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ANALYSIS OF BASKETBALL GAME STATES AND TRANSITION PROBABILITIES USING THE MARKOV CHAINS

Abstract

The abstract system of a basketball game has been established in the paper. Parts of the game are marked with the common characteristics; they are repeated, therefore, they can be denoted with the category: game states. The presented model enables the recognition and analysis of interaction between the set of system states. The discretization of the continuous course or flow of a basketball game and the definition of equivalence among game states have given the prerequisites for the determination of transition probabilities between system states. Discrete stochastic processes and Markov chains were used for events modeling and transition probabilities calculation between the states. The matrix of transition probabilities has been structured between particular states of the Markov chain. The proposed model differentiates game states within four phases of game flow and enables the prediction of the future states.

Key words: BASKETBALL / STATE OF THE GAME / GAME FLOW / POSITIONAL PLAY / TRANSITION PLAY / MARKOV CHAIN

INTRODUCTION

Worldwide popular team sport game with the ball known as basketball has its specific structural and functional characteristics which separate it from the other team sports, although they all belong to the same tree of ball sports games, like soccer, team handball, hockey, rugby, water-polo, the core of their nature being simultaneous existence of relationships of cooperation and opposition within the system of the game (Trninić, 1995; Trninić et al., 2010a, 2010b). Functional approach to the analysis of basketball game enables the decomposition of the system of interaction and interdependence of parameters within the structure of the game as well as the functional analysis of relationships and associations of offensive and defensive tactical intentions (Hernandez, 1987; Trninić, 1995). It also gives opportunity for mathematical formalization and for the analysis of complex interactions within the system of the game of basketball (Trninić, Perica, & Pavičić, 1994). Lapham and Bartlett (1995), McGarry et al. (2002) and Lees (2002) consider research into the complex interactions that occur in sport competition very important and propose further explorations of them.

The processes of cooperation and opposition emphasize the cognitive component of the game. They are treated as the process of interaction in which team-mates cooperate while performing innumerable

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tasks/jobs in the game. Simultaneously, players of the two intra cooperating systems on the court belong to the two opposing teams which employ the results of their cooperation to resist and outplay each other and win over each other (Trninić, 1995).

Therefore, basketball game must be observed as a complex phenomenon. In top-level sport that complexity, depending on the observer's standpoint, may be highlighted from the perspective of: players, expert coaches, managers, or scientists (Trninić, Jelaska, & Papić, 2009a). In this paper basketball is observed as a set of knowledge composed of several layers; it is called the body of basketball knowledge (Trninić, 1995; Trninić, Trninić, & Jelaska, 2010). In a competition, during a match, practical and conceptual knowledge of individual players and of the whole team is constantly scrutinized.

Basketball game can be observed, from the expert coaches and players' point of view, as the arranged, ordered sequence of game tasks each and every player must carry out relative to his/her playing position and role within a particular model of play tactics (Trninić, 1995; Trninić, et al., 2010a, 2010b). The realization of the game tasks implies successful application of individual technical-tactical knowledge and skills on the court in the actual game situations. Successful application of individual technical skills and tactics is not possible if not coordinated with team-mates' individual technical skills and tactics through the collective team tactics the aim of which is accomplishment of individual and common goals, that is, to win a match by counter playing the same intentions of the opposing team and players.

Trninić (1995) explains that game tasks individually classify motor activities and motor behavior of particular players in respect to playing position and the role within a team and a model of play tactics. That primarily regards basketball specific anthropological demands including: cognitive-motor (technical-tactical), energy-related (intensity of play) and socio-motor components of activities performed on the court i.e. coordination and opposition (Trninić, 1995). All the three components are manifested in solving and realizing particular game situations and in the flow of actions within game phases and models of play tactics, then in the intrinsic and extrinsic loads in training and competition activities as well as in the constellation of relevant sport-specific characteristics and state of players, being responsible for successful realization of particular playing position-specific tasks in a game.

Trninić, Perica and Pavičić (1994) as well as Perica, (2011) and Jelaska (2011) described mathematically the system "basketball game". The achievement of that model is the recognition of the two basic system states – *position* and *transition*, from the aspect of the kinematic description. These states are (1) *position*, or in the vocabulary of basketball practice the positional/set offense and the positional/set defense, and (2) *conversion* - *transition* (Knight, 1994), the state of transmuting defense into offense, and vice versa. Conversion – transition are interpreted as a switch, a connection between the phases of defense and offense. The tasks players of both (opposed) teams carry out on the court on particular playing positions, in relation to the position of the ball as the centre of communication, structure and generate various game states.

SYSTEM MODELLING

During the game the players recognize and anticipate events in play and, in accord with that, they make selective decisions and react (Trninić, 1995; Trninić, et al., 2010a, 2010b). The system in basketball can be understood as a set of all participants in a match. In consideration the limitation can be put on just the ball and the players. However, in the context of actual competition one must include also referees, coaches and other officials, bench players, and even spectators (Trninić, Perica, & Pavičić, 1994). Further, in the context of structural analysis of knowledge in the game of basketball the category of **game states** has been established using kinematic description of the game. The position of game states in the hierarchical structure of basketball tree has been defined in the space between game tasks (bellow) and play tactics (above) (Trninić, 1995, 2006).

Previous research using the Markov chains in sports

Team sports games are multilayer and complex sports activities in which a symbiosis of abundant cyclic and acyclic movements with the ball and without it can be seen (Trninić, 1995). They are determined by the relationships of cooperation between team-mates and of opposition between the opposing players and teams. A basketball match has its continuous course or *game flow*. It can be presented as an arranged sequence of tasks which, when realized on the court, generate game states (Trninić, Perica, & Pavičić, 1994). Within our model of basketball game states analysis it is assumed that the game flow has been discretized into definitely many moments or time parts. Also, the game flow has its three basic game states or phases: offense, defense and conversion (Jelaska, 2011; Knight & Newell, 1986; Perica, 2011).

Each phase of game flow has its specific characteristics conditioned by very specific and particularly defined goals within the complex collective tactical operation, which corroborate the notion that basketball game is high level tactical complexity sport.

The methods of artificial intelligence (Lapham & Bartlett, 1995), artificial neural networks (Lees, Barton, & Kershaw, 2003; Perl, 2001; Perl & Weber, 2004), the methods of theoretical computer science (Perl, 2005), dynamic systems theory (Gréhaigne, Bouthier, & David, 1997; McGarry, et al., 2002), stochastic processes and, particularly, the Markov chains (Forbes, & Clarke, 2004; Meyer, Forbes, & Clarke, 2006; Hirotsu, 2002), then nonlinear models (Trninić, Jelaska, & Papić, 2009b) are becoming unavoidable tools in the analyses of complex sports activities.

The Markov chains have appeared in kinesiology/sports science for the first time in 1977 applied in Bellman's paper. Norman (1999) made an overview of 17 scientific papers in which he analyzed the possibilities to use stochastic processes for modeling in kinesiology/sports sciences, whereas Clarke and Norman (1998) utilized stochastic techniques to investigate various decision-making processes in the game of cricket.

Lees (2002) advocates for new methods, for example, the use of artificial neural networks, to be used in kinesiology for establishing characteristics of the whole (biomechanical) skills, instead of quantitative analysis because these new methods can be a useful tool to overcome limitations of classical statistical methods.

For example, Hirotsu and Wright (2003b) analyzed the game of baseball using the Markov chains. They demonstrated how that approach might help to select optimal hitting strategies and how much the probability of winning increases if obtained strategy is followed. Also, the probability of winning in any state in the course of a game was calculated by using the Markov model – they solved the linear system of over one million simultaneous equations.

Bukiet, Harold and Palacios (1997) proposed an approach adoptable to be able to directly include the effects of pitching and defensive abilities. The approach can also be applied to find optimal batting orders, run distributions per half inning and per game, and so on…

Hirotsu and Wright (2003a) proposed, based on the actual data, a statistical model of an American football match that could be useful in providing deeper insights into team characteristics.

Also, there are empirical investigations embracing: scoring structures on the basis of probability, including the identification of optimal decision-making strategies in some instances, for squash (Ap Simon, 1951, 1957; Clarke, & Norman, 1979; Pollard, 1985; Schutz, & Kinsey, 1977; Wright, 1988), tennis (Croucher, 1982, 1986; Morris, 1977; Pollard, 1987; Schutz, 1970), badminton (Clarke, 1979; Renick, 1977), volleyball (Pfeifer, & Deutch, 1981) and baseball (Trueman, 1977). Following these authors, McGarry and Franks (1994, 1996a, 1996b) modeled the motor behavior (the shots) in squash games using the Markov chains.

Lames used the finite Markov chains as a model for game sports, including its calculus (Kemeny, & Snell, 1976). Simulations were undertaken to assess the usefulness of certain tactical behaviors, as well as to assess the performance of individual players in team games. This idea was applied to tennis (Lames, 1988; 1991), squash (McGarry, & Franks, 1994; 1996a; 1996b) and volleyball (Lames, & Hohmann, 1997; Lames, et al., 1997), table-tennis (Zhang, 2003) and team handball (Pfeiffer, 2003).

Shirley (2007) indicates the states of the Markov chain are defined in terms of three factors: 1. which team has the ball possession (2 factors): Home or Away (Host or Guest); 2. How that team gained the ball possession (5 factors): Inbound pass, Steal, Offensive Rebound, Defensive Rebound, Free Throws; 3. the number of points that were scored on the previous possession (4 factors): 0, 1, 2, or 3. the largest possible model would have $2 \cdot 5 \cdot 4 = 40$ states, but since certain combinations of the 3 factors are impossible, the largest model has 30 states. Also, making certain assumptions about the course of action in a basketball game can further reduce the number of game states. If one assumes, for example, that rare events like 4-point plays or loose ball fouls following missed free throws are impossible, then certain states can be eliminated without seriously affecting the usefulness of the model. The proposed model can provide a very detailed "micro simulation" of a basketball game. Quantities of interest can be computed via simulation. Some examples of these might be: 1. in-game win probabilities for a given team; 2. the expected number of points scored in a possession gained in different ways, such as offensive rebounds vs. defensive rebounds; and 3. the change in win probability as a function of the number of the ball possessions in a game; i.e. how useful a strategy is to "slow down the game?".

Basketball game modeling using the Markov chain

Within the context of the mentioned definition of basketball game states (Jelaska, 2011; Perica, 2011; Trninić, Perica, & Pavičić, 1994), it is obvious there are infinitely many different states of the game. Such definition of the basketball game states, although formal and scientific, is not practical to be submitted to the Markov chains analysis. Therefore, we have to get definite number of game states. It would be done by equivalency analysis. We define that two states are equivalent if they are alike in terms of space-time relationship. The feature of transitivity should be emphasized here, that is, if A and B are equivalent states and if C and D are also equivalent states, then A and C will be also equivalent states. Now the state of the Markov chain can be defined as the set of the entire states equivalent to a certain state.

Apparently, a single state of the Markov chain consists of infinitely many interequivalent states, as well as a particular game state can be found only in one state of the Markov chain. A single state of the Markov chain occurs in the interval $< t_i, t_i + \Delta t >$ where Δt is selected empirically, so that our consideration would have a practical purpose.

From the Markov chains' aspect, the momentary state of the Markov chain has all the information needed for the decision-making about the selection of its immediately following state, that is, for the calculation of transition probability into the future state. It is an adequate model for our approach to the analysis of basketball game states, since any action in the state of position/transition in the moment t is a consequence, result of previous game states, therefore, no any additional information is needed for the determination of transition probability into the future game state.

The assumption is that the realization of tasks of individual players within a particular play tactics model generates the total of $N = \sum_{i=1}^{6} n_i + 1$ game states divided into the four basic groups of game states:

1. We introduce states in four phases of game flow:

- positional/set defense (n_1 states) – the first set of states
- transition defense (n_2 states) – the second set of states
- transition offense (n_3 states) – the third set of states
- positional/set offense (n_4 states) – the fourth set of states

2. The states with the unsuccessful outcome (n_5 states) – the fifth set of states

3. The states with the successful outcome (n_6 states) – the sixth set of states

4. The state of a starting jump for the ball (one state) – the seventh set of states.

The standard assumption is that probability is $P(X_{t+1} = j | X_t = i)$ for the transition between the states of the Markov chains being independent of the moment t for all the game states i,j and for all moments t. The equation $P(X_{t+1} = j | X_t = i)$ denotes probability of the transition into the state j in the moment *(t+1)* if in the previous moment t the chain has been in the state i. We will use the following notation $P(X_{t+1} = j | X_t = i) = p_{i,j}$, $i, j \in \{1,..., N\}$.

Further, with $i \in \{1,..., n_1\}$ we will denote the state of positional/set defense, with $i \in \{n_1 + 1,..., n_1 + n_2\}$ the state of transition defense, with $i \in \{n_1 + n_2 + 1,..., n_1 + n_2 + n_3\}$ the state of transition offense, and with $i \in \{n_1 + n_2 + n_3 + 1,..., n_1 + n_2 + n_3 + n_4\}$ the state of positional/set offense. The states from the other sets of game states are noted accordingly.

Further, we assume that in the initial moment, $t = 0$, the starting jump for the ball is actively recognizable. That is, if with k we denote the state of the starting jump for the ball, then it would be valid that:

$$\begin{aligned} &P(X_0 = i) = 0, \quad i \neq k \\ &P(X_0 = k) = 1. \end{aligned} \tag{1}$$

Consequently, the Markov chain transition matrix is a block matrix P of the order N represented by:

$$P = \left[P_{i,j}\right]_{i,j=1,\dots,7} \tag{2}$$

where $P_{i,j}$, $i,j=1,\dots,7$ are matrices of transition probabilities from the n_i state of the i set states into the n_j state of the j set of states. In it, the single block matrix $P_{i,j}$ is the dimension $n_i \cdot n_j$.

As, for example, the following is true $P(X_{t+1} = j | X_t = i) = 0$, if the following is

$$i \in \{1,\dots,n_1\}, j \in \{n_1 + n_2 + 1,\dots,n_1 + n_2 + n_3\}$$

and $i \in \{n_1 + n_2 + 1,\dots,n_1 + n_2 + n_3\}, j \in \{1,\dots,n_1\}$ then in the proposed model it is not possible to transmute directly from the state of set/positional defense into the state of transition offense. That is because to transit from the set defense into the transition offense the decisive state of either success or failure of any previous play action should occur and, secondly, because it is not possible to transit directly from the transition offense into the set defense. Out of this follows that $P_{1,3} = P_{3,1} = 0$ (null matrix). Similar is valid for the transition from the set offense into the transition defense, and vice versa. Further analysis of game structure discloses the matrix P is the block matrix of the following form:

$$P = \left[\begin{array}{cccc|c|c|c} P_{1,1} & 0 & 0 & 0 & P_{1,5} & P_{1,6} & P_{1,7} \\ P_{2,1} & P_{2,2} & 0 & 0 & P_{2,5} & P_{2,6} & P_{2,7} \\ 0 & 0 & P_{3,3} & P_{3,4} & P_{3,5} & P_{3,6} & P_{3,7} \\ 0 & 0 & 0 & P_{4,4} & P_{4,5} & P_{4,6} & P_{4,7} \\ \hline P_{5,1} & P_{5,2} & P_{5,3} & 0 & 0 & 0 & 0 \\ \hline 0 & P_{6,2} & P_{6,3} & P_{6,4} & 0 & 0 & 0 \\ \hline p_{7,1} & p_{7,2} & p_{7,3} & p_{7,4} & 0 & 0 & 0 \end{array}\right] \tag{3}$$

The elements of the matrix P are interpreted in the following way:

1. Matrices $P_{i,5}$, $P_{i,6}$ and $P_{i,7}$ for $i=1,\dots,4$ are **performance indicators** within a particular game phase, because they contain probabilities of entering the states of success/failure.
2. $P_{2,1}$ and $P_{3,4}$ are the basketball game **intensity (tempo) control indicators**. In borderline instances, when blocks $P_{2,1}$ and $P_{3,4}$ converge to null matrix, that indicates high intensity and uncontrolled play.

A particular phase of a game can be divided into the initial, intermediate and final sub phases (Perica, 2011; Jelaska, 2011), where it is possible to have I_i initial states, M_i intermediate states and F_i final states, and where the following is valid $I_i + M_i + F_i = n_i$.

The matrix containing transition probabilities for the phases of positional/set and transition offense, $P_{i,i}$ $i=3,4$ (a diagonal elements of the block matrix P) will be a squared matrix with n_i rows and n_i columns of the following shape:

$$P_{i,i}=\left[\begin{array}{ccc|ccc|ccc}
0 & \cdots & 0 & p_{1,I_i+1} & \cdots & p_{1,I_i+M_i} & p_{1,I_i+M_i+1} & \cdots & p_{1,n_i}\\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & \cdots & 0 & p_{I_i,I_i+1} & \cdots & p_{I_i,I_i+M_i} & p_{I_i,I_i+M_i+1} & \cdots & p_{I_i,n_i}\\
\hline
0 & \cdots & 0 & p_{I_i+1,I_i+1} & \cdots & p_{I_i+1,I_i+M_i} & p_{I_i+1,I_i+M_i+1} & \cdots & p_{I_i+1,n_i}\\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & \cdots & 0 & p_{I_i+M_i,I_i+1} & \cdots & p_{I_i+M_i,I_i+M_i} & p_{I_i+M_i,I_i+M_i+1} & \cdots & p_{I_i+M_i,n_i}\\
\hline
0 & \cdots & 0 & 0 & \cdots & 0 & p_{I_i+M_i+1,I_i+M_i+1} & \cdots & p_{I_i+M_i+1,n_i}\\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & \cdots & 0 & 0 & \cdots & 0 & p_{n_i,I_i+M_i+1} & \cdots & p_{n_i,n_i}
\end{array}\right]$$

The first block matrix of transition probabilities is the null matrix according to the definition of variables of the initial states of the positional/set and transition offense (Jelaska, 2011; Perica, 2011).

The matrix containing transition probabilities for the phase of positional/set and set defense $P_{i,i}$ $i=1,2$ will be a squared matrix with n_i rows and n_i columns of the following shape:

$$P_{i,i}=\left[\begin{array}{ccc|ccc|ccc}
p_{1,1} & \cdots & p_{1,I_i} & p_{1,I_i+1} & \cdots & p_{1,I_i+M_i} & p_{1,I_i+M_i+1} & \cdots & p_{1,n_i}\\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
p_{I_i,1} & \cdots & p_{I_i,I_i} & p_{I_i,I_i+1} & \cdots & p_{I_i,I_i+M_i} & p_{I_i,I_i+M_i+1} & \cdots & p_{I_i,n_i}\\
\hline
0 & \cdots & 0 & p_{I_i+1,I_i+1} & \cdots & p_{I_i+1,I_i+M_i} & p_{I_i+1,I_i+M_i+1} & \cdots & p_{I_i+1,n_{i,i}}\\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & \cdots & 0 & p_{I_i+M_i,I_i+1} & \cdots & p_{I_i+M_i,I_i+M_i} & p_{I_i+M_i,I_i+M_i+1} & \cdots & p_{I_i+M_i,n_i}\\
\hline
0 & \cdots & 0 & 0 & \cdots & 0 & p_{I_i+M_i+1,I_i+M_i+1} & \cdots & p_{I_i+M_i+1,n_i}\\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & \cdots & 0 & 0 & \cdots & 0 & p_{n_i,I_i+M_i+F_i} & \cdots & p_{n_i,n_i}
\end{array}\right]$$

The matrix has a structure in accord with the definition of variables of the positional/set and transition defense (Jelaska, 2011; Perica, 2011).

The interpretation goes further as follows.

1. Diagonal block matrices $P_{i,i}$, $i=1,...,4$ depict the **game combinatory**.

The upper right sub block of the matrix $P_{i,i}$ indicate again game intensity or tempo of play. When it tends to the null block matrix, it indicates the controlled game flow („prolonged offense").

Our assumption is also that the process is stationary within the framework of one match, that is, particular probabilities do not change during a match.

The future experimental research studies should reveal preferred game models (combinatory) in every individual team as well as in the elite class of European basketball as a whole (Euroleague). In other words, the preferred „walks" along the Markov chain should be found within a particular phase and sub phase of game flow.

The process of in-season sports preparation and team play coordination improvement will probably change values of certain matrix elements, that is, transition probabilities will change. Efficiency, or successfulness increase of the preferred combinatory is expected in all phases of game flow.

CONCLUSIONS

Investigations, validation, and evaluation of events in the game of basketball are necessary for scientific foundation of applicative kinesiology in sport. In the present paper the concept of game states has been defined from the aspect of the Markov chains' application in the theory of complex sports activities. The continuous game flow has been discretized and explained as one characteristic sequence, order of game states. An abstract system of basketball game has been shaped in the paper as the theoretical foundation for its later detailed elaboration and verification of its operation. The discretization of the continuous course or flow of a basketball game and the definition of equivalence among game states have given the prerequisites for the determination of transition probabilities between system states. A mathematical model, the Markov chain and discrete stochastic process have been used to describe the interaction among the system states. The matrix of transition probabilities has been structured between particular states of the Markov chain. The proposed model differentiates game states within four phases of game flow and enables the prediction of the future states. The application of the model in kinesiology/sport science research studies will allow the recognition of prevailing game tendencies in the European elite basketball, thus enabling, through the calculated parameters, the recognition whether there are any tendencies to control game intensity, what are the levels of tactical combinatory, and what are the preferred tactical models of particular teams.

The future research guidelines should establish and explain the relationships and connections between particular game states and game phases, and the determination of numerical values in the proposed operational model. So, the analysis of transition probabilities matrix values should probably be a considerable contribution to the situational approach in the empirical verification of basketball regularities and balanced game principle.

From the aspect of sports games theory, the continuation of research is necessary on intrinsic set of information (internal states of players and the demands specific for a particular type of players), game states, game flow and balanced game as components of the system "basketball game". The final, eventual aim should be the determination of relations and correlations between kinematic parameters of game states and intrinsic parameters of balanced game, as well as the construction of a mathematical model which will embrace these associations also.

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