vdash \varphi \Rightarrow \Gamma \vdash \varphi$	v does not occur in $\Gamma \cup \{\varphi\}$

Proof

1		
2	$w : \neg !p$	assumption
3	$w : \neg\text{¡}\neg p$	assumption
4	$v \quad R_t(w,v)$	assumption
5	$v : \neg p$	assumption
6	$w : \text{¡}\neg p$	4, 5/ ¡ Intro
7	$w : \bot$	3, 6/ $\bot$Intro
8	$v : p$	5–7/ $\neg$Intro
9	$w : !p$	4–8/ !Intro
10	$w : \bot$	2, 9/ $\bot$Intro
11	$w : \neg\neg\,\text{¡}\neg p$	3–10/ $\neg$Intro
12	$w : \text{¡}\neg p$	11/ $\neg$Elim
13	$w : \neg !p \rightarrow \text{¡}\neg p$	2–12/ $\rightarrow$Intro

□

2.1.1 Multi-layered semantics

It has been argued in Section 1.2.4 that imperatives as being used in commands produce changes in the recipient's cognitive-motivational state and in the obligation pattern between the sender and the recipient. Therefore, imperative theory can isolate one or more semantic dimensions. If the theory focuses on changes in the recipient's motivational state, the bouletic dimension will be exploited. But if the theory concerns social relations, the deontic dimension will come to the fore. The multi-layered semantics explains the abundance of different systems, each convincing in its own right, but modeling different aspects: wishes of the speaker and imperative obligations (Chellas), will of the Imperator (Segerberg [41]), actions commanded (Belnap et al.), preferences of the hearer (Van Benthem and Liu [11]), obligations of the hearer (Yamada [54]), etc. Mimicking the style of the thesis on agentive imperative content by Belnap et al. [8], one could rightfully forward the thesis on multi-layered imperative semantics.

Thesis 2.1.1. *The semantics of imperatives is multi-layered.*

Deconstructive interpretation of Chellas's semantics shows that the optative reading 'Let it be the case that ...' does not mix well with the deontic interpretation. In addition, it is the theorem that $\neg !p \Leftrightarrow ¡\neg p$, but can it really be so that a negated optative equals permission?

Table 16: Semantic dimensions captured by the Chellas's system.

Semantic dimensions	Speaker	Hearer
bouletic	Yes	No
doxastic	No	No
deontic	No	Yes
agentive content	No	

2.1.2 Imperatives and semantics of action

According to this paper, when faced with the problem of modeling in theory of imperatives, one should follow 'imperative content thesis,' preserve multi-layered semantics (to a certain extent), and get "logical geography" right. Which logic of action to consider in order to incorporate it into logic of imperatives? There is number of logics of action to choose from. In particular, Krister Segerberg's (e.g. [32]), and Nuel Belnap's theories stand out, but we will turn to Georg Henrik von Wright. Why? Von Wright is said to have fathered the logic of action by prominent authors in the field (Segerberg [42], Hilpinen [26]) and rightly so. Von Wright's semantics of action is simple, reduced to basic elements, and yet strong enough to explicate important distinction. Von Wright's semantics can be easily adapted to dynamic semantics. We will discuss his logic of action in its final form [49].

Von Wright's action semantics

> To act is intentionally ("at will")to bring about or prevent a change in the world (in nature). On this definition, to forbear (omit) action is either to leave something unchanged or to let something happen. [49, p. 121]

There are two types of action, according to Von Wright: the productive action and the preventive action; and two types of forbearance (omission): letting something happen and leaving something unchanged. 'To act' refers both to "productive or preventive interference with the world" and to forbearance. G. H. von Wright gives a sequence of definitions (paraphrased here in Definitions 2.1.1), ending with 'state of affairs' as a primitive term.

Definitions 2.1.1. Action means bringing about or preventing a change in the world (in nature). Change is transformation of states (of affairs). Changes occur when a state of affairs cease to be or come to be or continues to be.

Remark. "Non-changes" are immediate progressions in time with the same initial and end-state, and they are classified as changes. Von Wright introduces the term 'state of affairs' by way of examples: "the sun is shining" is an example of a generic state of affairs, which can be "instantiated on a certain occasion in space and time," and "instantiated state of affairs" is an individual state of affairs. Still, one may go back to Wittgenstein's *Tracatus* for the definition of 'state of affairs':

> 2.01 A state of affairs (a state of things) is a combination of objects (things). [53]

Von Wright theoretically identifies the notion of a 'total state of the world on a given occasion' with a description that indicates "for every one of a finite number of n states $p_1,\dots,p_n$ whether it obtains or does not obtain on that occasion" [49, p. 122]. Formally, we will reduce a full state description to a set of non-negated propositional letters.

Definition 2.1.1. Let $A = \{p_1,\dots,p_n\}$ be a finite set of propositional letters. A subset $w \subseteq A$ is a Wittgenstein world, a total state, a state description.

The reduced description can be easily expanded to the full state description either in semantic or syntactic terms. In semantic terms, a truth assignment can be defined as binary function determined by w: $h(p,w) = t$ iff $p \in w$. In syntactic terms, set w can be expanded to a set of literals $\mathrm{lt}(w) = w \cup \{\neg p \mid p \in (A - w)\}$. The conjunction $\bigwedge \mathrm{lt}(w)$ of all the literals for a total state will be called 'state description' (assuming that the literals are listed in the conjunction according to their alphabetic order). It is well-known fact that a valuation of propositional letters determines the valuation of all sentences in the propositional language as well as the fact that any set of literals containing exactly one literal from the contradictory pair is syntactically complete. Therefore, a total state set w of propositional letters provides a minimal representative of a formally complete and consistent set.

The time of imperatives A model of time must be incorporated into semantics since actions are conceived of as "bringing about or preventing changes," and changes are identified with "state transformations." Therefore, an ordering between total states is needed for semantics of action. The formula $\bigwedge \mathrm{lt}(w_i)\mathrm{T} \bigwedge \mathrm{lt}(w_j)$ is a "change expression" showing that a change or a continuation of a total state has occurred. The T expressions, T is to be read 'and next,' can be concatenated as $\_\_\mathrm{T}(\_\_\mathrm{T}(\dots\mathrm{T}\_\_)\dots)$; if the empty places are filled with state descriptions, then a "history" (i.e. sequence of total states) will be depicted.[17]

If imperatives are commanded actions, then the action time is the time of imperatives. The concept of time presupposed in the understanding of action is the concept of a time with an open future and closed past sometimes called 'common-sense time.' The openness of the future figures prominently in Von Wright's action semantics. It comes as no surprise that in some theories the imperative is modeled against the ontological background of an indeterministic time, usually modeled as a tree-like structure like in Belnap's quote below.[18].

> (T3) Incomparable moments in Tree never have a common upper bound (No downward branching).
> By a past or a past history (the phrases are interchangeable) I mean a nonempty upper bounded set of moments that contains every moment below any moment it contains; and I let p range over pasts. Because of *No downward branching*, any past is a chain and thus can be extended to a history. The set of (either improper or proper) predecessors of each moment is a past. Thus, the phrase "the past" or "the past history of which the present moment is the last moment" is endowed by each context of utterance with a perfectly determinate meaning. [7, p. 142]

In Von Wright's T-syntax, the connective T must be indexed in order to enable the comparison of parallel histories. The time enters Von Wright's action semantics in two ways:

1. there is an ordering of time points,
2. there are orderings of total states (i.e. histories).

Hence, the action time is both empty and full. It is empty because it provides a frame of reference for histories occurring within it and making it full. Figuratively speaking, one could say that in Von Wright's and common sense concept of time there is one empty time plane for spreading of a number of concurrent histories' threads.

Example 2.1.4. The formula $(p\mathrm{T}p) \wedge (\neg p\mathrm{T}\neg p)$ is inconsistent if it is understood as a description of the same history. On the other hand, if $\underbrace{(p\mathrm{T}p)}_{h_1} \wedge \underbrace{(\neg p\mathrm{T}\neg p)}_{h_2}$ is taken to be a description of two histories, h_1 and h_2, their conjunction is consistent. Moreover, if T is understood as the border between same time points, e.g. $\underbrace{(p \overset{t_1/t_2}{\mathrm{T}} p)}_{h_1} \wedge \underbrace{(\neg p \overset{t_1/t_2}{\mathrm{T}} \neg p)}_{h_2}$, then the conjunction describes two concurrent histories.

[17] In this paper a similar syntax is used, but it is reduced to the atomic case and has '/' instead of 'T.' The similarity is only partial since in the schema $!(p/q)$ the part p/q is an act expression.

[18] In the literature the tree structure is usually defined as a well-founded partial order. E.g. "Definition 9.10. A tree is a partially ordered set $(T, <)$ with the property that for each $x \in T$, the set $\{y : y < x\}$ of all predecessors of x is well-ordered by $<$" [28, p. 114]

2.2 From states to actions

In Davidson's famous definition of 'action,' actions are defined as subset of events.

> ... an event is an action if and only if it can be described in a way that makes it intentional. [17, p. 229]

In short, according to Davidson, an action is an intentional event:

$$Action(e) \text{ iff } Event(e) \wedge Intentional(e)$$

At first glance it seems that in Davidson's approach no branching time is needed, no indeterminism presupposed.[19] But it is through the notion of intentionality that indeterminism might enter again. By adopting Davidson's definition one becomes committed to the ontology of events and to developing a theory of intention. The former is avoidable while the latter is not costless. In this paper the ontology of events is not followed, and consequently the theory of imperatives that rests upon ontology of actions will not be considered.[20]

Example 2.2.1. In Von Wright's semantics the events are identified with changes. Let C stand for 'the window is closed'. The event of opening the window is described by change expression $C\mathrm{T}\neg C$. In order to describe the action of opening the window, a notion of 'intentionality' will be needed, and, presumably, it will turn out to be a very complex, involving not only an appropriate mental state of the agent, but also a notion of causation. In the framework of Von Wright's semantics the latter is analyzed using the distinction between agent's and nature's concurrent histories (see Table 19).

Table 17: The empty time and a history within it.

The empty time linear ordering	...	-1		0		1		...	n	...
A history within it	...	$\bigwedge \mathrm{lt}(w_i)$	T	$\bigwedge \mathrm{lt}(w_j)$	T	$\bigwedge \mathrm{lt}(w_k)$	T	...	$\bigwedge \mathrm{lt}(w_l)$	...

Table 18: Divergent histories are required by the concept of action: $\varphi \leftrightarrow \neg\psi$.

	before	T	after
Agent's history h_{Ag}	$\bigwedge \mathrm{lt}(w)$	T	φ
Nature's history h_{Nt}	$\bigwedge \mathrm{lt}(w)$	T	ψ

[19] Indeed, Davidson himself adopts a deterministic ontology, see the quote here in subsubsection 1.2.

[20] In Mastop's theory of imperatives which rests upon the ontology of actions the impact of imperatives is modeled in terms of an act's addition to (or deletion from) a "to do list," where "A to do list ... is an assignment of do' and refrain' to atomic instructions" [33, p. 22].

"An act is not a change in the world" according to Von Wright. Each act has its corresponding change in the world, but, unlike Davidson's approach, the act is not identical to it.

> It would not be right, I think, to call acts a kind or species of events. An act is not a change in the world. But many acts may quite appropriately be described as the bringing about or effecting ('at will') of a change. To act is, in a sense, to interfere with 'the course of nature'. [49, p. 36]

A formal representation of an act requires taking into account concurrent histories (i.e. sequences of total states). The pretheoretical idea of causation (bringing it about, seeing to it that a generic state of affairs obtains) is captured by parallel-histories model. If p occurs in all agency histories at the instant $after$ and in no nature histories at the instant $after$, then agency is necessary and sufficient condition of p. At least

Table 19: The notion of agency causation of a state p as explicated by the relation between agent's and nature's histories.

	Agency histories	Nature histories	Condition		Theoretician
p obtains in	all	none	sufficient	necessary	G. H. von Wright
p obtains in	some	none		necessary	unknown
p obtains in	all	some but not all	sufficient		Belnap et al.

two histories must be taken into account to represent action:

1. agency history, which is the change for which the agent is responsible,
2. nature history, which is the contrafactual element in the concept of action, the change that would have occurred if the agent had not interfered with the world, the history in which the agent has been removed as an agent (i.e. has no intentions), but is still present as a physical object.

Thus, in its simplest form the action semantics consists of three different points lying on the branching lines of agent's and nature's histories:

1. initial-point is the same for agent's and nature's history lines, and that is the state where the act takes place,
2. end-point lies on a agent's history line, and it is the state that results from agent's action,
3. counter-point lies on a nature's history line, and that is the state which would have occurred had the agent remained passive.

There are two modes of action: acts and forbearances. While acts are characterized by the fact that end-point and counter-point are different, in forbearances they coincide.

Von Wright's theory of forbearance seems to be left in an unfinished form. Just like acts, forbearances stand in a need of a counterfactual element, which could have been introduced into the theory as another agent's history, the one lying within his ability. Von Wright in *Norm and Action* [47] uses $d(\_\mathrm{T}\_)$ and $f(\_\mathrm{T}\_)$ notation for acts and forbearances; while in 'The Logic of Action: A Sketch' [49] he uses connective I, to be read 'instead.' Let us use full state descriptions of the form $\bigwedge \mathrm{lt}(w)$ and let us assume that different indexes denote different descriptions:

act	$\bigwedge \mathrm{lt}(w_i)\mathrm{T}(\bigwedge \mathrm{lt}(w_j)\mathrm{I} \bigwedge \mathrm{lt}(w_k))$ where $j \neq k$
forbearance	$\bigwedge \mathrm{lt}(w_i)\mathrm{T}(\bigwedge \mathrm{lt}(w_j)\mathrm{I} \bigwedge \mathrm{lt}(w_j))$

Example 2.2.2. There are two useful distinctions that can be made on the basis of three point semantics, i.e. two concurrent shortest histories:

- the aforementioned distinction that gives us eight elementary modes of act and forbearance,
- the distinction regarding the range of possible results, see Table 20.

Table 20: The powers of nature and agent.

Determinism in nature	$\bigwedge \mathrm{lt}(w_i)\mathrm{T}(\top \mathrm{I} s)$	if only one total description s satisfies the formula
Indeterminism in nature	$\bigwedge \mathrm{lt}(w_i)\mathrm{T}(\top \mathrm{I} s)$	if more then one total description s satisfies the formula
Agent's impotence	$\bigwedge \mathrm{lt}(w_i)\mathrm{T}(s \mathrm{I} \top)$	if only one total description s satisfies the formula
Agent's omnipotence	$\bigwedge \mathrm{lt}(w_i)\mathrm{T}(s \mathrm{I} \top)$	if any total description s satisfies the formula

The ontology of imperative mood There is a strong ontological presupposition in the notion of action, and consequently in the use of imperative sentences. Acts include counterfactual element: if it were not for the agent's interference with nature, the proper change would not have occurred (in the case of productive act), or the proper change would have occurred (in the case of preventive act). So, the notion of time that lies at the bottom of the concept of action, the notion of time that makes our "imperative language practice" possible, is the notion of time with an "open future."

3 Logical Dynamics of Imperatives

Assume that there is a "picture relation" between the language and the mind and that speech acts project the semantic content of the sentence uttered to the human mind. When an imperative is used for commanding or requesting, its agentive content is projected to the cognitive-motivational state of the addressee and to the pattern of obligations between interlocutors. It is only structures that can stand in pictorial relation, i.e. be structurally similar. If the hypothesis on psychological projection holds, then one should be able to find the structural similarity between the semantic content of sentence and its projection to mental state according to the mode in which the content has been used (Table 21). From this perspective the sentence moods are distinguished according to their impact on psychological states. The imperatives used in issuing commands have an impact on the motivational state while the declaratives used in asserting typically act upon belief state of the hearer. If the agentive content of imperative is projected on the hearer's will, then one should be able to prove that logic of desire is a sublogic of imperative logic. This paper has proposed a technical solution to the problem of the structural similarity between the agentive content of imperative and its volitive psychological projection and a preliminary affirmative answer has been obtained in Theorem 1.3.12.

Table 21: Imperative mood and its psychological projection.

	Language		Mind
Mode	Imperative	acts on	motivational state.
Projection	Agentive content	is projected	to the hearer's desires.
		← structural similarity →	

3.1 Pictorial relation between language and mind

Dynamic semantics provides the formal tools for the understanding of how the agentive content of imperative can be projected to the hearer's beliefs and desires. The speaker's command performed by uttering the imperative $!\varphi$ with agentive content φ changes the hearer's cognitive-motivational state σ to state $\sigma[!\varphi]$ in which the hearer becomes motivated to perform action φ. In the weaker variant of Von Wright's semantics the successful performance of act is sufficient (possibly not necessary) for the result.[21]. It is the weaker variant that has been used for modeling in this paper.[22]

A simple update semantics The update semantics has been developed in Veltman's seminal paper [46]. A paraphrased summary of the basic ideas follows. Information is a set of valuations, valuations are sets of propositional letters, interpretations are functions taking a sentence and a set of valuations as their arguments and delivering a set of valuations as their values. The set W is the set of all valuations $W = \wp A$ possible with respect to a set A of propositional letters. An informational state is identified with set of valuations $\sigma \subseteq W$. Here, "less is more": lesser the number of valuations w in σ, greater the amount of information in σ. Limit cases are:

Minimal info-state If $\sigma = W$ (i.e. $|\sigma| = |W|$), then σ contains no information.

Maximal info-state If $|\sigma| = 1$, then σ gives full information.

Absurd info-state If $\sigma = \emptyset$ (i.e. $|\sigma| = 0$), then σ shows that learning (information acquisition process) has failed.

Definition 3.1.1 (Truth in a valuation)**.** Let $w \in W$, $p \in A$, $\varphi, \psi \in \mathcal{L}_A$.

- $w \models p$ iff $p \in w$.
- $w \models \neg\varphi$ iff $w \not\models \varphi$.
- $w \models (\varphi \wedge \psi)$ iff $w \models \varphi$ and $w \models \psi$.

Definition 3.1.2 (Updates)**.** Interpretation $\ldots[\ldots]$ is a function: $\cdot[\cdot] : \wp W \times \mathcal{L}_A \to \wp W$. An update-sentence φ^+ acts upon an info-state σ delivering the info-state in which it is accepted:

- $\sigma[\varphi^+] = \{w \in \sigma \mid w \models \varphi\}$,
- $\sigma[\varphi^+] = \sigma[\varphi^+][\varphi^+]$.

A structure for the three-point action semantics Following the simple update semantics kindred models for agentives can be built as pairs $\langle \rho, \pi \rangle$ where $\rho \subseteq W \times W$ is the set of ordered pairs representing the shortest agency-histories all starting at the same fixed instant called 'before,' while $\pi \subseteq W$ is the set of all total states which take place at the instant called *after* which succeeds the instant *before*. The set $\pi - mem_2(\rho)$ is the set of ending points of the shortest nature-histories starting at the instant *before*. It is required of the set $mem_2(\rho)$ of second members of ρ to be the subset of π since π

[21] The weak variant is also used in *stit* semantics. A comparison of notions of the agent's causation is given here in Table 19

[22] In this paper three variants ($\mathcal{L}_!$, $\mathcal{L}_{!might}$, $\mathcal{L}_{imp}^{act}$) of the language of imperative logic are discussed. The semantics for each of them relies on Von Wright's three-points action semantics alongside with the notion of weak agentive causation.

includes both agent's and nature's histories from *before* to *after* and thus includes all historically possible states at the instant *after*. That is the reason why in our model a relation and a superset of its second members is used instead of two relations, one for agent's and another for nature's histories. So, a basic semantics roughly corresponding to Von Wright's three-point semantics should contain (i) information on initial-point, (ii) information on end-point, (iii) information on counter-point. One way of building a semantics of the kind is the following:

- $[\![\varphi]\!] = \{w \in W \mid w \models \varphi\}$,
- $\langle \rho, \pi \rangle \models \mathit{Produce}\ \varphi$ iff (i) $\mathit{mem}_1(\rho) \subseteq [\![\neg\varphi]\!]$, (ii) $\mathit{mem}_2(\rho) \subseteq [\![\varphi]\!] \subseteq \pi$, and (iii) $\pi \cap [\![\neg\varphi]\!] \neq \emptyset$.
- $\langle \rho, \pi \rangle \models \mathit{Prevent}\ \varphi$ iff (i) $\mathit{mem}_1(\rho) \subseteq [\![\neg\varphi]\!]$, (ii) $\mathit{mem}_2(\rho) \subseteq [\![\neg\varphi]\!] \subseteq \pi$, and (iii) $\pi \cap [\![\varphi]\!] \neq \emptyset$.

Note that condition (iii) as stated corresponds to the notion of causation in which agent's activity is sufficient condition for end-state (cf. Table19).

Language, world and mind How can an imperative, a sentence that talks about an act create a motivation to perform that act? In my opinion, a plausible answer has been given in the *Tractatus*:

> 4.014 The gramophone record, the musical thought, the score, the waves of sound, all stand to one another in that pictorial internal relation, which holds between language and the world.
> To all of them the logical structure is common. [53]

But the scope of the pictorial relation must be extended: in case of imperative it is a relation between language and mind. The commanded acts are projected to "psychological models" as follows: the information on commanded act's end situation becomes the content of the hearer's will, the information on act's initial situation — the content of the hearer's belief, while information on end-situation and counter-situation becomes the content of the hearer's belief about the future possibilities. Given the fact that the hearer could already have some beliefs and motivating desires, the update with an imperative can produce a clash: e.g. if the hearer believes that the result of the commanded action will occur without his/her agency, or that the result is impossible to achieve. Therefore, imperative update will be composed of different internal "semantic actions:" first the hearer performs a consistency check (i.e. acceptability testing), and, second, if new information is consistent with the one he already has, the hearer updates beliefs about the acting situation and accepts new goals. Definition 3.1.3 gives a formal reconstruction of the idea.[23]

[23] An elaboration of a complete system for imperative updates, downdates and tests for the formal language $\mathcal{L}^{act}_{imp}$ has been given in my [57].

Definition 3.1.3 (Imperative update)**.**

$$\langle\rho,\pi\rangle[!(\varphi/\psi)^{+}] = \begin{cases} \langle(mem_1(\rho)\cap[\![\varphi]\!])\times(mem_2(\rho)\cap[\![\psi]\!]),\pi\rangle \\ \qquad \text{if } mem_2(\rho)\cap[\![\psi]\!]\subseteq\pi \text{ and } mem_2(\rho)\cap[\![\psi]\!]\neq\emptyset, \\ \qquad\qquad \text{and } [\![\neg\psi]\!]\cap\pi\neq\emptyset, \\ \text{failure} \quad \text{otherwise.} \end{cases}$$

3.1.1 Withdrawal: a case for dynamic approach

Is dynamic approach avoidable in a theory of imperatives? In several sections of this paper the dynamic approach has been used as a heuristic principle, so it appears that we may answer in the affirmative.[24] But it is the speech acts of unsaying that suggest the negative answer. The effects of a withdrawal cannot be described in any other way but dynamic. One can reduce positive speech acts to their "pictorial content," but the negative acts of unsaying are comprehensible only in terms of removing certain effects of the previous ones. One type of negative speech act is of special importance for theory of imperatives: permitting as withdrawing of an antecedent imperative or its entailments.

Problem 1. There are three main types of opposition for imperatives. For

Productive imperative Produce φ!

the following sentences stand in opposition:

Type of act opposition Preserve $\neg\varphi$!

Forbearance opposition Let $\neg\varphi$ remain!

Permissive opposition You don't have to produce φ.

The similar opposition, mutatis mutandis, holds for preventive imperatives. As of now, no semantical system has managed to incorporate all of the three types.

A number of authors has over the last decades drawn a distinction between two types of negation. E.g. the usual understanding of negation of assertion is that it is assertion too, but with a negative content; on the other hand, some authors discuss "denial in a non-derivative sense" [44], denial as a speech act sui generis (indicating a failure to obtain a reason for a certain assertion). The same goes for imperatives.

Example 3.1.1. In Searle's speech act theory (where illocutionary force indicator has the role similar to the role of modal element and propositional indicator corresponds to sentence radical) the term 'illocutionary negation' is used for external negation and

[24] In particular, Definition 1.3.16 and Definition 1.4.10 although formulated in a "static way" heavily depend on dynamic way of thinking.

permissions are classified as directives alongside other speech acts typically performed by uttering an imperative.

> "Permit" also has the syntax of directives, though giving permission is not strictly speaking trying to get someone to do something, rather it consists in removing antecedently existing restrictions on his doing it, and is therefore the illocutionary negation of a directive with a negative propositional content, its logical form is $\sim!(\sim p)$.[40, p. 22]

If permissions are conceived as "removal of antecedently existing restrictions," then the idea of downdate comes as a natural solution.[25] Still, the solution is not simple. To remove the motivational and obligation imposing effects of an imperative, which had been either explicitly uttered or implied, it is not enough to "move backwards" to a state where removed imperative is not accepted. The downdated state must be such as to enable update with an imperative with content opposite to the one being withdrawn.[26] So, in this case to model the semantics of withdrawn imperative, i.e. of permission-giving sentence, the opposition between 'act' and 'let it happen' imperatives must be correctly established. We hereby encounter a phenomenon of logical dynamics similar to the one elaborated in the logic of theory change, which has been elaborated by Alchourrón, Gärdenfors, and Makinson [1] . The withdrawal of imperative corresponds with a specific "logical act" of contraction. The research in [57] has shown that AGM theory of contraction together with downdate semantics entails the fact that external denial, instead od reducing, rises the degree of uncertainty thus bringing in an increase in communicative entropy.

3.2 Coda

Why study imperatives? The language of science of man (idiographic science) is characterized by the "language of intentionality" (e.g. beliefs, intentions, actions, ...). The fundamental methodological procedure in the science of man is rationalization formulated within the language of intentionality. An action becomes comprehensible if the agent's reasons make it rational. Rationalizations ("rational explanations," practical inferences) have complex logical structure. As yet there is no generally accepted logic for the language of intentionality. Imperatives open a rich semantic space, which can be grasped in terms of beliefs and desires of the speaker (the sender) and the hearer (the receiver) as well in terms of their commitments. On the syntactic side, imperatives embed "agentives" ("action radicals"). The rich semantic impact of imperatives results from projecting the structure of agentives to the structure of receiver's mental state (also to the intersubjective structure of obligations, which has not been discussed here). Dynamic semantics defines meaning of a sentence in terms

[25] A similar idea has been developed by Lewis in his [31].

[26] A fuller discussion of the problem can be found in my paper [57].

of its effects on mental state or on social relations. This way of thinking can be extended to imperatives in the following way: the content of every imperative is agentive, and agentive has its semantic structure, the utterance of imperative changes the mental state of the receiver in such a way that the semantic structure of agentive is projected on the mental state of the receiver so that he/she becomes motivated to perform the action described by the agentive. This approach imposes upon us a "holistic" methodology of checking the interconnections between logics: if the agentive content of imperative is adequately represented, then its psychological (or social) projection must conform to the corresponding logic (e.g. one must show that logic of desire is a sublogic of imperative logic). It is possible, under certain restrictions, to use Von Wright's semantics of action in the role of agentive content and to project its structure on the belief-desire model, so that doxastic and bouletic dimensions of meaning of imperatives are captured and that the pre-theoretical understanding of "logical geography" is accounted for and refined. Further research should address the projection of imperatives on normative social relations.

The mainstream in philosophical semantics of imperatives The investigations of logic of the language of intentionality (action, belief, desire) all are met in imperative logic. Therefore, an investigation in imperative logic is at the same time an investigation into foundations of human sciences. Speaking in technical terms, modal logic and dynamic semantics provide the tools needed:

1. agentive semantics can be modeled in modal logic,
2. psychological and social structures can be modeled in modal logic,
3. changes in psychological and social structures can be modeled as variations on modal logic models ("Kripkean variations").

Logical dynamics of imperatives In dynamic semantics an imperative appears as an action in a twofold sense: imperative is a "logical action" since the receiver in order to accept it must rearrange his mental state in a suitable a way, an imperative is about action since agentive is its content. A lot of philosophical questions arise in the investigation of imperative logic. The language shapes our mind and our social reality, but without certain pre-understanding of man and nature, our language practices would become futile and pointless. Let the world be deterministic and all imperatives will become meaningless! In our communication we frequently withdraw, cancel, unsay what we said before. The sentences we use for unsaying must have their meaning, don't they? This retractive move in the language game can be modeled within dynamic semantics. In communicative update, the receiver undergoes a transition where his/her mental state becomes more precise or at least as precise as before. (We may think of precision as a number of answers to questions 'what is the case' and 'what am I to do.') Update is uncertainty reduction. In downdate, triggered by the sender's withdrawal of that which he/she said or implied before, the transition goes backwards towards uncertainty escalation.

"One is a lonely number" as Van Benthem reminded us, and logic need not be a study of loneliness, or, even worse, of an universe of meaning where there is no-one for whom the words mean something. Now, one can observe the process of "shifting the logical perspective from valid argumentation to cooperative communication" [24, p. 62], and in that respect logic should restitute its core position in the trivium part of humanistic education, and reestablish itself not only as "ethics of reasoning," but also as "ethics of communication" thus helping us to preserve a human world for tomorrow. Logic of imperatives plays an important theoretical role in that process.

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