ESTIMATION OF FATIGUE STRESS CONCENTRATION FACTOR IN THE ZONE OF ELASTIC-PLASTIC STRAIN

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ABSTRACT

For the assumed computing model the simple explicit formula for fast estimation of fatigue stress concentration factor in the region of fatigue failure is derived in this paper. This formula enabled a derivation of a new relation for the product of stress and strain concentration factors. The magnitude of this product depends on amplitude stress level, and it is allways between the values calculated after original and modificated Neuber's rule.

KEYWORDS

Stress concentration factor; elastic-plastic strain region; quasistatic fracture region.

INTRODUCTION

The most popular method to estimate the approximate value of fatigue stress concentration factor in the region of elastic-plastic strain is based on empirical investigations of Hardrath and Ohman [1], Neuber [2] and Wetzel [3], who proposed the widely known relations between fatigue stress concentration factor K_σ and strain concentration factor K_ε. Intersection of the corresponding curve with the cyclic stress-strain curve determines the magnitude of local stress and strain and consequently the approximatelly real values of stress and strain concentration factor.

It was perceived that values of K_σ, obtained in described way after [1] and [2], are higher than the real ones, and those obtained after [3] less then real ones. This lack of precision makes this method unreliable. The method is also time-consuming. Therefore it became necessary to obtain a simple and reliable explicit formula for fast estimation of stress concentration factor in the region of elastic-plastic strain, or to modify the Neuber's rule once again.

DERIVATION OF FATIGUE STRESS CONCENTRATION FACTOR

On the basis of worldwide experimental investigations, including the author's investigations, *S-N* curves for smooth specimens and parts (same as those for notched specimens) might be desplayed as straight lines in $\log S$-$\log N$ coordinates (Fig.1). Every *S-N* curve is devided in three characteristic zones: The zone of quasistatic fracture ($N<N_q$), the zone of finite fatigue life ($N_q<N<N_{gr}$), and the zone of infinite life ($N>N_{gr}$), which is in the same time the zone of elastic strain. We shall limit our consideration at the zone of finite fatigue life, which is also a part of zone of elastic-plastic strain. The slope of any curve in this zone and its boundaries, depend upon material and shape of the part or

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specimen. The limit between finite and infinite life is taken to correspond with the limit between elastic and elastic-plastic strain region. For a materials without endurance limit, it might be taken $N_{gr}=10^7$. The same holds good for steels at elevated temperatures. For the notched specimen (same as for the component in the zone of stress concentration) the N_{gr} boundary is moved towards the region of less number of cycles. This boundary might be expressed as

$$N'_{gr}=k_{gr}N_{gr} \,. \tag{1}$$

The applied computing model after Fig.1 is designed on the base of the stress amplitude and fatigue life average values at the boundary points of great number of notched specimen *S-N* curves. Some possible aberrations are neglected. For example, it has long be known that both in the quasistatic fracture zone, and at the initial stage of low cycle fatigue failure of ductile metals, the strength of notched specimens is often higher then for the plain specimens. Also for brittle materials it is known, e.g. [4], that for some kinds of contrentrators, stress concentration factor is less of unity, i.e. the strength of notced specimen is then greater of that unnotched.
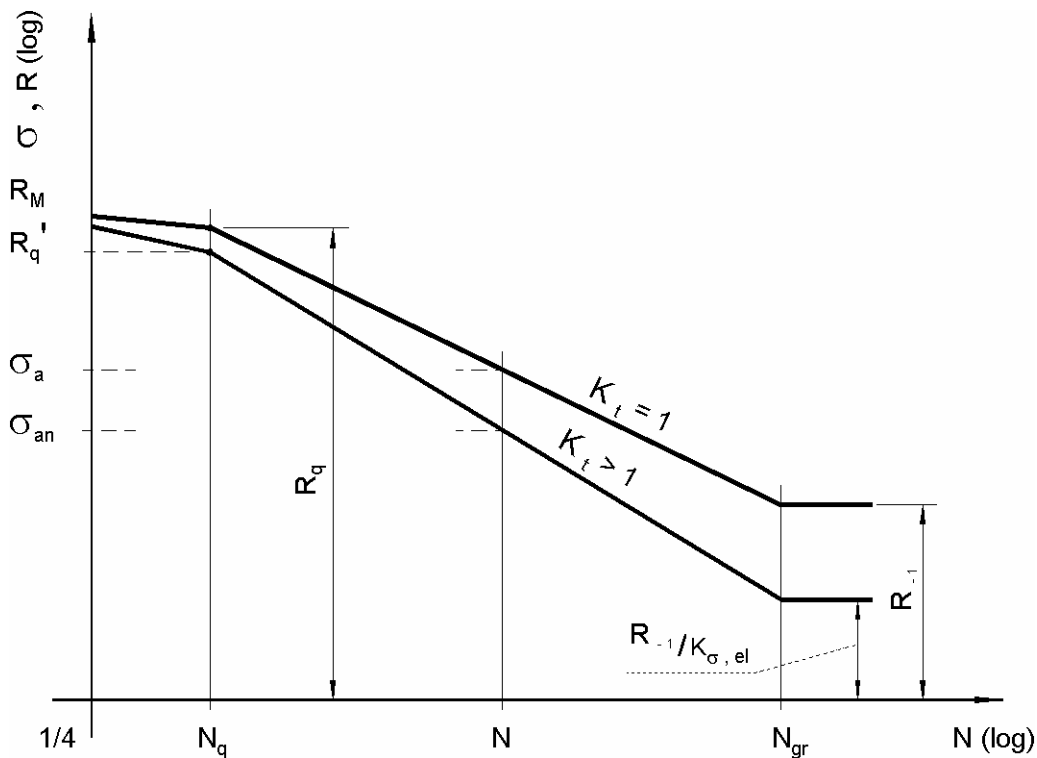

Fig. 1- Stress concentration factor computing model

It is evident that both stress concentration factor computing models for brittle and for ductile materials, might be reduced at unique computing model by involving $K_{\sigma,q}=1$ for ductile materials.

In the region of finite fatigue life the equations of both *S-N* curves might be written in the form:

$$\sigma_a N^{-b} = R_{-1} N_{gr}^{-b} = R_q N_q^{-b} \tag{2}$$

$$\sigma_{an} N^{-b'} = (R_{-1} / K_{\sigma,el})(N_{gr}')^{-b'} = (R_q / K_{\sigma,q})N_q^{-b'} \,. \tag{3}$$

According to definition

$$K_\sigma = \frac{\sigma_a}{\sigma_{an}} = \frac{R_{an}}{R_a} \,, \tag{4}$$

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from (2) and (3) it follows:

$$K_\sigma = k_{gr}^{b'} K_{\sigma,el} (N / N_{gr})^{b-b'},$$ (5)

or

$$K_\sigma = k_e K_{\sigma,el} (R_{-1} / K_{\sigma,el} \sigma_{an})^{1-b/b'}$$ (6)

From (2) and (3), according to Fig.1, it is easy to derive the ratio

$$\frac{b}{b'} = \frac{\log(N_{gr} / N_q) + \log k_{gr}}{\log(N_{gr} / N_q) - (1/b)\log(K_{\sigma,el} / K_{\sigma,q})} .$$ (7)

Whereas the ratio N_{gr} / N_q for most materials is close to 10^3, the exponent ratio might be approximated as

$$\frac{b}{b'} \cong \frac{3 + \log k_{gr}}{3 - \log(K_{\sigma,el} / K_{\sigma,q})^{1/b}} .$$ (7a)

Fatigue stress concentration factor in elastic region is usually available information. So, stress concentration factor $K_{\sigma,q}$ at quasistatic fracture boundary and k_{gr} factor remain two unknown in expressions (3) to (7a). On the basis of regression analysis of a great number of experimental investigations, e.g. [5,6,7,8,9], the stress concentration factor at quasistatic fracture limit for steels that fail in brittle manner might be estimated as

$$K_{\sigma,q} = 1 + (0,38 \cdot 10^{-3} R_M - 0,1)(K_{\sigma,el} - 1) .$$ (8)

and reducing coefficcient of N_{gr} boundary has been found to be

$$k_{gr} = 1,23 - 0,25 K_{\sigma,el} + 0,02 K_{\sigma,el}^2 .$$ (9)

The fatigue stress concentration factor K_σ obtained in this way, if substituted in cyclic stress-strain curve

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}}$$ (10)

enables the estimation of the strain concentration factor in the region of elastic-plastic strain:

$$K_\varepsilon = K_{\sigma,el} + E K'^{-1/n'} (K_{\sigma,el} \sigma_{an})^{1/n'-1}$$ (11)

For nine steels having the cyclic strenghtening characteristics K' and n' after [10] , the values of K_ε are calculated for various magnitudes of σ_{an}. It was observed that $K_\sigma K_\varepsilon$ product is not in accordance with either Neuber's hyperbola law $K_\sigma K_\varepsilon = K_t^2$, or with modified hyperbola low $K_\sigma K_\varepsilon = K_{\sigma,el}^2$.

For that reason, by means of "Mathematica" computer program, a new relation between stress and strain concentration factor in the region of finite fatigue life is obtained for the steels:

$$K_\sigma K_\varepsilon = K_{\sigma,el}^{2+0,57(\sigma_{an}/R_{-1}-1/K_{\sigma,el})} \tag{12}$$

This equation might be generalized for all materials, shape of parts and kind of joints:

$$K_\sigma K_\varepsilon = K_{\sigma,el}^{2+a(\sigma_{an}/R_{-1}-1/K_{\sigma,el})} \tag{12a}$$

where a is a constant, which should be determined experimentally.

From this equation it is clear that the product of stress and strain concentration factors in the region of elastic-plastic strain increases with stress amplitude σ_{an} and is equal to $K_{\sigma,el}^2$ (after [3]) only at N_{gr} limit. This new relation represents a hyperbola located between the original Neuber's hyperbola and the modified one, and it determines stress and strain concentration factor if the cyclic stress-strain curve is known (Fig.2).

For the assumeded exponential relation between stress and number of cycles of the notched and smooth specimens in the *region of quasistatic fracture*, it was easy to derive the expression for the approximate estimation of stress concentration factor:
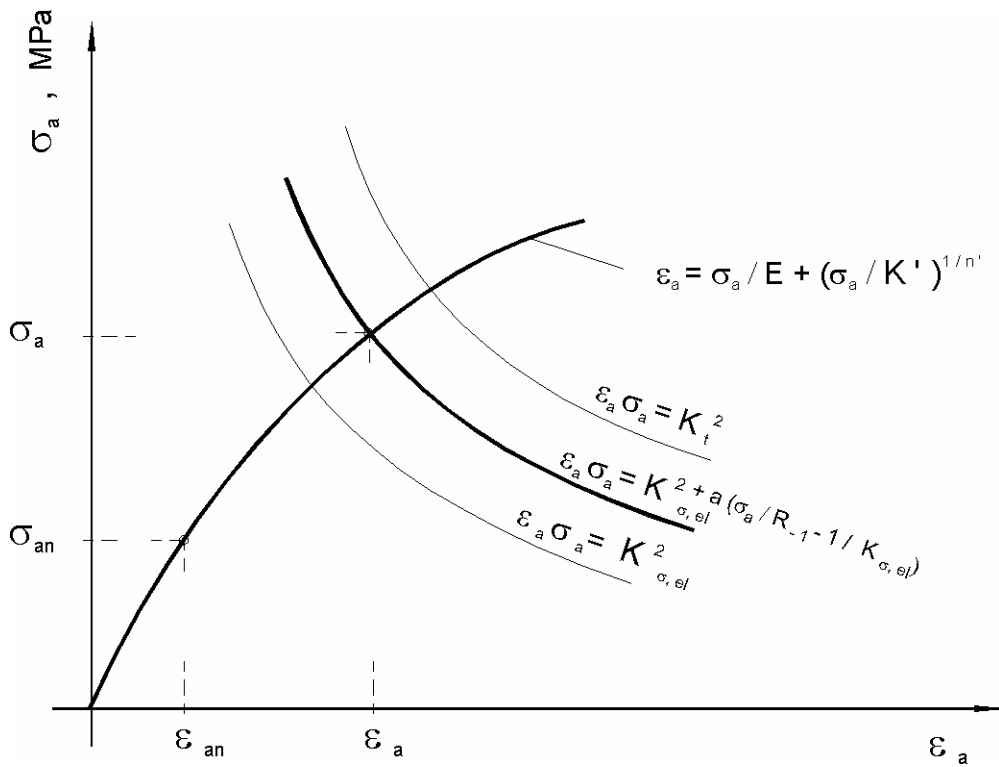

Fig.2- Determining of local stress and strain

For the assumeded exponential relation between stress and number of cycles of the notched and smooth specimens of brittle materials in the *region of quasistatic fracture*, it was easy to derive the expression for the approximate estimation of stress concentration factor:

$$K_{\sigma,qs} = K_{\sigma,s}(K_{\sigma,q} / K_{\sigma,s})^{\frac{\log 4N}{\log 4N_q}} \qquad (13)$$

where stress concentration factor $K_{\sigma,s}$ at static fracture depends upon K_t and on notch sensitivity related to the level of plastic deformation at static fracture, and it is inside limits $0,67 \le K_{\sigma,s} < K_{\sigma,q}$. For fully sensitive materials (such as titanium, beryllium and most of its alloys) is $K_{\sigma,s} = K_{\sigma,q} = K_t$. Nevertheless, for most even brittle materials at static fracture ($N=1/4$), it might be taken $K_{\sigma,s} = 1$, and consequently

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$$K_{\sigma,qs} = K_{\sigma,q}^{\frac{\log 4N}{\log 4N_q}} \quad . \qquad (13a)$$

N_q is an exsclusive characteristic of material and varies within limits $2,6 \cdot 10^2$ to 10^5 for metals. The same boundary values also apply to steels, but most frequently it is within limits 10^3 to 10^4.
The fatigue stress concentration factor relation for ductile materials is obtained by involving $K_{\sigma,q}=1$ in Eq. 12.

CONCLUDING REMARKS

Stress concentration factor calculated in the way described above, is as close to its true value, as the suggested computational model is close to the real one. It also depends on basic data reliability, particularly on stress concentration factor boundary values $K_{\sigma,q}, K_{\sigma,el}$ and $K_{\sigma,s}$. A special attention has to be paid to $K_{\sigma,q}$ factor for brittle materials, the suggested calculation of which is a good approximation only for steels. For other brittle materials additional investigations have to be made in order to obtain an applicable relation for determining of this factor. For that reason the author proposes to apply the above mentioned Neuber's method (Fig.2) using obtained relation (12), because this method becomes now much more precise.

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NOMENCLATURE

a- material constant

b - fatigue strength exponent

b'- fatigue strength exponent of the notched specimen

E- modulus of elasticity, MPa

K_t - theoretical stress concentration factor

K_ε- strain concentration factor

K_σ- stress concentration factor

K'- coefficient of cyclic strengthening, MPa

N - number of endurable cycles

n' - cyclic strengthening exponent

R- strength (generally), MPa

R_M - ultimate strength of a material, Mpa

R_q' - notched specimen strength at quasistatic fracture boundary, MPa

R_{-1} - endurance limit of material at -1 stress cycle asymmetry number, MPa

σ_a - local stress amplitude, MPa

σ_{an}-nominal stress amplitude, MPa

Subscripts

el- elastic region

gr- boundary between finite and infinite life region

s- static fracture

q- boundary betwen quasistatic fracture region and finite life fatigue region