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Tensor Product Transformation based Speed Control of Permanent Magnet Synchronous Motor Drives

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***Abstract*—Permanent magnet synchronous motor (PMSM), employing vector control, is used in high performance servo drive applications. It is highly non-linear plant which can be described by linear parameter varying (LPV) state-space model. In order to solve nonlinearity problem and syntesize speed controler, tensor product (TP) model transformation is proposed. Using this transformation, given linear parameter varying state-space model is transformed into polytopic model form, namely, to parameter varying convex combination of linear time invariant (LTI) systems. The main advantage of the TP model transformation is that it is executable in a relatively short time and the linear matrix inequality (LMI)-based control design frameworks can immediately be applied to the resulting polytopic models. Proposed control approach of nonlinear systems is applied to the speed control of permanent magnet synchronous motor drive (PMSM). Simulation results show benefits of the non-linear control applied to PMSM.**

***Keywords*—Tensor product model transformation, fuzzy logic control, vector control, permanent magnet motor**

I. INTRODUCTION

Permanent magnet synchronous motor (PMSM) has attracted increasing interest in recent years for industrial drive application. The several advantages (high efficiency, compact structure, high air-gap flux density, high power and torque to inertia ratio, high torque capability density, simpler controller compared with the induction motor drives, etc.) makes PMSM as a good alternative in certain applications. The two high-performance control strategies for PMSM, field-oriented control (FOC) and direct torque control (DTC), have been successfully implemented in industrial products. They aim both to control effectively the motor torque and flux in order to force motor to accurately track the command trajectory regardless of the machine and load parameter variation or any extraneous disturbances. [1]–[4]. FOC enables use of linear control theory to controlling PMSM, for example LQG optimal speed control is proposed, and proved effective by simulation in [2]. In this paper FOC is used, however non-linear controller is derived using TP model transformation methodology.

The TP model representation belongs to the class of polytopic models. It represents the linear parameter varying state-space models by the parameter varying combination of linear time invariant (LTI) models and was proposed as a uniform and automatic way to transform LPV model. The TP model transformation was introduced as the higher order singular value decomposition (HOSVD) of the linear parameter varying (LPV) state-space models and the result of the TP model transformation was defined as the HOSVD-based canonical form of LPV models [5], [6]. In [7] trade-off techniques between accuracy and complexity of the TP form are proposed.

Furthermore, the TP model transformation offers options to satisfy various convexity constrains on the type of the resulting parameter varying combination, which is suitable, for instance for the linear matrix inequality-based control design, [5], [8]. In [9] and [10] LMI control methodology is presented.

The main contributions of this paper merging field-oriented control and TP model transformation-based control design for application in PMSM motor control.

II. TENSOR PRODUCT MODEL TRANSFORMATION-BASED CONTROL DESIGN METHODOLOGY

Consider the linear parameter-varying state-space model

$$\begin{pmatrix} \dot{x}(t) \\ y(t) \end{pmatrix} = S(p(t)) \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \quad (1)$$

with input $u(t) \in R^k$, output $y(t) \in R^l$ and state vector $x(t) \in R^m$. The system matrix

$$S(p(t)) = \begin{pmatrix} A(p(t)) & B(p(t)) \\ C(p(t)) & D(p(t)) \end{pmatrix} \in R^{(m+k)\times(m+l)} \quad (2)$$

is a parameter-varying object, where $p(t) \in \Omega$ is time varying parameter vector, where Ω is a closed hypercube in R^N, $\Omega = [a_1, b_1] \times [a_2, b_2] \times \ldots \times [a_N, b_N]$. Parameter $p(t)$ can also include the elements of the state vector $x(t)$, therefore LPV system given in Eq. (1) is considered in the class of non-linear dynamic state space models. The main idea of TP model transformation is to discretize the given LPV model given in Eq. (1) over hyper rectangular grid M in Ω, then via executing higher order singular value decomposition, the tensor product structure of given model is obtained. By ignoring singular values, TP model of reduced complexity and accuracy can be obtained. For more details see [5] and [7]. Tensor product structure can be written as follows

$$\begin{aligned} S(p(t)) =& \mathcal{S} \boxtimes_{n=1}^{N} w_n(p_n) \\ =& \sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_n=1}^{I_N} \prod_{n=1}^{N} w_{n,i_n}(p_n) S_{i_1,i_2\cdots,i_N}, \end{aligned} \tag{3}$$

where $\mathcal{S} \in R^{I_1 \times I_2 \times \cdots I_N \times (m+k) \times (m+l)}$ denotes obtained tensor, I_n denotes number of LTI systems in n-th dimension of Ω, $\boxtimes$ denotes multiple n-mode product of a tensor by a matrix, w_n is row vector containing $w_{n,i_n}(p_n) \in [0,1]$ which is corresponding one variable weighting function defined on the n-th dimension of Ω and $S_{i_1,i_2\cdots i_N}$ is LTI system matrix obtained by TP model transformation. By using i as linear index, equivalent to the multi linear array index with the size of $I_1 \times I_2 \times \cdots I_N$, TP model (3) can be rewritten in standard polytopic form

$$S(p) = \sum_{i=1}^{N} w_i(p) S_i, \tag{4}$$

where S_i denotes

$$S_i = \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix}, \tag{5}$$

and w_i is corresponding weighting function. Controller is determined in same polytopic form as TP model. Control signal is given by

$$u = -\sum_{i=1}^{N} w_i(p) K_i x, \tag{6}$$

where the K_i are corresponding LTI feedback gains.

III. Controller Design

A. Linear Matrix Inequalities

A class of numerical optimization problems called linear matrix inequality LMI problems has received significant attention. These optimization problems can be solved in polynomial time and hence are tractable, at least in a theoretical sense. Interior-point methods developed for these problems have been found to be extremely efficient in practice. For systems and control, the importance of LMI optimization stems from the fact that a wide variety of system and control problems can be recast as LMI problems. Except for a few special cases, these problems do not have analytical solutions. However, the main point is that through the LMI framework they can be efficiently solved numerically.

A linear matrix inequality (LMI) has the form

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0, \tag{7}$$

where $x \in R^m$ is the variable and the symmetric matrices $F_i = F_i^T$ are given. The inequality symbol in (7) means that $F(x)$ is positive definite i.e.

$$z^T F(x) z > 0, \forall z \neq 0. \tag{8}$$

B. Control objective

The control objective is to find stabilizing controller, with prescribed decay rate with minimal overshoot and with constrained control signal. In order to obtain stabilizing controller, Lyapunov stability condition is considered. If there exist candidate quadratic Lyapunov function V(x) defined on some open set $D \in R^N$, containing the origin, such that

$$V(x) = x^T P x > 0, \tag{9}$$

and there exist derivation

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} < 0, \tag{10}$$

then origin of system $\dot{x} = f(x)$ is stable equilibrium point. The speed of response is related to decay rate, that is, the largest Lyapunov exponent α [10] (Stability corresponds to positive decay rate) such that

$$\lim_{t\to\infty} e^{-\alpha t} \|x(t)\| = 0 \tag{11}$$

A sufficient condition for desired decay rate can be written as

$$\dot{V}(x) \leq -2\alpha V(x), \tag{12}$$

for any initial point [10]. From (12) it follows that the equilibrium of the continuous system in polytopic form (4) is globally asymptotically stable if there exists a common positive definite matrix P such that

$$A_i^T P + P A_i + 2\alpha P < 0; \forall i \in (1, r). \tag{13}$$

Next, let us consider the stability of the closed-loop system (4) with control algorithm given in (6), which is globally asymptotically stable, with decay rate less than α, if there exists a common positive definite matrix P such that

$$\begin{aligned} G_{ii}^T P + P G_{ii} + 2\alpha P < 0, \\ \left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) + 2\alpha P \leq 0, i < j, \end{aligned} \tag{14}$$

where

$$G_{ij} = A_i + B_i K_j \tag{15}$$

denotes closed loop state matrix. The largest possible decay rate can be found by solving generalized eigenvalue minimization problem (GEVP).

maximize α
subject to

$$\begin{aligned}
&X > 0\\
&-XA_i^T - A_iX + M_i^T B_i^T + B_iM_i + 2\alpha X > 0\\
&-XA_i^T - A_iX - XA_j^T - A_jX + M_j^T B_i^T\\
&+ B_iM_j + M_i^T B_j^T + B_jM_i - 4\alpha X \geq 0,
\end{aligned} \tag{16}$$

where $X = P^{-1}$ and $M_i = K_iX$. In sequel predescribed value of α is used.

In order to satisfy the constraints on control value and output constraints initial conditions or its upper bound must be known. Assume that initial condition $x(0)$ is unknown, but its upper bound $\|x(0)\| \leq \phi$ is known, which can be recast as following LMI

$$\phi^2 I \leq X, \tag{17}$$

then the following LMIs are added to the Eq (16) [9] in order to satisfy constraints:

1) Constraint on the control value: The constraint $\|u\|_2 \leq \mu$ is enforced $\forall t \geq 0$ if the following LMI holds

$$\begin{pmatrix} X & M_i^T \\ M_i & \mu^2 I \end{pmatrix} \geq 0. \tag{18}$$

2) Constraint on the output: Assume that condition (17) is satisfied, the constraint $\|y(t)\|_2 \leq \lambda$ is enforced, $\forall t \geq 0$, if the following LMI holds

$$\begin{pmatrix} X & XC_i^T \\ C_iX & \lambda^2 I \end{pmatrix} \geq 0. \tag{19}$$

Furthermore, controller is obtained as follows:

$$K_r = M_r X^{-1}. \tag{20}$$

IV. TP Model-based Controller Design to the Permanent Magnet Synchronous Motor Drives

A. Dynamic model of Permanent Magnet Synchronous Motor Drive

Non-linear model of SMPM can be described by following equations

$$\begin{aligned}
\frac{d}{dt}i_d &= \frac{1}{L_d}u_d - \frac{R}{L_d}i_d + \frac{L_q}{L_d}\cdot p \cdot \omega_r i_q\\
\frac{d}{dt}i_q &= \frac{1}{L_q}u_q - \frac{R}{L_q}i_q + \frac{L_d}{L_q}\cdot p\omega_r \cdot i_d - \lambda\frac{p\cdot\omega_r}{L_q}\\
T_e &= 1.5\cdot p\cdot(\lambda\cdot i_q + (L_d - L_q)\cdot i_d\cdot i_q)\\
\frac{d}{dt}\omega_r &= \frac{1}{J}(T_e - b\cdot\omega_r - T_L)
\end{aligned} \tag{21}$$

where list of parameters is given in Table I.

TABLE I
PARAMETERS OF THE SMPM

Symbol	Description	Value	Unit
P_n	Nominal power	1.1	kW
n_n	Nominal speed	3000	rpm
R	Stator phase resistance	0.02588	Ω
L_d, L_q	Stator inductance	$8.5 \cdot 10^{-3}$	H
λ	Flux linkage established by magnets	0.125	Vs
J	Inertia	$0.8 \cdot 10^{-3}$	Kgm^2
b	Friction factor	0	Nms
p	Pole pairs	4	
ω_r	Angular rotor speed		rad/s
θ_e	Rotor position at electrical angle		rad
T_L	Motor load		Nm

The basic principle in controlling the PMSM is based on field orientation. This is obtained by letting the permanent magnet flux linkage to be aligned in d-axis and stator torque component vector, i_q is kept along q-axis direction. This means that the value of i_d is kept zero in order to achieve the field orientation. In order to implement field oriented control concept, PI controller is used to keep stator current i_d at zero value. PI controller is obtained using technical optimum.

$$u_d = K_R i_d + \frac{1}{T_I}\int_0^t i_d(t)dt \tag{22}$$

where

$$\begin{aligned}
K_R &= \frac{R}{2}\frac{T_I}{T_{mi}}\\
T_I &= \frac{L_d}{R}.
\end{aligned} \tag{23}$$

and T_{mi} is equivalent inverter time constant. Rotor speed ω_r and stator current i_q are controlled by TP controller given in (6). Proposed control structure is shown in Fig. 1

B. LPV model of Permanent Magnet Synchronous Motor Drive

By augumenting Eq. (21) with (22), and by letting $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T = \begin{bmatrix} i_d & i_q & \omega_r & \int_0^t i_d dt & \int_0^t w_r dt \end{bmatrix}^T$, the equations of motion in linear parameter varying state space form is

$$S = \begin{pmatrix} -\frac{R+Kp}{L_d} & 0 & \frac{L_q}{L_d} p i_q & -\frac{1}{T_I L_d} & 0 & 0 \\ 0 & \frac{-R}{L_q} & -\frac{L_d}{L_q} p i_d - \frac{\lambda p}{L_q} & 0 & 0 & \frac{1}{L_q} \\ \frac{1.5p(L_d - L_q) i_q}{J} & \frac{1.5p\lambda}{J} & \frac{-b}{J} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}. \quad (24)$$

V. Results

A. TP model representation of Permanent Magnet Synchronous Motor Drive

Operating area is selected as $\Omega = [i_{dmin}, i_{dmax}] \times [i_{qmin}, i_{qmax}] = [-10, 10] \times [-50, 50]$. The exact TP model (TP model obtained by keeping all singular values) representation is given by 4 LTI systems. Weighting functions of the TP model are given in Fig. 2 The LTI system matrices of the TP model are given in Eq. (25).

$$A_1 = \begin{pmatrix} -1058.0 & 0 & 200.0 & -39800.0 & 0 \\ 0 & -338.2 & -42.35 & 0 & 0 \\ 0 & 1312.0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \end{pmatrix}$$
$$A_2 = \begin{pmatrix} -1058.0 & 0 & 200.0 & -39800.0 & 0 \\ 0 & -338.2 & -122.4 & 0 & 0 \\ 0 & 1312.0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \end{pmatrix}$$
$$A_3 = \begin{pmatrix} -1058.0 & 0 & -200.0 & -39800.0 & 0 \\ 0 & -338.2 & -42.35 & 0 & 0 \\ 0 & 1312.0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \end{pmatrix} \quad (25)$$
$$A_4 = \begin{pmatrix} -1058.0 & 0 & -200.0 & -39800.0 & 0 \\ 0 & -338.2 & -122.4 & 0 & 0 \\ 0 & 1312.0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \end{pmatrix}$$

with

$$B_i = \begin{pmatrix} 0 & 117.6 & 0 & 0 & 0 \end{pmatrix}^T, \forall i \in [1, 4] \quad (26)$$

B. Control objective

The control objective is to find stabilizing controller, with prescribed decay rate $\alpha < 150$ with constrained control signal $u_q \leq 200V$ and constrained stator current $i_q \leq 30A$. By using the Yalmip [11] and Sedumi [12] the following feasible solution of (16) - (19) and feedback gains are obtained.

C. Obtained feedback gains

$$\begin{aligned} K_1 &= \begin{pmatrix} -0.004784 & 30.98 & 13.96 & -4.872 & 2118.0 \end{pmatrix}^T \\ K_2 &= \begin{pmatrix} -0.04661 & 36.08 & 16.53 & -47.51 & 2657.0 \end{pmatrix}^T \\ K_3 &= \begin{pmatrix} 0.004784 & 30.98 & 13.96 & 4.872 & 2118.0 \end{pmatrix}^T \\ K_4 &= \begin{pmatrix} 0.04661 & 36.08 & 16.53 & 47.51 & 2657.0 \end{pmatrix}^T \end{aligned} \quad (27)$$

In Fig. 3 simulation results are shown.

VI. Conclusion

In this paper, we investigated non-linear controller, obtained by tensor product metodology, in vector control of permanent magnet synchronous motor. Simulation results show better performance in terms of overshoot, speed and disturbance rejection when compared to the results of conventional cascade PI+PI control solution. PI controllers are tuned by symmetrical optimum and tehnical optimum, respectively. Switching effects of inverter and friction effects were neglected during controller design. Simulations were performed using the MATLAB/Simulink environment. In future work inverter dynamics, friction effects and testing of proposed algorithm in real experimental setup, will be considered.

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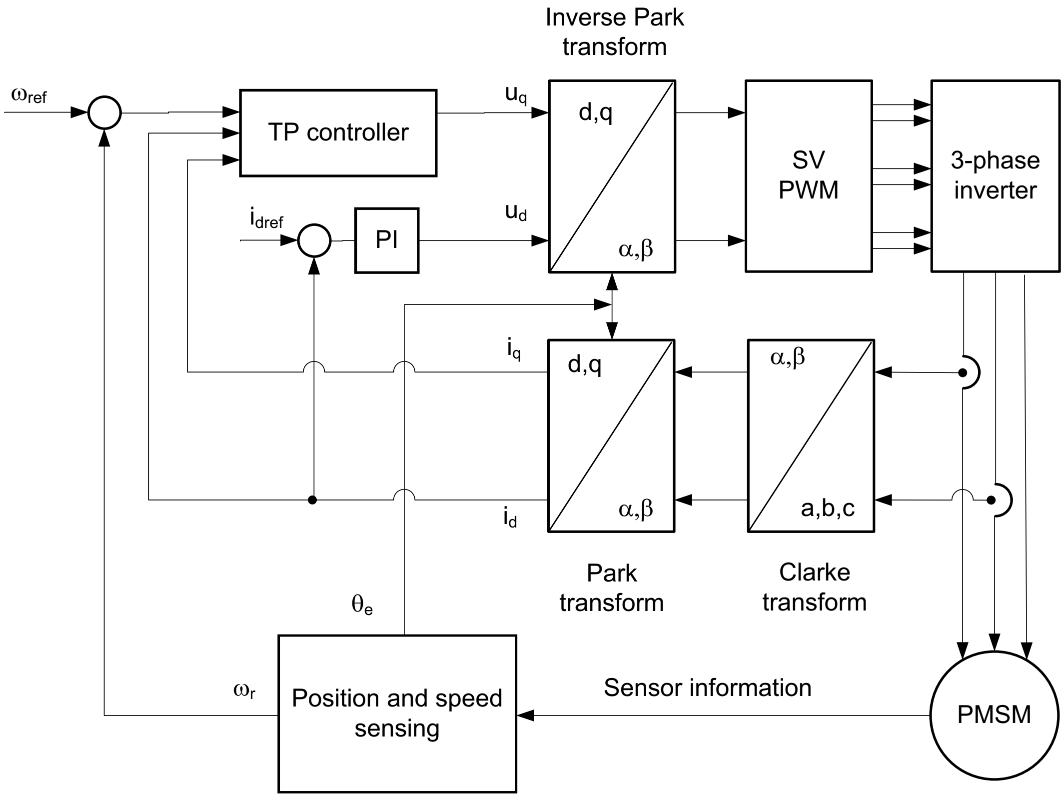

Fig. 1. Proposed control structure

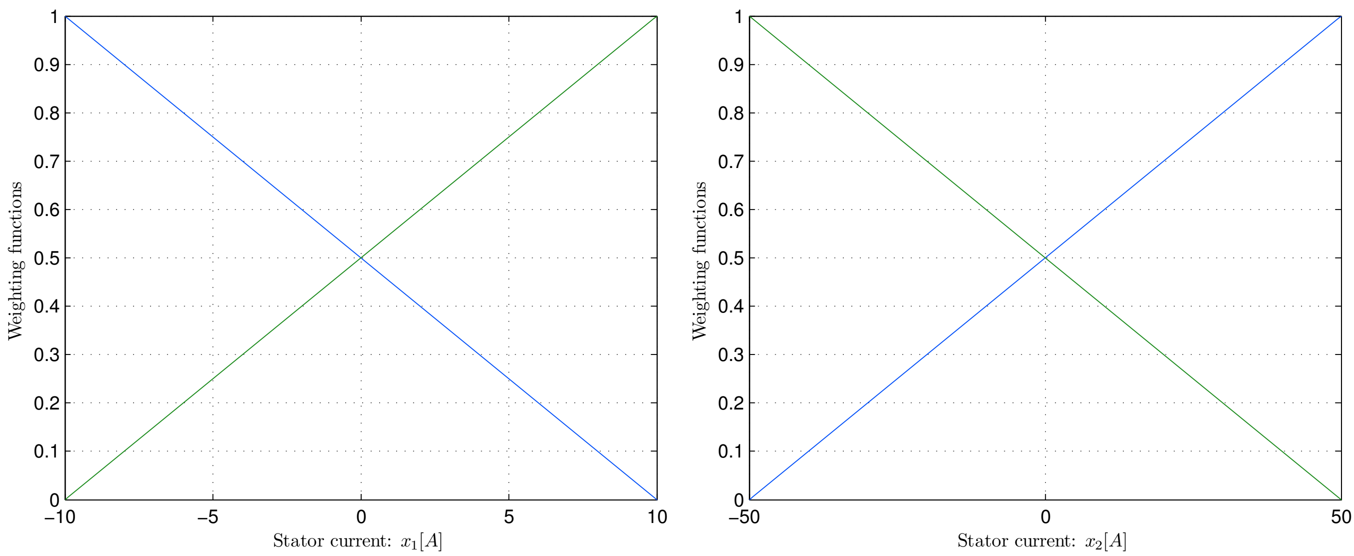

Fig. 2. Weighting functions obtained by tensor product transformation of non-linear model of permanent magnet synchronous motor

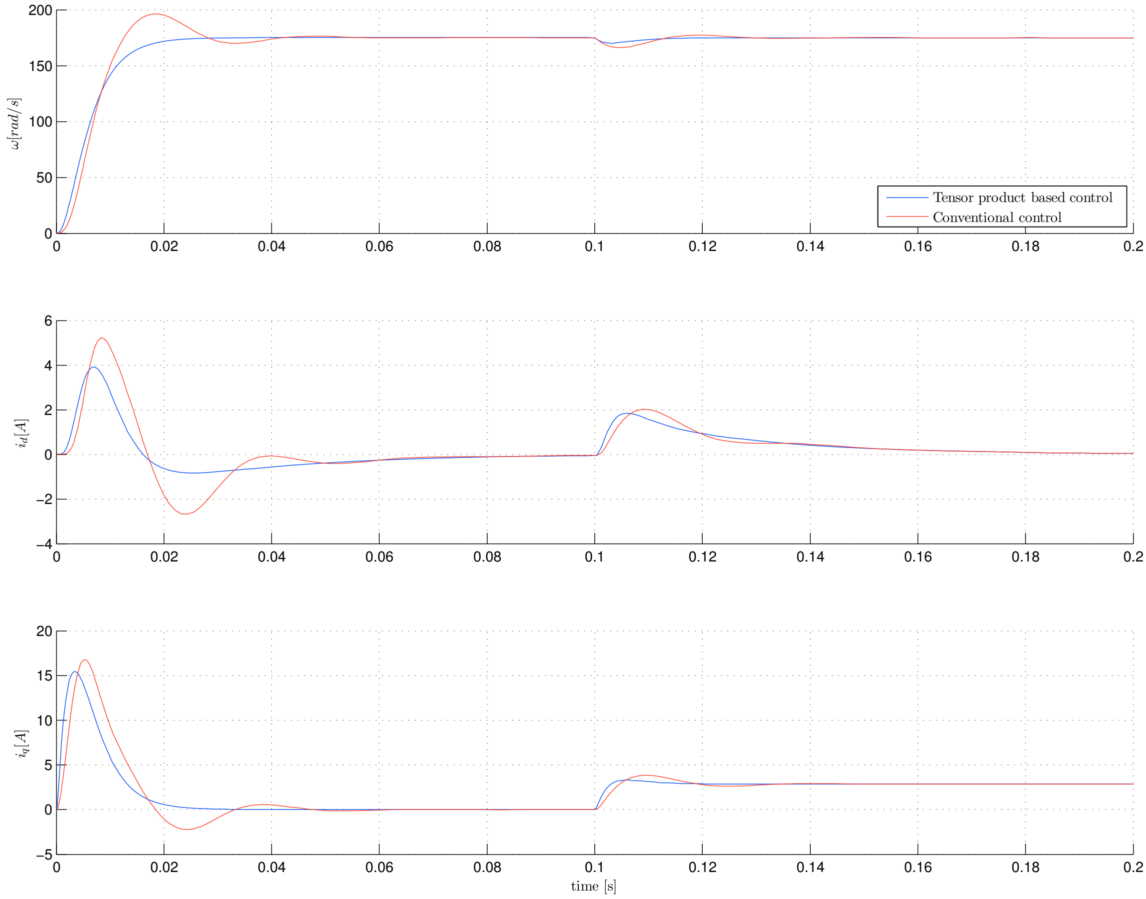

Fig. 3. Simulation results, speed step response to $\omega = 175$ rad/s, disturbance response to T_L = 3Nm at time $t = 0.1$ s of permanent magnet synchronous motor

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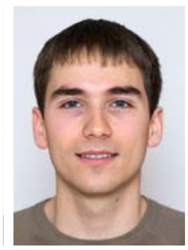

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