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REDISTRIBUTIVE EFFECTS OF DIRECT TAXES AND SOCIAL BENEFITS IN CROATIA

Doktorska disertacija

Ljubljana, 2010

Statement of authorship

I, Ivica Urban, hereby state that I'm the author of the doctoral dissertation titled "Redistributive effects of direct taxes and social benefits in Croatia".

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REDISTRIBUTIVNI UČINKI NEPOSREDNIH DAVKOV IN SOCIALNIH PREJEMKOV NA HRVAŠKEM

POVZETEK

Disertacija vključuje analizo uveljavljenih konceptov v merjenju dohodkovne neenakosti, kot je na primer Kakvanijeva dekompozicija redistributivnega učinka v vertikalni del in učinek prerangiranja. Poleg tega so v disertaciji razviti novi metodološki koncepti, ki so tudi uporabljeni za empirično oceno dohodkovne redistribucije na Hrvaškem za obdobje 2001 do 2006.

Prva hipoteza v disertaciji je, da prerangiranje ne more vplivati na redistributivni učinek na način, kot ga navajajo raziskovalci, ki so uporabili Kakwanijevo razčlenitev. Ti trdijo, da vsebuje vertikalni učinek (V^K) potencialni redistributivni učinek, ki bi bil dosežen, če bi bilo prerangiranje, merjeno z Atkinson-Plotnickovim indekom (R^{AP}) nekako eliminirano. V disertaciji je dokazano, da bi bila to nemogoče: izključitev prerangiranja vedno vodi k zmanjšanju vertikalnega učinka in redistributivni učinek ostane nespremenjen.

Disertacija prav tako pokaže, da ima kazalec vertikalnega učinka (V^K) kot merilo progresivnosti pomembne pomanjkljivosti prav zato, ker ni neodvisen od prerangiranja in ga pravzaprav vsebuje. Nekatera od teh stališč so predstavili že drugi raziskovalci, vendar so le-ta v disertaciji razširjena in analitično dokazana. Iz tega sledi naslednji sklep disertacije - izognitev uporabi V^K kot kazalca progresivnosti.

Izključevanje nekaterih interpretacij in načinov uporabe kazalcev je ustvarilo metodološko praznino, ki jo je bilo treba zapolniti. To je narejeno z novimi kazalci, ali raje z novimi interpretacijami obstoječih kazalcev: kazalcev fiskalne deprivacije, fiskalne dominacije, zmanjševanja razlik in drugih, ki jih je treba uporabljati in razvijati naprej v okviru novih kontekstov.

V disertaciji je predstavljena še ena metodološka inovacija: razčlenitev učinka prerangiranja za oceno doprinosa posameznih davkov in prejemkov. Posledica tega pristopa je očitna: iz podatkov

lahko izračunamo, v kakšni meri vsak davek in prejemek prispeva k redistributivnemu učinku in prerangiranju.

Kazalci in razčlenitve se nato uporabijo za preverjanje druge hipoteze, ki je empirična in povezana s hrvaškim fiskalnim sistemom. Analiziran je del fiskalnega sistema, ki sestoji iz prispevkov za socialno varnost, dohodnine, javnih pokojnin in denarnih socialnih prejemkov. Podatki so pridobljeni iz anket o porabi gospodinjstev, davčne spremenljivke pa so izračunane z mikrosimulacijskim modelom, izdelanim za namene disertacije.

Empirična analiza je potrdila hipotezo, da je fiskalni sistem ena od primarnih determinant dohodkovne neenakosti na Hrvaškem. Razpon zmanjšanja dohodkovne neenakosti, ki ga povzroča fiskalni sistem, sega od 10% za najmanj obširno definicijo fiskalnega sistema, ki vključuje samo dohodnino in denarne socialne prejemke, do 40% za najobširnejšo definicijo fiskalnega sistema, ki zajema javne pokojnine in prispevke za socialno varnost, skupaj z dohodnino in denarnimi socialnimi prejemki.

Če upoštevamo slednjo, najobširnejšo definicijo fiskalnega sistema, je največji del prerazdelitve dohodkov dosežen z javnimi pokojninami, prispevki za socialno varnost in dohodnino. Vendar pa raziskava ne nudi dokončnega zaključka glede zaporedja teh treh skupin fiskalnih instrumentov glede pomembnosti za zmanjšanje dohodkovne neenakosti. Ena od hipotez predpostavlja, da javne pokojnine v največji meri prispevajo k redistributivnemu učinku. Skupina razčlenitev, ki temelji na "odstopanju davkov in prejemkov od proporcionalnosti", jo je vsekakor dokazala. Na drugi strani pa so razčlenitve, ki temeljijo na "zneskih davkov in prejemkov", pokazale, da so prispevki za socialno varnost in dohodnina veliko pomembnejši od javnih pokojnin. Razlika v rezultatih je posledica kriterijev za definiranje progresivnosti in regresivnosti. Pristop "zneskov" teži k poudarku primarne vloge davkov, pristop "odstopanj" pa podpira prejemke. Posledica tega dognanja je zanimiva za raziskovalce, saj utegne nuditi novo perspektivo glede vloge davkov in prejemkov pri zmanjšanju dohodkovne neenakosti.

Nadaljnji rezultati empiričnega dela zadevajo prerangiranje dohodkovnih enot. V kakšni meri je prerangiranje vpeljano v hrvaški fiskalni sistem? Ocene prerangiranja so v veliki meri odvisne od načina definiranja fiskalnega sistema, in razpon meril sega od skromnega 1% od G_X (6% od RE) za ozko definiran sistem, ki vključuje samo dohodnino in denarne socialne prejemke, do

vrednosti nad 15% od G_X (34% od RE), ko fiskalni sistem vključuje javne pokojnine in prispevke za socialno varnost, skupaj z dohodnino in denarnimi socialnimi prejemki.

Če torej vzamemo najširšo definicijo fiskalnega sistema, ki vključuje javne pokojnine kot prejemke in prispevke za socialno varnost kot davke, so javne pokojnine nedvomno tiste, ki z več kot 75% deležem največ doprinesejo k prerangiranju. Sledijo jim prispevki za socialno varnost z 20% deležem. Na drugi strani pa dohodnina na prerangiranje vpliva le malo.

Prerangiranje splošno velja za nepravično in posledica teh ugotovitev je, da bi nosilci politik lahko zmanjšali neenakost, ki jo občuti javnost, s preoblikovanjem nekaterih delov fiskalnega sistema.

Ključne besede: davki in prejemki, redistributivni učinek, horizontalna enakost, vertikalna enakost, dekompozicija, mikrosimulacija

REDISTRIBUTIVE EFFECTS OF DIRECT TAXES AND SOCIAL BENEFITS IN CROATIA

SUMMARY

The dissertation analyses the most widely used concepts in measurement of income redistribution, such as Kakwani's decomposition of the redistributive effect into vertical and reranking terms. Certain methodological problems were recognized and solved upon an extensive study. The adapted and newly developed methodological concepts are then used to estimate the process of income redistribution in Croatia, during the period of 2001 to 2006.

The first hypothesis is that reranking cannot influence the redistributive effect (RE), in a manner proposed by researchers using Kakwani's decomposition. They claim that the Kakwani vertical effect (V^K) has a meaning of potential redistributive effect, which would be achieved if reranking, measured by the Atkinson-Plotnick index of reranking (R^{AP}), would be somehow eliminated. However, this dissertation proves that such a task is incomprehensible: 'elimination of reranking' always leads to decrease of the vertical effect leaving the redistributive effect unchanged.

The analysis also shows that V^K has major weaknesses as a measure of progressivity, exactly because it is not independent of reranking; actually, it contains reranking in itself. Some of these notions were proposed already by other researchers, but in this work they are extended and analytically proven. Thus, another implication of this research would be to avoid using V^K as an index of progressivity.

Ruling out some interpretations and uses of indices created a hole which had to be filled. Therefore, new indices – or rather new interpretations of existing indices - are proposed: indices of fiscal deprivation, fiscal domination, distance narrowing, and others. The implication is that the well-known indices do not have to be discarded; instead, they are given new meanings, and should be used and developed further in new contexts.

Another methodological innovation is presented: decomposition of the reranking effect to reveal contributions of individual taxes and benefits. The implication of this innovation is self-evident: we can learn from the data how much each tax and benefit contributes to the redistributive effect and reranking.

These new indices and decompositions are then applied to verify the second hypothesis, which is empirical and relates to the Croatian fiscal system. The study has analysed a section of the fiscal system in Croatia, consisting of social security contributions, personal income tax, public pensions and cash social benefits. The data are obtained from the household budget survey and tax variables are imputed using a microsimulation model designed purposefully for this study.

The empirical analysis has confirmed the hypothesis that the fiscal system is one of the prime determinants of disposable income inequality in Croatia. Reduction of income inequality caused by the fiscal system ranges from 10% for the least comprehensive definition of the fiscal system, including only personal income tax and cash social benefits, to 40% for the most inclusive definition of the fiscal system used in this research, capturing public pensions and social security contributions, together with personal income tax and cash social benefits.

If we assume the latter, all-inclusive definition of the fiscal system, the largest part of income redistribution is achieved by public pensions, social security contributions, and personal income tax. However, this study does not offer the definite conclusion about the order of importance of these three groups of fiscal instruments in achieving inequality reduction. One of the hypotheses claimed that public pensions are the main contributor to redistributive effect. Indeed, one set of decompositions; those based on "deviations of taxes and benefits from proportionality", provided evidence for that conclusion. On the other side, decompositions based on "amounts of taxes and benefits" have shown that social security contributions and personal income tax are far more important than public pensions. Those two approaches significantly differ in criterions by which progressivity and regressivity are defined. Therefore, it is not surprising that results are divergent. The "amounts" approach is inclined to stress the primary role of taxes, and "deviations" approach favours benefits. The implication of this finding is interesting for researchers as it may offer a new perspective on a role of taxes and benefits in inequality reduction. Whatever the order of importance of major fiscal instruments in fiscal distribution, the important finding of this research is that they all decrease income inequality.

Further findings of the empirical section are concerned with reranking of income units. How much reranking is introduced by Croatian fiscal system? The estimates of reranking are largely dependent on how we define the fiscal system, and the measures range from modest 1% of G_X (6% of RE) for the narrowly defined system, containing only personal income tax and cash social benefits, to vary large amounts of over 15% of G_X (34% of RE) when the fiscal system is widely defined, involving public pensions and social security contributions, together with personal income tax and cash social benefits.

Again, if we assume the widest definition of the fiscal system, which includes public pensions as benefits and all social security contributions as taxes, then, public pensions are undoubtedly the largest contributor to reranking, with a share of more than 75%. They are followed by social security contributions, whose share is about 20%. Personal income tax, on the other side, contributes only mildly to reranking.

Reranking is generally considered as inequitable, and implication of these findings is that the policy makers could reduce inequity felt by the public by redesigning some parts of the fiscal system. However, they have to be very cautious in interpretation of the results.

Keywords: taxes and benefits, redistributive effect, horizontal equity, vertical equity, decomposition, microsimulation

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1 INTRODUCTION

1.1 Subject and objectives of analysis

1.1.1 Measurement of income redistribution

The last three decades have seen a world-wide interest in the *measurement of redistributive effects of fiscal systems*. The research in this field is underpinned by a wide belief that the state has a major role in the determination of economic inequality in a society. The contention is undeniably proved by various empirical studies. Many researchers in the field of income redistribution posed further natural questions: what are the *contributions of individual taxes and benefits* to the redistributive effect?

The literature on the measurement of redistributive effect started to grow in the mid-seventies, perhaps half of decade after the seminal works in the closely related field of economic inequality measurement. Although the first famous paper on the redistributive effect was written by Musgrave and Thin (1948) sixty years ago, the works of Jakobsson (1976) and Kakwani (1977a, 1977b) have established the propositions fundamental for all further research. As Lambert (2001) points out: "Their central results are known […] as *Jakobsson/Kakwani* theorems, and they expose the links between progressive income taxation and concentration curve properties of the distributions of tax and of post-tax income".

Progressive tax pushes the income Lorenz curve toward the line of equality. The magnitude of this movement, measured as the difference between Gini coefficients of pre-tax and post-tax incomes, is nowadays known as the *redistributive effect*, and the greater the tax progressivity and the average tax rate, the larger the redistributive effect will be. All these notions were explained by Kakwani (1977b). However, one phenomenon remained hidden for some time, until Atkinson (1980) and Plotnick (1981) uncovered it. They noticed that taxation induces another process besides narrowing of income distances – income units reranking, which is simply measured as a difference between the Gini and the concentration index of post-tax income.

The famous synthesis of all these concepts arrived in Kakwani (1984) as a decomposition of redistributive effect into vertical or progressivity and reranking terms. It became and remained one of the most important tools in the income redistribution literature. The popularity of this decomposition rests on its comprehensiveness (capturing different notions of redistributive justice), simplicity and ease of computation as well as its availability for straightforward policy interpretation (redistributive power can be enhanced if horizontal inequity is reduced).

Methodological apparatus for the decomposition of redistributive effect in order to reveal relative contributions of individual taxes and benefits was first proposed by Lambert (1985). The model helped to reveal an important interaction between taxes and benefits: even if the tax system is regressive, taxes can still be regarded as a reinforcing factor of redistributive effect of the net fiscal system. However, this model assumed the absence of reranking, and in fact, it decomposed Kakwani (1984) vertical effect of fiscal system, and not the redistributive effect or reranking effect.

1.1.2 The original objective: income redistribution in Croatia

Several studies have confirmed that income inequality in Croatia is mild in international comparisons. Is the relatively low income inequality inherent to Croatian economy, or is it a consequence of fiscal activities? Given the experience of other countries and the fact that the share of government expenditure in GDP is high, we may anticipate that the government has a significant influence on the distribution of income in Croatia. However, since the distribution of taxes and benefits in Croatia was only partially investigated, we lack a confident proof for this assumption.

The primary objective of my research was to answer the following questions: (1) How does the fiscal system affect income inequality in Croatia, and (2) How do different fiscal instruments contribute to this effect? The research aimed to analyse the redistributive effects of several taxes and benefits in Croatia: social security contributions, personal income tax, public pensions, and means-tested and non means-tested cash benefits.

1.1.3 Methodological aspects of the research

In the beginning of the research the empirical aspects were of primary interest. The choice of the measurement models fell on the most widely used decompositions of redistributive effect, those proposed by Kakwani (1984) and Lambert (1985). However, at a certain point, two problems with the methodology became apparent, concerning: (a) interpretation of the measures of vertical and reranking effect, (b) decomposition of reranking effect. Before continuing with the empirical application, the methodological nuisances had to be resolved.

The first problem is related to the role of reranking in the redistributive process. Reviewing the literature again, it was discovered that the problem was anticipated by some scholars, but neglected later by others. On one side, there were Atkinson (1980) and Plotnick (1981), the inventors of reranking effect, who claimed that reranking of units does not affect the distribution of post-fiscal income. They warned that a measure of reranking effect should not be involved into some more comprehensive measurement system that would also attempt to measure progressivity. On the other hand, Kakwani (1984) did not follow the advice, but built a model that captured both reranking and progressivity. He also claimed the active role of reranking.

The second problem concerns the decomposition of reranking to reveal the contributions of taxes and benefits. As already mentioned, Lambert (1985) decomposition decomposes only vertical effect. There were two attempts to decompose reranking, by Jenkins (1988) and Duclos (1993). However, each of them has certain limitations.

Since the two issues mentioned above seemed crucially important, a decision was made to devote a greater part of research to finding solutions for these controversies. The research resulted in new indices and reinterpretations of the existing indices, and several new decompositions. Still, the empirical objective was not forgotten. All the various indices and decompositions are applied to Croatian data for the period 2001 to 2006.

A lot of effort was invested in data preparation and the evaluation of their relevance. Namely, the Croatian household budget data, which were selected as a source in this research, do not contain information on personal taxes. Therefore, they had to be imputed by purposefully designed microsimulation model.

1.2 Hypotheses

Based on the thoughts explained above, it was decided to propose one methodological hypothesis (1) and a set of empirical hypotheses (2).

(1) Kakwani (1984) offers the *mistaken interpretation* of the role of reranking in the redistributive process. The view taken here is that reranking of income units cannot influence the redistributive effect, RE. It was first proposed by Atkinson (1980), but neglected later by the above authors.

(2) Government redistributive policies are one of the prime determinants of disposable income inequality in Croatia. The largest part of income redistribution is achieved by public pensions but personal income tax and social security contributions also play an important role in it.

1.3 Methodology

As the section 1.1 announced, this dissertation, in its large part, deals with the methodology itself. The methodology in the field of redistributive effect measurement is examined, and the problems found are attempted to be proved and resolved. Naturally, certain *methodologies* were required for this task. The basic tool for the whole analysis is Gini coefficient. There are many different approaches to calculating Gini coefficient. In this research, three approaches are used, two of which provide a chance to watch how the Gini index is changed when small transfers between income units occur. Another method used to understand the redistributive process is the analysis of various transitions between income vectors ordered in different ways.

The most widely accepted models in the measurement of redistributive effect are based on Gini coefficients. Thus, the popularity of Gini coefficient in the measurement of economic inequality has also spread into the branch of income redistribution analysis. One of the reasons for the attractiveness of Gini coefficient is that it can be derived and interpreted in many different ways. The principal relationship stands between Gini coefficient and the

Lorenz curve. Consequently, the indices of redistributive effect are derived with respect to Lorenz and concentration curves.

The link between Gini coefficient and Lorenz curve is as natural and intuitive as it can be, but if we are interested in what happens to the Gini-based indices *when a small change occurs*, it is better to use some more straightforward analytical expressions. Formulas based on income ranks and income distances make it possible to see how a small transfer from one income unit to another affects the measure. This methodological approach is well-known in the literature, where it was often used to show that Gini coefficient satisfies the "principle of transfers", but does not satisfy the "principle of diminishing transfers" (for example, in Lambert, 2001:35).

In fact, we will show that the whole redistributive process can be imagined as a series of small transfers between income units. The indices of redistributive effect and reranking can be thus reconstructed scrutinizing their changes induced by these small transfers. Principally, the approach helps us realize why reranking cannot influence distance narrowing. When an analyst refers only to "aggregate" indices, important problems in interpretation may remain concealed. Therefore, all the indices of redistributive effect are expressed in terms of three different approaches besides Lorenz curves approach: "distance from mean", "distance between units" and "Gini welfare function" approach.

Another method used in this work is something we could call the *analysis of income vectors' transitions*. The population (or sample) income data are written into separate vectors for pre-fiscal income and post-fiscal income. The incomes of units can be sorted in any possible way resulting in multiple vectors, but we are interested in two orderings: by pre-fiscal and post-fiscal income. Thus, both pre-fiscal and post-fiscal incomes can be sorted either by pre-fiscal or post-fiscal income ordering, giving us four different income vectors. Now, we can analyse different transitions between these vectors to reveal their properties. Some of them are factual and the others are counterfactual. These transitions underlie the main indices of redistributive effect and reranking. The method is useful because knowing the nature and composition of each vector transition we can make judgements about important properties of indices based on them.

The third and probably the most lucrative method for understanding the nature and significance of the indices of redistributive effect and reranking goes down to the level of individual units again. We ask ourselves: What is the income unit's A "feeling" about its current income status in relation to unit B? Unit A may feel "advantage" or "supremacy" toward B if its income is higher, but this can be changed during the fiscal process: the distance may be lowered, "harming A's feelings" and B may even outrank A, having higher post-fiscal income, and deepening A's "deprivation" even further. Concepts like these facilitate understanding the meaning of various numerical indicators measuring the redistributive effect.

All the three methods mentioned above can be found in the literature on income redistribution. However, there is the fourth one that seems to be new. It originates in "distance between units" approach to the calculation of Gini coefficient, and is used to decompose the redistributive effects in order to estimate the contributions of individual taxes and benefits. This new approach uses specifically designed matrices containing differences between the values of a chosen variable, for each pair of income units. The variables are pre-fiscal and post-fiscal incomes, and individual taxes and benefits. By combining and comparing these matrices in different ways, and aggregating over all the pairs of units, we obtain main indices and their decompositions.

1.4 Structure

The dissertation is composed of six chapters. *Chapter 1* contains pre-defined sections explaining the subject, objectives, hypotheses and methodology of the dissertation. *Chapter 2* reviews the literature on the measurement of redistributive effects. It concentrates on Kakwani indices and decompositions which have received most attention among the scholars and researchers. This overview is not only a summary of methods used in the field; it is a critical exposition of the development of the methodological apparatus that reveals certain problems with the current approach in the measurement of redistributive effect. In this sense, it serves as an introduction to the subsequent, methodology part of the dissertation.

Chapter 3 is a methodological one and represents a core of the dissertation. In the *first* part, measurement frameworks are developed based on "distance from mean" and "distance between units" approaches to the calculation of Gini coefficient. The indices of redistributive effects, distance narrowing and reranking are derived and their connection with the existing indices in the literature is established. These new concepts are then used to interpret Kakwani-based decompositions of redistributive effect in order to prove the existing problems and suggest new measures of redistributive and reranking effects. In the *second* part of the chapter, the methodology is presented that decomposes these measures to reveal the contributions of individual fiscal instruments to the redistributive effect and reranking.

The following two chapters are devoted to the application of newly developed methodologies on the case of the Croatian fiscal subsystem consisting of individual taxes and cash benefits. *Chapter 4* deals with the data used in the empirical analysis. As a prelude to data issues, a brief description of the actual fiscal instruments is given followed by an extensive description of the microsimulation model used for imputation of taxes since these are not available in the original household survey data. The chapter ends with an evaluation of the quality of the data used in this study, which is made by comparing the adapted survey data with those from the administrative sources. *Chapter 5* analyses the empirical results for Croatia in the period from 2001 to 2006.

Chapter 6 concludes the dissertation, indicating limitations of the current research and suggesting the venues for further investigation.

2 LITERATURE OVERVIEW

2.1 Introduction

The literature on the measurement of redistributive effect started to grow in the mid-seventies, perhaps half a decade after the seminal works in the closely related field of economic inequality measurement. Although the first famous paper on the redistributive effect was written by Musgrave and Thin back in 1948, the works of Jakobsson (1976) and Kakwani (1977b) established the propositions fundamental for all further research. As Lambert (2001) points out: “Their central results are known […] as *Jakobsson/Kakwani* theorems, and they expose the links between progressive income taxation and concentration curve properties of the distributions of tax and of post-tax income”.

Progressive tax pushes the income Lorenz curve toward the line of equality. The magnitude of this movement, measured as the difference between Gini coefficients of pre-tax and post-tax incomes, is nowadays known as the redistributive effect, and it is larger, the greater are the tax progressivity and the average tax rate. All these notions were explained by Kakwani (1977b). However, one phenomenon remained hidden for some time, until Atkinson (1980) and Plotnick (1981) uncovered it. They noticed that taxation induces another process, besides narrowing of income distances – income units reranking, which is simply measured as a difference between the Gini and concentration index of post-tax income.

The great synthesis of all these concepts arrived in Kakwani (1984) as a decomposition of redistributive effect into *vertical* or progressivity term and *reranking* term. It remained one of the most important tools in the income redistribution literature. The popularity of this decomposition rests on its comprehensiveness (capturing different notions of redistributive justice), simplicity and ease of computation, and its availability for straightforward policy interpretation (redistributive power can be enhanced if horizontal inequity is reduced).

This chapter is primarily devoted to the Kakwani decomposition with a critical overview of its development in the first part of the chapter. The context is described in which the component terms emerged: the vertical and the reranking index. Atkinson’s and Plotnick’s sceptical views on introducing reranking into the context of progressivity and the redistributive effect measurement, which have been forgotten in the meantime, are revealed here. Lerman and Yitzhaki's (1995) criticism of Kakwani vertical term – that it contains reranking in itself – has remained underdeveloped, and further investigation of the problem is called for.

In the second part of the chapter, the main "upgrades" of Kakwani decomposition are reviewed. Aronson, Johnson and Lambert (1994) developed a model that separately measures vertical inequity, horizontal inequity and reranking; Duclos, Jalbert and Araar (2003) set the similar approach into the wider context of social welfare functions, and used more refined statistical estimation techniques. Lambert (1985) extended the Kakwani vertical effect to measure the individual contributions of tax and benefit instruments, while Pfähler (1990) decomposed it to show the contributions of tax schedule, allowances and deductions to the redistributive effect. Kakwani and Lambert (1998) propose a new measurement system, based on three axioms of equitable tax; corresponding indices are constructed which measure the reduction of redistributive effect due to violation of the axioms. Duclos (2000) redefines the well-known indices of inequality, progressivity, vertical effect and reranking, basing them on the theory of relative deprivation as well as notions such as fiscal harshness, looseness of the tax system, and ill-fortune.

The third part reviews the empirical research in the measurement of redistributive effect. The studies are presented chronologically and then classified by the methodology used. The review reveals that, although many different indices and methods were employed, the majority of research is based on Kakwani indices and decompositions.

2.2 Kakwani decomposition of redistributive effect

2.2.1 Redistributive effect

The standard and the most popular measure of redistributive strength of a fiscal (tax, or benefit) system is called the *redistributive effect* (RE). It is defined as a difference between Gini coefficients of pre-fiscal (-tax, -benefit) income (G_X) and post-fiscal (-tax, -benefit) income (G_N), as shown in (2.1).

$$RE = G_X - G_N \tag{2.1}$$

Some other indices are also called "redistributive effect", but should be distinguished from the redistributive effect as defined in (2.1). Musgrave and Thin (1948) proposed a different index (RE^{MT}), which also relates the indices G_X and G_N, but in the way shown by (2.2).

$$RE^{MT} = \frac{1 - G_N}{1 - G_X} \tag{2.2}$$

Reynolds and Smolensky's (1977) index of redistributive effect (RE^{RS}) is very similar to RE in (2.1), but instead of the Gini coefficient of post-fiscal income (G_N), we find here a concentration coefficient of post-fiscal income (D_N^x), as presented in (2.3).

$$RE^{RS} = G_X - D_N^x \tag{2.3}$$

Observe the important difference between the Gini (G_N) and concentration (D_N^x) indices of post-fiscal income. G_N is obtained for the Lorenz curve of post-fiscal incomes, $L_N(p)$, while D_N^x is calculated for the concentration curve of post-fiscal incomes, $C_N^x(p)$. While for $L_N(p)$ the units are sorted in ascending order of post-fiscal income, for $C_N^x(p)$ the income units are ordered by *pre*-fiscal income; hence, x in the superscript of $C_N^x(p)$. It is proved (e.g. in Lambert, 2001) that the concentration curve, such as $C_N^x(p)$, never lies below the corresponding Lorenz curve, here $L_N(p)$, and therefore, G_N can never be lower than D_N^x.

2.2.2 Kakwani decomposition

2.2.2.1 Origins

The decomposition of redistributive effect (RE) that is central to this investigation is first presented in Kakwani (1984:159-163) and repeated with minor differences, mostly in notation, in Kakwani (1986:82-86). The aim of the model is to capture two well-known theoretical concepts – horizontal and vertical equity – into a unified measurement framework. The original methodology used a different presentation (and notation) from the one that is usual nowadays, but it will be useful to compare these two. Kakwani expressed the main components in terms of the pre-tax Gini coefficient, as follows. The redistributive effect (R) is the difference between the Gini coefficients of pre-tax and post-tax income, as in (2.1), but divided by G_X, as in (2.4).

$$R = \frac{G_X - G_N}{G_X} \tag{2.4}$$

The redistributive effect (R) is decomposed into the sum of horizontal inequity (H) and vertical equity (V) terms, as shown in (2.5).

$$R = H + V \tag{2.5}$$

The horizontal inequity index (H) is a difference between the concentration coefficient of post-tax income (D_N^x) and G_N, normalized by G_X, as in (2.6). The vertical equity index (V) is equal to Kakwani progressivity index (P_T^K) scaled by the average tax rate (t^x), and normalized by G_X, as shown by (2.7).

$$H = \frac{D_N^x - G_N}{G_X} \tag{2.6}$$

$$V = \frac{t^x P_T^K}{(1 - t^x) G_X} \tag{2.7}$$

Modern exposition is different in several respects. The notion "horizontal inequity" is changed into "reranking" for reasons that will be discussed below. The Kakwani horizontal inequity term (H), based on the difference $D_N^x - G_N$, is replaced by its negative correspondent, $G_N - D_N^x$, and named more conveniently as the Atkinson-Plotnick index of reranking (R^{AP}). In addition, the components are presented as "absolute" values of coefficients, and not in terms of relative to G_X. Thus, what we have in (2.8) is representing the decomposition of redistributive effect (RE) into the Kakwani vertical effect (V^K) and the Atkinson-Plotnick index of reranking (R^{AP}).

$$RE = V^K - R^{AP} \tag{2.8}$$

The differences between two presentations are shown in Table 2.1. The Kakwani horizontal inequity term (H) can never be positive. On the other hand, the Atkinson-Plotnick index of reranking ($R^{AP}(=-H)$) is always non-negative. The minus sign in front of the reranking term, in the modern expression (2.8), better reflects the common notion that reranking reduces the redistributive effect (more on this, below).

Table 2.1: Presentation of Kakwani decomposition

Original	**Modern**
Redistributive effect	
$R=\frac{G_X-G_N}{G_X}$ (2.4)	$RE=G_X-G_N$ (2.1)
Index of progressivity	
$P_T^K=D_T^x-G_X$ (2.9)	
Horizontal / Reranking effect	
$H=\frac{D_N^x-G_N}{G_X}$ (2.10)	$R^{AP}=G_N-D_N^x$ (2.11)
Vertical effect	
$V=\frac{t^x P_T^K}{(1-t^x)G_X}$ (2.12)	$V^K=\frac{t^x P_T^K}{1-t^x}=G_X-D_N^x$ (2.13)
Decomposition of the redistributive effect	
$G_X-G_N=(D_N^x-G_N)+\frac{t^x P_T^K}{1-t^x}$ (2.14)	$G_X-G_N=\frac{t^x P_T^K}{1-t^x}-(G_N-D_N^x)$ (2.15) $G_X-G_N=(G_X-D_N^x)-(G_N-D_N^x)$ (2.16)
$R=H+V$ (2.5)	$RE=V^K-R^{AP}$ (2.8)

2.2.2.2 Horizontal inequity or reranking?

The principle of vertical equity requires that people with larger income pay higher taxes than those with lower income. The standard definition of horizontal equity in taxation requires that people with equal income pay equal taxes. Violation of this principle gives rise to horizontal inequity. Following several other authors, Kakwani identifies horizontal inequity with reranking. However, it seems that by decomposition of redistributive effect into H (R^{AP}) and V (V^{K}) we obtain two measures that both deal with unequal treatment of unequals. The following example will prove this contention.

If we want to measure the violation of equality, we must first be able to define equals. People with different ranks are usually not in *equal* positions. For example, person A has income of 100$ and B has 500$. Imagine that the fiscal process reversed their incomes, and now A has 500$ and B only 100$: reranking occurred. Everybody will agree that inequity has happened, but should we call it *horizontal* inequity? A and B were not in equal positions before the fiscal action: B had five times larger income than A. The followers of Kakwani decomposition have realized the problem, and today it is common to name the effect reranking instead of horizontal inequity.[1]

2.2.3 The components

2.2.3.1 Origins of vertical effect

The Kakwani index of tax progressivity (P_T^K) has emerged as a reaction to certain inadequacies of Musgrave and Thin "index of progressivity" (RE^{MT}) and the index of redistributive effect (RE). For Kakwani (1977b), two tax systems with equal elasticity of tax liability to pre-tax income, over the whole income distribution, should be judged as *equally progressive*. As Kakwani shows, the proposed index (P_T^K) satisfies this requirement, while the indices RE^{MT} and RE fail in this task.

Kakwani (1977a, 1977b) proved the following relationship between the index of progressivity and the redistributive effect, shown in (2.17).[2]

$$RE = G_X - G_N = \frac{t^x P_T^K}{1 - t^x} \qquad (2.17)$$

[1] Obvious contribution to renaming came from Aronson, Johnson and Lambert (1994) who invented a decomposition of redistributive effect into vertical, horizontal *and* reranking distinctive effects (see more on this below). Also, see the serious criticism of research on horizontal inequity by Kaplow (1989; 2000).

[2] Observe that (2.17) differs from (2.15): the former equation did not recognize the existence of reranking.

According to equation (2.17), the redistributive effect (RE) represents the reduction of inequality of taxation, but it *does not* measure tax progressivity. Two equally progressive tax systems with different average tax rates would therefore be erroneously observed as differently progressive by RE or RE^{MT}.

2.2.3.2 Origins of reranking effect

The concept of reranking effect that appears in the Kakwani decomposition was derived almost simultaneously and independently by two scholars, Atkinson (1980) and Plotnick (1981).[3] Analyzing empirical literature, Atkinson concluded that some studies used the *concentration* coefficient of post-fiscal income (D_N^x), which understates the true post-fiscal inequality, as would be measured by the corresponding Gini coefficient of post-fiscal income (G_N). Consequently, a measure $G_X - D_N^x$ (identical to Reynolds-Smolensky index, RE^{RS}) overstates the redistributive effect of fiscal system ($RE^{RS} > RE$). Therefore, for Atkinson, one motive for measurement of reranking would be a correct assessment of income redistribution. The other is to estimate the magnitude of mobility along the income scale induced by the fiscal process.

Atkinson explained several ways in which the new concept of reranking could be measured, but left the issue of choosing the best one somewhat open. He was not very inclined toward single index measures, as “any such summary statistic involves assumptions that may be little more than arbitrary”. Use of the transition matrix[4] or Lorenz and concentration curves was considered much more preferable.

Contrary to that, a single index measure of reranking is the most widely used form of presentation. This approach started with Plotnick, whose index is $R^P = 100\% \cdot R^{AP} / G_N$. Expressing reranking as a percentage of the post-fiscal Gini coefficient has logical

[3] Hence the superscript AP in R^{AP}, defined in (2.11). Both works contain “horizontal (in)equity” in their titles. In the previous section we have seen the problem with the term “horizontal”, and why “reranking” is a better choice.
[4] The transition matrix shows complete and detailed transformation of income vector from one ordering to another in steps, where each step involves changing the ranks of two units. For example, before step *t*, A has income *a* and rank *v*; B has income *b* and rank *w*. After step *t*, A has income *b* and rank *w*, while B has income *a* and rank *v*.

background: the maximum value of R^{AP} is exactly G_N.[5] Plotnick also suggested stating R^{AP} as a percentage of RE, which has become standard practice in empirical studies.

Plotnick also discusses the issue of the appropriate benchmark ranking of income units. In order to evaluate inequity, we must first establish what is equitable. The pre-fiscal ranking is said to be a natural choice, supported by both theoretical works and common belief that the ordering of people emerging as a result of activities on the market should not be disturbed in the fiscal process.[6] Of course, we should be able to properly calculate pre-fiscal incomes, i.e. incomes in the absence of fiscal activities, and this is likely an impossible task. Still, relying on certain fiscal incidence assumptions and existing data we can produce some estimates of fiscal inequities.

2.2.3.3 The role of reranking: advice to future developers

In the view of Atkinson (1980), reranking does not influence overall redistributive effect, RE (see also Jenkins (1988), and Duclos (1993), who shared this view). He states: "Changes in the ranking of observations as a result of taxation do not in themselves affect the degree of inequality in the post-tax distribution. They do, however, influence certain ways of representing the redistribution and of calculating summary measures of inequality." This is a very important statement, with the following messages that can be deduced. Researchers should be aware of the extent of reranking caused by the fiscal system in order to make judgments about the quality of the redistributive process. Reranking is a by-product or a consequence of an income redistribution process; it does not contribute, positively or negatively, to the redistributive effect.

It is interesting that Plotnick (1981) had similar thoughts on the relationship between the measures of reranking and the redistributive effect. In his model "the structure of post-redistribution income inequality is taken as a datum by the measure". As in Atkinson (1980), this implies that the reranking should not be interpreted as a factor that produces redistribution. Instead, "...given the change in inequality, the measure should tell us how

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[5] In this extreme case, when ranks of all units are changed, $C_N^x(p)$ lies above the line of absolute equality and is symmetrical to $L_N(p)$.

[6] In the last passage, Atkinson (1980:18) briefly mentions several different conceptual views on the overall issue of (re)ranking and horizontal inequity.

seriously the redistributive activities violated the norms of horizontal equity." Also, Plotnick warns that a measure of reranking "should not attempt to compare the actual extent of redistribution or change in inequality to some exogenous criterion. Doing so would be an exercise in measuring vertical inequity."

Thus, both Atkinson and Plotnick, more or less explicitly, avoid setting the reranking effect into any context other than the measurement of "horizontal inequity". Although aware of the strong connection between the concepts of vertical and horizontal equity, they do not attempt to build a comprehensive model capturing both of them. They implicitly suggest to future users and developers to be cautious about the introduction of a reranking measure into some broader frameworks.

2.2.3.4 Advice taken?

It seems that the message did not reach Kakwani, whose "horizontal inequity" term H ($H = R^{AP}$) is given a specific interpretation: it measures a reduction of redistributive effect("increase in inequality"). New interpretation of the decomposition has later emerged in literature, which usually goes as in the following passage:

> *A progressive fiscal system is "good" because it reduces inequality. In case of progressivity, Kakwani vertical effect (V^K) is positive, and the larger it is, the "better". On the other hand, reranking increases inequality. In the presence of reranking, the Atkinson-Plotnick index (R^{AP}) is positive and the larger it is, the "worse". Furthermore, V^K measures a "potential" redistributive effect, attainable in the absence of reranking. Because of reranking, actual redistribution amounts "only" to $RE = V^K - R^{AP}$. If reranking could be somehow eliminated, redistributive effect would be equal to the potential amount: $RE^{potential} = V^K$.*

This interpretation contributed significantly to the popularity of Kakwani decomposition among applied researchers and scholars as well. It is straightforward, capturing some of the desired qualities of a fiscal system (achievement of vertical and horizontal equity), and offers deceptively simple advice to policymakers (simply by reduction of horizontal inequity – without additional resources – you can increase the redistributive effect to a certain extent).

Although the interpretation based on "potential redistributive effect" was not present in Kakwani (1984; 1986), we may say it was firmly inspired by these contributions.

2.2.4 Criticism by Lerman and Yitzhaki and their new framework

Substantive criticism of the Kakwani decomposition is presented in the article by Lerman and Yitzhaki (1995) (henceforth LY). They develop their own decomposition of the redistributive effect, following, in fact, the philosophy of Kakwani, but arriving at different conclusions about the role of reranking. In their view, reranking positively contributes to the creation of overall inequality reduction, together with another component, "gap narrowing", which corresponds to vertical effect in Kakwani's model.

Lerman and Yitzhaki (1995) claim that their method enables a decomposition of redistributive effect "into two exclusive, exhaustive terms". As Kakwani, they too regard reranking as an independent source of the redistributive effect. Perhaps, this is most explicitly stated in the abstract of the article: "...policies may reduce inequality by rearranging rankings as well."[7] They even claim that Atkinson supported this view, saying that he indicated "that the reranking effect might be important in explaining a proportion of the impact of taxes on inequality." However, we have already seen that, in fact, Atkinson regarded that reranking could only explain the discrepancy between two measures of the redistributive effect, namely RE and RE^{RS}; he explicitly noted that reranking does not affect the redistributive effect.

Table 2.2: Comparison of Kakwani and Lerman-Yitzhaki decompositions

Lerman-Yitzhaki	**Kakwani**
Index of progressivity	
$P_T^{LY} = D_T^n - G_N$ (2.18)	$P_T^K = D_T^x - G_X$ (2.9)
Horizontal / Reranking effect	
$R^{LY} = G_X - D_X^n$ (2.19)	$R^{AP} = G_N - D_N^x$ (2.11)

[7] Other places are: (a) p.46, „one might well care about whether inequality reductions *result* from reranking or from gap-narrowing". (b) p.46, „In this paper we ask: to what extent is the overall redistributive impact of U.S. taxes and transfers *the result* of rerankings of income versus pure reductions in income gaps (holding rankings constant)?" (c) p.55, „For taxes, the reranking component amounted to nearly 40 percent of the total reduction in inequality."

Vertical effect	
$V^{LY}=\dfrac{t^n P_T^{LY}}{1-t^n}=D_X^n-G_N$ (2.20)	$V^K=\dfrac{t^x P_T^K}{1-t^x}=G_X-D_N^x$ (2.13)
Decomposition of the redistributive effect	
$G_X-G_N=\dfrac{t^n P_T^{LY}}{1-t^n}+(G_X-D_X^n)$ (2.21) $G_X-G_N=(D_X^n-G_N)+(G_X-D_X^n)$ (2.22) $RE=V^{LY}+R^{LY}$ (2.23)	$G_X-G_N=\dfrac{t^x P_T^K}{1-t^x}-(G_N-D_N^x)$ (2.15) $G_X-G_N=(G_X-D_N^x)-(G_N-D_N^x)$ (2.16) $RE=V^K-R^{AP}$ (2.8)

Table 2.2 presents the LY system in comparison with Kakwani's. The main difference in the two approaches lies in the "reference" income: in the former model, it is the post-fiscal income, whereas in the latter, it is the pre-fiscal income. Observe that the LY index of progressivity (P_T^{LY}) and vertical effect (V^{LY}), defined in (2.18) and (2.23), compare the concentration coefficients of tax (D_T^n) and pre-fiscal income (D_X^n) with the Gini coefficient of *post-tax* income (G_N). Here, D_T^n and D_X^n are derived from the concentration curves for which the units are sorted in the ascending order of post-fiscal income (hence *n* in the superscript), namely $C_T^n(p)$ and $C_X^n(p)$. Also, the average tax rate (t^n) is expressed in terms of post-fiscal income. Decomposition of redistributive effect is shown by (2.23), and the noticeable difference in comparison with (2.8) lies in the sign in front of the reranking term – it is positive. Recall that R^{AP} is non-negative due to its construction; the same is true for R^{LY} as well.

LY do not say *explicitly* of what would happen with the redistributive effect if reranking disappeared or if it were somehow eliminated. However, their text does imply the interpretation already mentioned in many instances, based on "actual" and "potential" redistributive effect, which goes as follows:

> *A progressive fiscal system is "good" because it reduces inequality. In case of progressivity, Lerman-Yitzhaki vertical effect (V^{LY}) is positive, and the larger it is, the "better". Reranking is another factor that decreases inequality. The higher the reranking and Lerman-Yitzhaki index (R^{LY}), the "better" it is. Actual redistribution is*

equal to $RE = V^{LY} + R^{LY}$. *Therefore, in case that reranking is somehow reduced or eliminated, the redistributive effect would fall below* RE, *down to* $RE^{no\_reranking} = V^{LY}$.

Why have LY decided to abandon the Kakwani decomposition of redistributive effect and invent a new one? *First*, they criticize the Kakwani vertical effect: for given redistributive effect, V^K increases automatically when reranking is increased. *Second*, they suggest that "the after-tax ranking is the appropriate ranking for calculating progressivity", because they believe it is the correct ranking in the analysis of marginal changes in the tax system. They illustrate this on an example of the rich and the poor taxpayer whose places on the income scale are reversed due to taxation. This would result in an increase of Kakwani progressivity (V^K) that is based on pre-tax rankings: this suggests that *even more* taxation of the "now poor" is desired. However, and unfortunately for the advancement of the field, they do not delve into any deeper technical elaboration of their criticism.

2.3 Upgrades of methodology

2.3.1 Frameworks capturing vertical and horizontal inequity and reranking

2.3.1.1 Model by Aronson, Johnson and Lambert

We have objected to identifying reranking with horizontal inequity: the latter should be concerned with unequal treatment of equals, while the former considers changing the order on the income scale of the unequals. The issue was reconciled by Aronson, Johnson and Lambert (1994) (henceforth AJL) with much praised methodology that decomposes the redistributive effect into vertical equity, horizontal inequity and reranking effects. The constraint of the model, from an empirical application perspective, is that it works only with true equals – units with identical income x.

Suppose we can partition the population (sample) into J groups, such that in each group j, all K_j units have equal pre-tax income x_j. Each member k of group j has pre-tax income x_j and post-tax income $n_{j,k}$, after paying tax of $t_{j,k}$. Average tax paid by the group j is $\bar{t}_j$.

If everybody within each group j had identical post-tax income, it would mean that the system is horizontally equitable and there is no reranking. In reality, members within groups pay different taxes – this leads to horizontal inequity. It is the inequality of post-tax income within the groups that is the basis for AJL horizontal effect. It is equal to:

$$H^{AJL} = \sum_{j=1}^{J} \alpha_j \beta_j G_{N,j} \tag{2.24}$$

where α_j, β_j and $G_{N,j}$ are, respectively, population share, post-tax income share and the Gini coefficient of post-tax income for the group j.

The basis for the calculation of vertical effect is a vector of taxes that would occur if all members within the group j paid $\bar{t}_j$ instead of $t_{j,k}$; call it $\widetilde{T}^x$. This counterfactual tax is deemed free of horizontal inequity (but, is it free of reranking?). The AJL vertical effect is the Kakwani vertical effect obtained for $\widetilde{T}^x$ in (2.25), with $P^K_{\widetilde{T}}$ explained in (2.26), where $D^x_{\widetilde{N}}$ is the concentration coefficient for counterfactual post-income vector $\widetilde{N}^x = X - \widetilde{T}^x$.

$$V^{AJL} = \frac{t^x}{1-t^x} P^K_{\widetilde{T}} = G_X - D^x_{\widetilde{N}} \tag{2.25}$$

$$P^K_{\widetilde{T}} = D^x_{\widetilde{T}} - G_X \tag{2.26}$$

Some unit z in group j may end with post-tax income $n_{j,z}$, such that $n_{j,z} < \max(n_{e,k})$, $k = 1,...,K_j$, and $e < j$; in other words, there may be at least one unit in a lower pre-tax income group e that has higher post-tax income than the unit z in group j. It means that the latter unit is reranked. AJL model enables measurement of reranking, and the corresponding term is R^{AJL}, obtained as a residual $R^{AJL} = V^{AJL} - H^{AJL} - RE$. In the AJL model, R^{AJL} is identical to R^{AP}.

Finally, we have the complete decomposition:

$$RE = V^{AJL} - H^{AJL} - R^{AJL} \tag{2.27}$$

The main constraint of the AJL model is its non-conformability with real empirical data, where one can hardly find pre-tax exact equals. In the first applications, the researchers created artificially those equals, rounding pre-tax incomes of close equals.[8] This procedure has obviously distorted information about individual effects.

Subsequent work by van de Ven, Creedy and Lambert (2001) transformed the original model to avoid the creation of *artificial pre-tax equals*. Instead, close equals groups are used.[9] However, another problem remained unenvisaged by AJL model and this was the issue of whole-group reranking. The recent model of Urban and Lambert (2008) enables decomposition of RE accounting for all these issues.

AJL set their decomposition in the context of a welfare function, thus giving the indices a normative interpretation. They write:

$$\begin{aligned} W_{X-T} - W_{X-F} &= \mu^x (1-t^x) RE = \mu^x \left[t^x P^K_{\tilde{T}} - (1-t^x) H^{AJL} - (1-t^x) R^{AJL} \right] \\ &= \mu^x (1-t^x)(V^{AJL} - H^{AJL} - R^{AJL}) \qquad (2.28) \end{aligned}$$

Here, W_{X-T} and W_{X-F} are respectively social welfares after actual tax and after an equal-yield proportional one; μ^x is the mean pre-tax income. The difference $W_{X-T} - W_{X-F}$ represents a welfare premium due to tax progressiveness. Minuses standing before the horizontal and reranking effects mean that they "provide subtractions from the welfare superiority of the actual tax code over a flat (distributionally neutral) one".

Lambert (2001) shows how these welfare indices are derived and obtains a corresponding welfare result for the more simple Kakwani (1984) decomposition, which shows how "rerankings detract from the welfare-enhancing property of an otherwise progressive tax system".

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[8] For example, if units *i*, *i*+1 and *i*+2 have pre-tax incomes of \$101, \$102 and \$103, the researcher may decide to transform all of them into pre-tax income \$102. Now, imagine that all of them paid zero tax; their post-tax incomes are \$101, \$102 and \$103. The AJL model would show the appearance of horizontal inequity, despite the fact that it does not occur.

[9] In the example from the previous footnote, the researcher might form the close pre-tax equals group of units *i*, *i*+1 and *i*+2; their pre-tax incomes thus remain \$101, \$102 and \$103. Nonetheless, this procedure also requires certain amount of artificial business, which seems to be inevitable when measurement of horizontal inequity is in question.

$$W_{X-T} - W_{X-F} = \mu^x(1-t^x)RE = \mu^x\left[t^x P_T^K - (1-t^x)R^{AP}\right]$$
$$= \mu^x(1-t^x)(V^K - R^{AP}) \qquad (2.29)$$

We have shown these results serve not only to offer another way of presenting decompositions, but also to discuss the various interpretations of the indexes. It is seen that horizontal and reranking effect are contributing negatively to overall redistributive effect. Does this also mean that RE would be higher in the absence of horizontal inequity and reranking? The authors think it would, and explicitly confirm it: "If differences in tax treatment... could be eliminated..., the redistributive effect would have been increased, to around $[100(1+V^{AJL})]$% of its actual value..." (AJL:268). This interpretation of the results has become particularly popular, as it gives an apparently straightforward message which the researcher can send to policy-makers: "Through alignments of taxes and benefits, without additional resources, you can achieve two popular goals at the same time – eliminate horizontal injustices and further reduce post-fiscal inequality".

2.3.1.2 Model by Duclos, Jalbert and Araar

The model suggested by Duclos, Jalbert and Araar (2003) follows the philosophy of AJL, in which vertical effect enhances the reduction of inequality, while horizontal and reranking effects diminish it. The redistributive effect is decomposed as in (2.30), and in (2.31) an extended version is presented.

$$RE = V^{DJA} - H^{DJA} - R^{DJA} \qquad (2.30)$$
$$RE = (I_X - I_N) = (I_X - I_N^E) - (I_N^P - I_N^E) - (I_N - I_N^P) \qquad (2.31)$$

This model works with more general inequality indices, unlike AJL, which is based on ordinary Gini coefficients. The pre-tax income social welfare function is defined in the following way:

$$W_X(\varepsilon, v) = \int_0^1 U_\varepsilon\big(X(p)\big)w(p,v)dp \qquad (2.32)$$

where $w(p,v)$ is a weighting scheme dependent on the ranks of income units, p, and a parameter of inequality aversion, v; in this case we have $w(p,v) = v(1-p)^{(v-1)}$; note that $\int_0^1 w(p,v) = 1$. $X(p)$ is the pre-tax income of an income unit with rank p in the pre-tax income distribution, and $U_\varepsilon(y)$ is a utility function with ε as a parameter of relative risk aversion; $U_\varepsilon(y) = y^{1-\varepsilon}/(1-\varepsilon)$ for $\varepsilon \neq 1$ and $U_\varepsilon(y) = \ln(y)$ for $\varepsilon = 1$.

If everybody in society received identical income equal to $\xi_X(\varepsilon,v)$ (the term is called the "equally distributed equivalent", EDE), welfare would be the same as the actual. Then, by definition:

$$W_X(\varepsilon,v) \equiv W_{\xi_X(\varepsilon,v)}(\varepsilon,v) = \int_0^1 U_\varepsilon(\xi_X(\varepsilon,v))w(p,v)dp = U_\varepsilon(\xi_X(\varepsilon,v)) \qquad (2.33)$$

ξ_X is then obtained from W_X by inversion of the utility function:

$$\xi_X = U_\varepsilon^{-1}(W_X) \qquad (2.34)$$

where $U_\varepsilon^{-1}(y) = ((1-\varepsilon)y)^{(1/(1-\varepsilon))}$ for $\varepsilon \neq 1$ and $U_\varepsilon^{-1}(y) = e^y$ for $\varepsilon = 1$.

Finally, the index of inequality, I_X, is given as:

$$I_X = 1 - \frac{\xi_X}{\mu^x} \qquad (2.35)$$

Returning to the model, the decomposition equation consists of several inequality indices, which can all be derived analogously to I_X. Here, I_N is simply inequality of post-tax income. In one particular case, when $\varepsilon = 0$ and $v = 2$, it follows that $I_X = G_X$ and $I_N = G_N$, the ordinary Gini indexes. On the other hand, I_N^E and I_N^P are based on counterfactual incomes and utilities. I_N^E is obtained for the distribution of conditional incomes, $\overline{N}(p) = \int_0^1 N(q \mid p)dq$, which are the expected post-tax incomes of those at rank p in the distribution of pre-tax income. I_N^P is obtained for the distribution of expected utilities, $\overline{U}_\varepsilon(p) = \int_0^1 U_\varepsilon(N(q \mid p))dq$.

2.3.2 Extension to the net fiscal system

2.3.2.1 The methodological challenge

Analysts in the area of income redistribution are naturally interested in capturing the widest possible picture of a fiscal system, given data limitations and theoretical assumptions concerning fiscal incidence. Since the fiscal systems include at least several tax and benefit instruments, it is useful to know how each of them influences the redistribution, and what the interactions between them are.

The Kakwani index of vertical effect can be readily applied to benefits, and the simplest way to estimate the effects of single taxes and benefits is to calculate the redistributive effect of each instrument, one by one, after choosing an appropriate reference income base. However, contributions obtained in such way do not simply add to the total redistributive effect of the net tax system; results from different data sources cannot be combined to get the overall redistribution (not even in case of “fiscal balance”); the interaction of different instruments obfuscates the situation.

2.3.2.2 Lambert’s approach

All these problems were envisaged and solved by Lambert (1985; 1988) who decomposed the redistributive effect of the net tax system into parts that explain the contributions of taxes and benefits individually. As (2.36) demonstrates, the Kakwani vertical effect for the combined system of taxes and benefits ($V_{T\&B}^K$) is a weighted average of vertical effects (or indices of progressivity) obtained for taxes (V_T^K) and benefits (V_B^K), where the weights are shares of taxes and benefits in pre-fiscal income (t^x and b^x).

$$V_{T\&B}^K = \frac{(1-t^x)V_T^K + (1+b^x)V_B^K}{1-t^x+b^x} \tag{2.36}$$

$$V_{T\&B}^K = \frac{t^x P_T^K + b^x P_B^K}{1-t^x+b^x} \tag{2.37}$$

Table 2.3: Lambert's decomposition for the net fiscal system

Auxiliary terms and decompositions	
$V_{T\&B}^{K} = G_X - D_N^x$ (2.38)	
$P_T^K = D_T^x - G_X$ (2.39)	$P_B^K = G_X - D_B^x$ (2.40)
$V_T^K = G_X - D_{X-T}^x = \frac{t^x P_T^K}{1-t^x}$ (2.41)	$V_B^K = G_X - D_{X+B}^x = \frac{b^x P_B^K}{1+b^x}$ (2.42)

The first important thing exposed by this decomposition is that the vertical effect of a net fiscal system is different from the simple sum of vertical effects of taxes and benefits, i.e. $V_{T\&B}^{K} \neq V_T^K + V_B^K$. This helps us to reveal an interesting interaction: even if the overall tax system is regressive ($V_T^K < 0$), taxes can reinforce the redistributive effect of the net fiscal system, so that $V_{T\&B}^{K} > V_B^K$. For example, take that $V_T^K = -0.05$ and $V_B^K = 0.10$ and assume that $t^x = b^x = 0.4$. Then, we have that $V_{T\&B}^{K} = 0.6 \cdot (-0.05) + 1.4 \cdot 0.1 = 0.11 > V_B^K$.

However, this framework does not tell us how different tax-benefit instruments contribute to overall reranking. "This would introduce severe analytical complications", as Lambert (1985: footnote 2) points out. As we will see shortly, the two other scholars decided to deal with this intricate task.

2.3.2.3 Jenkins' decomposition

Following the derivational grounds of Lambert (1985), who dealt with the vertical effect and admittedly ignored reranking, Jenkins (1988) decomposed the overall redistributive effect of the net fiscal system into tax and benefit contributions (RE). Equation (2.43) replicates Jenkins' formula 10, extending some of the terms.

$$RE = \frac{(1-t^x)(G_X - G_{X-T})}{1-t^x+b^x} + \frac{(1+b^x)(G_X - G_{X+B})}{1-t^x+b^x} + \\ + (D_N^x - G_N) - \frac{(1-t^x)}{1-t^x+b^x}(D_{X-T}^x - G_{X-T}) - \frac{(1+b^x)}{1-t^x+b^x}(D_{X+B}^x - G_{X+B}) \qquad (2.43)$$

The last three terms on the right side of (2.43) represent reranking effects. The term $(D_N^x - G_N)$ is simply $-R^{AP}$ or a measure of the overall reranking induced by a fiscal system consisting of taxes and benefits. The other two terms are reranking effects that should reflect the contributions of taxes (the fourth term) and benefits (the fourth term). The decomposition (2.43) fully decomposes the redistributive effect (RE). However, the contributions of taxes and benefits to reranking do not add up to total reranking (as measured by R^{AP}), and this may be judged as unsatisfactory. One may also pose a question, why in a measurement of reranking due to taxes (benefits) is the term $D_{X-T}^x - G_{X-T}$ $(D_{X+B}^x - G_{X+B})$ used instead of $D_{X-T}^x - G_X$ $(D_{X+B}^x - G_X)$? Furthermore, the first two terms on the right side of (2.43) do not represent Kakwani vertical effects of taxes and benefits (as V_T^K and V_B^K in (2.36)), but something else.

2.3.2.4 Duclos' decomposition

In another attempt to cope with the difficulties of the reranking effect decomposition was undertaken by Duclos (1993) and the formula (2.44) shows the result.

$$RE \quad = (G_X - D_N^x) - (G_N - D_N^x) =$$

$$= \frac{\sum_{m=1}^{M} t_m^x (D_{T_m}^x - G_X)}{1 - \sum_{m=1}^{M} t_m^x} - \left(\sum_{k=1}^{M} \left(D_{N, X - \sum_{m=0}^{k} T_m} - D_{N, X - \sum_{m=0}^{k-1} T_m} \right) \right) \tag{2.44}$$

In the start, one has to somehow order the tax/benefit variables and call them $T_1, T_2, \ldots, T_M$ (with $T_0 = 0$). Any combination is allowed,[10] leading to a number of final results which are conveniently presented by a tree-root diagram (we will turn to this issue later again). Thus, for the first tax/benefit, T_1 $(k = 1)$ the variables in question should be $X - T_1$ and X; for the last tax/benefit, T_M $(k = M)$, the variables are N and $N + T_M$.

[10] For example, if the fiscal system consists of instruments A, B and C, each of these can be T_1, T_2 or T_3, resulting in 6 possible orders.

The first row of (2.44) is the already familiar Kakwani decomposition of redistributive effect into the vertical and reranking effects. The first term in the second row is a decomposition of vertical effect from (2.36), but applied to M individual tax or benefit instruments, each with its own concentration coefficient $D_{T_m}^x$, and a share in pre-fiscal income t_m^x.[11] The second term is then a corresponding decomposition of the reranking effect, which is separated into M differences between specifically defined concentration coefficients.

For all tax/benefit instruments, $k = 1,\ldots,M$, both concentration coefficients are obtained for (final) post-fiscal income (N), as suggested by the subscripts N. Thus, the same variable is used to determine the contributions of all instruments. What makes the difference between the concentration coefficients is the ordering of units in the construction of the concentration curve. However, this matter is not very clear in the paper.

Duclos (1993:356) says: "$D_{N,X-\sum_{m=0}^{k} T_m}$ indicates that the concentration curve used to build D employs the ranking of units based on $\sum_{m=0}^{k} T_m$". What about $D_{N,X-\sum_{m=0}^{k-1} T_m}$? It must be supposed that in this case the ordering variable is $\sum_{m=0}^{k-1} T_m$. But, if this is so, for $k=1$, the ordering is not defined. Also, the sums of taxes/benefits do not seem as intuitive variables for ranking income units. From the overall text, it may rather be guessed that the actual ordering variables are $X-\sum_{m=0}^{k} T_m$ and $X-\sum_{m=0}^{k-1} T_m$.

Let us show how it would work on a simple example of one tax ($T_1 = T$) and one benefit ($T_2 = B$) instrument ($M = 2$). For $k=1$, the ordering variable for the first concentration coefficient is $X-\sum_{m=0}^{1} T_m = X - T$ and for the second it is $X-\sum_{m=0}^{0} T_m = X$. For $k=2$, we obtain $X-\sum_{m=0}^{2} T_m = X - T + B = N$ and $X-\sum_{m=0}^{1} T_m = X - T$, respectively. The decomposition of reranking effect for this one-tax-one-benefit case is then written as:

$$R^{AP} = G_N - D_N^x = (D_N^{x-t} - D_N^x) + (G_N - D_N^{x-t}) \tag{2.45}$$

[11] Duclos (1993) designates both taxes and benefits as *taxes*, using letter *T*. This made his presentation more simple, but at a loss of convenience.

2.3.3 Other developments

2.3.3.1 Kakwani-Lambert "new approach"

The measurement system proposed by Kakwani and Lambert (1998) arises from three axioms of equitable taxation that deal with both horizontal and vertical equity considerations. The first axiom requires „minimal progression": tax should increase monotonically with respect to income. The second axiom is based on the „progressive principle", demanding that higher income people are faced with higher tax rates. The third axiom presents the „no reranking" criterion: the marginal tax rate should not exceed 100 percent. The axioms are designed in such way as to be independent.

The indices S_1, S_2 and S_3 are then constructed, whose zero value means that the respective axiom is upheld, or violated if the value is positive: $S_1 = t^n R_T$, $S_2 = t^n (R_A - R_T)$ and $S_3 = R_N (= R^{AP})$, where $R_T = G_T - D_T^x$, $R_A = G_A - D_A^x$ and $R_N = G_N - D_N^x$ are obtained for taxes (T_i), average taxes ($A_i = T_i / X_i$) and post-tax income ($N_i = X_i - T_i$), respectively. The redistributive effect can now be decomposed, as in (2.46).

$$RE = t^n (P^K + R_A) - S_1 - S_2 - S_3 \qquad (2.46)$$

Recall that the Kakwani decomposition is $RE = t^n P^K - S_3$. The term $t^n (P^K + R_A)$ is a measure of potential redistributive effect that "might be achieved if all inequities could be abolished." It is analogous to V^K, which is the potential redistributive effect achievable if reranking could be eliminated. However, as Kakwani and Lambert note, "there is no uniquely well-defined way to abolish axiom violations from a tax system, and thereby to say what maximal value of [RE] might be achieved". The decomposition was applied to Australian income tax data, resulting in an estimate of $RE = 0.0240$, while $t^n (P^K + R_A)$ was remarkably high at 0.1382. The authors conclude that RE could be improved by removal of inequities "without change to the marginal rate structure which governs incentives."

2.3.3.2 Approach based on relative deprivation

The relative deprivation of a person is a sum of the incomes of all people who are richer than that person. Using the concept of relative deprivation as a principle, Duclos (2000) invents the concepts of "fiscal harshness", "fiscal looseness" and "ill-fortune", and afterwards combines their measures to reinvent all the terms in the Kakwani decomposition(s).

Kakwani's index of progressivity (P^K) is obtained as a difference between the mean-normalized average fiscal harshness and average relative deprivation in the population. Kakwani's index of vertical inequity (V^K) is a difference between the mean-normalized average relative deprivation and average fiscal looseness. The Atkinson-Plotnick index of reranking is the mean-normalized average of ill-fortune in the population.[12]

2.4 Empirical research in the field of income redistribution

2.4.1 Overview of empirical studies

Since the 70's, when the major methodological innovations in the measurement of inequality, progressivity, and redistributive effect emerged, there was a huge empirical interest in evaluating how fiscal systems affect income distribution. Table 2.4 presents a summary of studies that measure redistributive effects of fiscal instruments and overall fiscal systems. The aim is not to provide a full review of the research in this field. Instead, we have primarily attempted to illustrate the variety of methodological approaches used in the estimation of redistributive effects. It can be easily noted that most of the analyses are based on the works of Kakwani (1977b, 1984) – his progressivity index and his decomposition of RE.

Another dimension shown in Table 2.4 is fiscal coverage of the studies. Many of them concentrate on single tax or benefit instruments. However, researchers are typically aware that *all* fiscal activities affect income distribution. Therefore, the studies often cover whole fiscal subsystems – personal taxes, indirect taxes, cash and in-kind benefits, and even complete fiscal systems. However, it must be noted that the selection in Table 2.4 is biased toward the

[12] Loose definitions of the concepts follow. For person i with pre-fiscal income X_i, relative deprivation (fiscal harshness; fiscal looseness) is a sum of $X_j - X_i$ ($T_j - T_i$; $N_j - N_i$) for all j with $X_j > X_i$. Ill-fortune occurs for person i, if $X_i > X_j$ and $N_i < N_j$, and is measured in terms of N_j.

studies covering both taxes and benefits, since the empirical part of this dissertation deals with taxes and benefits in Croatia. Concerning the countries included in the research, the high-income countries like UK, USA, Canada, Australia and Sweden are the most represented. Only a few studies are devoted to low-income countries.

Table 2.4: Overview of empirical studies in the field of income redistribution

Authors	**Indices / decompositions used**	**Countries / Fiscal coverage**	**Data sources / equivalence scales**
Kakwani (1977b)	P^K	(a) *Australia, Canada, United Kingdom, United States*: PIT; (b) *Australia, Canada, U.S.*: wide range of direct/indirect taxes and public expenditures	(a) Official income-tax statistics, grouped data ITP (b) different, not fully comparable sources
Reynolds & Smolensky (1977)	RE	*United States:* wide range of direct/indirect taxes and public expenditures	"Survey of Consumer Finances" (1950), "Survey of Consumer Expenditures" (1961), "Current Population Survey" (1970)
Plotnick (1981)	R^{AP}	*United States:* PIT, payroll tax, public cash benefits, food stamps	"Michigan Panel Study of Income Dynamics"
Dilnot, Kay & Norris (1984)	MT	*United Kingdom*: PIT, SSC, indirect taxes	"Family Expenditure Survey" four groups of households
Berliant & Strauss (1985)	BS, 16 other measures of VE and 1 of HI	*United States:* PIT	"Statistics of Income Individual Tax Returns"; ITP
Plotnick (1985)	R^{AP}, 4 other measures of HI	*United States:* PIT, SSC, social benefits	"Current Population Survey"
Kakwani (1986)	K84	*Australia*: PIT, property taxes, social benefits	"Survey of Consumer Finances and Expenditures"; $E_3(.7,.4)$, $E_1(.4)$
Nolan (1987)	Tm	*United Kingdom*: PIT, SSC, social benefits	"Family Expenditure Survey" E : implied by certain benefit programs
Jenkins (1988)	J88	*United States:* wide range of direct/indirect taxes and public expenditures	"Current Population Survey", other sources
Norregaard (1990)	P_T^K	*17 OECD countries*: PIT, SSC	OECD data base (decile groups); ITP
Ankrom (1993)	K84; L85	*Sweden, United States, United Kingdom*: PIT, SSC, property taxes, other direct taxes, indirect taxes, social benefits	Household budget surveys; $E_1(.54)$
Duclos (1993)	K84, D93	*United Kingdom*: PIT, SSC, social benefits	"Family Expenditure Survey" and author's own tax-benefit model
Aronson, Johnson, Lambert (1994)	AJL	*United Kingdom*: PIT	"Family Expenditure Survey" and Tax-benefit model of the Institute for Fiscal Studies; $E_2([0,1],[0,1])$
Lerman & Yitzhaki (1994)	LY94	*United States:* PIT, SSC, social benefits	"Current Population Survey"; $E_1(1)$
Bishop, Chow & Formby (1995)	LC	*Australia, Canada, Sweden, West Germany, United States, United Kingdom:* PIT, payroll taxes,	"Luxembourg Income Study"

Authors	Indices / decompositions used	Countries / Fiscal coverage	Data sources / equivalence scales
		other direct taxes	
Lerman & Yitzhaki (1995)	LY95	*United States:* PIT, SSC, property taxes, social benefits	"Current Population Survey"
Jännti (1997)	S82	*Canada, the Netherlands, Sweden, United Kingdom, United States:* PIT, SSC, social benefits	"Luxembourg Income Study"; E : implied by the U.S. poverty line
Ervik (1998)	RE	*8 high-income countries:* PIT, SSC, social benefits	"Luxembourg Income Study"; $E_1(.5)$
Kakwani & Lambert (1998)	KL	*Australia*: PIT	"Household Expenditure Survey"; $E_4(1,.2,.4,.7,.1)$
Aronson, Lambert & Trippeer (1999)	AJL	*United States:* PIT	"Statistics of Income Panel of Individual Returns"; $E_2(.5,.5)$
Fellman, Jäntti & Lambert (1999)	V^K	*Finland:* personal taxes and social benefits	Household Budget Survey; $E_3(.7,.5)$
van Doorslaer et al. (1999)	AJL	*12 OECD countries*: various sources of financing health expenditures: taxes, social and private insurance, direct payments	Household budget surveys; $E_2(.5,.5)$
Wagstaff et al. (1999a)	AJL	*12 OECD countries*: PIT	Household budget surveys; $E_2(.5,.5)$
Wagstaff et al. (1999b)	P_T^K	*12 OECD countries*: various sources of financing health expenditures: taxes, social and private insurance, direct payments	Household budget surveys; $E_2(.5,.5)$
Duclos (2000)	K84; L85	*Canada*: personal taxes and social benefits	"Survey of Consumer Finances"; $E_3(.7,.5)$
Förster (2000)	LC, S82	*21 OECD countries:* PIT, SSC, social benefits	national household budget surveys and tax administration databases; $E_1(.5)$
Duclos & Lambert (2000)	DL	*Canada*: personal taxes and social benefits	"Survey of Consumer Finances"; $E_3(.7,.5)$
Iyer, Seetharaman (2000)	AJL	*United States:* PIT	"Statistics of Income Panel of Individual Returns"; ITP
Creedy & van de Ven (2001)	VCL	*Australia:* personal taxes and social benefits	Simulated incomes over the life cycle $E_2([0,1],[0,1])$
Dardanoni & Lambert (2001)	DL01	*United Kingdom, Israel, Canada:* cash benefits and direct taxes	"Family Expenditure Survey" and EBORTAX (for UK), "Family Expenditure Survey" (Israel), "Survey of Consumer Finances" (Canada); $E_2(.5,.5)$
Decoster & Van Camp (2001)	P^K, V^K	*Belgium*: PIT and indirect taxes	Administrative data (IPCAL) with microsimulated taxes; household budget survey (ASTER); $E_3(.7,.5)$

Authors	Indices / decompositions used	Countries / Fiscal coverage	Data sources / equivalence scales
Heady, Mitrakos & Tsakloglou (2001)	PCF	*13 EU countries*: social benefits, public pensions	"European Community Household Panel"; $E_3(.5,.3)$
Smith (2001)	P^K	*Australia*: PIT	Aggregated data from annual statistics (1917-1997); ITP
van de Ven, Creedy & Lambert (2001)	VCL	*Australia*: personal taxes and social benefits	"Income Distribution Survey"
Wagstaff & van Doorslaer (2001)	PL (P^K)	*15 OECD countries:* PIT	OECD data base (decile groups); ITP
Creedy (2002)	VCL	*Australia*: Goods and Services Tax	"Household Expenditure Survey"; $E_2(\theta,\alpha)$, wide range of θ - and α -values
Dardanoni & Lambert (2002)	DL02	*see* Dardanoni, Lambert (2001)	*see* Dardanoni, Lambert (2001)
Duclos, Jalbert & Araar (2003)	DJA	*Canada:* PIT, SSC, social benefits	"Survey of Consumer Finances"; $E_3(.7,.5)$, $E_2(.5,.5)$
Thoresen (2004)	P^K	*Norway*: PIT	"Income Distribution Survey"; $E_1(.5)$
Verbist (2004)	PL (P^K)	*EU-15 countries*: PIT, SSC paid by employees, other income taxes	EUROMOD data; $E_3(.5,.3)$
Dyck (2005)	RSA	*Canada:* overall fiscal system	"Social Policy Simulation Database and Model" census family groups
Hyun, Lim (2005)	AJL	*South Korea*: PIT	Administrative data – microsimulated taxes; $E_2(.5,.5)$
Immervoll et al. (2005)	RE	*EU-15 countries*: PIT, SSC paid by employees, other income taxes, social benefits, public pensions	EUROMOD data; $E_3(.5,.3)$
Johannes, Akwi, Anzah (2006)	RE , P^K	*Cameroon*: wide range of direct and indirect taxes, expenditures for health and education	"ECAM2" (household budget survey), 2001; additional government sources
Mahler & Jesuit (2006)	RE	*13 high-income countries*: PIT, SSC, social benefits	"Luxembourg Income Study"; $E_1(.5)$
Urban (2006)	PL (V^K)	*Croatia*: PIT	Administrative tax data; ITP
Cissé, Luchini & Moatti (2007)	P^K	*Abidjan* (Ivory Coast), *Bamako* (Mali), *Conakry* (Guinea) and *Dakar* (Mali): health care payments	questionnaires
Čok & Urban (2007)	UL	*Slovenia* and *Croatia*: PIT and SSC	Administrative tax data; ITP
Creedy et al. (2008)	RE , P^K	*New Zealand*: PIT, Family Tax Credit, unemployment benefit, superannuation, Working for Families, accommodation supplement	"Household Expenditure Survey" $E_2(.7,.6)$
Urban (2008)	K84; LY95; L85	*Croatia*: PIT, SSC, social benefits, public pensions	"APK" (household budget survey); $E_3(.5,.3)$

Authors	Indices / decompositions used	Countries / Fiscal coverage	Data sources / equivalence scales
Urban & Lambert (2008)	UL	*Croatia*: PIT	Administrative tax data; ITP
Kim & Lambert (2009)	UL; L85	*United States*: PIT, Earned Income Tax Credit, property tax, payroll tax, social benefits	"Current Population Survey" and "Annual Social and Economic Supplement"; $E_2(.5,.5)$
Lambert, Thoresen (2009)	BD, KJ; DL01, DL02; DL	*Norway*: PIT	"Income Distribution Survey"; $E_?(\cdot)$
Zaidi (2009)	RE	*Slovakia, Slovenia, Poland, Czech R., Estonia, Lithuania, Hungary, Latvia:* social benefits, PIT, SSC, taxes on wealth	"EU-SILC" databases; $E_3(.5,.3)$

Notes: (a) The term "social benefits", if not specified differently, denotes a wide range of cash and near-cash direct transfers from government to the households (near-cash transfers are in-kind benefits whose values are easily determined); (b) ITP = unit of observation is individual taxpayer; VE = vertical equity; HI = horizontal inequity; (c) See sections 2.4.2.1 and 2.4.2.2 for other terms and abbreviations.

2.4.2 Classification of studies

2.4.2.1 By equivalence scales

The third class of information presented in Table 2.5 relates to data sources and equivalence scales. Almost all studies deal with household budget survey data, while administrative data are used only in the studies of PIT progressivity. Household data require some form of aggregation / averaging at the household level – equivalence scales are used for this purpose. The two most frequent types of scales are: the so-called "Cutler and Katz" scale (denoted with E_2) and the "OECD" scale (denoted with E_3). Both of them take into account economies of scale and recognize the difference between children and adults.

Table 2.5: Equivalence scales

Symbol	Formula	Name
$E_1(\sigma)$	$m=(n)^{\sigma}$	Power/root scale
$E_2(\theta,\alpha)$	$m=(n_a+\theta n_c)^{\alpha}$	"Cutler and Katz" scale
$E_3(\delta,\varepsilon)$	$m=1+\delta(n_a-1)+\varepsilon n_c$	"OECD" scale
$E_4(\eta,\varepsilon_1,\varepsilon_2,\varepsilon_3,\phi)$	$m=(\eta n_a+\varepsilon_1 n_{c1}+\varepsilon_2 n_{c2}+\varepsilon_3 n_{c3})+\phi n_w$	Kakwani scale

Notes: m = equivalent adults; n = adults and children; n_a = adults; n_c = children; $n_{c1}/n_{c2}/n_{c3}$ = children aged 0-5/6-14/15-17 years; n_w = working adults.

2.4.2.2 By methodologies used

In this part we present the classification of empirical studies from Table 2.4 according to the methodology employed. Table 2.6 is divided into two major parts, distinguishing two groups of approaches: (a) those based on Kakwani (1977b), and (b) other approaches. Indices and decompositions rooted in Kakwani (1977b) were used 49 times, while other methodologies in the measurement of redistributive effects were employed in 15 papers. It was already mentioned that the selection of studies is not exhaustive and is biased toward the research capturing both taxes and benefits, but anyway, the ratio of 3:1 evidences that Kakwani's methodologies pervade the field.

Many studies have estimated only one of the main indices. Thus, the index of redistributive effect (RE), the Kakwani index of progressivity (P^K), the Kakwani index of vertical effect (V^K) and the Atkinson-Plotnick index of reranking (R^{AP}) are used in 6, 9, 2 and 2 studies, respectively. Among the various decompositions of these indices, we have to mention Kakwani (1984) decomposition of redistributive effect, used in 5 studies. This may seem a small number, but we must remember another decomposition originating from Kakwani's, namely of Aronson, Johnson and Lambert (1994). It served as a tool in 6 studies, and if we add studies based on adaptations of this decomposition (VCL and UL) we arrive at the number of 12.

What about the studies that attempted to reveal the contributions of individual taxes and benefits to the redistributive effect? Three methodologies were developed to cope with this task, as we have seen above. Lambert (1985) attracted most attention, with 5 studies employing it, while the approaches by Jenkins (1988) and Duclos (1993) were not adopted in later studies.

Table 2.6: Overview of methodologies used in measurement of income redistribution

Symbol / Abbrev.	Approach	Studies using the methodology
(a) Approaches based on Kakwani (1977b)		
RE	redistributive effect (a difference between Gini coefficients of pre-TB and post-TB income)	Reynolds & Smolensky (1977), Ervik (1998), Immervoll et al. (2005), Johannes, Akwi & Anzah (2006), Mahler & Jesuit (2006), Creedy et al. (2008) [6]

Symbol / Abbrev.	**Approach**	**Studies using the methodology**
P^K	Kakwani (1977b) index of progressivity	Kakwani (1977b), Norregaard (1990), Wagstaff et al. (1999b), Decoster & Van Camp (2001), Smith (2001), Thoresen (2004), Johannes, Akwi & Anzah (2006), Cissé, Luchini & Moatti (2007), Creedy et al. (2008) [9]
V^K	Kakwani (1977b) index of vertical effect	Fellman, Jäntti & Lambert (1999), Decoster & Van Camp (2001) [2]
R^{AP}	Atkinson (1980) / Plotnick (1981) index of reranking	Plotnick (1981), Plotnick (1985) [2]
K84	Kakwani (1984) decomposition of redistributive effect	Kakwani (1986), Ankrom (1993), Duclos (1993), Duclos (2000), Urban (2008) [5]
L85	Lambert (1985) decomposition of vertical effect for the net fiscal system	Ankrom (1993), Duclos (1993), Duclos (2000), Urban (2008), Kim & Lambert (2009) [5]
PL	Pfähler (1990) / Lambert (2001) decomposition of P^K and V^K	Wagstaff & van Doorslaer (2001), Verbist (2004), Urban (2006) [3]
LY95	Lerman & Yitzhaki (1995) decomposition of redistributive effect	Lerman & Yitzhaki (1995), Urban (2008) [2]
KL	Kakwani & Lambert (1998) decomposition of redistributive effect	Kakwani & Lambert (1998)
AJL	Aronson, Johnson & Lambert (1994) decomposition of redistributive effect, as originally applied	Aronson, Johnson & Lambert (1994), Aronson, Lambert & Trippeer (1999), van Doorslaer et al. (1999), Wagstaff et al. (1999a), Iyer & Seetharaman (2000), Hyun, Lim (2005) [6]
VCL	van de Ven, Creedy & Lambert (2001) adaptation of AJL decomposition	van de Ven, Creedy & Lambert (2001), Creedy & van de Ven (2001), Creedy (2002) [3]
UL	Urban & Lambert (2005; 2008) adaptation of AJL decomposition	Čok & Urban (2007), Urban & Lambert (2008), Kim & Lambert (2009) [3]
D93	Duclos (1993) decomposition of vertical and reranking effect for the net fiscal system	Duclos (1993)
J88	Jenkins (1988) decomposition of vertical and reranking effect for the net fiscal system	Jenkins (1988)
	(b) Other approaches	
R^{MT}	Musgrave & Thin (1948) index of redistributive effect	Dilnot, Kay & Norris (1984)
BS	Berliant & Strauss (1984) measures of vertical and horizontal inequity	Berliant & Strauss (1985)
DJA	Duclos, Jalbert & Araar (2003) decomposition of the change in inequality	Duclos, Jalbert & Araar (2003)
DL	Duclos & Lambert (2000) measurement of horizontal and vertical inequity	Duclos & Lambert (2000)
DL01	Dardanoni & Lambert (2001) horizontal inequity comparisons	Dardanoni & Lambert (2001)

Symbol / Abbrev.	Approach	Studies using the methodology
DL02	Dardanoni & Lambert (2002) progressivity comparisons	Dardanoni & Lambert (2002)
LC	Lorenz curves comparisons	Bishop, Chow & Formby (1995), Förster (2000) [2]
LY94	Lerman & Yitzhaki (1994) decomposition of Gini coefficient	Lerman & Yitzhaki (1994)
PCF	Pyatt, Chen & Fei (1980) decomposition of Gini coefficient	Heady, Mitrakos & Tsakloglou (2001)
RSA	Baum (1987) relative share adjustment index	Dyck (2005)
S82	Shorrocks (1982) decompositions	Jännti (1997), Förster (2000) [2]
Tm	Atkinson (1980) analysis of reranking using transition matrices	Atkinson (1980), Nolan (1987) [2]
KJ	King (1983) / Jenkins (1994) "no reranking procedure"	Lambert, Thoresen (2009)
BD	Blackorby, Donaldson (1984) index of progressivity	Lambert, Thoresen (2009)

Note: A number in the square brackets appearing in the third column shows total number of studies in each subgroup.

3 METHODOLOGY

3.1 Introduction

This chapter makes the core of the dissertation. It is the bridge between the preceding chapter on the literature about the redistributive effects and the subsequent two chapters which aim to estimate these effects for Croatia. When we say that, we mean the following. In the literature overview we have set some doubts about the existing and widely used methodologies for the assessment of the redistributive effect, which are concerned with the role of reranking in the redistributive process. We have also seen that decompositions of redistributive effect did not fully achieve their aim. This chapter solves these issues and lays down new methods necessary for accomplishment of the mentioned empirical task.

One of the main hypotheses of this dissertation is that reranking of income units cannot influence the redistributive effect RE. The *first part* of this chapter (section 3.2) is fully devoted to defending this contention. For that purpose, measurement systems are carefully built based on income vector transitions, and income and rank distances between units. All these approaches are already known in the literature on Gini coefficient, but here they are extended to other indices and measures of redistributive effect and reranking. New/old concepts of distance narrowing, fiscal deprivation and domination are also presented. After all the measures are derived, we compare them to indices and decomposition existing in the literature, and establish the relationships.

The methodological apparatus developed here helps to develop important propositions about fiscal process, which are then used to prove the hypothesis that Kakwani and Lerman and Yitzhaki (1995) gave the *mistaken interpretation* of the role of reranking in the redistributive process. After obtaining the proof, the natural question is how to proceed: which measures should be used? The answers are offered in the end of the first part of the chapter.

In the *second part* of this chapter we continue the investigation of the properties of the redistributive process, scrutinizing it further. In the first part we have only discussed the starting and ending states in the process, pre-fiscal and post-fiscal income, abstracting what is there between them: taxes and benefits. These are two sorts of instruments by which the income redistribution is driven. Instinctive desire of many researchers in this field was to evaluate how different taxes and benefits contribute to overall redistributive effect. Some attempts were made, but as we have seen in the second chapter, they are not fully satisfactory. Here we derive new decompositions of redistributive effect and reranking.

Two methods are used in derivation. The first uses Lorenz and concentration curves and is already known in the literature ever since the first such work by Lambert (1985). The other method, based on "distances between units" approach to calculation of Gini coefficient, is newly applied in this area. The advantage over the former method is that it is able to also decompose reranking. Its further quality is that the contribution of each tax and benefit instrument is fixed, i.e. independent of the order in which taxes (benefits) are subtracted from (added to) the pre-fiscal income, as in Duclos (1993).

3.2 Measurement of income redistribution

3.2.1 Gini and concentration coefficient

3.2.1.1 Variables, vectors and ordering of units

First, we will define the ranking function $r(\mathbf{a})$, which returns a rank for each unit a_k in vector $\mathbf{a}$, such that the smallest element receives rank 1, etc. Let $\mathbf{y} = [y_1, \ldots, y_s]^{\mathrm{T}}$ be an income vector with s units and mean value $\bar{y}$. Vector $y^y = [y_1^y, \ldots, y_s^y]^{\mathrm{T}}$ contains values $y_i^y \in \mathbf{y}$ such that $i = r(y_k)$. Thus, $\mathbf{y}^y$ has the same values as the original vector $\mathbf{y}$, but sorted in ascending order of $\mathbf{y}$.

Vector $\mathbf{z} = [z_1, \ldots, z_s]^{\mathrm{T}}$ presents another variable. We define $\mathbf{z}^z = [z_1^z, \ldots, z_s^z]^{\mathrm{T}}$, with values $z_i^z \in \mathbf{z}$ such that $i = r(z_k)$. However, we may also define $\mathbf{z}^y = [z_1^y, \ldots, z_s^y]^{\mathrm{T}}$ with values $z_i^y \in \mathbf{z}$ such that $i = r(y_k)$. Observe the following distinction: $\mathbf{z}^y$ has the same values as $\mathbf{z}$, but they are sorted in ascending order of $\mathbf{y}$. Yet another variable, $\mathbf{y}^z$, can be defined analogously.

3.2.1.2 "Distance from mean" approach

Equation (3.1) specifies Gini coefficient (G_y). For each income unit with **y**-rank i and value y_i^y, the distance from the mean value $\bar{y}$ is weighted by $(s - i + \frac{1}{2})$, where s is the highest rank. The weighted distances from mean are then averaged by s^2, and expressed as a share in the mean income.

$$G_y = \frac{2}{s^2 \bar{y}} \sum_{i=1}^{n} (s - i + \tfrac{1}{2})(\bar{y} - y_i^y) \qquad (3.1)$$

The concentration coefficient is defined analogously. As equation (3.2) shows, for each income unit with $\mathbf{z}$-rank i and value y_i^z, the distance from the mean value $\bar{y}$ is weighted. Remember that y_i^z contains values of $\mathbf{y}$ sorted using ranks from the vector $\mathbf{z}$.

$$D_y^z = \frac{2}{s^2 \bar{y}} \sum_{i=1}^{s} (s - i + \tfrac{1}{2})(\bar{y} - y_i^z) \qquad (3.2)$$

The Gini coefficient and concentration coefficient can be seen as members of a class of single-parameter or S-Gini coefficients, for which the parameter takes value $\rho = 2$. In a discrete case, S-Gini is $G_y = 2s^{-2}\bar{y}^{-1} \sum_i^s \omega(i;\rho)(\bar{y} - y_i)$, where $\omega(i;\rho) = \left((s-i+1)^\rho - (s-i)^\rho\right)/s^{\rho-1}$. In (3.1) and (3.2), the term $s - i + \frac{1}{2}$ is actually the weighting scheme $\omega(p_i;\rho)$ obtained for $\rho = 2$. Since the number $\frac{1}{2}$ in the term $s - i + \frac{1}{2}$ does not affect the estimate of Gini and concentration coefficients, we will ignore it for simplicity. We will introduce it again in welfare analysis. Thus, we can write:

$$G_y = \frac{2}{s^2 \bar{y}} \sum_{i=1}^{s} (s - i)(\bar{y} - y_i^y) \qquad (3.3)$$

$$D_y^z = \frac{2}{s^2 \bar{y}} \sum_{i=1}^{s} (s - i)(\bar{y} - y_i^z) \qquad (3.4)$$

3.2.1.3 "Distance between units" approach

Another way to calculate Gini and concentration coefficients is based on the differences between income pairs (Lambert, 2001:34) and is even more straightforward. Instead of summing distances from the mean, formula (3.6) sums *income distances* between units, for all possible pairs (i, j).

$$G_y = \frac{1}{2s^2\overline{y}} \sum_{i=1}^{s} \sum_{j=1}^{s} \left| y_i - y_j \right| \tag{3.5}$$

Notice the distinction between the terms of *income difference* and *income distance*. The former is presented by $y_i - y_j$, and can be either positive or negative. The latter term, $\left| y_i - y_j \right|$, is always positive as a result of absolute signs. Now, if instead of values y_i and y_j, we decide to use y_i^y and y_j^y, and if we take only the values such that i is always greater or equal to j, then we can rewrite (3.5) to obtain the Gini coefficient of y, as shown by (3.6). Analogously, replacing y_i^y and y_j^y in (3.6) with y_i^z and y_j^z, we obtain the concentration coefficient of y with respect to z, as in (3.7).

$$G_y = \frac{1}{s^2\overline{y}} \sum_{i=1}^{s} \sum_{j=1}^{i} (y_i^y - y_j^y) \tag{3.6}$$

$$D_y^z = \frac{1}{s^2\overline{y}} \sum_{i=1}^{s} \sum_{j=1}^{i} (y_i^z - y_j^z) \tag{3.7}$$

For illustration purposes and easier derivation of other indices later, we draw matrices of the following form: $\mathbf{M}(i, j) = y_i^y - y_j^y$, defined only for $i \geq j$. Because the numbers in these matrices fill only the space on one side of the diagonal, we call them triangular. It is shown in general form by Figure 3.1 Since diagonal elements are always equal to zero, the presentation of the matrix can be reduced to the form presented in Figure 3.2.

Figure 3.1: "Full" triangular matrix

y_1^y	$y_1^y - y_1^y$						
y_2^y	$y_2^y - y_1^y$	$y_2^y - y_2^y$					
$\dots$	$\dots$	$\dots$	$\dots$				
y_p^y	$y_p^y - y_1^y$	$y_p^y - y_2^y$	$\dots$	$y_p^y - y_p^y$			
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$		
y_{s-1}^y	$y_{s-1}^y - y_1^y$	$y_{s-1}^y - y_2^y$	$\dots$	$y_{s-1}^y - y_p^y$	$\dots$	$y_{s-1}^y - y_{s-1}^y$	
y_s^y	$y_s^y - y_1^y$	$y_s^y - y_2^y$	$\dots$	$y_s^y - y_p^y$	$\dots$	$y_s^y - y_{s-1}^y$	$y_s^y - y_s^y$
	y_1^y	y_2^y	$\dots$	y_p^y	$\dots$	y_{s-1}^y	y_s^y

Figure 3.2: Compact triangular matrix

y_2^y	$y_2^y - y_1^y$							
$\dots$	$\dots$							
y_p^y	$y_p^y - y_1^y$	$y_p^y - y_2^y$	$\dots$	$y_p^y - y_{p-1}^y$				
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$				
y_{s-1}^y	$y_{s-1}^y - y_1^y$	$y_{s-1}^y - y_2^y$	$\dots$	$y_{s-1}^y - y_{p-1}^y$	$\dots$	$y_{s-1}^y - y_{s-2}^y$		
y_s^y	$y_s^y - y_1^y$	$y_s^y - y_2^y$	$\dots$	$y_s^y - y_{p-1}^y$	$\dots$	$y_s^y - y_{s-2}^y$	$\dots$	$y_s^y - y_{s-1}^y$
	y_1^y	y_2^y	$\dots$	y_{p-1}^y	$\dots$	y_{s-2}^y	$\dots$	y_{s-1}^y

Equations (3.6) and (3.7) can be rewritten in the light of this reduced form of the matrix presentation.

$$G_y = \frac{1}{s^2 \bar{y}} \sum_{i=2}^{s} \sum_{j=1}^{i-1} \left(y_i^y - y_j^y \right) \tag{3.8}$$

$$D_y^z = \frac{1}{s^2 \bar{y}} \sum_{i=2}^{s} \sum_{j=1}^{i-1} \left(y_i^z - y_j^z \right) \tag{3.9}$$

In all subsequent analysis we will present the formulas in the matrix presentation. Therefore, it will always be assumed that $i \geq j$. A useful property should be remembered, presented in (3.10).

$$\left| y_i - y_j \right| = y_i^y - y_j^y \text{, for all } (i, j) \text{ such that } i \geq j \tag{3.10}$$

3.2.1.4 Lorenz and concentration curves approach

As the third way of presenting Gini and concentration coefficients, we mention the original approach that uses Lorenz and the concentration curves. Lorenz curve abscissas are cumulative proportions of units, p_i, and ordinates are cumulative proportions of the variable considered, $L_y(i)$. Equations (3.11) and (3.12) are used to obtain them.

$$p_i = \frac{i}{s} \tag{3.11}$$

$$L_y(p_i) = \frac{1}{s\bar{y}} \sum_{j=1}^{i} y_j^y \tag{3.12}$$

The Gini coefficient is defined as double the area between the line of absolute equality and Lorenz curve. The line of absolute equality presents a situation in which all values of **y** are equal to $\bar{y}$. Notice that in this case Lorenz curve would be equal to:

$$L_{\bar{y}}(p_i) = \frac{1}{s\bar{y}} \sum_{j=1}^{i} \bar{y} = \frac{1}{s\bar{y}} \bar{y}i = \frac{i}{s} = p_i \tag{3.13}$$

In the discrete case we deal here with, the Gini can be approximated as double the average of distances between the line of absolute equality and Lorenz curve, $L_y(p_i)$.

$$G_y = \frac{2}{s}\left(\frac{1}{s\bar{y}} \bar{y}i - \frac{1}{s\bar{y}} \sum_{j=1}^{i} y_j^y \right) =$$
$$= \frac{2}{s^2 \bar{y}} \sum_{i=1}^{s} \left(\bar{y}i - \sum_{j=1}^{i} y_j^y \right) = \frac{2}{s} \sum_{i=1}^{s} \left(\frac{i}{s} - \sum_{j=1}^{i} \frac{y_j^y}{s\bar{y}} \right) = \frac{2}{s} \sum_{i=1}^{s} \left(p_i - L_y(p_i) \right) \tag{3.14}$$

Similarly, the concentration coefficient can be calculated using the concentration curve $C_y^z(p_i)$ instead of Lorenz curve, as presented in (3.15).

$$D_y^z = \frac{2}{s}\left(\frac{1}{s\bar{y}} \bar{y}i - \frac{1}{s\bar{y}} \sum_{j=1}^{i} y_j^z \right) =$$

$$= \frac{2}{s^2 \bar{y}} \sum_{i=1}^{s} \left(\bar{y}i - \sum_{j=1}^{i} y_j^z \right) = \frac{2}{s} \sum_{i=1}^{s} \left(\frac{i}{s} - \sum_{j=1}^{i} \frac{y_j^z}{s\bar{y}} \right) = \frac{2}{s} \sum_{i=1}^{s} \left(p_i - C_y^z(p_i) \right) \quad (3.15)$$

3.2.2 Analysis of income transitions

3.2.2.1 Income variables

$\mathbf{X}$ and $\mathbf{N}$ are vectors of pre-fiscal and post-fiscal income, respectively; the jth entry of $\mathbf{X}$, X_j, and the jth entry of $\mathbf{N}$, N_j, present income information for the particular income unit j. Vectors $\mathbf{X}^x$, $\mathbf{X}^n$, $\mathbf{N}^n$ and $\mathbf{N}^x$ are different sortings of vectors $\mathbf{X}$ and $\mathbf{N}$, as explained in the previous section.

Table 3.1 shows a hypothetical population of five and their income vectors $\mathbf{X}$ and $\mathbf{N}$. In these original vectors, the units take either random or alphabetic (perhaps, according to family names) or some other order, independent of incomes. Columns $r(\mathbf{X})$ and $r(\mathbf{N})$ present ranks of units according to pre-fiscal and post-fiscal income. We observe they are not identical: indeed each unit changes its rank in the transition from pre-fiscal to post-fiscal income.

Table 3.1 Hypothetical data set

Unit	$\mathbf{X}$	$\mathbf{N}$	$r(\mathbf{X})$	$r(\mathbf{N})$
A	180	80	5	4
B	30	100	3	5
C	70	20	4	1
D	8	40	1	2
E	12	60	2	3

Unit	$\mathbf{X}^x$	$\mathbf{N}^x$
D	8	40
E	12	60
B	30	100
C	70	20
A	180	80

Unit	$\mathbf{X}^n$	$\mathbf{N}^n$
C	70	20
D	8	40
E	12	60
A	180	80
B	30	100

In the second step, we sort units in ascending order of pre-fiscal income and create vectors $\mathbf{X}^x$ and $\mathbf{N}^x$. Notice that the 1st place in $\mathbf{X}^x$ and $\mathbf{N}^x$ is taken by the unit with pre-fiscal rank 1 (unit D), the 2nd place is taken by the unit with pre-fiscal rank 2 (unit E) etc. In the similar way, but using **N**-ranks, we create vectors $\mathbf{X}^n$ and $\mathbf{N}^n$. The 1st place is taken by the unit with post-fiscal rank 1 (C), …, the 5th place is occupied by the unit with post-fiscal rank 5 (B).

We can see from this example that pre-fiscal and post-fiscal rankings of units, represented by $r(\mathbf{X})$ and $r(\mathbf{N})$, are not necessarily identical, and in reality they are certainly not. The difference in them is a consequence of the "process" we will call reranking to which we will devote a great deal of attention in what follows.

3.2.2.2 Transitions from pre-fiscal to post-fiscal income

By means of redistributive effects we mean various transitions from pre-fiscal to post-fiscal income, but also the transitions from pre-fiscal to pre-fiscal income, and post-fiscal to post-fiscal income. For each transition we derive a specific index of the redistributive effect. Later we will develop further distinct concepts of income distance narrowing, fiscal deprivation (domination), and deprivation (domination) due to reranking, and see how these are connected with the redistributive effects.

From pre-fiscal vector $\mathbf{X}$ and post-fiscal vector $\mathbf{N}$, we have derived four ordered vectors: $\mathbf{X}^x$, $\mathbf{X}^n$, $\mathbf{N}^x$ and $\mathbf{N}^n$, which form the basis of the analysis. We will first concentrate on the transitions from pre-fiscal to post-fiscal incomes, and leave the transitions from pre-fiscal to pre-fiscal and from post-fiscal to post-fiscal income for the next section. We have four possible transitions from pre-fiscal to post-fiscal income:

(a) $\mathbf{X}^x \rightarrow \mathbf{N}^x$ ($X_i^x \rightarrow N_i^x$); (b) $\mathbf{X}^n \rightarrow \mathbf{N}^n$ ($X_i^n \rightarrow N_i^n$)

(c) $\mathbf{X}^x \rightarrow \mathbf{N}^n$ ($X_i^x \rightarrow N_i^n$); (d) $\mathbf{X}^n \rightarrow \mathbf{N}^x$ ($X_i^n \rightarrow N_i^x$)

In transitions $\mathbf{X}^x \rightarrow \mathbf{N}^x$ and $\mathbf{X}^n \rightarrow \mathbf{N}^n$, the pre-fiscal income of one unit is compared to the post-fiscal income of the same unit. In transition $\mathbf{X}^x \rightarrow \mathbf{N}^n$, the pre-fiscal income of the unit with *pre-fiscal* rank i is compared to the post-fiscal income of the unit with *post-fiscal* rank i. In transition $\mathbf{X}^n \rightarrow \mathbf{N}^x$, the pre-fiscal income of the unit with *post-fiscal* rank i is compared to the post-fiscal income of the unit with *pre-fiscal* rank i. In the presence of reranking, these are different units. Thus, for transitions (c) and (d), the link between pre-fiscal and post-fiscal income will not be factual but counterfactual. In the rest of the analysis, we will concentrate on the first three transitions.

These aspects are illustrated in Table 3.2, based on the hypothetical data set from the previous table. For the first two transitions, the pre-fiscal income of unit D is transformed into the post-fiscal income of unit D (and so for the other four units). However, for the third transition, the pre-fiscal income of D is transformed into the post-fiscal income of unit C; E is translated into D, B into E, etc.

Table 3.2: Transitions from pre-fiscal to post-fiscal income

$\mathbf{X}^x \rightarrow \mathbf{N}^x$				$\mathbf{X}^n \rightarrow \mathbf{N}^n$				$\mathbf{X}^x \rightarrow \mathbf{N}^n$			
Unit	X_i^x	Unit	N_i^x	Unit	X_i^n	Unit	N_i^n	Unit	X_i^x	Unit	N_i^n
D	8	D	40	C	70	C	20	D	8	C	20
E	12	E	60	D	8	D	40	E	12	D	40
B	30	B	100	E	12	E	60	B	30	E	60
C	70	C	20	A	180	A	80	C	70	A	80
A	180	A	80	B	30	B	100	A	180	B	100

3.2.2.3 Transitions from pre-fiscal to pre-fiscal income and from post-fiscal to post-fiscal income

In the previous section we have analyzed transitions from pre-fiscal to post-fiscal income. It was indicated that other transitions are also possible: from pre-fiscal to pre-fiscal income, and from post- to post-fiscal income. The former occurs between $\mathbf{N}^n$ and $\mathbf{N}^x$ and the latter between $\mathbf{X}^x$ and $\mathbf{X}^n$ as follows:

(a) $\mathbf{N}^n \rightarrow \mathbf{N}^x$ $(N_i^n \rightarrow N_i^x)$

(b) $\mathbf{X}^x \rightarrow \mathbf{X}^n$ $(X_i^x \rightarrow X_i^n)$

In transition $\mathbf{N}^n \rightarrow \mathbf{N}^x$, the post-fiscal income of the unit with *post-fiscal* rank i is compared to the post-fiscal income of the unit with *pre-fiscal* rank i. In presence of reranking, these are different units. The same relates to the transition $\mathbf{X}^x \rightarrow \mathbf{X}^n$, where the pre-fiscal income of the unit with *pre-fiscal* rank i translates into the pre-fiscal income of the unit with *post-fiscal* rank i.

This is illustrated in Table 3.3. The post-fiscal income of unit C is transformed into post-fiscal income of unit D, D is translated into E, E into B, etc. The pre-fiscal income of unit D is translated into pre-fiscal income of unit C, etc.

Table 3.3: Transitions from pre-fiscal to pre-fiscal and from post- to post-fiscal income

$\mathbf{N}^n \rightarrow \mathbf{N}^x$			
Unit	N_i^n	Unit	N_i^x
C	20	D	40
D	40	E	60
E	60	B	100
A	80	C	20
B	100	A	80

$\mathbf{X}^x \rightarrow \mathbf{X}^n$			
Unit	X_i^x	Unit	X_i^n
D	8	C	70
E	12	D	8
B	30	E	12
C	70	A	180
A	180	B	30

3.2.2.4 Indices of redistributive effect

All measurement in this study is based upon the concepts of Gini and the concentration coefficients. There are many different ways to calculate them; three methods are used here, which have been explained above. Redistributive effect and other indices, are also formed on the basis of Gini and the concentration coefficients.

Throughout the text, we assume that average post- and pre-fiscal incomes are equal, $\overline{N} = \overline{X}$. This enables easier derivation of the formulas and later we make adaptations to account for the case where $\overline{N} \neq \overline{X}$.

For the first three transitions from pre-fiscal to post-fiscal income explained in the previous section, we have the following three indices of the redistributive effect, shown in (3.16), (3.17) and (3.18). For the two other transitions, from pre-fiscal to pre-fiscal and post- to post-fiscal income, we have two additional indices of the redistributive effect, presented in (3.19) and (3.20).

For transition $\mathbf{X}^x \rightarrow \mathbf{N}^x$, the index RE^x:

$$RE^x \quad = 2c\sum_{i=1}^{s}(s-i)\left(N_i^x - X_i^x\right) \quad = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(X_i^x - X_j^x\right) - \left(N_i^x - N_j^x\right)\right)$$

$$= \frac{2}{s}\sum_{i=1}^{s}\left(C_N^x(p_i) - L_X(p_i)\right) \quad = G_X - D_N^x \qquad (3.16)$$

For transition $\mathbf{X}^n \rightarrow \mathbf{N}^n$, the index RE^n:

$$RE^n = 2c\sum_{i=1}^{s}(s-i)\left(N_i^n - X_i^n\right) = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(X_i^n - X_j^n\right)-\left(N_i^n - N_j^n\right)\right)$$

$$= \frac{2}{s}\sum_{i=1}^{s}\left(L_N(p_i) - C_X^n(p_i)\right) = D_X^n - G_N \qquad (3.17)$$

For transition $\mathbf{X}^x \rightarrow \mathbf{N}^n$, the index RE^{xn}:

$$RE^{xn} = 2c\sum_{i=1}^{s}(s-i)\left(N_i^n - X_i^x\right) = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(X_i^x - X_j^x\right)-\left(N_i^n - N_j^n\right)\right)$$

$$= \frac{2}{s}\sum_{i=1}^{s}\left(L_N(p_i) - L_X(p_i)\right) = G_X - G_N \qquad (3.18)$$

For transition $\mathbf{N}^n \rightarrow \mathbf{N}^x$, the index RE^{rx}:

$$RE^{rx} = 2c\sum_{i=1}^{s}(s-i)\left(N_i^x - N_i^n\right) = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(N_i^n - N_j^n\right)-\left(N_i^x - N_j^x\right)\right)$$

$$= \frac{2}{s}\sum_{i=1}^{s}\left(C_N^x(p_i) - L_N(p_i)\right) = G_N - D_N^x \qquad (3.19)$$

For transition $\mathbf{X}^x \rightarrow \mathbf{X}^n$, the index RE^{rn}:

$$RE^{rn} = 2c\sum_{i=1}^{s}(s-i)\left(X_i^n - X_i^x\right) = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(X_i^x - X_j^x\right)-\left(X_i^n - X_j^n\right)\right)$$

$$= \frac{2}{s}\sum_{i=1}^{s}\left(C_X^n(p_i) - L_X(p_i)\right) = G_X - D_X^n \qquad (3.20)$$

where the value of c is equal to $c = 1/s^2\overline{N}$.

3.2.3 Income distance, fiscal deprivation and domination

3.2.3.1 Fiscal deprivation

At the same time transitions from pre-fiscal to post-fiscal income induce changes in income distances and changes of income ranks. In this section, we will scrutinize the redistributive process at the level of two income units, and afterwards, the relations will be aggregated to the level of the whole population. This will result in new indices of distance narrowing and reranking.

Suppose that two income units have pre-fiscal incomes X_i^x and X_j^x, such that $X_i^x > X_j^x$ and $i > j$. Their respective post-fiscal incomes are N_i^x and N_j^x. First, let us define *distance narrowing* ($\Delta_{i,j}$).

$$\Delta_{i,j} = |X_i - X_j| - |N_i - N_j| \tag{3.21}$$

If the distance between units is narrowed, we have that $\Delta_{i,j} > 0$; if it is widened, there is $\Delta_{i,j} < 0$. Next, we will define the *deprivation due to reranking* ($r_{i,j}^x$) of the unit with pre-fiscal rank i, that may be reranked by the unit with rank j.

$$r_{i,j}^x = \frac{1}{2}\left(\left|N_i^x - N_j^x\right| - \left(N_i^x - N_j^x\right)\right) \tag{3.22}$$

If $N_i^x > N_j^x$, there is no reranking and $r_{i,j}^x = 0$. However, if $N_i^x < N_j^x$, it means that reranking occurred, and $r_{i,j}^x = N_j^x - N_i^x$.[13] Finally, let us define *fiscal deprivation* ($\varsigma_{i,j}^x$) of the unit with pre-fiscal income rank i, over the unit with pre-fiscal income rank j.

$$\varsigma_{i,j}^x = \left(X_i^x - X_j^x\right) - \left(N_i^x - N_j^x\right) \tag{3.23}$$

The three measures defined above, $\Delta_{i,j}$, $\varsigma_{i,j}^x$ and $r_{i,j}^x$, are connected as shown by the following equation.

$$\varsigma_{i,j}^x = \Delta_{i,j} + 2r_{i,j}^x \tag{3.24}$$

[13] Notice that "deprivation due to reranking" closely resembles the concept of „fiscal looseness", presented by Duclos (2000).

How to interpret all these terms notions intuitively? First, we may say that the difference $X_i^x - X_j^x$ denotes "income supremacy" of i over j. Say that i worked harder, and now enjoys having higher income than j, and $X_i^x - X_j^x$ measures the intensity of this "feeling". However, the fiscal process occurs, and i's "income supremacy" changes to $N_i^x - N_j^x$. Thus, the term $\varsigma_{i,j}^x$ (3.23) signifies the *change* of income advantage of i over j, in the transition from pre-fiscal to post-fiscal income. If i loses a part of this advantage or supremacy ($\varsigma_{i,j}^x > 0$), we say that she is "fiscally deprived", and hence the name for the term. Fiscal deprivation can be divided (3.24) into two components: distance narrowing ($\Delta_{i,j}$) and reranking ($2r_{i,j}^x$).

Now, assume that the units with ranks i and j are informed that, in order to improve social welfare, the income distance between i and j will be reduced by $\Delta_{i,j}^T \leq |X_i - X_j|$. What may be the consequences of this action on the "income supremacy" of i, i.e. how large could her fiscal deprivation be? In the case of no reranking, fiscal deprivation will be equal to $\Delta_{i,j}^T$. In the presence of reranking, it increases to $\Delta_{i,j}^T + 2r_{i,j}^x$.[14]

Assume that "society" agrees that certain distance narrowing is desirable between i and j, i.e. i must sacrifice part of her "income supremacy". However, it is also required that pre-fiscal rankings should not be altered, i.e. i must remain "the rich", and j "the poor". In this light, we may treat the reranking component of fiscal deprivation ($2r_{i,j}^x$) as an ***excess*** *fiscal deprivation* felt by i.

[14] Is it "just" that i must sacrifice $\Delta_{i,j}^T$ of her income supremacy? For a "libertarian", the only permissible situation is $\Delta_{i,j}^T = 0$. For an "equalitarian", the perfect situation would be that $\Delta_{i,j}^T = |X_i - X_j|$, so that $N_i = N_j$. Usually, we would say that it is "just" that $\Delta_{i,j}^T > 0$, but the allowed magnitude of $\Delta_{i,j}^T$ would vary.

3.2.3.2 Fiscal domination

Two income units have post-fiscal incomes N_i^n and N_j^n, such that $N_i^n > N_j^n$ and $i > j$. Their respective pre-fiscal incomes are X_i^n and X_j^n. *Distance narrowing* ($\Delta_{i,j}$) is already defined in (3.21). Here we also define *distance widening* as negative *distance narrowing*.

$$\nabla_{i,j} = -\Delta_{i,j} \tag{3.25}$$

Let us define the *domination due to reranking* ($r_{i,j}^n$) of the unit with post-fiscal rank i, that might have reranked the unit with post-fiscal rank j.

$$r_{i,j}^n = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left(X_i^n - X_j^n\right)\right) \tag{3.26}$$

If $X_i^n \geq X_j^n$, there was no reranking and $r_{i,j}^n = 0$. However, if $X_i^n < X_j^n$, it means that reranking occurred, whereby $r_{i,j}^n = X_j^n - X_i^n$. Finally, we will define the *fiscal domination* ($\varsigma_{i,j}^n$) of the unit with post-fiscal income rank i, over the unit with post-fiscal income rank j, as in (3.27).

$$\varsigma_{i,j}^n = \left(N_i^n - N_j^n\right) - \left(X_i^n - X_j^n\right) \tag{3.27}$$

The relationship between the measures is represented by the following equation.

$$\varsigma_{i,j}^n = -\Delta_{i,j} + 2r_{i,j}^n = \nabla_{i,j} + 2r_{i,j}^n \tag{3.28}$$

The difference $N_i < N_j$ denotes the post-fiscal "income supremacy" of i over j. The former unit enjoys higher income, either because she worked harder (whereby earning higher pre-fiscal income), or as a consequence of the fiscal process. Fiscal domination ($\varsigma_{i,j}^n$) measures a change of i's "income supremacy" in the transition from pre-fiscal to post-fiscal income. This change can be arrived at through two channels: distance widening ($\nabla_{i,j}$) and reranking ($2r_{i,j}^{n}$). On the other hand, distance narrowing ($\Delta_{i,j} = -\nabla_{i,j}$) reduces fiscal domination.

The decomposition (3.28) also tells us that, for given $\nabla_{i,j}$, fiscal domination will be larger, the higher reranking is. Therefore, we may treat $2r_{i,j}^{n}$ as an ***augmented*** *fiscal domination* of the unit with post-fiscal rank i.

3.2.3.3 Comparison of the approaches

Compare the role of reranking in this and the previous section: it *increases* both fiscal domination (3.28) and fiscal deprivation (3.24). Since domination and deprivation are opposite terms, it means that reranking plays a reverse role in the two approaches: it is "bad" when causing excess fiscal deprivation (3.24), but it is "good" when it enhances fiscal domination (3.28).

The concept of fiscal domination is somewhat odd because it favours (assuming that a positive value of a measure means "good") both distance widening and reranking, two concepts that are usually disapproved of.

3.2.3.4 Indices of change in income distance, fiscal deprivation and domination

In the previous two sections, we have defined exactly five new terms related to distances and ranks of income units. All these terms were defined for pairs of units (i, j). Fortunately, we can easily aggregate them to obtain indices that reflect these concepts for the whole population of units.

The index of *distance narrowing*, Δ, is derived from (3.21). By rule (3.10), we have that $\left|X_i - X_j\right| = \left(X_i^x - X_j^x\right)$ and $\left|N_i - N_j\right| = \left(N_i^n - N_j^n\right)$ for all (i, j) where $i \geq j$.

$$\Delta = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left|X_i - X_j\right| - \left|N_i - N_j\right|\right) = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(X_i^x - X_j^x\right) - \left(N_i^n - N_j^n\right)\right) \qquad (3.29)$$

The index of *deprivation due to reranking*, R^x, is derived from (3.22).

$$R^x = \frac{c}{2}\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left|N_i^x - N_j^x\right| - \left(N_i^x - N_j^x\right)\right) \tag{3.30}$$

The *fiscal deprivation* index, V^x, is derived from (3.23).

$$V^x = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(X_i^x - X_j^x\right) - \left(N_i^x - N_j^x\right)\right) \tag{3.31}$$

The index of *domination due to reranking*, R^n, is derived from (3.26).

$$R^n = \frac{c}{2}\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left|X_i^n - X_j^n\right| - \left(X_i^n - X_j^n\right)\right) \tag{3.32}$$

The *reverse fiscal domination* index, V^n, is derived from (3.27). The "true" index of *fiscal domination* would be $-V^n$, but this reversal was done for easier alignment with other indices as will be witnessed later.

$$V^n = -c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(\left(N_i^n - N_j^n\right) - \left(X_i^n - X_j^n\right)\right) \tag{3.33}$$

To demonstrate how these indices are obtained, we use hypothetical data shown in Table 3.4, which are based on the hypothetical data set from Table 3.1. In the left part of Table 3.4, we have vectors sorted by pre-fiscal income, and in the right part, by the post-fiscal income. However, you should notice that they relate to the same hypothetical population of five units. We assume that there is only one tax and one benefit, and total tax is equal to total benefit. Here, we calculate the basic indicators, those presented by equations (3.29) through (3.33).

Table 3.4: Hypothetical data set

Unit	i	X_i^x	$T_{i,1}^x$	$B_{i,1}^x$	N_i^x
D	1	8	0	32	40
E	2	12	0	48	60
B	3	30	10	80	100
C	4	70	60	10	20
A	5	180	100	0	80

Unit	k	X_k^n	$T_{k,1}^n$	$B_{k,1}^n$	N_k^n
C	1	70	60	10	20
D	2	8	0	32	40
E	3	12	0	48	60
A	4	180	100	0	80
B	5	30	10	80	100

For income vectors in Table 3.4, we first construct triangular matrices, M_1 to M_6, shown in Figure 3.3. The matrix M_5 contains values of $r_{i,j}^x$ as defined in (3.22), and M_6 has the values of $r_{i,j}^n$ obtained by (3.26). For each matrix, the sum of its elements is calculated, which we may denote simply as $\Sigma\,\mathrm{M}_1$, $\Sigma\,\mathrm{M}_2$, etc. Thus, by $\Sigma\,\mathrm{M}_1$ we mean $\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^x - X_j^x\right)$, while $\Sigma\,\mathrm{M}_5$ denotes $\sum_{i=2}^{s}\sum_{j=1}^{i-1} r_{i,j}^x$, etc.

For indicators Δ, V^x and V^n, we combine $\Sigma\,\mathrm{M}_1$ to $\Sigma\,\mathrm{M}_4$ and multiply them by $c = 5^{-2}\cdot 60^{-1}$. They are calculated as follows.

$$\Delta \quad = c\left(\Sigma\mathrm{M}_1 - \Sigma\mathrm{M}_4\right) = c\cdot(804-400) = 0.2693$$

$$V^x \quad = c\left(\Sigma\mathrm{M}_1 - \Sigma\mathrm{M}_2\right) = c\cdot(804-80) = 0.4827$$

$$V^n \quad = -c\left(\Sigma\mathrm{M}_4 - \Sigma\mathrm{M}_3\right) = -c\cdot(400-184) = -0.1440$$

To obtain R^x we use $\Sigma\,\mathrm{M}_5$, and $\Sigma\,\mathrm{M}_6$ for R^n.

$$R^x \quad = c\left(\Sigma\mathrm{M}_5\right) = c\cdot 160 = 0.1067$$

$$R^n \quad = c\left(\Sigma\mathrm{M}_6\right) = c\cdot 310 = 0.2067$$

Figure 3.3: Matrices with hypothetical data

$M_1(i,j) = X_i^x - X_j^x$

12	4			*804*
30	22	18		
70	62	58	40	
180	172	168	150	110
	8	12	30	70

$M_2(i,j) = N_i^x - N_j^x$

60	20			*80*
100	60	40		
20	-20	-40	-80	
80	40	20	-20	60
	40	60	100	20

$M_3(i,j) = X_i^n - X_j^n$

8	-62			*184*
12	-58	4		
180	110	172	168	
30	-40	22	18	-150
	70	8	12	180

$M_4(i,j) = N_i^n - N_j^n$

40	20			*400*
60	40	20		
80	60	40	20	
100	80	60	40	20
	20	40	60	80

$M_5(i,j) = r_{i,j}^x$

60	0			*160*
100	0	0		
20	20	40	80	
80	0	0	20	0
	40	60	100	20

$M_6(i,j) = r_{i,j}^n$

8	62			*310*
12	58	0		
180	0	0	0	
30	40	0	0	150
	70	8	12	180

3.2.4 Relationships between new and existing indices

We have already defined a number of different concepts, terms and indices above. At this point we have to reveal the relationships between them, and with measures already present in the literature. As we will see, all the new indices have their traditional correspondents. Urban (2009) provides detailed overview of the latter indices, and here we briefly summarize them. The "classical" or standard index of redistributive effect (RE), the Kakwani (1977; 1984) index of vertical effect (V^K), the Lerman and Yitzhaki (1995) index of "gap narrowing" (V^{LY}), the Atkinson (1980) and Plotnick (1981) index of reranking (R^{AP}), and the Lerman and Yitzhaki (1995) index of reranking (R^{LY}), are respectively defined in equations (3.34) through (3.38).

$$RE = G_X - G_N \tag{3.34}$$

$$V^K = G_X - D_N^x \tag{3.35}$$

$$V^{LY} = D_X^n - G_N \tag{3.36}$$

$$R^{AP} = G_N - D_N^x \tag{3.37}$$

$$R^{LY} = G_X - D_X^n \tag{3.38}$$

The index of redistributive effect RE^{xn} in (3.18), has the same content as the index of distance narrowing Δ in (3.29), and is identical to the standard redistributive effect RE in (3.34).

$$RE^{xn} = \Delta = RE \tag{3.39}$$

The index of redistributive effect RE^{x} (3.16) and the fiscal deprivation index V^{x} (3.31) correspond to the Kakwani index of vertical effect V^{K} (3.35).

$$RE^{x} \equiv V^{x} \equiv V^{K} \tag{3.40}$$

The index of redistributive effect RE^{n} (3.17) and the reverse fiscal domination index V^{n} (3.33) correspond to the Lerman-Yitzhaki index of "gap narrowing" V^{LY} (3.36).

$$RE^{n} \equiv V^{n} \equiv V^{LY} \tag{3.41}$$

The index of redistributive effect RE^{rx} (3.19) equals twice the index of deprivation due to reranking R^{x} (3.30), and is identical to the Atkinson-Plotnick index of reranking R^{AP} (3.37).

$$RE^{rx} = 2R^{x} = R^{AP} \tag{3.42}$$

The index of redistributive effect RE^{rn} (3.20) is equal to twice the index of domination due to reranking R^{n} (3.32), and has the same content as Lerman-Yitzhaki index of reranking R^{LY} (3.38).

$$RE^{rn} = 2R^{n} = R^{LY} \tag{3.43}$$

3.2.5 Properties of redistributive effects, distance narrowing and reranking

3.2.5.1 Arguments to prove

Above, we have defined indices of redistributive effect (section 3.2.2.4), and of distance narrowing and reranking (section 3.2.3.4). Now we reveal their interrelatedness and present several important properties. The sections that follow aim to explain and prove the following three arguments. They are important for deriving the main conclusions about the Kakwani and Lerman-Yitzhaki decompositions in section 3.2.6.

(1) *Distance narrowing and reranking are independent* (see section 3.2.5.2)

(2) *Elimination of reranking cannot change the extent of distance narrowing* (see 3.2.5.3)

(3) *Redistributive effects can be presented as combinations of distance narrowing and reranking* (see 3.2.5.4)

3.2.5.2 Distance narrowing and reranking as separate effects

In this section, we prove the Argument 1 that distance narrowing and reranking are distinct and independent concepts. Imagine that we have two lottery boxes, one with balls representing pre-fiscal and the other post-fiscal incomes. We draw the balls one by one randomly and simultaneously from both boxes, and write the *combination* on the board, creating vectors $\mathbf{X}$ and $\mathbf{N}$, with pairs (X_i, N_i), as in Table 3.1.

Now, observe the formula (3.29) for distance narrowing and imagine that we repeat the lottery, obtaining many combinations. The fact is that, for each combination, the index Δ will be the same. The distance narrowing index does not depend on the order in which the units are drawn (sorted, ranked). Recall now the two formulas for deprivation / domination due to reranking, (3.30) and (3.32). The situation is quite different for reranking: each combination will result in different values of the indices RE^{rx} and RE^{rn}.

For given vectors $\mathbf{X}$ and $\mathbf{N}$, imagine a process of income swapping within any pair of units, so that the first unit obtains the post-fiscal income of the other, and vice versa. Referring to the above, we conclude that such swapping will affect reranking, but not distance narrowing.

We have seen that identity exists between indices RE^{xn} and Δ. We may conclude that the redistributive effect RE^{xn}, except that depicting the transition $\mathbf{X}^x \rightarrow \mathbf{N}^n$, is also a true measure of distance narrowing. This has some important implications: RE^{xn}, in the same way as Δ, is not sensitive to income ranks. Given the elements of the vectors $\mathbf{X}$ and $\mathbf{N}$, for any actual permutation of $\mathbf{N}^x$ and $\mathbf{X}^n$, the indices Δ and RE^{xn} will have the same values.

Thus, RE^{xn} fully registers the distance narrowing effect induced by the fiscal system. At the same time, it is completely indifferent about rank changes of the units in the transition from pre-fiscal to post-fiscal income. These are important messages to users of the index, having a normative significance that should not be neglected. Thus, if we use RE^{xn} as our *sole* measure of the redistributive effect, it means the following:

(a) We do not care about the reranking of units in the transition from pre-fiscal to post-fiscal income;
(b) Any final or post-fiscal ranking of units is equally good;
(c) Reranking is neither good nor bad: it does not improve nor does it weaken inequality reduction.

To illustrate the meaning of these conclusions, imagine a case of three units A, B and C, with pre-fiscal incomes 10, 20 and 60. An "equalitarian" would like to see the following post-fiscal incomes: 30, 30 and 30. In this case, maximum distance narrowing Δ and redistributive effect RE^{xn} would be achieved, $\Delta = RE^{xn} = G_X$.

In an alternative setting, let the post-fiscal incomes of A, B and C be the following: 60, 20 and 10. Thus, C transferred 50 money units to A, and became the "new poor" member of society, while A became the "new rich". In this case we have that $RE^{xn} = 0$, and obviously, everything that one would conclude solely through inspecting RE^{xn} is that the fiscal system did not change the inequality. On the other hand, quite a lot of redistribution has occurred, probably beyond what many observers would regard as acceptable or sustainable. But, RE^{xn} is completely silent about reranking between A and C. How do the other two redistributive effects react?

3.2.5.3 Impact of the reranking-eliminating transfer

This section and the next one aim to prove Argument 2, regarding the following question: If reranking is somehow eliminated, what would be the impact of that change on the redistributive effect? To answer the question, we must first determine how the reranking could be eliminated. Unfortunately, we are not offered the recipe. However, there is one very intuitive way to achieve this: through transfer of post-fiscal income from the outranking unit to the unit that was outranked. Let us see how such transfer would affect different measures of income redistribution: vertical effect, reranking and redistributive effect.

A and B are units with pre-fiscal incomes X_u^x and X_v^x and pre-fiscal ranks u and v, such that $X_u^x < X_v^x$ and consequently $u < v$. The fiscal process has resulted in reranking, and A has higher post-fiscal income than B: $N_{u,0}^x > N_{v,0}^x$. The post-fiscal ranks of units A and B are y and z, where $y > z$, because of reranking. Their pre-fiscal incomes are $X_{y,0}^n$ and $X_{z,0}^n$, $X_{y,0}^n < X_{z,0}^n$.

Assume that we want to eliminate reranking between these two units through a transfer of post-fiscal income from A to B equal to $\tau = N_{u,0}^x - N_{v,0}^x$. After the transfer we have new post-fiscal incomes $N_{u,1}^x = N_{v,0}^x$ and $N_{v,1}^x = N_{u,0}^x$, and new pre-fiscal incomes $X_{y,1}^n = X_{z,0}^n$ and $X_{z,1}^n = X_{y,0}^n$.

Proposition 1

A transfer $\tau = N_{u,0}^x - N_{v,0}^x$ of post-fiscal income *from* unit A with pre-fiscal (post-fiscal) rank u (y) to unit B with pre-fiscal (post-fiscal) rank v (z) induces a change of:

(a) The Kakwani vertical effect V^K and Atkinson-Plotnick reranking effect R^{AP} by $2c\tau(u-v)$.

(b) The Lerman-Yitzhaki vertical effect V^{LY} by $2c(z-y)(X_{y,0}^n - X_{z,0}^n)$ and the Lerman-Yitzhaki reranking effect R^{LY} by $-2c(z-y)(X_{y,0}^n - X_{z,0}^n)$.

Proof.

(a) First, observe that reranking-inducing transfer of post-fiscal income does not change the order of units in N_i^x. The changes in post-fiscal incomes are equal to:

$$\Delta N_u^x = N_{u,1}^x - N_{u,0}^x = N_{v,0}^x - N_{u,0}^x = -\tau ;$$

$$\Delta N_v^x = N_{v,1}^x - N_{v,0}^x = N_{u,0}^x - N_{v,0}^x = +\tau .$$

Recall formulas (3.16) for $RE^x = V^K$, and (3.19) for $RE^{rx} = R^{AP}$. We may abstract from all the fixed elements and concentrate only on the changes ΔN_u^x and ΔN_v^x. The changes of V^K and R^{AP} are then equal to:

$$\Delta V^K = 2c\big((s-u)(-\tau)+(s-v)\tau\big) = 2c\tau(u-v).$$

$$\Delta R^{AP} = 2c\big((s-u)(-\tau)+(s-v)\tau\big) = 2c\tau(u-v).$$

(b) Notice that the order of units in vector X_i^n changes because of the reranking-inducing transfer of post-fiscal income. The changes in pre-fiscal income are:

$$\Delta X_y^n = X_{y,1}^n - X_{y,0}^n = X_{z,0}^n - X_{y,0}^n;$$

$$\Delta X_z^n = X_{z,1}^n - X_{z,0}^n = X_{y,0}^n - X_{z,0}^n.$$

For easier presentation, define the counterfactual transfer $\tilde{\tau} = X_{y,0}^n - X_{z,0}^n$. Recall formulas (3.17) for $RE^n = V^{LY}$, and (3.20) for $RE^{rn} = R^{LY}$. The changes of V^{LY} and R^{LY} are as follows:

$$\Delta V^{LY} = 2c\big((s-y)(-\Delta X_y^n)+(s-z)(-\Delta X_z^n)\big) = 2c\big((s-y)(-(-\tilde{\tau}))+(s-z)(-\tilde{\tau})\big) =$$

$$\Delta V^{LY} = 2c(z-y)\tilde{\tau} = 2c(z-y)(X_{y,0}^n - X_{z,0}^n).$$

$$\Delta R^{LY} = 2c\big((s-y)(\Delta X_y^n)+(s-z)(\Delta X_z^n)\big) = 2c\big((s-y)(-\tilde{\tau})+(s-z)\tilde{\tau}\big) =$$

$$\Delta R^{LY} = -2c(z-y)\tilde{\tau} = -2c(z-y)(X_{y,0}^n - X_{z,0}^n).$$

From Proposition 1 we conclude that this transfer of post-fiscal income between the two units, which is equal to the difference between their post-fiscal incomes, does not affect the redistributive effect. Let us see how:

(a) The change of Kakwani vertical effect is identical to the change of Atkinson-Plotnick reranking index: $\Delta V^K = \Delta R^{AP} = 2c\tau(u-v)$. Therefore $\Delta RE = \Delta V^K - \Delta R^{AP} = 0$.

(b) The change of Lerman-Yitzhaki vertical effect is the same in absolute amount, but of opposite sign from, the change in reranking effect: $\Delta V^{LY} = -\Delta R^{LY} = 2c(w-v)(X^n_{v,0} - X^n_{w,0})$. Therefore $\Delta RE = \Delta V^{LY} + \Delta R^{LY} = 0$.

Now, imagine a series of reranking-eliminating transfers τ between different units in the population. Each transfer has impact on vertical and reranking indices as shown above, and the total effect is equal to the sum of single impacts. If the transfer process is guided in a specific way, full values of reranking indices can be restored.

Robin Hood regards the current post-fiscal situation, presented in Table 3.5, as unacceptable, because there is too much reranking. Pre-fiscal income is already earned and cannot be changed or influenced (this is a usual assumption in the analysis of income redistribution). Also, assume that at the moment additional taxes cannot be collected and neither do there exist some reserve funds from which cash benefits could be paid. In this situation, in order to fix the problem, Robin Hood must rely on transfers of post-fiscal income between reranked units: to take from the undeservingly rich and give to the harmed poor.

Table 3.5 presents incomes of five hypothetical units from Table 3.1. According to Robin Hood's report, the harmed units are C, who had pre-fiscal rank $i = 4$ and post-fiscal rank of only $k = 1$, and A, with pre-fiscal rank $i = 5$ and post-fiscal rank $k = 4$. Three units (D, E and B) outranked C, while A was outranked by one unit (B).

A series of transfers occurred in four steps described in Table 3.6 and Table 3.7. We will concentrate on the former table, while for the latter, the interpretation is analogous. As can be seen in column 2 of Table 3.6, in the first step a transfer of $\tau_t = 20$ goes from D to C, enlarging the income of C by 20, and decreasing the income of D by the same amount. The consequence is a decrease of R^{AP} by $\Delta R^{AP} = -2c \cdot 60$ (observe that incomes of units participating in transfers are in bold letters).

During the first three steps, C's income has grown to 100, 20 more than he 'deserves'. Thus, in the fourth step, a transfer of 20 goes from C to A, and in column 6 we see the final vector of post-fiscal incomes. We reveal what was Robin Hood's idea: to achieve that pre-fiscal rankings are preserved in the final state. Summing the values in the last row of Table 3.6, we can see that during the transfer process the index R^{AP} fell by $\Delta R^{AP} = -2c \cdot 160$ in total, which is exactly the starting value of R^{AP}: in the end there is no reranking.

Notice also that by Proposition 1(a), V^K must have also been changed by the same amount of $\Delta V^K = -2c \cdot 160$, leaving the redistributive effect RE unchanged. The Lerman-Yitzhaki index of reranking has changed by $\Delta R^{LY} = -2c \cdot 310$, as shown in the bottom row of Table 3.7, while according to Proposition 1(b), the vertical effect increased by $\Delta V^{LY} = +2c \cdot 310$.

Table 3.5: Hypothetical case

Unit	i	X_i^x	N_i^x
D	1	8	40
E	2	12	60
B	3	30	100
C	4	70	20
A	5	180	80

Unit	k	X_k^n	N_k^n
C	1	70	20
D	2	8	40
E	3	12	60
A	4	180	80
B	5	30	100

Table 3.6: A series of transfers and a change in Atkinson-Plotnick reranking

i	$N_{i,1}^x = N_i^x$	$N_{i,2}^x$	$N_{i,3}^x$	$N_{i,4}^x$	$N_{i,1}^x = N_i^n$
1	2	3	4	5	6
1	**40**	20	20	20	20
2	60	**60**	40	40	40
3	100	100	**100**	60	60
4	**20**	**40**	**60**	**100**	80
5	80	80	80	**80**	100
v_t	1	2	3	4	
$N_{v,t}^x$	40	60	100	100	
w_t	4	4	4	5	
$N_{w,t}^x$	20	40	60	80	
τ_t	20	20	40	20	
$(v-w)\tau$	–60	–40	–40	–20	$= -160$

Table 3.7: A series of transfers and a change in Lerman-Yitzhaki reranking

k	$X^n_{k,1} = X^n_k$	$X^n_{k,2}$	$X^n_{k,3}$	$X^n_{k,4}$	$X^n_{k,5} = X^x_k$
1	2	3	4	5	6
1	**70**	8	8	8	8
2	**8**	**70**	12	12	12
3	12	**12**	**70**	30	30
4	180	180	180	**180**	70
5	30	30	**30**	**70**	180
$\widetilde{v}_t$	1	2	3	4	
$X^n_{v,t}$	70	70	70	180	
$\widetilde{w}_t$	2	3	5	5	
$X^n_{w,t}$	8	12	30	70	
$\widetilde{\tau}$	62	58	40	110	
$-(\widetilde{w}-\widetilde{v})\widetilde{\tau}$	–62	–58	–80	–110	$=-310$

However, one may wonder: is there any other model of change in the income distribution that would show something different? We can experiment with the following option: A and B are units with pre-fiscal incomes $X^x_a < X^x_{a+1}$, ranks a and $a+1$, and post-fiscal incomes $N^n_a > N^n_{a+1}$. One way of eliminating reranking between them would be to transfer $\tau_0 = (N^n_a - N^n_{a+1})/2$ from A to B, in which case they would have the same incomes. It can be shown that this process would decrease R^{AP} by $4c\tau_0$, while the decrease of V^K would be only $2c\tau_0$, with the final consequence: a rise in RE by $2c\tau_0$!

However, a careful analysis reveals that the above process can be divided into two parts:

(1) A transfer of $\tau_1 = N^n_a - N^n_{a+1} = 2\tau_0$ from A to B that eliminates reranking and reduces both R^{AP} and V^K by $2c\tau_1 = 4c\tau_0$ (thus, $\Delta RE = 0$), and

(2) An additional transfer of $\tau_2 = (N^n_a - N^n_{a+1})/2 = \tau_0$ from B to A, that equalizes their incomes, and increases both V^K and RE by $2c\tau_2 = 2c\tau_0$.

The crucial point is that the increase of redistributive effect caused by transfer τ_0 is not a consequence of reranking elimination, but of income equalization or distance narrowing between units A and B.

3.2.5.4 Decompositions of redistributive effects

This section is devoted to Argument 3, which claimed that redistributive effects can be presented as combinations of distance narrowing and reranking. First, we deal with the redistributive effect RE^x and after that with RE^n. We also establish a relationship between these and other indices presented earlier in the text.

The redistributive effect RE^x depicts the transition $\mathbf{X}^x \to \mathbf{N}^x$. The same superscript x in both $\mathbf{X}^x$ and $\mathbf{N}^x$ symbolizes that the transition preserves pre-fiscal income ranks. Let us break this transition into two sub-transitions:

$$\mathbf{X}^x \to \mathbf{N}^x \Leftrightarrow \left(\mathbf{X}^x \to \mathbf{N}^n\right) \to \left(\mathbf{N}^n \to \mathbf{N}^x\right) \tag{3.44}$$

The first sub-transition, $\mathbf{X}^x \to \mathbf{N}^n$, ascribes to each unit with pre-fiscal income rank i and pre-fiscal income X_i^x its counterfactual post-fiscal income N_i^n; N_i^n is a post-fiscal income of the unit with rank i on the post-fiscal ranking scale. Thus, the sub-transition $\mathbf{X}^x \to \mathbf{N}^n$ breaks the ranking link. Another sub-transition, $\mathbf{N}^n \to \mathbf{N}^x$, restores the ranking link between pre-fiscal and post-fiscal income.

We can write: $X_i^x - N_i^x = \left(X_i^x - N_i^n\right) + \left(N_i^n - N_i^x\right)$. Summing over (i, j) and multiplying by c we obtain:

$$\begin{aligned}
&c\left(\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^x - X_j^x\right) - \sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(N_i^x - N_j^x\right)\right) = \\
&c\left(\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^x - X_j^x\right) - \sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(N_i^n - N_j^n\right)\right) + \\
&+ c\left(\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(N_i^n - N_j^n\right) - \sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(N_i^x - N_j^x\right)\right)
\end{aligned} \tag{3.45}$$

Comparing (3.45) with (3.16), (3.18), (3.19), (3.29), (3.30) and (3.40) we reach several conclusions. First, the redistributive effect RE^x can be decomposed into a sum of redistributive effects RE^{xn} and RE^{rx}.

$$RE^x = RE^{xn} + RE^{rx} \tag{3.46}$$

Second, the redistributive effect RE^x, which corresponds to the fiscal deprivation index V^x, can be decomposed into distance narrowing and deprivation due to reranking effects.

$$RE^x (= V^x) = \Delta + 2R^x \tag{3.47}$$

Third, when (3.46) or (3.47) is translated into terms of traditional indices, we obtain that the Kakwani vertical effect V^K $(= RE^x = V^x)$ is the sum of redistributive effect RE $(= RE^{xn} = \Delta)$ and the Atkinson-Plotnick index of reranking R^{AP} $(= RE^{rx} = 2R^x)$.

$$V^K = RE + R^{AP} \tag{3.48}$$

We conclude that V^K is composed of distance narrowing and reranking. The identification of V^K with V^x results in further interesting conclusions. $V^K(= V^x)$ now also represents total fiscal deprivation, and should be compared to total reduction of income distance Δ. The difference between these two is the *excess* fiscal deprivation ($R^{AP} = V^K - RE$), the part of total V^K not necessary to achieve actual distance narrowing Δ.

The redistributive effect RE^n explains the transition $\mathbf{X}^n \rightarrow \mathbf{N}^n$. The superscript n in both vectors means that the transition preserves post-fiscal ranking. As in the previous section, we break this transition into two sub-transitions. The decomposition is slightly more complicated, with minus signs meaning the transition goes in the opposite direction.

$$\begin{aligned}
&\mathbf{X}^n \rightarrow \mathbf{N}^n \Leftrightarrow -\left(\mathbf{N}^n \rightarrow \mathbf{X}^n\right) \\
&\Leftrightarrow -\left(\left(\mathbf{N}^n \rightarrow \mathbf{X}^x\right) + \left(\mathbf{X}^x \rightarrow \mathbf{X}^n\right)\right) \Leftrightarrow -\left(-\left(\mathbf{X}^x \rightarrow \mathbf{N}^n\right) + \left(\mathbf{X}^x \rightarrow \mathbf{X}^n\right)\right) \\
&\Leftrightarrow \left(\mathbf{X}^x \rightarrow \mathbf{N}^n\right) - \left(\mathbf{X}^x \rightarrow \mathbf{X}^n\right)
\end{aligned} \tag{3.49}$$

The first sub-transition, $\mathbf{X}^x \to \mathbf{N}^n$, is distance narrowing and breaks the ranking link. However, another sub-transition, $\mathbf{X}^x \to \mathbf{X}^n$, restores it. We can write: $X_i^n - N_i^n = \left(X_i^x - N_i^n\right)$ $-\left(X_i^x - X_i^n\right)$. Summing over (i,j) and multiplying by c we obtain:

$$c\left(\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^n - X_j^n\right) - \sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(N_i^n - N_j^n\right)\right) =$$

$$c\left(\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^x - X_j^x\right) - \sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(N_i^n - N_j^n\right)\right) -$$

$$-c\left(\sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^x - X_j^x\right) - \sum_{i=2}^{s}\sum_{j=1}^{i-1}\left(X_i^n - X_j^n\right)\right) \qquad (3.50)$$

Comparing (3.50) with (3.17), (3.19), (3.20), (3.32), (3.33) and (3.41) we arrive at several conclusions. Firstly, the redistributive effect RE^n can be decomposed into difference of RE^{xn} and RE^{rx}.

$$RE^n = RE^{xn} - RE^{rn} \qquad (3.51)$$

Secondly, the redistributive effect RE^n, which is identical to the reverse fiscal domination index V^n, can be decomposed into effects of distance narrowing and domination due to reranking.

$$RE^n (= V^n) = \Delta - 2R^n \qquad (3.52)$$

Finally, "translating" (3.51) and (3.52), we obtain a decomposition of the Lerman-Yitzhaki index of "progressivity" V^{LY} $(= RE^n = V^n)$ into the redistributive effect RE $(= RE^{xn} = \Delta)$ and the Lerman-Yitzhaki index of reranking R^{LY} $(= RE^{rn} = 2R^n)$.

$$V^{LY} = RE - R^{LY} \qquad (3.53)$$

It can be seen that V^{LY}, just as V^K, can be decomposed into distance narrowing and reranking.

3.2.5.5 A series of small transfers and redistributive effects

This section again relies on an experiment with transfers, but this time we deal with a series of small transfers. Up to now, we have not considered the meaning of the weights in the Gini index, $\omega(p_i;2) = s - i$, described earlier. Interpretation is straightforward: the units with lower positions i receive larger weights, and vice versa. It can be shown that a small transfer σ from the unit with rank v to the unit with rank $w < v$ will decrease the Gini coefficient by $2c\sigma(v - w)$.

A *small* transfer from the rich to the poor decreases inequality and increases welfare because the sacrifice felt by the rich is valued as less important than the marginal benefit to the poor. We must stress that the terms “poor” and “rich” correspond to the relative positions of persons involved, *before* and *after* the transfer.

Now, imagine a *series of small transfers* from the rich B to the poor A. Obviously, after each of these transfers B will be becoming less rich and A will be getting less poor: the income distance between them will be narrowing and the income supremacy of B will be falling. In one moment, these persons’ incomes will be equalized. After that point, the next small transfer from B to A will reverse the situation: the “poor” A will become the rich one, and the “rich” B will become the poor. Reranking occurs. Suppose that the transfers continue to the point where B and A completely swap their incomes. How do the measurement concepts analyzed in this study respond to the challenge? We analyze the changes of our indices during a series of small transfers between two hypothetical units in the following example.

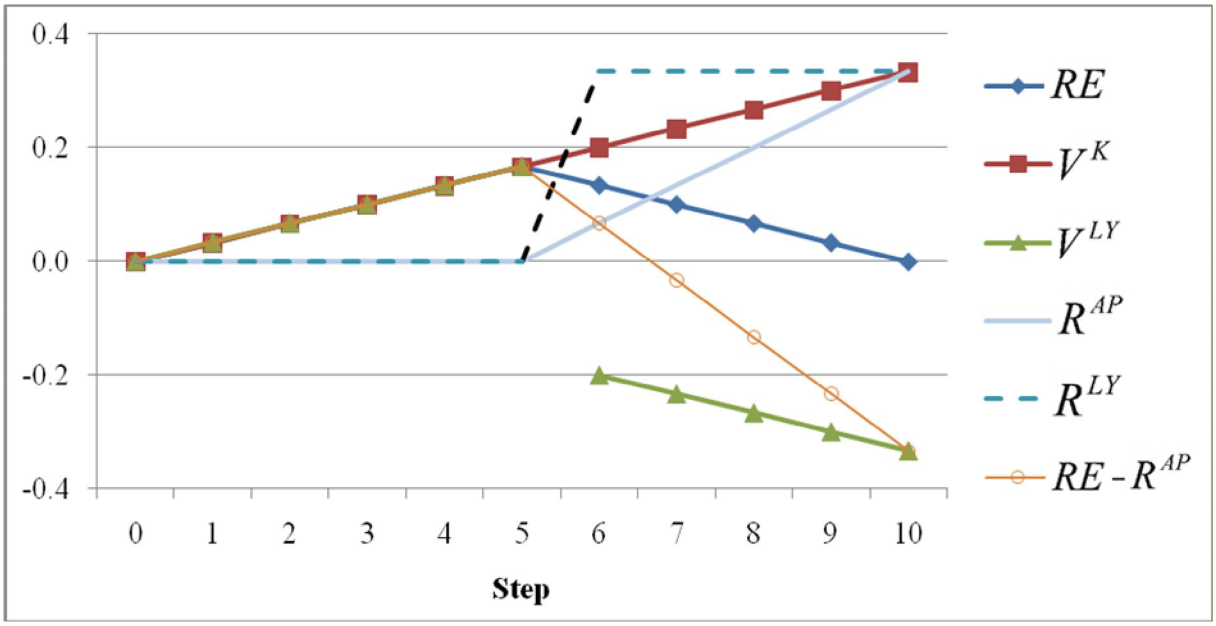
The transfer process is presented both in Table 3.8 and Figure 3.4. Unit A starts with income of 10 and ends with 20, while it is the opposite for B. There are ten steps, each presenting a small transfer of 1 monetary unit (not all steps are shown in the table, for better visibility). Indices of redistributive effect and reranking for each step are all calculated with respect to step 0. Thus, for example, indices in step 7 are based on pre-fiscal incomes 10 and 20, and post-fiscal incomes 17 and 13, for A and B respectively.

Table 3.8: Small transfers and indices of the redistributive effect

Step	0	1	3	5	6	7	9	10
Income of A	10	11	13	15	16	17	19	20
Income of B	20	19	17	15	14	13	11	10
RE	0	0.033	0.100	0.167	0.133	0.100	0.033	0
V^K	0	0.033	0.100	0.167	0.200	0.233	0.300	0.333
R^{AP}	0	0	0	0	0.067	0.133	0.267	0.333
V^{LY}	0	0.033	0.100	0.167	-0.200	-0.233	-0.300	-0.333
R^{LY}	0	0	0	0	0.333	0.333	0.333	0.333
$RE - R^{AP}$	0	0.033	0.100	0.167	0.067	-0.033	-0.233	-0.333

In the first 5 steps, there is no reranking ($R^{AP} = R^{LY} = 0$), and therefore the three redistributive effects are identical: $RE = V^K = V^{LY}$. In the 5th step, the incomes are equalized and the distance narrowing ($RE = \Delta$) reaches its maximum of 0.167. After this point, the redistributive effects completely diverge: (a) RE falls back toward zero; (b) V^K continues to grow; (c) V^{LY} has a breaking point at the 5th step, when it drops significantly and continues to fall in later steps. Reranking effects also behave differently. R^{LY} is equal for all steps after the 5th, while R^{AP} grows toward the value of V^K in the 10th step.

Figure 3.4: Small transfers and indices of the redistributive effect



As a potpourri to the discussion of the above hypothetical results, we cite a lucid argument delivered by Lerman and Yitzhaki (1995), in their critique of Kakwani vertical effect:

"Imagine a rich taxpayer who becomes poor because of heavy taxation. According to before-tax rankings, the taxpayer will continue to be considered as rich even if the tax causes him to become poor. Reliance on the before-tax ranking may lead the analyst to recommend increasing a tax on progressivity grounds even though the additional tax will be paid by the poor."

And this is exactly what we can conclude observing the development of V^K in our example, after the reranking has occurred in the 5th step . In subsequent steps, unit B, who was rich, now becomes poorer and poorer, but V^K increases yet further. Thus, the measure V^K "rewards" reranking, which looks contradictory since we know that it is based on pre-fiscal ranks, and given that fact, it should "protect" the pre-fiscally richer. This *is* achieved by V^{LY}, the measure based on post-fiscal ranks. V^{LY} falls as we go to the right from the 5th step, thus "penalizing" reranking.

Our example confirms that Lerman and Yitzhaki were right when saying that dependence on pre-fiscal ranks would lead the analyst to recommend more redistribution even when reranking has occurred and the formerly rich became the poor. V^K continues to rise even when the "rich" person is left with zero or negative income. This was one of the reasons which caused them to propose their index V^{LY}, which is attractive, but also has a deficiency. Observe in the example that between steps 5 and 6 there is only a small difference, but the index falls drastically, from 0.167 to -0.200. The reason for such a plunge lies in R^{LY}, which appears as a deducting element in $V^{LY} = RE - R^{LY}$. Recall that R^{LY} is based exclusively on pre-fiscal incomes, which do not change in our experiment and are the same all the way, once reranking has occurred.

One intuitive choice, although not based on algebraic facts, was to draw a curve that also deducts reranking from the redistributive effect, but using R^{AP} instead of R^{LY}. We obtained a measure $RE - R^{AP}$ (recall that $V^K = RE + R^{AP}$), which does have a quality of falling when the outranked person further loses her income, but there is no break in the turning point at the 5th step. The latter is due to the fact that R^{AP} is based on post-fiscal incomes.

3.2.6 Setting the new context for existing indices

3.2.6.1 Problems with Kakwani and Lerman-Yitzhaki decompositions

Kakwani (1984) and Lerman and Yitzhaki (1995) derived two different, but conceptually related decompositions of redistributive effect (RE) into vertical and reranking effects. The former became one of the most widely used tools in the analysis of the redistributive effect, while the latter aimed to replace it, but without success. *Chapter 2* thoroughly describes their origins and debates on certain unsolved issues. The decompositions are respectively represented by the following two equations.

$$RE = V^{K} - R^{AP} \quad (2.8)$$

$$RE = V^{LY} + R^{LY} \quad (2.23)$$

Kakwani decomposes RE into a *difference* between vertical and reranking effects, while Lerman and Yitzhaki decompose RE into a *sum* of vertical and reranking effects. By construction, reranking effects, R^{AP} and R^{LY}, are always positive, while vertical effects may be either positive or negative.

Based on the algebraic constructions of the formulas, the authors respectively concluded that R^{AP} contributes negatively, while R^{LY} contributes positively to the redistributive effect RE. For them, reranking plays a distinctive role in the determining the magnitude of RE. For Kakwani, reranking deteriorates RE, while for Lerman-Yitzhaki it improves RE. For both Kakwani and Lerman-Yitzhaki, the respective vertical effects V^{K} and V^{LY} are also standalone concepts, completely independent of reranking. Kakwani (1984) identifies V^{K} with *potential* redistributive effect, interpreted as the amount of RE that would be achieved in the absence of reranking. Thus, RE could be increased through *elimination* of reranking, while at the same time V^{K} would remain unchanged. Lerman and Yitzhaki (1995) follow this interpretation, but in their version, RE could be enlarged through *enhancement* of reranking, while V^{LY} would stay the same.

In the foregoing sections, we have provided a lot of material to answer the problem with these interpretations of indices. The principal concern is a specific connection between vertical and reranking effects. Each attempt to decrease (increase) overall reranking R^{AP} (R^{LY}), automatically leads to a decrease (decrease) of vertical effect V^{K} (V^{LY}). The consequence is that RE remains unchanged.

The most illustrative proof of this contention was the analysis of the impact of a series of transfers between population units which eliminate reranking. Further evidence about the relation between reranking and vertical effect is that V^{K} (V^{LY}) is a sum (difference) of distance narrowing and reranking, as shown by equations (3.48) and (3.53). Recall that it was proven that distance narrowing and reranking are separate and independent concepts.

These conclusions support Atkinson's (1980) views that "changes in the ranking of observations as a result of taxation do not in themselves affect the degree of inequality in the post-tax distribution". In other words, since the distribution of pre-tax income is also assumed to be unchanged by taxation, Atkinson claimed that reranking does not influence the redistributive effect (RE). However, the suggestion was ignored in the subsequent work of both Kakwani and Lerman and Yitzhaki.

We have demonstrated another problem with the Kakwani decomposition, advanced by Lerman and Yitzhaki (1995), using an appealing example of taxation which makes a rich person poor. The Kakwani vertical effect (V^{K}) rewards reranking, "asking for" an ever larger take from the formerly rich, now poor, and giving to the formerly poor, now the rich. At the same time, proponents of the Kakwani decomposition blame reranking for this trouble. If reranking were eliminated, the redistributive effect would increase to V^{K}. But, as we have already seen, there is no practicable scheme that would tell us how to achieve this.

3.2.6.2 Which indices to use?

After a thorough discussion of the existing methodologies and criticism of their contemporary interpretations, a course for future research should be provided. A straight answer to the question posed by this section title will perhaps sound surprising: the same indices we used before; however, with an important distinction: they must be interpreted properly. In this section we discuss acceptable interpretations for each of these indices.

Recall that we analyzed properties of the indices (of redistributive, vertical and reranking effects) using different approaches (vector transitions and income units' "feelings"). Each of them revealed a certain interesting aspect of the measure the researchers should have in mind when clarifying the meaning of their estimated indicators. In Table 3.9 we summarize these aspects for five indices and two approaches, and then explain how each index should be treated.

Table 3.9: Interpretation of indices

	Vector transitions	Income units' "feelings"
RE	$\mathbf{X}^x \to \mathbf{N}^n$; breaks the link between pre-fiscal and post-fiscal incomes	distance narrowing; Δ
V^K	$\mathbf{X}^x \to \mathbf{N}^x$; preserves the link between pre-fiscal and post-fiscal incomes. Decomposable into $\mathbf{X}^x \to \mathbf{N}^n$ and $\mathbf{N}^n \to \mathbf{N}^x$	fiscal deprivation; V^x
R^{AP}	$\mathbf{N}^n \to \mathbf{N}^x$, reranks post-fiscal incomes	deprivation due to reranking; R^x
V^{LY}	$\mathbf{X}^n \to \mathbf{N}^n$; preserves the link between pre-fiscal and post-fiscal incomes. Decomposable into $\mathbf{X}^x \to \mathbf{N}^n$ and $\mathbf{X}^x \to \mathbf{X}^n$	reverse fiscal domination; V^n
R^{LY}	$\mathbf{X}^x \to \mathbf{X}^n$; reranks pre-fiscal incomes	domination due to reranking; R^n

Redistributive effect (RE). This will remain the main indicator of the redistributive effect. RE is synonymous with distance narrowing and is indifferent about rank changes. For two systems with equal distance narrowing ($\Delta = RE$) and different amounts of reranking, RE will be identical. Thus, analysts who do think that reranking has a negative or positive normative significance will consider the indices below as a supplement to RE.

Kakwani vertical effect V^K. We have seen different problems with the index itself, and also with its contemporary interpretation. Should we completely avoid the use of V^K? In one of its forms, the index can still be interesting: as a measure of fiscal deprivation ($V^x = V^K$).

Take an analyst who holds that the fiscal system should preserve *differences in incomes*. In other words, this principle says that everybody should pay (receive) *equal* amounts of taxes (benefits). Then the index V^x measures the violation of this principle: positive fiscal deprivation means that the richer lost their income advantages over the poorer. Additionally, in case of reranking, the richer people not only use their income supremacy, but end up poorer, and this notion is captured by V^x as compared to RE.

Atkinson-Plotnick reranking effect R^{AP}. In the context of fiscal deprivation, $R^{AP} = 2R^{x}$ is titled excess fiscal deprivation. It is a part of total fiscal deprivation (V^{x}), that stands above the fraction of fiscal deprivation that is necessary to achieve actual distance narrowing (Δ).

This is perhaps an opportunity to divorce R^{AP} from V^{K}, with whom it was unhappily married during the last 25 years. Unlike the other term, R^{AP} remains what it was since its appearance: an index measuring the extent of reranking caused by the fiscal process. It is a perfect complement of RE in judging the redistributive performance of the fiscal system.

Lerman-Yitzhaki vertical effect V^{LY} *and reranking effect* R^{LY}. Lerman and Yitzhaki (1995) called V^{LY} the index of "gap-narrowing", assuming that it quantifies a process that is independent of reranking. We have seen that the contention was wrong: V^{LY} decreases with the increase of reranking. In this paper, the similar term "distance narrowing" is used for a truly independent concept, measured by RE.

Nevertheless, V^{LY} can be an interesting choice for the analyst who appreciates distance narrowing, but believes that pre-fiscal rankings should be preserved. V^{LY} is a single measure that combines both of these notions and is suitable for a comparison of performance of different fiscal systems. It is higher the larger the distance narrowing and the lower the reranking.

R^{LY} can be used as a measure of reranking in the same way as R^{AP}. Remember that the difference between the two lies in the income vector on which they are built: in the former case it is pre-fiscal income, and in the latter, post-fiscal income, which makes it slightly more intuitive.

Analogously to $V^{K}(V^{x})$ and $R^{AP}(2R^{x})$, there are alternative interpretations for V^{LY} and R^{LY}, in terms of fiscal domination. The reverse fiscal domination index $V^{n} = V^{LY}$ is a counterpart to the index of fiscal deprivation, and suitable for analysts who consider that the fiscal process should insist on reranking of units, disrespecting pre-fiscal ranks.

3.3 Redistributive effect: contributions of individual fiscal instruments

3.3.1 Lorenz and concentration curves based approaches

3.3.1.1 Decomposition of V^x

This decomposition was first derived by Lambert (1985), but only for one tax / one benefit (or taxes / benefits as a whole). Duclos (1993, 2000) extended the methodology to cover many taxes and benefits. The studies mentioned above related only to V^x, while decomposition of V^n was used in Urban (2008). In this and the next section, we present a detailed derivation of decompositions of both V^x and V^n.

According to (3.16) and (3.40), V^x is a double area between Lorenz curve of pre-fiscal income $L_X(u_i)$ and the concentration curve of post-fiscal income for which the units are sorted in ascending order of pre-fiscal income, $C_N^x(u_i)$, thus, $V^x = (2/s)\sum_i^s \left(C_N^x(u_i) - L_X(u_i)\right)$.

Suppose the fiscal system consists of P tax and Q benefit instruments. For the unit with pre-fiscal income rank i, the amount of the pth tax paid is $T_{i,p}^x$, and the amount of the qth benefit received is $B_{i,q}^x$. Respective concentration curves will be $C_{T,p}^x(u_i)$ and $C_{B,q}^x(u_i)$. Also, let t_p^x and b_q^x be the average tax and benefit rates expressed in terms of post-fiscal income; $t_p^x = \sum_i^s T_{i,p}^x / \sum_i^s X_i^x$ and $b_q^x = \sum_i^s B_{i,q}^x / \sum_i^s X_i^x$.

$$V^x = V^K = \frac{\sum_{p=1}^{P} t_p^x (D_{T,p}^x - G_X) + \sum_{q=1}^{Q} b_q^x (G_X - D_{B,q}^x)}{1 - t_p^x + b_q^x} \qquad (3.54)$$

The decomposition (3.54) is derived as follows. For purposes of simplification, and to avoid cumbersome notation, let there be only one tax, $T_i^x = T_{i,p=1}^x$, and one benefit, $B_i^x = B_{i,q=1}^x$; therefore, $t^x = t_{p=1}^x$, $b^x = b_{q=1}^x$. Also, in denoting Lorenz and concentration curves we drop "(u_i)".

$X_i^x = N_i^x + T_i^x - B_i^x \Rightarrow X_i^x = (X_i^x - T_i^x + B_i^x) + T_i^x - B_i^x$, for all i

$L_x = (1 - t^x + b^x)C_N^x + t^x C_T^x - b^x C_B^x$

Add $-t^x L_X + b^x L_X$ on both sides:

$$L_x - t^x L_X + b^x L_X = (1 - t^x + b^x)C_N^x + t^x C_T^x - b^x C_B^x - t^x L_X + b^x L_X$$

$$(1 - t^x + b^x)L_X - (1 - t^x + b^x)C_N^x = t^x C_T^x - t^x L_X + b^x L_X - b^x C_B^x$$

$$(1 - t^x + b^x)(L_X - C_N^x) = t^x (C_T^x - L_X) + b^n (L_X - C_B^x)$$

Finally, we have:

$$L_X - C_N^x = \frac{t^x (C_T^x - L_X) + b^x (L_X - C_B^x)}{1 - t^x + b^x}$$

which multiplied by -1 gives:

$$C_N^x - L_X = \frac{t^x (L_X - C_T^x) + b^x (C_B^x - L_X)}{1 - t^x + b^x}$$

This decomposition can be analogously derived for P taxes and Q benefits.

$$C_N^x - L_X = \frac{\sum_{p=1}^{P} t_p^x (L_X - C_{T,p}^x) + \sum_{q=1}^{Q} b_q^x (C_{B,q}^x - L_X)}{1 - t_q^x + b_q^x}$$

In terms of Gini and concentration coefficients, the last equation can be written as (3.54).

Contributions of pth tax and qth benefit to fiscal deprivation are presented in Table 3.10, respectively by $\lambda_p^{x,T}$ and $\lambda_q^{x,B}$, as defined in (3.55) and (3.57). Equations (3.56) and (3.58) represent their relative contributions to overall fiscal deprivation.

Table 3.10: Components of fiscal deprivation and their contributions

Component	Contribution to V^x
$\lambda_p^{x,T} = \dfrac{t_p^x(D_{T,p}^x - G_X)}{1 - t_p^x + b_q^x}$ (3.55)	$\dfrac{\lambda_p^{x,T}}{\sum_p^P \lambda_p^{x,T} + \sum_q^Q \lambda_q^{x,B}}$ (3.56)
$\lambda_q^{x,B} = \dfrac{b_q^x(G_X - D_{B,q}^x)}{1 - t_p^x + b_q^x}$ (3.57)	$\dfrac{\lambda_q^{x,B}}{\sum_p^P \lambda_p^{x,T} + \sum_q^Q \lambda_q^{x,B}}$ (3.58)

3.3.1.2 Decomposition of V^n

According to (3.17) and (3.41), V^n is a double area between Lorenz curve of post-fiscal income, $L_N(u_i)$, and the concentration curve of pre-fiscal income for which the units are sorted in ascending order of post-fiscal income, $C_X^n(u_i)$, thus, $V^n = (2/s)\sum_{i=1}^{s}\left(L_N(u_i) - C_X^n(u_i)\right)$.

Again we have a fiscal system that consists of P tax and Q benefit instruments. For the unit with post-fiscal income rank i, the amount of the p th tax paid is $T_{i,p}^n$, and the amount of the q th benefit received is $B_{i,q}^n$. Respective concentration curves will be $C_{T,p}^n(u_i)$ and $C_{B,q}^n(u_i)$. Also, let t_p^n and b_q^n be the average tax and benefit rates expressed in terms of post-fiscal income; $t_p^n = \sum_i^s T_{i,p}^n / \sum_i^s N_i^n$ and $b_q^n = \sum_i^s B_{i,q}^n / \sum_i^s N_i^n$.

$$V^n = V^{LY} = \frac{\sum_{p=1}^{P} t_p^n(D_{T,p}^n - G_N) + \sum_{q=1}^{Q} b_q^n(G_N - D_{B,q}^n)}{1 + t_p^n - b_q^n} \qquad (3.59)$$

The decomposition (3.59) is derived as follows. Let there be only one tax, $T_i^n = T_{i,p=1}^n$, and one benefit, $B_i^n = B_{i,q=1}^n$; therefore, $t^n = t_{p=1}^n$, $b^n = b_{q=1}^n$. Also, in denoting Lorenz and concentration curves we drop "(u_i)".

$X_i^n = N_i^n + T_i^n - B_i^n \Rightarrow N_i^n = (N_i^n + T_i^n - B_i^n) - T_i^n + B_i^n$, for all i

$L_N = (1 + t^n - b^n)C_X^n - t^n C_T^n + b^n C_B^n$

Add $t^n L_N - b^n L_N$ on both sides:

$$L_N + t^n L_N - b^n L_N = (1 + t^n - b^n) C_X^n - t^n C_T^n + b^n C_B^n + t^n L_N - b^n L_N$$

$$(1 + t^n - b^n) L_N - (1 + t^n - b^n) C_X^n = t^n L_N - t^n C_T^n + b^n C_B^n - b^n L_N$$

$$(1 + t^n - b^n)(L_N - C_X^n) = t^n (L_N - C_T^n) + b^n (C_B^n - L_N)$$

Finally, we have:

$$L_N - C_X^n = \frac{t^n (L_N - C_T^n) + b^n (C_B^n - L_N)}{1 + t^n - b^n}$$

The decomposition can be analogously derived for P taxes and Q benefits.

$$L_N - C_X^n = \frac{\sum_{p=1}^{P} t_p^n (L_N - C_{T,p}^n) + \sum_{q=1}^{Q} b_q^n (C_{B,q}^n - L_N)}{1 + t_q^n - b_q^n}$$

In terms of Gini and concentration coefficients, the last equation can be written as (3.59).

Contributions of *p*th tax and *q*th benefit to reverse fiscal domination are presented in Table 3.11, respectively by $\lambda_p^{T,n}$ and $\lambda_q^{B,n}$, defined in (3.60) and (3.62). Equations (3.61) and (3.63) represent their relative contributions to overall reverse fiscal domination.

Table 3.11: Components of reverse fiscal domination and their contributions

Component		Contribution to V^n	
$\lambda_p^{n,T} = \frac{t_p^n (D_{T,p}^n - G_N)}{1 + t_p^n - b_q^n}$	(3.60)	$\frac{\lambda_p^{n,T}}{\sum_p^P \lambda_p^{n,T} + \sum_q^Q \lambda_q^{n,B}}$	(3.61)
$\lambda_q^{n,B} = \frac{b_q^n (G_N - D_{B,q}^n)}{1 + t_p^n - b_q^n}$	(3.62)	$\frac{\lambda_q^{n,B}}{\sum_p^P \lambda_p^{n,T} + \sum_q^Q \lambda_q^{n,B}}$	(3.63)

3.3.2 Approach based on distance narrowing and amounts of taxes and benefits

3.3.2.1 Basic terms

Post-fiscal income is equal to pre-tax income minus all taxes and plus all benefits. For two different orderings of units, by pre-fiscal and post-fiscal income, we have that:

$$X_i^x - N_i^x = \sum_{p=1}^{P} T_{i,p}^x - \sum_{q=1}^{Q} B_{i,q}^x \tag{3.64}$$

$$X_i^n - N_i^n = \sum_{p=1}^{P} T_{i,p}^n - \sum_{q=1}^{Q} B_{i,q}^n \tag{3.65}$$

where $T_{i,p}^x$ and $T_{i,p}^n$ ($B_{i,q}^x$ and $B_{i,q}^n$) and are amounts of pth tax paid (qth benefit received) by the ith unit, with units arranged in increasing order of pre-fiscal income (hence x in superscript) and post-fiscal income (hence n in superscript), respectively. Overall, there is P tax and Q benefit instruments.

In an analysis of the real fiscal system, for a chosen mix of tax and benefit instruments, it will always be either that $\sum N_i < \sum X_i$ or $\sum N_i > \sum X_i$ ($\sum N_i = \sum X_i$ is merely accidental). In the former case, total taxes are larger than total benefits, and vice versa. However, the models that will be presented in the next sections require that on aggregate post-fiscal income equals pre-fiscal income (benefits equal taxes), the so-called fiscal balance assumption. We will achieve this by introducing an additional benefit or tax variable, depending on whether $\sum N_i < \sum X_i$ or $\sum N_i > \sum X_i$.

We define these "fill-in" variables as:

$$B_{i,Q+1}^x = \begin{cases} \overline{X} - \overline{N} & \text{if } \overline{N} < \overline{X} \\ 0 & \text{otherwise} \end{cases} \qquad T_{i,P+1}^x = \begin{cases} \overline{N} - \overline{X} & \text{if } \overline{N} > \overline{X} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{i,Q+1}^n = B_{i,Q+1}^x \qquad T_{i,P+1}^n = T_{i,P+1}^x$$

The next step is forming new variables of pre-fiscal and post-fiscal incomes. In case of the "pre-fiscal income-oriented decomposition", we will add (subtract) a fictive tax (benefit) to (from) the both sides of (3.64); after substituting $\widetilde{X}_i^x$ for $X_i^x - B_{i,Q+1}^x + T_{i,P+1}^x$, we obtain (3.66).

In case of the "post-fiscal income-oriented decomposition", we will subtract (add) a fictive tax (benefit) to (from) the both sides of (3.65), after substitution of $\widetilde{N}_i^n$ for $N_i^n + B_{i,Q+1}^n - T_{i,P+1}^n$, we obtain (3.67).

$$\widetilde{X}_i^x - N_i^x = \left(T_{i,P+1}^x + \sum_{p=1}^{P} T_{i,p}^x \right) - \left(B_{i,Q+1}^x + \sum_{q=1}^{Q} B_{i,q}^x \right) \tag{3.66}$$

$$X_i^n - \widetilde{N}_i^n = \left(T_{i,P+1}^n + \sum_{p=1}^{P} T_{i,p}^n \right) - \left(B_{i,Q+1}^n + \sum_{q=1}^{Q} B_{i,q}^n \right) \tag{3.67}$$

3.3.2.2 Decomposition of R^x and V^x

For purposes of derivation it is useful to imagine that in transition from pre-fiscal to post-fiscal income all the incomes, taxes and benefits undergo a change. In reality they do not, but the counterfactual values of these variables in the "periods" before and after the fiscal process are very helpful analytical constructs, as we shall see very soon.

Thus, we will decompose a *change* of the index of deprivation due to reranking R^x occurring in transition from pre-fiscal to post-fiscal income. In a situation before (after) fiscal actions, we have that: $T_{i,p}^{x,0} = 0$ ($T_{i,p}^{x,1} = T_{i,p}^x$), $B_{i,q}^{x,0} = 0$ ($B_{i,q}^{x,1} = B_{i,q}^x$) and $N_i^{x,0} = \widetilde{X}_i^x$ ($N_i^{x,1} = N_i^x$). Thus, the changes between the two situations are: $\Delta T_{i,p}^x = T_{i,p}^x$, $\Delta B_{i,q}^x = B_{i,q}^x$, $\Delta \widetilde{X}_i^x = 0$, $\Delta N_i^x = N_i^{x,1} - N_i^{x,0} = N_i^x - \widetilde{X}_i^x$. We also know that $\widetilde{X}_i^x - N_i^x = \sum_p^{P+1} T_{i,p}^x - \sum_q^{Q+1} B_{i,q}^x$.

Recall that (3.22) presents deprivation due to reranking felt by the unit with pre-fiscal rank i that may be reranked by the unit with rank j.

$$r_{i,j}^x = \frac{1}{2}\left(\left| N_i^x - N_j^x \right| - \left(N_i^x - N_j^x \right) \right) \tag{3.22}$$

Now, the *change* of deprivation due to reranking ($\dot{r}_{i,j}^x$) during the fiscal process is a difference between deprivation's values after ($r_{i,j}^{x,1}$) and before ($r_{i,j}^{x,0}$) fiscal action, or $\dot{r}_{i,j}^x = r_{i,j}^{x,1} - r_{i,j}^{x,0}$.

Substituting (3.22) into the last equality, and defining $N_i^{x,1}$ and $N_i^{x,0}$ analogously to $r_{i,j}^{x,1}$ and $r_{i,j}^{x,0}$, we obtain (3.68).

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left|N_i^{x,1} - N_j^{x,1}\right| - \left(N_i^{x,1} - N_j^{x,1}\right)\right) - \frac{1}{2}\left(\left|N_i^{x,0} - N_j^{x,0}\right| - \left(N_i^{x,0} - N_j^{x,0}\right)\right) \tag{3.68}$$

From (3.68) we derive (3.69), in the way presented as follows. Substituting $N_i^{x,0} = \widetilde{X}_i^x$ and $N_i^{x,1} = N_i^x$ into (3.68), we obtain:

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left|N_i^x - N_j^x\right| - \left(N_i^x - N_j^x\right)\right) - \frac{1}{2}\left(\left|\widetilde{X}_i^x - \widetilde{X}_j^x\right| - \left(\widetilde{X}_i^x - \widetilde{X}_j^x\right)\right)$$

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left|N_i^x - N_j^x\right| - \left|\widetilde{X}_i^x - \widetilde{X}_j^x\right|\right) - \frac{1}{2}\left(\left(N_i^x - N_j^x\right) - \left(\widetilde{X}_i^x - \widetilde{X}_j^x\right)\right)$$

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left(\widetilde{X}_i^x - \widetilde{X}_j^x\right) - \left(N_i^x - N_j^x\right)\right) - \frac{1}{2}\left(\left|\widetilde{X}_i^x - \widetilde{X}_j^x\right| - \left|N_i^x - N_j^x\right|\right)$$

Notice that the last equation presents the relation $R^{AP} = RE^x - RE^{xn}$.

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left(\widetilde{X}_i^x - N_i^x\right) - \left(\widetilde{X}_j^x - N_j^x\right)\right) - \frac{1}{2}\left(\left|\widetilde{X}_i^x - \widetilde{X}_j^x\right| - \left|N_i^x - N_j^x\right|\right)$$

Substituting (3.66) into the equation above, we obtain:

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left(\sum_p^{P+1} T_{i,p}^x - \sum_q^{Q+1} B_{i,q}^x\right) - \left(\sum_p^{P+1} T_{j,p}^x - \sum_q^{Q+1} B_{j,q}^x\right)\right) - \frac{1}{2}\left(\left|\widetilde{X}_i^x - \widetilde{X}_j^x\right| - \left|N_i^x - N_j^x\right|\right)$$

$$\dot{r}_{i,j}^{x} = \frac{1}{2}\left(\left(\sum_p^{P+1} T_{i,p}^x - \sum_p^{P+1} T_{j,p}^x\right) + \left(\sum_q^{Q+1} B_{j,q}^x - \sum_q^{Q+1} B_{i,q}^x\right)\right) - \frac{1}{2}\left(\left|\widetilde{X}_i^x - \widetilde{X}_j^x\right| - \left|N_i^x - N_j^x\right|\right)$$

Finally, multiplying the last equation by 2, we obtain the expression (3.69).

$$2\dot{r}_{i,j}^{x} = \left(\sum_p^{P+1}\left(T_{i,p}^x - T_{j,p}^x\right) + \sum_q^{Q+1}\left(B_{j,q}^x - B_{i,q}^x\right)\right) - \left(\left|\widetilde{X}_i^x - \widetilde{X}_j^x\right| - \left|N_i^x - N_j^x\right|\right) \tag{3.69}$$

Remember that $T^x_{i,P+1} = T^x_{j,P+1}$ and $B^x_{i,Q+1} = B^x_{j,Q+1}$ for all (i, j). Let $v^{x,T}_{i,j,p} = T^x_{i,p} - T^x_{j,p}$ and $v^{x,B}_{i,j,q} = B^x_{j,q} - B^x_{i,q}$. Therefore, (3.69) can be written as:

$$2\dot{r}^x_{i,j} = \left(\sum_p^P v^{x,T}_{i,j,p} + \sum_q^Q v^{x,B}_{i,j,q} \right) - \left(\left| \widetilde{X}^x_i - \widetilde{X}^x_j \right| - \left| N^x_i - N^x_j \right| \right) \tag{3.70}$$

Let $\Phi^x_{i,j} = \sum_p^P v^{x,T}_{i,j,p} + \sum_q^Q v^{x,B}_{i,j,q}$. The contribution of tax p to the change in deprivation due to reranking of the unit with pre-fiscal rank i, outranked by the unit with rank j, is obtained as follows.

$$\rho^{x,T}_{i,j,p} = \dot{r}^x_{i,j} \frac{v^{x,T}_{i,j,p}}{\Phi^x_{i,j}} \tag{3.71}$$

The contribution of benefit q to the change of deprivation due to reranking for a pair of units with pre-fiscal ranks (i, j) is obtained as follows.

$$\rho^{x,B}_{i,j,q} = \dot{r}^x_{i,j} \frac{v^{x,B}_{i,j,q}}{\Phi^x_{i,j}} \tag{3.72}$$

Summing the contributions of all taxes and benefits from (3.71) and (3.72), we obtain:

$$\dot{r}^x_{i,j} = \sum_p^P \rho^{x,T}_{i,j,p} + \sum_q^Q \rho^{x,B}_{i,j,q} \left(= \frac{\dot{r}^x_{i,j} \sum_p^P v^{x,T}_{i,j,p} + \dot{r}^x_{i,j} \sum_q^Q v^{x,B}_{i,j,q}}{\sum_p^P v^{x,T}_{i,j,p} + \sum_q^Q v^{x,B}_{i,j,q}} \right)$$

Substituting $\dot{r}^x_{i,j} = \sum_p^P \rho^{x,T}_{i,j,p} + \sum_q^Q \rho^{x,B}_{i,j,q}$ into (3.70), and rearranging, we get:

$$\left| \widetilde{X}^x_i - \widetilde{X}^x_j \right| - \left| N^x_i - N^x_j \right| = \left(\sum_{p=1}^P v^{x,T}_{i,j,p} + \sum_{q=1}^Q v^{x,B}_{i,j,q} \right) - 2\left(\sum_{p=1}^P \rho^{x,T}_{i,j,p} + \sum_{q=1}^Q \rho^{x,B}_{i,j,q} \right) \tag{3.73}$$

Finally, integrating for all pairs (i, j), we obtain the decomposition (3.74). Here, $V_p^{x,T}$ is a contribution of *p*th tax to overall fiscal deprivation V^x, also defined in (3.79). The relative contribution of *p*th tax to fiscal deprivation is a share of $V_p^{x,T}$ in the sum of all taxes' and benefits' contributions, as in (3.80). Similarly, $V_p^{x,B}$, $R_p^{x,T}$ and $R_q^{x,B}$, and their relative contributions are defined.

$$G_{\hat{X}} - G_N = \left(\sum_{p=1}^{P} V_p^{x,T} + \sum_{q=1}^{Q} V_q^{x,B} \right) - 2 \left(\sum_{p=1}^{P} R_p^{x,T} + \sum_{q=1}^{Q} R_q^{x,B} \right) \qquad (3.74)$$

Table 3.12: Components of the fiscal deprivation and their contributions

Component	**Contribution to overall effect**
$R_p^{x,T} = c \sum_{i=2}^{s} \sum_{j=1}^{i-1} \dot{r}_{i,j}^{x} \frac{v_{i,j,p}^{x,T}}{\Phi_{i,j}^{x}}$ (3.75)	$\frac{R_p^{x,T}}{\sum_p^P R_p^{x,T} + \sum_q^Q R_q^{x,B}}$ (3.76)
$R_q^{x,B} = c \sum_{i=2}^{s} \sum_{j=1}^{i-1} \dot{r}_{i,j}^{x} \frac{v_{i,j,q}^{x,B}}{\Phi_{i,j}^{x}}$ (3.77)	$\frac{R_q^{x,B}}{\sum_p^P R_p^{x,T} + \sum_q^Q R_q^{x,B}}$ (3.78)
$V_p^{x,T} = c \sum_{i=2}^{s} \sum_{j=1}^{i-1} v_{i,j,p}^{x,T}$ (3.79)	$\frac{V_p^{x,T}}{\sum_p^P V_p^{x,T} + \sum_q^Q V_q^{x,B}}$ (3.80)
$V_q^{x,B} = c \sum_{i=2}^{s} \sum_{j=1}^{i-1} v_{i,j,q}^{x,B}$ (3.81)	$\frac{V_q^{x,B}}{\sum_p^P V_p^{x,T} + \sum_q^Q V_q^{x,B}}$ (3.82)

To show how the above decompositions of R^x, V^x and Δ work, we use a hypothetical example from Table 3.4. Again, we draw triangular matrices needed to obtain the contributions of taxes and benefits, M_1 to M_4, presented in the Figure 3.5, and calculate their sums $\Sigma\,\mathrm{M}_1$ to $\Sigma\,\mathrm{M}_4$.

According to (3.79), to obtain $V_1^{x,T}$, we need to sum the values $v_{i,j,p}^{x,T} = T_{i,p}^{x} - T_{j,p}^{x}$, presented by the matrix M_1 and multiply the sum by c. Thus, we obtain a contribution of tax to fiscal deprivation V^x.

$$V_1^{x,T} = c(\Sigma \mathrm{M}_1) = c \cdot 520 = 0.3467$$

Similarly, a contribution of benefit to fiscal deprivation V^x is obtained by (3.81). The matrix M_2 contains values $v_{i,j,q}^{x,B} = B_{j,q}^x - B_{i,q}^x$, and ΣM_2 is their sum.

$$V_1^{x,B} = c(\Sigma M_2) = c \cdot 204 = 0.1360$$

Contribution of tax to reranking effect R^x is obtained by (3.75). Using (3.22) we first need to calculate the values $\dot{r}_{i,j}^x = r_{i,j}^x$, already presented in M_5 of Figure 3.3. Multiplying each $\dot{r}_{i,j}^x$ with corresponding $v_{i,j,1}^{x,T}$, and dividing by $\Phi_{i,j}^x = v_{i,j,1}^{x,T} + v_{i,j,1}^{x,B}$, we obtain the matrix M_3. Finally, as (3.75) commands, we multiply ΣM_3 by c.

$$R_1^{T,x} = c \cdot (\Sigma M_3) = c \cdot 83.05 = 0.0554$$

Similarly, from (3.77), we obtain the contribution of benefit to reranking effect R^x, after construction of M_4 and calculation of ΣM_4.

$$R_1^{B,x} = c \cdot (\Sigma M_4) = c \cdot 76.95 = 0.0513$$

Finally, full decomposition of distance narrowing for the hypothetical case is shown by the following equation.

$$\Delta = (V_1^{x,T} + V_1^{x,B}) - 2(R_1^{x,T} + R_1^{x,B}) = (0.3467 + 0.1360) - 2 \cdot (0.0554 + 0.0513) = 0.2693$$

Figure 3.5: Matrices with hypothetical data

$\mathrm{M}_1(i,j) = v^{x,T}_{i,j,1} = T^x_{i,1} - T^x_{j,1}$

0	0				*520*
10	10	10			
60	60	60	50		
100	100	100	90	40	
	0	0	10	60	

$\mathrm{M}_2(i,j) = v^{x,B}_{i,j,1} = B^x_{j,1} - B^x_{i,1}$

48	-16				*204*
80	-48	-32			
10	22	38	70		
0	32	48	80	10	
	32	48	80	10	

$\mathrm{M}_3(i,j) = \dot{r}^x_{i,j} v^{x,T}_{i,j,1} / \Phi^x_{i,j}$

					83.05
	14.6	24.5	33.3		
			10.59		

$\mathrm{M}_4(i,j) = \dot{r}^x_{i,j} v^{x,B}_{i,j,1} / \Phi^x_{i,j}$

					76.95
	5.4	15.5	46.7		
			9.4		

3.3.2.3 Decomposition of R^n and V^n

Similarly, we can decompose a change in the index of domination due to reranking. We have to go in opposite direction: starting with post-fiscal situation and ending with pre-fiscal one. In the beginning (end), we have that: $T^{n,0}_i = T^n_i$ ($T^{n,1}_i = 0$), $B^{n,0}_i = B^n_i$ ($B^{n,1}_i = 0$) and $X^{n,0}_i = \tilde{N}^n_i$ ($X^{n,1}_i = X^n_i$). Thus, the changes are: $\Delta T^n_i = -T^n_i$, $\Delta B^n_i = -B^n_i$, $\Delta \tilde{N}^n_i = 0$, $\Delta X^n_i = X^{n,1}_i - X^{n,0}_i = X^n_i - \tilde{N}^n_i$, which is also equal to $X^n_i - \tilde{N}^n_i = \sum_p^{P+1} T^n_{i,p} - \sum_q^{Q+1} B^n_{i,q}$.

Remember that (3.26) presents *domination due to reranking* felt by the unit with post-fiscal rank i, that might have reranked the unit with post-fiscal rank j.

$$r^n_{i,j} = \frac{1}{2}\left(\left|X^n_i - X^n_j\right| - \left(X^n_i - X^n_j\right)\right) \tag{3.26}$$

The *change* of domination ($\dot{r}^n_{i,j}$) during the fiscal process is a difference between the values of domination after ($r^{n,1}_{i,j}$) and before ($r^{n,0}_{i,j}$) fiscal actions took place, or $\dot{r}^n_{i,j} = r^{n,1}_{i,j} - r^{n,0}_{i,j}$. Substituting (3.26) into the last formula, and denoting $X^{n,1}_i$ and $X^{n,0}_i$ analogously to ($r^{n,1}_{i,j}$) and ($r^{n,0}_{i,j}$), we obtain (3.83).

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^{n,1} - X_j^{n,1}\right| - \left(X_i^{n,1} - X_j^{n,1}\right)\right) - \frac{1}{2}\left(\left|X_i^{n,0} - X_j^{n,0}\right| - \left(X_i^{n,0} - X_j^{n,0}\right)\right) \tag{3.83}$$

From (3.83), we derive (3.84) as follows. Substituting $X_i^{n,0} = \widetilde{N}_i^n$ and $X_i^{n,1} = X_i^n$ into (3.83), we obtain:

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left(X_i^n - X_j^n\right)\right) - \frac{1}{2}\left(\left|\widetilde{N}_i^n - \widetilde{N}_j^n\right| - \left(\widetilde{N}_i^n - \widetilde{N}_j^n\right)\right)$$

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left|\widetilde{N}_i^n - \widetilde{N}_j^n\right|\right) - \frac{1}{2}\left(\left(X_i^n - X_j^n\right) - \left(\widetilde{N}_i^n - \widetilde{N}_j^n\right)\right)$$

Notice that the last equation presents the relation $R^{LY} = RE^{xn} - RE^n$.

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left|\widetilde{N}_i^n - \widetilde{N}_j^n\right|\right) - \frac{1}{2}\left(\left(X_i^n - \widetilde{N}_i^n\right) - \left(X_j^n - \widetilde{N}_j^n\right)\right)$$

Substituting (3.67) into the equation above, we obtain:

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left|\widetilde{N}_i^n - \widetilde{N}_j^n\right|\right) - \frac{1}{2}\left(\left(\sum_p^{P+1} T_{i,p}^n - \sum_q^{Q+1} B_{i,q}^n\right) - \left(\sum_p^{P+1} T_{j,p}^n - \sum_q^{Q+1} B_{j,q}^n\right)\right)$$

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left|\widetilde{N}_i^n - \widetilde{N}_j^n\right|\right) - \frac{1}{2}\left(\left(\sum_p^{P+1} T_{i,p}^n - \sum_p^{P+1} T_{j,p}^n\right) - \left(\sum_q^{Q+1} B_{i,q}^n - \sum_q^{Q+1} B_{j,q}^n\right)\right)$$

$$\dot{r}_{i,j}^{n} = \frac{1}{2}\left(\left|X_i^n - X_j^n\right| - \left|\widetilde{N}_i^n - \widetilde{N}_j^n\right|\right) - \frac{1}{2}\left(\left(\sum_p^{P+1} T_{i,p}^n - \sum_p^{P+1} T_{j,p}^n\right) + \left(\sum_q^{Q+1} B_{j,q}^n - \sum_q^{Q+1} B_{i,q}^n\right)\right)$$

Finally, multiplying the last equation by 2, we obtain the expression (3.84).

$$2\dot{r}_{i,j}^{n} = \left(\left|X_i^n - X_j^n\right| - \left|\widetilde{N}_i^n - \widetilde{N}_j^n\right|\right) - \left(\left(\sum_p^{P+1} T_{i,p}^n - \sum_p^{P+1} T_{j,p}^n\right) + \left(\sum_q^{Q+1} B_{j,q}^n - \sum_q^{Q+1} B_{i,q}^n\right)\right) \tag{3.84}$$

Remember that $T_{i,P+1}^n = T_{j,P+1}^n$ and $B_{i,Q+1}^n = B_{j,Q+1}^n$ for all (i, j). Let $v_{i,j,p}^{n,T} = T_{i,p}^n - T_{j,p}^n$ and $v_{i,j,q}^{n,B} = B_{j,q}^n - B_{i,q}^n$. Therefore, (3.84) can be written as:

$$2\dot{r}_{i,j}^{n} = \left(\left|X_i^n - X_j^n\right| - \left|\tilde{N}_i^n - \tilde{N}_j^n\right|\right) - \left(\sum_{p}^{P} v_{i,j,p}^{n,T} + \sum_{q}^{Q} v_{i,j,q}^{n,B}\right) \tag{3.85}$$

Let $\Phi_{i,j}^{n} = \sum_{p}^{P} v_{i,j,p}^{n,T} + \sum_{q}^{Q} v_{i,j,q}^{n,B}$. Contribution of tax p to the change of domination due to reranking of the unit with post-fiscal rank i, that outranked the unit with rank j, is obtained as follows.

$$\rho_{i,j}^{n,T} = \dot{r}_{i,j}^{n} \frac{v_{i,j,p}^{n,T}}{\Phi_{i,j}^{n}} \tag{3.86}$$

Contribution of benefit q to the change of domination due to reranking for a pair of units with post-fiscal ranks (i, j) is obtained as in the next equation.

$$\rho_{i,j}^{n,B} = \dot{r}_{i,j}^{n} \frac{v_{i,j,q}^{n,B}}{\Phi_{i,j}^{n}} \tag{3.87}$$

Summing the contributions of all taxes and benefits from (3.86) and (3.87), we obtain:

$$\dot{r}_{i,j}^{n} = \sum_{p}^{P} \rho_{i,j,p}^{n,T} + \sum_{q}^{Q} \rho_{i,j,q}^{n,B} \left(= \frac{\dot{r}_{i,j}^{n} \sum_{p}^{P} v_{i,j,p}^{n,T} + \dot{r}_{i,j}^{n} \sum_{q}^{Q} v_{i,j,q}^{n,B}}{\sum_{p}^{P} v_{i,j,p}^{n,T} + \sum_{q}^{Q} v_{i,j,q}^{n,B}} \right)$$

Substituting $\dot{r}_{i,j}^{n} = \sum_{p}^{P} \rho_{i,j,p}^{n,T} + \sum_{q}^{Q} \rho_{i,j,q}^{n,B}$ into (3.85), and rearranging, we obtain:

$$\left|X_i^n - X_j^n\right| - \left|\tilde{N}_i^n - \tilde{N}_j^n\right| = \left(\sum_{p=1}^{P} v_{i,j,p}^{n,T} + \sum_{q=1}^{Q} v_{i,j,q}^{n,B}\right) + 2\left(\sum_{p=1}^{P} \rho_{i,j,p}^{n,T} + \sum_{q=1}^{Q} \rho_{i,j,q}^{n,N}\right) \tag{3.88}$$

Finally, integrating for all pairs (i, j), we obtain (3.89). The term $V_p^{n,T}$ shows a contribution of pth tax to reverse fiscal domination V^x, defined in (3.94). The relative contribution of pth tax is a share of $V_p^{n,T}$ in the sum of all taxes' and benefits' contributions, as in (3.95). Similarly, $V_p^{n,T}$, $R_p^{n,T}$ and $R_q^{n,B}$, and their relative contributions are defined.

$$G_X - G_{\tilde{N}} = \left(\sum_{p=1}^{P} V_p^{n,T} + \sum_{q=1}^{Q} V_q^{n,B} \right) + 2\left(\sum_{p=1}^{P} R_p^{n,T} + \sum_{q=1}^{Q} R_q^{n,B} \right) \qquad (3.89)$$

Table 3.13: Components of reverse fiscal domination and their contributions

Component	Contribution to overall effect
$R_p^{n,T} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \dot{r}_{i,j}^{n} \frac{v_{i,j,p}^{n,T}}{\Phi_{i,j}^{n}}$ (3.90)	$\frac{R_p^{n,T}}{\sum_p^P R_p^{n,T} + \sum_q^Q R_q^{n,B}}$ (3.91)
$R_q^{n,B} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \dot{r}_{i,j}^{n} \frac{v_{i,j,q}^{n,B}}{\Phi_{i,j}^{n}}$ (3.92)	$\frac{R_q^{n,B}}{\sum_p^P R_p^{n,T} + \sum_q^Q R_q^{n,B}}$ (3.93)
$V_p^{n,T} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} v_{i,j,p}^{n,T}$ (3.94)	$\frac{V_p^{n,T}}{\sum_p^P V_p^{n,T} + \sum_q^Q V_q^{n,B}}$ (3.95)
$V_q^{n,B} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} v_{i,j,q}^{n,B}$ (3.96)	$\frac{V_q^{n,B}}{\sum_p^P V_p^{n,T} + \sum_q^Q V_q^{n,B}}$ (3.97)

According to (3.94), to obtain $V_1^{n,T}$ we only need to sum the values $v_{i,j,1}^{n,T}$, presented by the matrix M_1 of the Figure 3.6 and multiply $\Sigma\mathrm{M}_1$ by c. Thus, we obtain a contribution of tax in the redistributive effect V^n. Similarly, we obtain the contribution of benefit to V^n.

$$V_1^{n,T} = c(\Sigma\mathrm{M}_1) = c \cdot 0 = 0$$
$$V_1^{n,B} = c(\Sigma\mathrm{M}_2) = c \cdot (-216) = -0.1440$$

Contribution of tax to reranking effect R^n is obtained by (3.90). Values $\dot{r}_{i,j}^{n} = r_{i,j}^{n}$ are already shown in M_6 of Figure 3.3. Multiplying each $\dot{r}_{i,j}^{n}$ with corresponding $v_{i,j,1}^{n,T}$, and dividing by $\Phi_{i,j}^{n} = v_{i,j,1}^{T,n} + v_{i,j,1}^{B,n}$, we obtain the matrix M_3. Finally, as (3.90) commands, we obtain $\Sigma\,\mathrm{M}_3$ and multiply by c. In the same way, contribution $R_1^{n,B}$ is calculated.

$$R_1^{n,T} = c \cdot (\Sigma\mathrm{M}_3) = c \cdot 176.95 = 0.1180$$

$$R_1^{n,B} = c \cdot (\Sigma \mathrm{M}_4) = c \cdot 133.05 = 0.0887$$

Full decomposition of redistributive effect for the hypothetical case is shown by the following equation:

$$\Delta = (V_1^{n,T} + V_1^{n,B}) + 2 \cdot (R_1^{n,T} + R_1^{n,B}) = (0 - 0.1440) + 2 \cdot (0.1180 + 0.0887) = 0.2693$$

Figure 3.6: Matrices with hypothetical data

$\mathrm{M}_1(i,j) = v_{i,j,1}^{n,T} = T_{i,1}^n - T_{j,1}^n$

0	-60			*0*
0	-60	0		
100	40	100	100	
10	-50	10	10	-90
	60	0	0	100

$\mathrm{M}_2(i,j) = v_{i,j,1}^{n,B} = B_{j,1}^n - B_{i,1}^n$

32	-22			*-216*
48	-38	-16		
0	10	32	48	
80	-70	-48	-32	-80
	10	32	48	0

$\mathrm{M}_3(i,j) = \dot{r}_{i,j}^n v_{i,j,1}^{n,T} / \Phi_{i,j}^n$

	45.4			*177*
	35.5			
	16.7			79.4

$\mathrm{M}_4(i,j) = \dot{r}_{i,j}^n v_{i,j,1}^{n,B} / \Phi_{i,j}^n$

	16.6			*133*
	22.5			
	23.3			70.6

3.3.3 Approach based on distance narrowing and deviations of taxes and benefits from proportionality

3.3.3.1 Basic terms

As in the previous sections, we have to create "fill in" variables that will account for differences in total post- and pre-fiscal incomes. This time the scheme is different for the reasons that will be shown later.

$$B_{i,Q+1}^x = \begin{cases} X_i^x \left(1 - \overline{N} / \overline{X}\right) & \text{if } \overline{N} < \overline{X} \\ 0 & \text{otherwise} \end{cases} \qquad T_{i,P+1}^x = \begin{cases} \ldots & \text{if } \overline{N} > \overline{X} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{i,Q+1}^n = \begin{cases} X_i^x \left(\overline{X} / \overline{N} - 1\right) & \text{if } \overline{N} < \overline{X} \\ 0 & \text{otherwise} \end{cases} \qquad T_{i,P+1}^n = \begin{cases} \ldots & \text{if } \overline{N} > \overline{X} \\ 0 & \text{otherwise} \end{cases}$$

We have to form new variables of post-fiscal income as:

$$\hat{X}_i^x = X_i^x - B_{i,Q+1}^x + T_{i,P+1}^x$$

$$\hat{N}_i^n = N_i^n + B_{i,Q+1}^n - T_{i,P+1}^n$$

We obtain the following equation.

$$\hat{X}_i^x - N_i^x = \left(T_{i,P+1}^x + \sum_{p=1}^{P} T_{i,p}^x \right) - \left(B_{i,Q+1}^x + \sum_{q=1}^{Q} B_{i,q}^x \right) \tag{3.98}$$

$$X_i^n - \hat{N}_i^n = \left(T_{i,P+1}^n + \sum_{p=1}^{P} T_{i,p}^n \right) - \left(B_{i,Q+1}^n + \sum_{q=1}^{Q} B_{i,q}^n \right) \tag{3.99}$$

Let us now define new variables which will measure distances of taxes and benefits from the counterfactual proportional ones. As already mentioned, $T_{i,p}^x$ is the p th tax paid by the i th unit; $t_p^x = \sum_i^s T_{i,p}^x / \sum_i^s \hat{X}_i^x$ is a share of p th tax in (adjusted) pre-fiscal income; $B_{i,q}^x$ is the q th benefit received by the i th unit; $b_q^x = \sum_i^s B_{i,q}^x / \sum_i^s \hat{X}_i^x$ is a share of q th benefit in pre-fiscal income. Now, we have that:

$$\widehat{T}_{i,p}^x = T_{i,p}^x - t_p^x \hat{X}_i^x ; \qquad \widehat{T}_{i,P+1}^x = T_{i,P+1}^x - t_{P+1}^x \hat{X}_i^x \tag{3.100}$$

$$\widehat{B}_{i,q}^x = B_{i,q}^x - b_q^x \hat{X}_i^x ; \qquad \widehat{B}_{i,Q+1}^x = B_{i,Q+1}^x - b_{Q+1}^x \hat{X}_i^x \tag{3.101}$$

The term $t_p^x \hat{X}_i^x$ in (3.100) is the amount of p th tax paid by unit i if this tax instrument were proportional with pre-fiscal income. Then, the term $\widehat{T}_{i,p}^x$, obtained as a difference between actual p th tax paid by units i, $T_{i,p}^x$ and $t_p^x \hat{X}_i^x$, measures how the actual system deviates from proportionality. The same relates to distances $\widehat{B}_{i,q}^x$, between $B_{i,q}^x$ and $b_q^x \hat{X}_i^x$.

From the perspective of post-fiscal income, we have that $T_{i,p}^n$ is the p th tax paid by the i th unit; $t_p^n = \sum_i^s T_{i,p}^n / \sum_i^s \hat{N}_i^n$ is a share of p th tax in (adjusted) post-fiscal income; $B_{i,q}^n$ is the q th benefit received by the i th unit; $b_q^n = \sum_i^s B_{i,q}^n / \sum_i^s \hat{N}_i^n$ is a share of q th benefit in post-fiscal income. We calculate the following variables:

$$\widehat{T}_{i,p}^{n} = T_{i,p}^{n} - t_{p}^{n}\hat{N}_{i}^{n} \tag{3.102}$$

$$\widehat{B}_{i,q}^{n} = B_{i,q}^{n} - b_{q}^{n}\hat{N}_{i}^{n} \tag{3.103}$$

The term $t_p^n \hat{N}_i^n$ in (3.102) is an amount of p th tax that would be paid by unit i if this tax instrument were proportional to post-fiscal income. The difference $\widehat{T}_{i,p}^{n}$, between actual p th tax paid, $T_{i,p}^{n}$ and $t_p^n \hat{N}_i^n$ measures how the actual system deviates from proportionality defined in terms of post-fiscal income distribution. The same relates to $\widehat{B}_{i,q}^{n}$, $B_{i,q}^{n}$ and $b_q^n \hat{N}_i^n$.

3.3.3.2 Decomposition of R^x and V^x

From (3.100) and (3.101), it is obtained:

$$\widehat{T}_{i,p}^{x} = T_{i,p}^{x} - t_{p}^{x}\hat{X}_{i}^{x} \Rightarrow T_{i,p}^{x} = \widehat{T}_{i,p}^{x} + t_{p}^{x}\hat{X}_{i}^{x}$$

$$\widehat{B}_{i,q}^{x} = B_{i,q}^{x} - b_{q}^{x}\hat{X}_{i}^{x} \Rightarrow B_{i,q}^{x} = \widehat{B}_{i,q}^{x} + b_{q}^{x}\hat{X}_{i}^{x}$$

Remember (3.69), which calculates a change in deprivation due to reranking:

$$2\dot{r}_{i,j}^{x} = \left(\sum_{p}^{P+1}\left(T_{i,p}^{x} - T_{j,p}^{x}\right) + \sum_{q}^{Q+1}\left(B_{j,q}^{x} - B_{i,q}^{x}\right) \right) - \left(\left|\widetilde{X}_{i}^{x} - \widetilde{X}_{j}^{x}\right| - \left|N_{i}^{x} - N_{j}^{x}\right| \right) \tag{3.69}$$

We are particularly interested in the first term on the right side: $\sum_{p}^{P+1}\left(T_{i,p}^{x} - T_{j,p}^{x}\right)$ $+\sum_{q}^{Q+1}\left(B_{j,q}^{x} - B_{i,q}^{x}\right)$. It is equal to:

$$\sum_{p}^{P+1}\left(T_{i,p}^{x} - T_{j,p}^{x}\right) + \sum_{q}^{Q+1}\left(B_{j,q}^{x} - B_{i,q}^{x}\right) = \sum_{p}^{P+1}\left(\widehat{T}_{i,p}^{x} + t_{p}^{x}\hat{X}_{i}^{x} - \widehat{T}_{j,p}^{x} - t_{p}^{x}\hat{X}_{j}^{x}\right) + \sum_{q}^{Q+1}\left(\widehat{B}_{j,q}^{x} + b_{q}^{x}\hat{X}_{j}^{x} - \widehat{B}_{i,q}^{x} - b_{q}^{x}\hat{X}_{i}^{x}\right) =$$

$$= \left(\sum_{p}^{P+1}\left(\widehat{T}_{i,p}^{x} - \widehat{T}_{j,p}^{x}\right) + \sum_{q}^{Q+1}\left(\widehat{B}_{j,q}^{x} - \widehat{B}_{i,q}^{x}\right) \right) + \left(\sum_{p}^{P+1}\left(t_{p}^{x}\hat{X}_{i}^{x} - t_{p}^{x}\hat{X}_{j}^{x}\right) + \sum_{q}^{Q+1}\left(b_{q}^{x}\hat{X}_{j}^{x} - b_{q}^{x}\hat{X}_{i}^{x}\right) \right)$$

The last term in the above identity can be rearranged as $\sum_{p}^{P+1} t_p^x \hat{X}_i^x - \sum_{q}^{Q+1} b_q^x \hat{X}_i^x +$ $+ \sum_{q}^{Q+1} b_q^x \hat{X}_j^x - \sum_{p}^{P+1} t_p^x \hat{X}_j^x$, and it is easy to see that its value is zero, because $\sum_{q}^{Q+1} b_q^x = \sum_{p}^{P+1} t_p^x$.

Therefore, we have that:

$$\sum_{p}^{P+1}\left(\widehat{T}_{i,p}^x - \widehat{T}_{j,p}^x\right) + \sum_{q}^{Q+1}\left(\widehat{B}_{j,q}^x - \widehat{B}_{i,q}^x\right) \equiv \sum_{p}^{P+1}\left(T_{i,p}^x - T_{j,p}^x\right) + \sum_{q}^{Q+1}\left(B_{j,q}^x - B_{i,q}^x\right) \quad (3.104)$$

The identity which we have just derived means that we can apply decompositions formulated in the previous sections to variables that present differences from the proportionality as well.

Observe that $\widehat{T}_{i,P+1}^x - \widehat{T}_{j,P+1}^x = 0$ and $\widehat{B}_{j,Q+1}^x - \widehat{B}_{i,Q+1}^x = 0$ for all (i,j). This is proved in the following way. We know that $\widehat{B}_{i,Q+1}^x = B_{i,Q+1}^x - b_{Q+1}^x X_i^x$ and $b_{Q+1}^x = 1 - \overline{N}/\overline{X}$. Also, we know that $B_{i,Q+1}^x$ is defined as $X_i^x(1 - \overline{N}/\overline{X})$. Therefore, $\widehat{B}_{i,Q+1}^x = X_i^x\left(1 - \overline{N}/\overline{X}\right) - \left(1 - \overline{N}/\overline{X}\right)X_i^x = 0$. Similarly we can prove the $\widehat{T}_{i,P+1}^x = 0$.

Replace $\widehat{v}_{i,j,p}^{x,T} = \widehat{T}_{i,p}^x - \widehat{T}_{j,p}^x$ and $\widehat{v}_{i,j,q}^{x,B} = \widehat{B}_{j,q}^x - \widehat{B}_{i,q}^x$. Let $\Theta_{i,j}^x = \sum_{p}^{P} \widehat{v}_{i,j,p}^{x,T} + \sum_{q}^{Q} \widehat{v}_{i,j,q}^{x,B}$. Now, analogously to (3.71) and (3.72), define the following two terms:

$$\widehat{\rho}_{i,j,p}^{T,x} = \dot{r}_{i,j}^x \frac{\widehat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^x} \quad (3.105)$$

$$\widehat{\rho}_{i,j,q}^{B,x} = \dot{r}_{i,j}^x \frac{\widehat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^x} \quad (3.106)$$

with $\dot{r}_{i,j}^x$ already defined in (3.69). The sum of all contributions to $\dot{r}_{i,j}^x$ is $\dot{r}_{i,j}^x = \sum_{p}^{P} \widehat{\rho}_{i,j,p}^{x,T} + \sum_{q}^{Q} \widehat{\rho}_{i,j,q}^{x,B}$, and analogously to (3.73), we have that:

$$\left|\hat{X}_i^x - \hat{X}_j^x\right| - \left|N_i^x - N_j^x\right| = \left(\sum_{p=1}^{P} \widehat{v}_{i,j,p}^{x,T} + \sum_{q=1}^{Q} \widehat{v}_{i,j,q}^{x,B}\right) - 2\left(\sum_{p=1}^{P} \widehat{\rho}_{i,j,p}^{x,T} + \sum_{q=1}^{Q} \widehat{\rho}_{i,j,q}^{x,B}\right) \tag{3.107}$$

In the end, integrating for all pairs (i, j), we obtain (3.108). The contributions of individual taxes and benefits to fiscal deprivation ($V_p^{x,\hat{T}}$, $V_q^{x,\hat{B}}$) and reranking ($R_p^{x,\hat{T}}$, $R_q^{x,\hat{B}}$) are defined in equations (3.109) through (3.116), and presented in Table 3.14.

$$G_{\hat{X}} - G_N = \left(\sum_{p=1}^{P} V_p^{x,\hat{T}} + \sum_{q=1}^{Q} V_q^{x,\hat{B}}\right) - 2\left(\sum_{p=1}^{P} R_p^{x,\hat{T}} + \sum_{q=1}^{Q} R_q^{x,\hat{B}}\right) \tag{3.108}$$

Table 3.14: Components of fiscal deprivation and their contributions

Component	Contribution to overall effect
$R_p^{x,\hat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \dot{r}_{i,j}^x \frac{\widehat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^x}$ (3.109)	$\frac{R_p^{x,\hat{T}}}{\sum_p^P R_p^{x,\hat{T}} + \sum_q^Q R_q^{x,\hat{B}}}$ (3.110)
$R_q^{x,\hat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \dot{r}_{i,j}^x \frac{\widehat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^x}$ (3.111)	$\frac{R_q^{x,\hat{B}}}{\sum_p^P R_p^{x,\hat{T}} + \sum_q^Q R_q^{x,\hat{B}}}$ (3.112)
$V_p^{x,\hat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \widehat{v}_{i,j,p}^{x,T}$ (3.113)	$\frac{V_p^{x,\hat{T}}}{\sum_p^P V_p^{x,\hat{T}} + \sum_q^Q V_q^{x,\hat{B}}}$ (3.114)
$V_q^{x,\hat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \widehat{v}_{i,j,q}^{x,B}$ (3.115)	$\frac{V_q^{x,\hat{B}}}{\sum_p^P V_p^{x,\hat{T}} + \sum_q^Q V_q^{x,\hat{B}}}$ (3.116)

The calculations for hypothetical data are analogous to previous examples. Respective triangular matrices are shown in Figure 3.7, while contributions are obtained using (3.109), (3.111), (3.113) and (3.115).

$$V_1^{x,\hat{T}} = c(\Sigma \mathrm{M}_1) = c \cdot 64.4 = 0.0429$$

$$V_1^{x,\hat{B}} = c(\Sigma \mathrm{M}_2) = c \cdot 659.6 = 0.4397$$

$$R_1^{x,\hat{T}} = c \cdot (\Sigma \mathrm{M}_3) = c \cdot 35.95 = 0.0240$$

$$R_1^{x,\hat{B}} = c \cdot (\Sigma \mathrm{M}_4) = c \cdot 124.05 = 0.0827$$

$$\Delta = (V_1^{x,\hat{T}} + V_1^{x,\hat{B}}) - 2 \cdot (R_1^{x,\hat{T}} + R_1^{x,\hat{B}}) = (0.0429 + 0.4397) - 2 \cdot (0.0240 + 0.0827)$$

Figure 3.7: Matrices with hypothetical data

$\mathrm{M}_1(i,j)=\widehat{v}^{x,T}_{i,j,1}=\widehat{T}^x_{i,1}-\widehat{T}^x_{j,1}$

-6.8	-2.3				*64.4*
-7.0	-2.5	-0.2			
20.3	24.9	27.1	27.3		
-2.0	2.5	4.8	5.0	-22.3	
	-4.5	-6.8	-7.0	20.3	

$\mathrm{M}_2(i,j)=\widehat{v}^{x,B}_{i,j,1}=\widehat{B}^x_{j,1}-\widehat{B}^x_{i,1}$

41.2	-13.7				*659.6*
63.0	-35.5	-21.8			
-29.7	57.1	70.9	92.7		
-102.0	129.5	143.2	165.0	72.3	
	27.5	41.2	63.0	-29.7	

$\mathrm{M}_4(i,j)=\dot{r}^x_{i,j}\widehat{v}^{x,T}_{i,j,1}/\Theta^x_{i,j}$

					36.0
	6.1	11.1	18.2		
			0.6		

$\mathrm{M}_4(i,j)=\dot{r}^x_{i,j}\widehat{v}^{x,B}_{i,j,1}/\Theta^x_{i,j}$

					124.1
	14.0	28.9	61.8		
			19.4		

3.3.3.3 Decomposition of R^n and V^n

From (3.102) and (3.103), it is obtained that:

$$\widehat{T}^n_{i,p}=T^n_{i,p}-t^n_pN^n_{i,p}\Rightarrow T^n_{i,p}=\widehat{T}^n_{i,p}+t^n_pN^n_{i,p}$$

$$\widehat{B}^n_{i,q}=B^n_{i,q}-b^n_qN^n_{i,q}\Rightarrow B^n_{i,q}=\widehat{B}^n_{i,p}+b^n_qN^n_{i,q}$$

Analogously to (3.104), we can establish that

$$\sum_p^{P+1}\left(\widehat{T}^n_{i,p}-\widehat{T}^n_{j,p}\right)+\sum_q^{Q+1}\left(\widehat{B}^n_{j,q}-\widehat{B}^n_{i,q}\right)\equiv\sum_p^{P+1}\left(T^n_{i,p}-T^n_{j,p}\right)+\sum_q^{Q+1}\left(B^n_{j,q}-B^n_{i,q}\right) \tag{3.117}$$

It can be proven, similarly as in the previous section, that $\widehat{T}^n_{i,P+1}-\widehat{T}^n_{j,P+1}=0$ and $\widehat{B}^n_{j,Q+1}-\widehat{B}^n_{i,Q+1}=0$. Replace $\widehat{v}^{n,T}_{i,j,p}=\widehat{T}^n_{i,p}-\widehat{T}^n_{j,p}$ and $\widehat{v}^{n,B}_{i,j,q}=\widehat{B}^n_{j,q}-\widehat{B}^n_{i,q}$. Let $\Theta^n_{i,j}=\sum_p^P\widehat{v}^{n,T}_{i,j,p}+\sum_q^Q\widehat{v}^{n,B}_{i,j,q}$ Analogously to (3.86) and (3.87), define the following two terms:

$$\widehat{\rho}^{n,T}_{i,j,p}=\dot{r}^n_{i,j}\frac{\widehat{v}^{n,T}_{i,j,p}}{\Theta^n_{i,j}} \tag{3.118}$$

$$\widehat{\rho}^{n,B}_{i,j,q}=\dot{r}^n_{i,j}\frac{\widehat{v}^{n,B}_{i,j,q}}{\Theta^n_{i,j}} \tag{3.119}$$

with $\dot{r}_{i,j}^{n}$ already defined in (3.83). The sum of all contributions to $\dot{r}_{i,j}^{n}$ is $\dot{r}_{i,j}^{n} = \sum_{p}^{P} \widehat{\rho}_{i,j,p}^{n,T} + \sum_{q}^{Q} \widehat{\rho}_{i,j,q}^{n,B}$, and analogously to (3.88), we have that:

$$\left|X_i^n - X_j^n\right| - \left|\hat{N}_i^n - \hat{N}_j^n\right| = \left(\sum_{p=1}^{P} \widehat{v}_{i,j,p}^{n,T} + \sum_{q=1}^{Q} \widehat{v}_{i,j,q}^{n,B}\right) + 2\left(\sum_{p=1}^{P} \widehat{\rho}_{i,j,p}^{n,T} + \sum_{q=1}^{Q} \widehat{\rho}_{i,j,q}^{n,B}\right) \quad (3.120)$$

Finally, integrating for all pairs (i, j), we obtain (3.121). The contributions of individual taxes and benefits to fiscal deprivation ($V_p^{n,\hat{T}}$, $V_q^{n,\hat{B}}$) and reranking ($R_p^{n,\hat{T}}$, $R_q^{n,\hat{B}}$) are defined in equations (3.122) through (3.129), and presented in Table 3.14.

$$G_X - G_{\hat{N}} = \left(\sum_{p=1}^{P} V_p^{n,\hat{T}} + \sum_{q=1}^{Q} V_q^{n,\hat{B}}\right) + 2\left(\sum_{p=1}^{P} R_p^{n,\hat{T}} + \sum_{q=1}^{Q} R_q^{n,\hat{B}}\right) \quad (3.121)$$

Table 3.15: Components of reverse fiscal domination and their contributions

Component	Contribution to overall effect
$R_p^{n,\hat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \dot{r}_{i,j}^{n} \frac{\widehat{v}_{i,j,p}^{n,T}}{\Theta_{i,j}^{n}}$ (3.122)	$\frac{R_p^{n,\hat{T}}}{\sum_p^P R_p^{n,\hat{T}} + \sum_q^Q R_q^{n,\hat{B}}}$ (3.123)
$R_q^{n,\hat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \dot{r}_{i,j}^{n} \frac{\widehat{v}_{i,j,q}^{n,B}}{\Theta_{i,j}^{n}}$ (3.124)	$\frac{R_q^{n,\hat{B}}}{\sum_p^P R_p^{n,\hat{T}} + \sum_q^Q R_q^{n,\hat{B}}}$ (3.125)
$V_p^{n,\hat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \widehat{v}_{i,j,p}^{n,T}$ (3.126)	$\frac{V_p^{n,\hat{T}}}{\sum_p^P V_p^{n,\hat{T}} + \sum_q^Q V_q^{n,\hat{B}}}$ (3.127)
$V_q^{n,\hat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1} \widehat{v}_{i,j,q}^{n,B}$ (3.128)	$\frac{V_q^{n,\hat{B}}}{\sum_p^P V_p^{n,\hat{T}} + \sum_q^Q V_q^{n,\hat{B}}}$ (3.129)

The calculations for hypothetical data are analogous to previous examples. Respective triangular matrices are shown in Figure 3.8, while contributions are obtained using (3.122), (3.124), (3.126) and (3.128).

$$V_1^{n,\hat{T}} = c(\Sigma \mathrm{M}_1) = c \cdot (-226.7) = -0.1511$$

$$V_1^{n,\hat{B}} = c(\Sigma \mathrm{M}_2) = c \cdot 10.7 = 0.0071$$

$$R_1^{n,\hat{T}} = c \cdot (\Sigma \mathrm{M}_3) = c \cdot 224.05 = 0.1494$$

$$R_1^{n,\hat{B}} = c \cdot (\Sigma \mathrm{M}_4) = c \cdot 85.95 = 0.0573$$

$$\Delta = (V_1^{n,\hat{T}} + V_1^{n,\hat{B}}) + 2(R_1^{n,\hat{T}} + R_1^{n,\hat{B}}) = (-0.1511 + 0.0071) + 2 \cdot (0.1494 + 0.0573)$$

Figure 3.8: Matrices with hypothetical data

$\mathrm{M}_1(i,j) = \hat{v}^{n,T}_{i,j,1} = \hat{T}^n_{i,1} - \hat{T}^n_{j,1}$

-22.7	-71.3			*-226.7*
-34.0	-82.7	-11.3		
54.7	6.0	77.3	88.7	
-46.7	-95.3	-24.0	-12.7	-101.3
	48.7	-22.7	-34.0	54.7

$\mathrm{M}_2(i,j) = \hat{v}^{n,B}_{i,j,1} = \hat{B}^n_{j,1} - \hat{B}^n_{i,1}$

9.3	-10.7			*10.7*
14.0	-15.3	-4.7		
-45.3	44.0	54.7	59.3	
23.3	-24.7	-14.0	-9.3	-68.7
	-1.3	9.3	14.0	-45.3

$\mathrm{M}_3(i,j) = \dot{r}^n_{i,j}\hat{v}^{T,n}_{i,j,1} / \Theta^n_{i,j}$

	53.9			*224.05*
	48.9			
	31.8			89.4

$\mathrm{M}_4(i,j) = \dot{r}^n_{i,j}\hat{v}^{T,n}_{i,j,1} / \Theta^n_{i,j}$

	8.1			*85.95*
	9.1			
	8.2			60.6

3.3.4 Comparison of different methods

3.3.4.1 Introduction

In this chapter, we have derived several new decompositions which aim at estimating contributions of different tax and benefit instruments to the redistributive, vertical and reranking effect of the overall fiscal system. With (a) different methods relating to seemingly the same thing, (b) several dozens of formulas at hand, and (c) no particular explanation of their meaning in the text above, it is more than urgent to provide a summary and comparison of them, to which this section is devoted.

3.3.4.2 Methods decomposing vertical effects

The first two decompositions, of V^x and V^n, are adaptations of Lambert (1985) methodology. Section 3.3.1.1 only extends the original model of total taxes and total benefits to the model of p tax and q benefit instruments. Section 3.3.1.2 does the same, but for Lerman-Yitzhaki vertical effect. The final decompositions of these two sections, (3.54) and (3.59), are very similar, both comparing concentration indices of taxes/benefits with Gini coefficients of post-TB (pre-TB) income: the larger the differences between these coefficients are, and the larger the share of tax/benefit in post-TB (pre-TB) income is, the greater the contribution of tax or benefit is.

The reference that serves to determine a contribution is *inequality* of income (pre-TB or post-TB). If a tax is *more* (less) concentrated than income, it will be regarded as *positively* (negatively) contributing to the redistributive effect. It is opposite for a benefit: it will be regarded as *positive* (negative) contributor if it is *less* (more) concentrated than income. If tax/benefit is "proportional" with income (i.e. with constant average tax/benefit rate), it will be distributed equally as income and contribution will be zero whatever the total amount of tax/benefit. Thus, the formulas (3.54) and (3.59) determine contributions of taxes through their *deviations from proportionality* ($D^x_{T,p} - G_X$ and $D^n_{T,p} - G_N$).

Now, recall the decompositions in section 3.3.3, and the section title itself consisting of the words "based on…*deviations* of taxes and benefits *from proportionality*". In (3.100) through (3.103), we have first derived new variables, where each unit's value of tax or benefit is a distance or deviation from the value that would be obtained if income were taxed/benefited by average tax/benefit rate. For the purpose of determining contributions to the redistributive effect, the differences of these distances for all pairs of units (i, j) are aggregated for each tax and benefit, as shown by (3.113), (3.115), (3.126) and (3.128).

The phrase "deviations from proportionality", appearing both in sections 3.3.1 and 3.3.3 is not incidental, but intentional: the decompositions in these sections – based on different approaches – *lead to the same results*! However, this is not proven algebraically, but established only through comparison of different empirical results and from hypothetical data. Therefore, we introduce an *ad hoc* invented symbol "$\doteqdot$" instead of using "$\equiv$". We have following "identities" that should be provable:

$$V_p^{x,\widehat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\widehat{v}_{i,j,p}^{x,T} \doteqdot \lambda_p^{x,T} = \frac{t_p^x(D_{T,p}^x - G_X)}{1-t_p^x+b_q^x} \qquad (3.130)\ [\text{from } (3.113) \text{ and } (3.55)]$$

$$V_q^{x,\widehat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\widehat{v}_{i,j,q}^{x,B} \doteqdot \lambda_q^{x,B} = \frac{b_q^x(G_X - D_{B,q}^x)}{1-t_p^x+b_q^x} \qquad (3.131)\ [\text{from } (3.115) \text{ and } (3.57)]$$

$$V_p^{n,\widehat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\widehat{v}_{i,j,p}^{n,T} \doteqdot \lambda_p^{n,T} = \frac{t_p^n(D_{T,p}^n - G_N)}{1+t_p^n-b_q^n} \qquad (3.132)\ [\text{from } (3.126) \text{ and } (3.60)]$$

$$V_q^{n,\hat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\hat{v}_{i,j,q}^{n,B} \cong \lambda_q^{n,B} = \frac{b_q^n(G_N - D_{B,q}^n)}{1+t_p^n - b_q^n} \qquad (3.133)\ [\text{from } (3.128) \text{ and } (3.62)]$$

It remains now to compare the approaches given in 3.3.1 and 0, with the third one, from section 3.3.2, based on "amounts of taxes and benefits", where "amounts" stands in contrast to "deviations from proportionality". The contribution to the redistributive effect of a tax/benefit is obtained by aggregation of the differences between the amount of the relevant tax or benefit, for all pairs of units (i, j), as shown by (3.79), (3.81), (3.94) and (3.96) (repeated below). Notice that in the previous case we had tax and benefit variables that were "distances from proportionality", while here we have the amounts of taxes and benefits alone.

$$V_p^{x,T} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}v_{i,j,p}^{x,T} \qquad (3.79)$$

$$V_q^{x,B} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}v_{i,j,q}^{x,B} \qquad (3.81)$$

$$V_p^{n,T} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}v_{i,j,p}^{n,T} \qquad (3.94)$$

$$V_q^{n,B} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}v_{i,j,q}^{n,B} \qquad (3.96)$$

There is a large difference in the two approaches. Why this difference? Let us use some hypothetical examples again, to illustrate how the two methods give diverging results on the contributions of taxes and benefits to V^x and V^n. In the first example, shown by Table 3.16, tax and benefit are obviously regressive, meaning that the shares of tax and benefit in pre-TB income are decreasing (see τ_i^x and β_i^x). There is a difference between tax and benefit, however, that will be shown as crucial: values of a benefit are equal for all units, while tax values are increasing in income.

Table 3.16: Hypothetical data

	X_i^x	T_i^x	B_i^x	N_i^x	t_i^x	b_i^x	$T_i^x - t^x X_i$	$B_i^x - b^x X_i$
H	5	1	3	7	0.20	0.60	0.6	2.6
G	12	2	3	13	0.17	0.25	1.1	2.1
J	30	3	3	30	0.10	0.10	0.7	0.7
K	50	4	3	49	0.08	0.06	0.2	-0.8
L	100	5	3	98	0.05	0.03	-2.6	-4.6

Figure 3.9: Matrices with hypothetical data

$\mathrm{M}_1(i,j) = X_i^x - X_j^x$

12	7			
30	25	18		
50	45	38	20	
100	95	88	70	50
	5	12	30	50

$\mathrm{M}_2(i,j) = N_i^x - N_j^x$

13	6			
30	23	17		
49	42	36	19	
98	91	85	68	49
	7	13	30	49

$\mathrm{M}_3(i,j) = v_{i,j,1}^{x,T} = T_{i,1}^x - T_{j,1}^x$

2	1			
3	2	1		
4	3	2	1	
5	4	3	2	1
	1	2	3	4

$\mathrm{M}_4(i,j) = v_{i,j,1}^{x,B} = B_{j,1}^x - B_{i,1}^x$

3	0			
3	0	0		
3	0	0	0	
3	0	0	0	0
	3	3	3	3

$\mathrm{M}_5(i,j) = \widehat{v}_{i,j,1}^{x,T} = \widehat{T}_{i,1}^x - \widehat{T}_{j,1}^x$

1.1	0.5			
0.7	0.1	-0.4		
0.2	-0.4	-0.9	-0.5	
-2.6	-3.2	-3.7	-3.3	-2.8
	0.6	1.1	0.7	0.2

$\mathrm{M}_6(i,j) = \widehat{v}_{i,j,1}^{x,B} = \widehat{B}_{j,1}^x - \widehat{B}_{i,1}^x$

2.1	0.5			
0.7	1.9	1.4		
-0.8	3.4	2.9	1.5	
-4.6	7.2	6.7	5.3	3.8
	2.6	2.1	0.7	-0.8

The system in Table 3.16 reduces inequality as can be seen from the comparison of matrices M_1 and M_2 of Figure 3.9: all the values in M_2 are lower than the values in M_1; thus, the *distances are narrowed* and the Gini coefficient of post-TB income will be smaller (there is no reranking in this case, so $N_i^x - N_j^x = N_i^n - N_j^n$ for all pairs of units).

What is the role of tax and benefit in this reduction? According to M_3 and M_4, a whole redistributive effect is due to a tax! (observe zero-sum of M_4). This contradicts severely our common notions about tax regressivity and income redistribution: the tax in our example is regressive, but still it collects all contribution to Δ. On the other hand, the benefit is regressive and normally it means inequality reducing, but in our case it has zero contribution to the redistributive effect.

To understand the "problem" with the result above, we must remember the formulas for "amounts". From Figure 3.9, M_4, we conclude that if benefit vector contains equal values for all units, the sum of the "triangular" matrix, which determines $V_q^{x,B}$ in (3.81), will also be zero. In other words, benefit does not contribute at all to achieved overall distance narrowing – everything is attributed to tax, which has different (increasing) values, and M_3 in Figure 3.9 and $V_p^{x,T}$ in (3.79) are therefore positive.

The problem is actually well recognized: whether we shall say that tax or benefit is progressive or regressive (inequality reducing or increasing) depends on how we define these terms.[15] The reference point to determine whether tax/benefit is progressive or regressive for methodologies presented in sections 3.3.1 and 3.3.3, is *cumulative share of income*. On the other hand, the benchmark for the methodology in 3.3.2 is *cumulative population share*.

For the former approach, based on "deviations of taxes and benefits from proportionality", we have the following conditions for inequality reducing (henceforth INER). Tax p is INER if average rate of this tax is *increasing* in income; in that case, the concentration curve of tax p lies *below* the Lorenz curve of income (see condition A). Benefit q is INER if the average rate of this benefit is *decreasing* in income; the concentration curve of benefit q lies *above* the Lorenz curve of income (see condition B).

A: Condition for INER of tax p – "deviations from proportionality" approach

(a) $V_p^{\bar{T},x} > 0$ if $\dfrac{T_{i,p}^x}{X_i^x} < \dfrac{T_{j,p}^x}{X_j^x}$ for all i, j s.t. $X_i^x < X_j^x$

L/C curves: $V_p^{x,\bar{T}} > 0$ if $C_{T,p}^x(u) < L_X(u)$ for all $u \in]0,1[$

(b) $RE_p^{\bar{T},n} > 0$ if $\dfrac{T_{i,p}^n}{N_i^n} < \dfrac{T_{j,p}^n}{N_j^n}$ for all i, j s.t. $N_i^n < N_j^n$

[15] As Duclos and Araar (2006:137) explain: "Note that these progressivity comparisons have as a reference point the initial Lorenz curve. In other words, a tax is progressive if the poorest individuals bear a share of the total tax burden that is less than their share in total gross income. As mentioned above, an alternative reference point would be the cumulative shares in the population. This is often argued in the context of state support – the reference point to assess the equity of public expenditures is population share. The analytical framework above can easily allow for this alternative view – for instance, simply by replacing $L_X(u)$ by u in the above definitions of *TR* progressivity. This will make more stringent the conditions to declare a benefit to be progressive, but it will also make it easier for a tax to be declared progressive…"

L/C curves: $V_p^{n,\hat{T}} > 0$ if $C_{T,p}^n(u) < L_N(u)$ for all $u \in]0,1[$

B: Condition for INER of benefit q – "deviations from proportionality" approach

(a) $RE_q^{\hat{B},x} > 0$ if $\frac{B_{i,q}^x}{X_i^x} > \frac{B_{j,q}^x}{X_j^x}$ for all i, j s.t. $X_i^x < X_j^x$

L/C curves: $V_q^{x,\hat{B}} > 0$ if $C_{B,q}^x(u) > L_X(u)$ for all $u \in]0,1[$

(b) $RE_q^{\hat{B},n} > 0$ if $\frac{B_{i,q}^n}{N_i^n} > \frac{B_{j,q}^n}{N_j^n}$ for all i, j s.t. $N_i^n < N_j^n$

L/C curves: $V_q^{n,\hat{B}} > 0$ if $C_{B,q}^n(u) > L_N(u)$ for all $u \in]0,1[$

For the latter approach, based on "amounts of taxes and benefits", the following conditions are valid. Tax p is INER if its amounts are *increasing* in income; in that case, the concentration curve of tax lies *below* the curve of absolute equality (see condition C). Benefit q is INER if its amounts are *decreasing* in income; the concentration curve of benefit q lies *above* the curve of absolute equality (see condition D).

C: Condition for INER of tax p – "amounts" approach

(a) $RE_p^{\hat{T},x} > 0$ if $T_{i,p}^x < T_{j,p}^x$ for all i, j s.t. $X_i^x < X_j^x$

L/C curves: $V_p^{x,\hat{T}} > 0$ if $C_{T,p}^x(u) < u$ for all $u \in]0,1[$

(b) $RE_p^{\hat{T},n} > 0$ if $T_{i,p}^n < T_{j,p}^n$ for all i, j s.t. $N_i^n < N_j^n$

L/C curves: $V_p^{n,\hat{T}} > 0$ if $C_{T,p}^n(u) < u$ for all $u \in]0,1[$

D: Condition for INER of benefit q – "amounts" approach

(a) $RE_q^{\hat{B},x} > 0$ if $B_{i,q}^x > B_{j,q}^x$ for all i, j s.t. $X_i^x < X_j^x$

L/C curves: $V_q^{x,\hat{B}} > 0$ if $C_{B,q}^x(u) > u$ for all $u \in]0,1[$

(b) $RE_q^{\hat{B},n} > 0$ if $B_{i,q}^n > B_{j,q}^n$ for all i, j s.t. $N_i^n < N_j^n$

L/C curves: $V_q^{n,\hat{B}} > 0$ if $C_{B,q}^n(u) > u$ for all $u \in]0,1[$

3.3.4.3 Methods decomposing reranking effects

As we have seen, two approaches are laid out to decompose overall redistributive effect into vertical and reranking effect, and also into contributions of individual taxes and benefits: one based on "amounts of taxes and benefits" (section 3.3.2) and the other based on "deviations of taxes and benefits from proportionality" (section 3.3.3). In the previous section, we have dealt with decompositions of vertical effect; here we want to answer which approach is appropriate to decompose reranking effect. We have two sets of formulas:

Table 3.17: The formulas for decomposition of reranking effect

Amounts of T&B	**Deviations of T&B from proportionality**
$R_p^{x,T} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{x}\frac{v_{i,j,p}^{x,T}}{\Phi_{i,j}^{x}}$ (3.75)	$R_p^{x,\widehat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{x}\frac{\widehat{v}_{i,j,p}^{x,T}}{\Theta_{i,j}^{x}}$ (3.109)
$R_q^{x,B} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{x}\frac{v_{i,j,q}^{x,B}}{\Phi_{i,j}^{x}}$ (3.77)	$R_q^{x,\widehat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{x}\frac{\widehat{v}_{i,j,q}^{x,B}}{\Theta_{i,j}^{x}}$ (3.111)
$R_p^{n,T} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{n}\frac{v_{i,j,p}^{n,T}}{\Phi_{i,j}^{n}}$ (3.90)	$R_p^{n,\widehat{T}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{n}\frac{\widehat{v}_{i,j,p}^{n,T}}{\Theta_{i,j}^{n}}$ (3.122)
$R_q^{n,B} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{n}\frac{v_{i,j,q}^{n,B}}{\Phi_{i,j}^{n}}$ (3.92)	$R_q^{n,\widehat{B}} = c\sum_{i=2}^{s}\sum_{j=1}^{i-1}\dot{r}_{i,j}^{n}\frac{\widehat{v}_{i,j,q}^{n,B}}{\Theta_{i,j}^{n}}$ (3.124)

Comparing the formulas on the left and the right side in each row of Table 3.17, we easily observe that "deprivation terms", $\dot{r}_{i,j}^{x}$ or $\dot{r}_{i,j}^{n}$, appear in all equations. It is the weights by which the deprivation terms are multiplied, which make the difference: the shares of v-terms in total sums of vs. Let us remind of the meaning of these terms on the example of the first row (others are analogously explained). In case of (3.75) [(3.109)], the weight $v_{i,j,p}^{x,T}/\Phi_{i,j}^{x}$ [$\widehat{v}_{i,j,p}^{x,T}/\Theta_{i,j}^{x}$] has the following meaning: $v_{i,j,p}^{x,T}$ [$\widehat{v}_{i,j,p}^{x,T}$] is a difference between amount [deviation from proportionality] of pth tax paid by unit i and unit j, $v_{i,j,p}^{x,T} = T_{i,p}^{x} - T_{j,p}^{x}$ [$\widehat{v}_{i,j,p}^{x,T} = \widehat{T}_{i,p}^{x} - \widehat{T}_{j,p}^{x}$]; $\Phi_{i,j}^{x}$ [$\Theta_{i,j}^{x}$] is a sum of these differences for all taxes and benefits:

$$\Phi_{i,j}^{x} = \sum_{p}^{P} v_{i,j,p}^{x,T} + \sum_{q}^{Q} v_{i,j,q}^{x,B} \quad [\Theta_{i,j}^{x} = \sum_{p}^{P}\widehat{v}_{i,j,p}^{x,T} + \sum_{q}^{Q}\widehat{v}_{i,j,q}^{x,B}].$$

Now we pose the key question: are the two approaches equally suitable in decomposing reranking effect? We have seen that the difference is in the weights, and here we must search for the answer. In decomposing vertical effect, we concluded that both approaches are appropriate, each in its own regard. However, in case of reranking, the intuition tells us that the approach based on "amounts of taxes and benefits" is the appropriate one, because the weights that determine contributions of taxes and benefits are based on pure values and not on their derivatives – deviations from proportionality. Therefore, to estimate contributions to reranking effect we recommend the formulas on the left side of Table 3.17.

3.3.4.4 Methods decomposing distance narrowing

Sections 3.3.2 and 3.3.3 have presented decompositions of vertical and reranking effects to reveal contributions of different taxes and benefits. However, the decomposition formulas offer ready available elements to do an additional operation: obtaining decomposition of the redistributive effect, RE, or distance narrowing effect. This is already envisaged by (3.74), (3.89), (3.108) and (3.123), which decompose the difference between Gini coefficients of pre-fiscal and post-TB income. In order to obtain a contribution of tax p or benefit q to distance narrowing, we simply need to combine their contributions to vertical and reranking effect. Considering the conclusion from the previous section, that decomposition of reranking should follow the approach based on the "amounts of taxes and benefits", we ignore the other approach and concentrate on the following decompositions:

$$G_{\hat{X}} - G_N = \left(\sum_{p=1}^{P} V_p^{x,T} + \sum_{q=1}^{Q} V_q^{x,B} \right) - 2\left(\sum_{p=1}^{P} R_p^{x,T} + \sum_{q=1}^{Q} R_q^{x,B} \right)$$

$$G_X - G_{\tilde{N}} = \left(\sum_{p=1}^{P} V_p^{n,T} + \sum_{q=1}^{Q} V_q^{n,B} \right) + 2\left(\sum_{p=1}^{P} R_p^{n,T} + \sum_{q=1}^{Q} R_q^{n,B} \right)$$

The components by which we determine the contribution of tax p and benefit q to distance narrowing are as follows:

$$\Delta_p^{x,T} = V_p^{x,T} - 2R_p^{x,T} \qquad (3.134)\ [\text{from } (3.75) \text{ and } (3.79)]$$

$$\Delta_q^{x,B} = V_q^{x,B} - 2R_q^{x,B} \qquad (3.135)\ [\text{from } (3.77) \text{ and } (3.81)]$$

$$\Delta_p^{n,T} = V_p^{n,T} + 2R_p^{n,T} \qquad \text{(3.136) [from (3.90) and (3.94)]}$$

$$\Delta_q^{n,B} = V_q^{n,B} + 2R_q^{n,B} \qquad \text{(3.137) [from (3.92) and (3.96)]}$$

The contribution of tax p or benefit q to overall distance narrowing is determined as a ratio between the component (defined in (3.134) through (3.137)) and sum of all contributions, analogously to formulas in the right column of Table 3.12 and Table 3.13. It is very interesting that Kakwani and Lerman-Yitzhaki approaches result in the *identical sets of contributions* to distance narrowing! We have the following identities:

$$\frac{\Delta_p^{x,B}}{\sum_p^P \Delta_p^{x,T} + \sum_q^Q \Delta_q^{x,B}} \doteqdot \frac{\Delta_p^{n,B}}{\sum_p^P \Delta_p^{n,T} + \sum_q^Q \Delta_q^{n,B}} \qquad (3.138)$$

$$\frac{\Delta_p^{x,T}}{\sum_p^P \Delta_p^{x,T} + \sum_q^Q \Delta_q^{x,B}} \doteqdot \frac{\Delta_p^{n,T}}{\sum_p^P \Delta_p^{n,T} + \sum_q^Q \Delta_q^{n,B}} \qquad (3.139)$$

3.3.5 Application issues

The decompositions based on distance narrowing, as developed in this chapter, are ready-for-use only when observations have equal weights (for example, in case of a random sample drawn from a population of individual taxpayers). The reason is the following: the basic operation underlying these decompositions is the calculation of the difference in income (tax, benefit) of some units i and j; now, if they have different weights, *what should be the weight of the difference*? Unfortunately, most empirical applications in this field employ household budget survey data and observation units are households, this leading to two kinds of weights: (a) the sampling weights; (b) the weights arising from the use of equivalence scales. Therefore, some adaptations of original data sets are needed before the decompositions are applied.

Each sample household i represents w_i households drawn from the total population of $W = \sum_i w_i$. Household i has m_i equivalent adults; thus, there are $M = \sum_i m_i w_i$ equivalent adults in the population. Here, values w_i are sampling weights defined by sample designers and values m_i are obtained by equivalence scale formulas. The approach is as follows. Let y_i

be the income of household i (the same procedure goes for taxes and benefits). Then, $y_i^e = y_i / m_i$ is the equivalent income of this household. Now, we form a new sample of incomes y_i^e replicating each income $m_i w_i / k$ times, where k serves as a parameter that will decide about the total number of observations in the new sample. Smaller the value of k, the larger the sample: greater precision is achieved, but at the cost of computation time.

For example, the 2006 APK sample consists of 2,790 households and 5,090 equivalent adults if $E_3(.5,.3)$ is used; for given sampling weights we obtain W of about 2.6 million. This size of the sample is prohibitively large, concerning the amount of needed calculations. Therefore, we set $k \approx 100$ and form the sample of about 25,000 observations. However, despite the fact that only 1% of "population" is covered, the 25k samples gave quite close values of the main indicators and comparable decompositions to those obtained from the original sample. Exercises with 50k samples have shown little improvement in precision, at the large cost of additional computation time. The author does not presume that other method is not conceivable; quite the contrary: perhaps relatively little manipulation with the decomposition formulas is needed to avoid the whole issue of unit replication (through inclusion of weights into them). However, for the sake of avoiding further complications in exposition of formulas, and limited time that can be devoted to this work, we leave it to future upgrades.

4 DATA

4.1 Introduction

The research on the redistributive effects of individual taxes and cash benefits in Croatia started in 2006 with acquisition of micro data. Two data sources were available: Tax Administration data on PIT and household budget survey (HBS) data. The former is ruled out since it contains only the data on PIT and surtax, and covers only the population of individual PIT payers. On the other hand, HBS data do not contain data on personal taxes, but include much information upon which it is possible to impute amounts of personal taxes. This required a building of a microsimulation model that transforms net to gross incomes.

This chapter explains the content and origins of datasets used in the research on income redistribution in Croatia. It provides a basis for judgment of the relevance of the results that emerged in subsequent analyses. *Section two* is a brief overview of Croatian system of personal taxes, cash benefits and public pensions, and serves as a background to further developments. The largest *section three* describes Croatian HBS and thoroughly explains the microsimulation model by which the amounts of PIT and SSC are imputed to each sample individual. *Section four* attempts to reveal how well the aggregate figures obtained from these new data sets fare in comparison with official and administrative figures on incomes, taxes and benefits.

The first issue concerning data appropriateness lies in the quality of transformation of net into gross incomes. The description of the microsimulation model in this chapter will show its merits. A larger problem with suitability of data is whether the HBS data properly reflect the true population data. It is a well-known fact that survey data usually under-represent the households at the bottom and at the top of income distribution. Comparison of income distributions based on HBS and Tax Administration data on individual PIT payers confirms the standard view that high incomes are not captured well by the survey data.

4.2 Croatian system of personal taxes and social benefits

4.2.1 Structure of fiscal revenue and expenditure in Croatia

The structure of revenue and expense of general government in Croatia is briefly presented by Table 4.1. Value added tax and excise taxes combined collect about 40% of all revenue. One of the most important revenue items are SSC with contribution of 30%. Income taxes, PIT and CIT, together deliver about one sixth of total revenue. The structure of revenue did not significantly change between 2002 and 2008, except of the growing importance of CIT, and the opposite trend is for excise taxes.

Table 4.1: General government revenue in Croatia

	Billion HRK			*Share (%)*		
	2002	2005	2008	2002	2005	2008
Total revenue	83.5	104.0	136.3	100.0	100.0	100.0
1. Tax revenue	75.9	91.8	120.4	90.8	88.3	88.3
Personal income tax	7.2	7.8	10.8	8.6	7.5	7.9
Corporate income tax	3.7	5.6	10.6	4.4	5.4	7.8
Property taxes	0.6	0.8	1.2	0.7	0.7	0.9
Value added tax	26.0	32.2	41.3	31.1	31.0	30.3
Excise taxes	9.8	10.9	11.9	11.8	10.5	8.7
Taxes on foreign trade and transactions	2.1	1.6	1.9	2.5	1.5	1.4
Social security contributions	25.2	31.3	40.7	30.2	30.1	29.9
Other taxes	1.3	1.6	2.0	1.5	1.6	1.5
2. Property revenue	2.1	3.4	3.1	2.6	3.3	2.3
3. Revenue from sale of goods and services	3.3	4.8	7.6	3.9	4.6	5.6
4. Other revenue	1.6	3.1	3.7	2.0	2.9	2.7
5. Disposal of non-financial assets	0.7	0.9	1.5	0.8	0.8	1.1

Source: *Ministry of Finance, Croatia*

The largest group of expenditures is social transfers to the citizens, representing about 40% of total expenditure, as shown in Table 4.2. The main item in this group is public pensions, followed by other cash transfers and transfers in kind, such as health services; we will search deeper into this category later. One third of expense relates to government production of goods and services, namely wages of employees and use of goods and services. The structure of expense is rather stable over the period.

Table 4.2: General government expense in Croatia

	Billion HRK			Share (%)		
	2002	2005	2008	2002	2005	2008
Total expense	89.2	110.4	139.1	100.0	100.0	100.0
1. Wages of employees	22.2	26.7	33.6	24.9	24.2	24.2
2. Use of goods and services	9.8	10.9	16.5	11.0	9.8	11.9
3. Interest	3.8	5.1	5.0	4.2	4.6	3.6
4. Subsidies to state and private enterprises	4.1	6.0	8.1	4.6	5.4	5.8
5. Transfers to other levels of government	0.5	1.4	2.3	0.6	1.3	1.7
6. Social transfers	36.1	42.5	53.3	40.5	38.5	38.3
7. Other expense	4.8	8.0	11.3	5.4	7.2	8.1
8. Acquisition of non-financial assets	7.9	9.9	8.9	8.9	9.0	6.4

Source: *Ministry of Finance, Croatia*

4.2.2 Taxes

4.2.2.1 Social security contributions

As we have seen in Table 4.1, social security contributions are a major source of government revenue. They are used to cover the costs of pension, health and unemployment protection systems, with each of these systems having its own designated revenue source. SSC for pension system are paid by employees as a percentage of their gross wage. The legal incidence of SSC for health and SSC for unemployment protection system lies on employers, and they are also obtained as percentages of gross wage. Self-employed people pay contributions to the pension and health fund, but their obligation is determined in lump-sum terms. Contractual workers pay contributions as a percentage of their income. More details on SSC can be found in section 4.3.6.

4.2.2.2 Personal income tax

PIT is an important source of revenue in Croatia, although not as central to the fiscal system as in some other countries. Income taxable by PIT is a sum of individual's incomes from different sources: wages and salaries, pensions, income from self-employment and contractual work, rental activities and royalties. Tax base is equal to total income less personal allowances and deductions. General tax schedule with four rates is applied to tax base in order to obtain tax obligation. Many more details on PIT are left to sections 4.3.3 and 4.3.4.

Two further features of PIT are important. First, income from capital is not included into taxable income unless it is part of business income of the self-employed people. Second, for several income sources 'cedular' taxation is enabled, meaning that not the whole income faces the general tax schedule, but a range of single rate schedules with rates depending on the source of income.

PIT in Croatia is accompanied by surtax, which is obtained as a product of PIT obligation and the surtax rate. Surtax is entirely a revenue of local units, which themselves proscribe the surtax rate. Currently, the rates range from 0% to 18%.

4.2.3 Social transfers

4.2.3.1 Overview of the social benefit system

Croatia lacks some kind of a systematized overview or registry of social benefits paid by the central government budget. This overview is based on the data and descriptions available in the executive's budget proposal for 2007, and also on the laws governing the disbursal of benefits. There are more than 70 different social benefits at the level of the central government, but here we will concentrate on the major ones, while only mentioning those less relevant, using the budget amounts as a criterion for differentiation. Also, we will neglect social benefits disbursed at lower levels of government.

Social benefits are categorized into five groups.

(A) Basic support. These benefits should provide subsistence level of living standard to all those in need. They are mostly disbursements of money going directly into the pockets of beneficiaries. Prime instrument in this group is the "Subsistence income allowance".
(B) War related support. It provides basic and supplemental cash income to Croatian Homeland War Soldiers (HWS) with disabilities and to members of family of HWS killed in the Homeland War (hence HWSF denotes both *s*oldiers and their *f*amily members).
(C) Unemployment and sick-leave benefits. These are standard benefits based on insurance from unemployment and health insurance.

(D) Family policies. These are money transfers to families with children, designed to provide an incentive to increase fertility. The main instrument here is the "Child allowance".

An overview of the most important benefits is presented in Table 4.3. For each benefit, a short description is given, together with the amount expended in 2005 budget.

Table 4.3: Most important cash social benefits in Croatia, 2005

	Benefit	**Amount**	**%**
(A) Basic support			
1	Subsistence income allowance (SIA) For people whose overall income is smaller than the "basic" amount that depends on the number of children, persons with disabilities, etc.; greater for single-parent families. A beneficiary obtains a difference between the basic and actual income.	499	7.6
2	One-time money allowance Paid in special situations (illness, death, child birth, etc.) if a person or family cannot satisfy their basic needs.	58	0.9
3	Personal invalidity allowance For persons with physical or mental illness, or health problems, if they do not obtain support or income from other sources. The amount obtained similarly as for SIA, but the base is higher.	169	2.6
4	Allowance for help and care For persons with physical or mental illness, or health problems, if they need constant help of other people, and if they do not obtain support or income from other sources.	307	4.7
5	Other social benefits Various compensations for costs made on food, clothing, etc.		
(B) War related support			
1	Support for military and civil people with disabilities A package of benefit instruments providing basic income support to persons with disabilities: military personnel and civil victims of war (during the IIWW, peace time and HW). These benefits are similar in design to A1 to A4.	386	5.9
2	Support for HW veterans with disabilities A package of benefit instruments providing income support for HWV with disabilities and families of killed and missing HW soldiers, and HW prisoners. These benefits are similar in design to A1 to A4.	774	11.7
(C) Unemployment and sick-leave assistance			
1	Unemployment benefit Paid to unemployed persons during the first 3 to 15 months of unemployment, depending on their employment record.	889	13.5
2	Sick-leave benefit Paid to the persons on sick leave for longer than 42 days.	1,000	15.2
3	Benefits for HW veterans	58	0.9
4	Other payments to the health insured	189	2.9
(D) Family policies			
1	Child allowance Intended for families with children and income lower than certain threshold. Total amount of benefit depends on income and the number of children;	1,435	21.8

	Benefit	Amount	%
	additional amount for children with health problems.		
2	Maternity allowance and layette supplement Maternity allowance is income supplement for a parent with a newly-born child. It lasts up to 1 year for the 1st and the 2nd child and up to 3 years for the 3rd (4th, etc.) child or in case of twins. Available to employed, self-employed and unemployed mothers. Layette supplement is one-time compensation for a family with a newly-born child and health insured parent.	718	10.9
3	Allowance for parental care For a parent of a child with physical or mental illness, or health problems, aged up to 7, who leaves full-time work completely or works half-time. The benefit covers the loss of earnings because not working full-time, up to certain amount.	114	1.7
Total		**6,596**	**100.0**

Notes: amounts in millions HRK; IIWW = World War II; HW = Homeland War; HWV = HW veterans; HWVD = HWV with disability

Source: Executive's budget proposal for 2007, Ministry of Finance, Croatia

4.2.3.2 Public pensions

Public pensions are the major cash social transfer in Croatia, with total amount about four times higher than all other cash social benefits mentioned in the previous section. We have divided them into three groups: *(A) Insurance based, (B) Military and police servants, and (C) Other*. Each group consists of several types of pensions, which is indicated in Table 4.4, together with their 2005 figures.

Table 4.4: Public pensions in Croatia, 2005

		Amount	%
(A) Insurance based			
1	Old-age	11,924	46.0
2	Invalidity	4,238	16.4
3	Family	3,167	12.2
4	Supplement to pensions	1,487	5.7
(B) Military and police servants			
1	Invalidity and family pensions of HWV and HWVD	2,943	11.4
2	Croatian army servants	355	1.4
3	Ministry of interior	328	1.3
4	ex-JNA servants	396	1.5
5	IIWW freedom fighters	666	2.6
6	IIWW homeland fighters	168	0.6
(C) Other			
1	Members of parliament	39	0.2
2	Public officials and servants from ex-Yugoslavia	17	0.1

3	Political ex-prisoners	121	0.5
4	Croatian academy of sciences and arts	13	0.1
5	Other	32	0.1
Total		**25,894**	**100.0**

Notes: amounts in millions HRK; IIWW = World War II; HW = Homeland War; HWV = HW veterans; HWVD = HWV with disabilities

Source: *Executive's budget proposal for 2007, Ministry of Finance, Croatia*

4.3 Original data and tax microsimulation model

4.3.1 Household budget survey data

As noted in the introduction, the empirical part of the research started with the acquisition of microdata from the national household budget survey, called Anketa o potrošnji kućanstava (APK). APK is designed and collected by Croatian statistical office (Državni zavod za statistiku, DZS). The survey has been conducted since 1998. However, DZS's recommendation was to start with a year 2001, because older surveys are not fully compatible with the new ones due to sampling problems. Thus, six yearly samples, relating to the single years from 2001 to 2006 are taken into account. APK contains the relevant data on incomes at individual level, on consumption at household level, and many other indicators, for a sample of Croatian households. The size of the sample gradually declined from 9.460 persons in 2001 to 7.730 in 2005 and 7.860 in 2006.

The data on incomes are registered net of PIT and SSC. Therefore, in order to make the analysis of these two instruments possible, we had to develop a microsimulation model that applies tax code to the data and transforms net incomes into gross incomes, identifying the amounts of PIT and SSC for each individual. This model uses all the data on individuals and their household members available in APK: working status, number of children and dependent spouses; place of living; net incomes by source (wages, pensions, self-employment income, capital income, rents, etc.); outlays on items such as mortgage interest rate, life insurance (needed for calculation of PIT deductions). A detailed presentation of the model is offered in the next section. The data on social transfers are already available in APK.

When the research project started, APK was the best choice for given purpose. A superior choice would be a database compiled by merging datasets from various *official* sources (for example, population survey, tax administration, pension fund, relevant welfare state ministries and agencies, etc.). Such database has been created for Slovenia (see Čok et al, 2008). In section four, we will reveal some comparisons between APK and official data aggregates, in order to judge how appropriate the APK data are for the subsequent analysis.

4.3.2 A sketch of the microsimulation model

The rest of this section is fully devoted to explanations of the microsimulation model. The presentation interchangeably discusses three groups of facts: on APK, on determinants of PIT and SSC systems in Croatia, and on the model itself. A sketch of the whole model is made, so it will be easier to follow the rest of presentation. As can be seen in Figure 4.1, a process may be divided into four steps, each of them containing several procedures.

The four steps are respectively covered in sections 4.3.3, 4.3.4, 4.3.5 and 4.3.6.

Figure 4.1: Tax microsimulation model in four steps

Step	Procedures
1	(a) Identify all income variables in APK (b) Classify incomes according to tax treatment into taxable and non-taxable (c) Obtain total net incomes by income sources (d) Obtain total net *taxable* incomes by income sources ↓
2	(a) Obtain amounts of allowances (b) Obtain amounts of deductions ↓
3	(a) Use algorithm to transform net taxable incomes into gross incomes (b) Obtain amounts of PIT as residuals ↓
4	(a) Obtain gross wages and other SSC tax bases (b) Calculate amounts of various SSC

4.3.3 Incomes and taxable incomes

4.3.3.1 Overview

Table 4.5 lists all the income variables from APK, divided into seven groups, and explains the treatment of each income source by PIT Law – whether this kind of income is taxed or not. Some incomes are taxable by PIT but not at a full amount. These types of income are shown in Table 4.6, accompanied by the yearly amounts that are not taxable. Thus, for example, if person X's compensation for retirement in 2006 was 12,000 HRK (1,640 EUR), only 4,000 HRK (550 EUR) was taxable, while the rest was not. All these facts will be used in calculation of total taxable income from wages and salaries, as will be seen in the following paragraphs.

Table 4.5: Classification of personal incomes and their PIT treatment

Item	Status in PIT Law	APK variable
1 Wages and salaries		
1.1 Basic wages and salaries	FA	p25
1.2 Allowance for vacation	AAT	p26
1.3 Food stamps	FA	p27
1.4 Transportation allowance	*ni*	p28
1.5 Allowance for living separately	AAT	p29
1.6 Additional money receipts	FA	p32
1.7 Additional receipts in kind	FA	p34
1.8 Jubilee and other awards	AAT	p31
1.9 Compensation for retirement	AAT	p30
1.10 Compensation in case of injury at work or professional illness	AAT	
1.11 Compensation in case of dismissal	AAT	
1.12 Net wage from abroad	*ni*	p33
2. Income from part-time and contractual work		
2.1 Income from contractual work	FA	p37
2.2 Income from student's work	*ni*	p38
2.3 Income from work done after "direct negotiation"	*ni*	p39; p40
3. Income from self-employment		
3.1 Income of personal enterprise, made from selling at the market		
3.1.1 Non-agriculture	FA	p35
3.1.2 Agriculture	FA	p80
3.2 Income of personal enterprise, equal to consumption for own use		
3.2.1 Non-agriculture	*ni*	p36
3.2.2 Agriculture	*ni*	p81; p82
4. Income from capital and property rights		
4.1 Rental income		
4.1.1 Apartments, houses and rooms	FA	p76
4.1.2 Business premises	FA	p77
4.1.3 Land	FA	p78
4.1.4 Movable property	FA	p79
4.2 Income from royalties	FA	p73
4.3a. Dividends (2001-2004)	FA	p72
4.3b. Dividends (2005-2006)	*ni*	p72
4.4 Interest		
4.4.1 Saving deposits	*ni*	p74
4.4.2 Bonds and other securities	*ni*	p75
5. Pensions		
5.1 Public pensions – old-age	FA	p50
5.2 Public pensions – family	FA	p52
5.3 Public pensions – invalidity	FA	p53
5.4 Pensions from abroad – old-age	*ni*	p51

Item	Status in PIT Law	APK variable
5.5 Pensions from abroad – family	*ni*	-
5.6 Pensions from abroad – invalidity	*ni*	p54
6. Social benefits		
6.1 Basic support allowances	*ni*	p48
6.2 Child allowance	*ni*	p41
6.3 Unemployment benefit	*ni*	p55
6.4 Supplement for the injured	*ni*	p47
6.5 Maternity allowance	*ni*	p42
6.6 Layette supplement	*ni*	p43
6.7 Sick-leave benefit	*ni*	p46
6.8 Support for rehabilitation and employment of people with disabilities	*ni*	p49
6.9 Housing allowance	*ni*	p45
7. Other transfers		
7.1 Skills improvement aid	*ni*	p56
7.2 Education aid	*ni*	p57
7.3 Awards for successful learning	*ni*	p58
7.4 Alimonies	*ni*	p44
7.5 Gifts in money and in kind	*ni*	p62; p63

Notes: FA = at full amount; AAT = at amount above the threshold; see Table 4.6; *ni* = not included
[a] Variable p52 contains both domestic and foreign family pensions. As we are unable to separate a domestic component from the foreign one, we assume that full amount of the variable belongs to the item 5.2: "public pensions – family".

Table 4.6: Thresholds above which the receipt enters taxable income, in thousands HRK (EUR)

		2001	2002	2003	2004	2005	2006
Allowance for vacation	yearly	1 (0.1)	1 (0.1)	1 (0.1)	1 (0.1)	2 (0.3)	2 (0.3)
Allowance for living separately	yearly	18 (2.5)	18 (2.5)	18 (2.5)	18 (2.5)	19.2 (2.6)	19.2 (2.6)
Compensation for retirement	1-time	7 (1.0)	8 (1.1)	8 (1.1)	8 (1.1)	8 (1.1)	8 (1.1)
Compensation in case of dismissal [a]	1-time	* [c]	5 (0.7)	6 (0.8)	6 (0.8)	6.4 (0.9)	6.4 (0.9)
Compensation in case of injury at work or professional illness [a]	1-time	* [c]	6.3 (0.9)	7.5 (1.0)	7.5 (1.0)	8 (1.1)	8 (1.1)
Jubilee and other awards [b]	1-time	1-3 (0.1-0.4)	1.5-5 (0.2-0.7)	1.5-5 (0.2-0.7)	1.5-5 (0.2-0.7)	1.5-5 (0.2-0.7)	1.5-5 (0.2-0.7)

Notes: [a] For each full year of work for the relevant employer; [b] Depending on total number of years at work; [c] Average monthly wage or salary paid to employee for the last 3 months at work; the amounts in brackets are in EUR. All conversions are made according to the exchange rate: 1 EUR=7.3 HRK.

4.3.3.2 Total net incomes by sources of income

In this section, we explain how the income sub-totals are obtained for different sources of income. We will make difference between *total net income* and *net taxable income* for each group A to E. For groups F and G, we do not calculate the sub-totals, while for H, only *total net income* is obtained.

A. Wages and salaries

Total net income from wages and salaries is equal to the sum of "basic" wages and salaries (item 1.1 in Table 4.5; henceforth "i-*1.1*") and various other receipts obtained by employees (i-*1.2* to i-*1.11*), plus the wages and salaries from abroad (i-*1.12*).

Net taxable income from wages and salaries is equal to "basic" wages and salaries (i-*1.1*) and those from abroad (i-*1.12*), food stamps (i-*1.3*), additional receipts in money (i-*1.6*) and in kind (i-*1.7*), *plus* the incomes that enter taxable income only in amounts above the thresholds from Table 4.6.

For six incomes – allowance for vacation (i-*1.2*), Allowance for living separately (i-*1.5*), jubilee and other awards (i-*1.8*), compensation for retirement (i-*1.9*), compensation in case of injury at work or professional illness (i-*1.10*) and Compensation in case of dismissal (i-*1.11*) – we may not use full original amounts, but have to create new variables. These variables take only the cut-offs above the threshold.

B. Pensions

For convenience, pensions from abroad (i-*5.4* to i-*5.6*) are included into "income from capital and property rights" (see below). Private pension funds in Croatia were established several years ago and it is too early for them to make any proportion of total pensions. Henceforth, under the term "pensions" we assume public pensions, i.e. those paid by the state pension fund.

Total net income from pensions covers three kinds of pensions: old-age (i-*5.1*), family (i-*5.2*) and invalidity (i-*5.3*). We assume that all public pensions are taxable by PIT[16] and therefore the *net taxable income* from pensions is identical to the total income from pensions.

C. Self-employment

Net taxable income from self-employment is identical to *total net income* from self-employment. It consists of market income made by the small private enterprise (i-*3.1*). The "small private enterprise" may be involved in different kinds of production: services, manufacturing, transportation and agriculture. A part of income that is consumed for own use (i-*3.2*) is not included here, but into a specially defined category; see below.

D. Part-time and contractual work

Total net income from part-time and contractual work consists of income from contractual work (i-*2.1*), student's work (i-*2.2*) and work done after "direct negotiation" (i-*2.3*).

The model assumes that only the main item, income from contractual work (i-*2.1*), enters taxable income. Income from student's work (i-*2.2*) and income from work done after "direct negotiation" (i-*2.3*) are ignored; the former one because of high threshold of 6,800 EUR, and the latter due to the assumption that this kind of work usually escapes taxation. Thus, the *net taxable income* from part-time and contractual work is equal to income from contractual work (i-*2.1*).

E. Capital and property rights

This category includes various kinds of income based on property or capital ownership. *Total net income* from capital and property rights includes rental income (i-*4.1*), income from royalties (i-*4.2*), dividends (i-*4.3*), interest (i-*4.4*) and pensions from abroad (i-*5.5* and i-*5.6*).

Net taxable income from capital and property rights covers only rental income (i-*4.1*) and income from royalties (i-*4.2*).[17]

[16] The exceptions are the 'family pensions of HWV and HWVD' (see Table 4.4). However, in APK we cannot distinguish them from other pensions.

F. Social benefits

The data cover eight social benefits which are conveniently grouped into: basic support allowances (i-*6.1*, i-*6.9*), child allowance (i-*6.2*), unemployment benefit (i-*6.3*), supplement for the injured and support for rehabilitation and employment of persons with disabilities (i-*6.4*, i-*6.8*), maternity allowance and layette supplement (i-*6.5*, i-*6.6*), and sick-leave benefit (i-*6.7*).

G. Non-government transfers

This category includes transfers from donors other than the government. Total income from non-government transfers is a sum of alimonies (i-*7.4*), housing allowance (i-*7.5*), skill improvement aid (i-*7.1*), education aid (i-*7.2*), and gifts in money and in kind (i-*7.6*).

H. Production for own use

As already mentioned, the part of self-employment income consumed by the producing household, represents a special category. *Total net income* from production for own use includes non-agricultural (i-*3.2.1*) and agricultural (i-*3.2.2*) part. Although this income is taxable, we treat it as non-taxable in the model under assumption that in reality it mostly escapes taxation.

4.3.4 Allowances and deductions

4.3.4.1 Personal allowance

Personal allowance is the main tax-base-reducing element in the Croatian PIT system. For analytical purposes, we may say that it consists of two elements:

(a) *Basic personal allowance* (BPA), which may be general and specific. Taxpayers who are not pensioners, and live outside the particular areas of the country, obtain *general basic*

[17] This is not in full accordance with PIT system. From 2001 to 2004, dividends were taxed ‘cedularly’, at a rate of 15%. However, the share of dividends in total income (for APK data) is only 0.1%. Simulating taxation of dividends would only complicate the overall model, with insignificant improvements in the final result.

personal allowance (GBPA). If they are pensioners, or living in particular areas, they obtain *specific basic personal allowance* (SBPA), which is larger than the GBPA.[18]
(b) *Additional personal allowance* (APA). It is received by taxpayers with children and other dependants.

Yearly amounts of GBPA are shown in Table 4.7. Amounts of SBPA are calculated as products of GBPA and a factor that depends on the group a taxpayer belongs to. These factors are shown in Table 4.8. Amount of APA is determined in a similar way, as a product of GBPA and a factor that depends on the number of children and dependants. These factors are presented in Table 4.9. For those living in particular areas, APA is a product of this factor and SBPA, not GBPA.

If both spouses are taxpayers, it must be decided which one will use additional personal allowance on account of children and adult dependants. The obvious choice is a member with larger income.

Table 4.7: General basic personal allowance, in thousands HRK (EUR)

	2001	**2002**	**2003**	**2004**	**2005 / 2006**
Yearly amount	15 (2.1)	15 (2.1)	18 (2.5)	18 (2.5)	19.2 (2.6)

Note: The amounts in brackets are in EUR.

Table 4.8: Factors of specific basic personal allowance

	2001	**2002**	**2003**	**2004**	**2005 / 2006**
Pensioners	2.0	2.0	1.7	1.7	1.875
ASGC 1	3.0	3.0	2.5	2.5	2.4
ASGC 2	2.5	2.5	2.0	2.0	2.0
ASGC 3	2.0	2.0	1.5	1.5	1.5
MHA	1.0	1.0	1.0	1.0	1.5

Notes: ASGC – Areas of special government concern, three different groups are defined; MHA – Mountain and hill areas

[18] These areas are the „Areas of special government concern” which were especially damaged during the Homeland War in the 1990s. According to intensity of war damage they are divided into three groups. Another sort of areas is “Mountain and hill areas”.

Table 4.9: Factors of additional personal allowance

	2001	2002	2003	2004	2005 / 2006
Dependant [a]	0.5	0.5	0.40	0.40	0.5
Children					
1	0.5	0.5	0.42	0.42	0.5
2	1.2	1.2	1.01	1.01	1.2
3	2.2	2.2	1.85	1.85	2.2
4	3.6	3.6	3.02	3.02	3.6
5	5.5	5.5	4.61	4.61	5.5
6	8.0	8.0	6.70	6.70	8.0
7	11.2	11.2	9.37	9.37	11.2
8	15.2	15.2	12.71	12.71	15.2
9	20.1	20.1	16.80	16.80	20.1
10	26.0	26.0	21.80	21.80	26.0
11	33.0	33.0	27.80	27.80	33.0

Note: [a] For each adult dependant

4.3.4.2 Deductions

Since 2001 PIT in Croatia has been introduced with more than a dozen deductions. Some of them are intended for self-employed people only, while there are several deductions for all taxpayers. Since the former cannot be modelled, due to lack of data, the model deals with the latter group. We have the following five deductions, as shown in Table 4.10.

Table 4.10: Deductions

Symbol	Item
D_1	Premiums for voluntary pension insurance
D_2	Premiums for additional health insurance
D_3	Premiums for life insurance characterized as savings
D_4	Expenditures for health services
D_5	Costs incurred to meet housing needs – mortgage interest rate

Deductions are limited for each taxpayer to the amounts given in Table 4.11. From 2001 to 2002, only first three deductions were available and their total amount was limited to 12,000 HRK (1,640 EUR) per taxpayer. In 2003 and 2004 two new deductions were introduced, and the combined amount of all five deductions could reach 36,600 HRK (5,000 EUR). In 2005 this amount was restricted to 12,000 HRK (1,640 EUR). The data about deductions are derived from APK data on consumption and personal investment.

Table 4.11: Maximum yearly amount per taxpayer, in thousands HRK (EUR)

	2001	2002	2003	2004	2005 / 2006
D_1, D_2 and D_3	12 (1.6)	12 (1.6)	12.6 (1.7)	12.6 (1.7)	12 (1.6)
D_4	x	x	12 (1.6)	12 (1.6)	
D_5	x	x	12 (1.6)	12 (1.6)	

Note: The amounts in brackets are in EUR.

4.3.5 Tax base, rate schedule and amounts of tax

4.3.5.1 Basic and intermediate variables

In the previous three sections we have described the variables which the microsimulation model utilizes to calculate the final targets, amounts of PIT and consequently, amounts of pre-tax incomes. In this section, we show how this is achieved. In APK questionnaire, the respondents report their net incomes. We assume that all incomes categorized as taxable were indeed taxed by PIT.

First, we have to calculate the *total net taxable income* (N_i^T) for each taxpayer *i*, as a sum of net taxable incomes (NTI) from all sources.

$$N_i^T = NTI(wages\ and\ salaries) + NTI(pensions) + \\ + NTI(self\text{-}employment) + NTI(contractual\ work) + \\ + NTI(rental\ income) \quad (4.1)$$

Next, we determine the maximum amount of allowances and deductions a taxpayer may achieve (A_i).[19] It is a sum of basic personal allowance (BPA; which may be general or specific, as we have seen above), additional personal allowance and five deductions, as presented in (4.2). For calculation of each element, special procedures are developed using various data on household structure, area of living, consumption, etc.

[19] Note that this amount may be higher than taxpayer's income. In this case, the taxpayer will effectively use only the amount of allowances and deductions equal to her income.

$$A_i = BPA + APA + D_1 + D_2 + D_3 + D_4 + D_5 \quad (4.2)$$

Prior to the previous step, we have to determine a member with the largest total net taxable income for each household, so that we can determine which member will use the right on additional personal allowance for children and adult dependants.

In reality, calculation of tax amount goes from pre-tax (gross) to post-tax (net) income. Allowances and deductions are deducted from gross income to obtain a tax base which is then divided into several parts (in Croatia: three parts in 2001-2002; four parts in 2003-2006) using predetermined thresholds, and each part is multiplied by corresponding marginal rate. The sum of these products makes the amount of PIT. Surtax is obtained by multiplying PIT with the surtax rate.

The thresholds are denoted with P_1, P_2 and P_3, and the parameters over the period are shown in Table 4.12. The marginal rates are marked with m_1, m_2, m_3, m_4, and their values can be found in Table 4.14. For example in 2004, the first 36,000 HRK (5,000 EUR) of tax base is multiplied by the marginal rate of 15%; the part of tax base between 36,000 HRK (5,000 EUR) and 81,000 HRK (11,100 EUR) is multiplied by 25%; the next part by 35%; finally, the part of tax base above 252,000 HRK (34,500 EUR) is multiplied by 45%.

Table 4.12: Variables for calculation of PIT using the general tax schedule

N_i^T	total net taxable income of taxpayer i
A_i	upper limit of allowances and deductions of taxpayer i
P_1, P_2, P_3	thresholds
B_i	tax base of taxpayer i
m_1, m_2, m_3, m_4	marginal PIT rates
r_i	surtax rate of taxpayer i
T_i^T	imputed tax and surtax of taxpayer i
X_i^T	imputed pre-PIT income of taxpayer i

Table 4.13: Thresholds, yearly amounts, in thousands HRK (EUR)

	2001	2002	2003	2004	2005 / 2006
P_1	30 (4.1)	30 (4.1)	36 (4.9)	36 (4.9)	38.4 (5.3)
P_2	75 (10.3)	75 (10.3)	81 (11.1)	81 (11.1)	96 (13.2)
P_3	∞	∞	252 (34.5)	252 (34.5)	268.8 (36.8)

Note: The amounts in brackets are in EUR.

Table 4.14: Marginal PIT rates

	2001	2002	2003	2004	2005 / 2006
m_1	0.15	0.15	0.15	0.15	0.15
m_2	0.25	0.25	0.25	0.25	0.25
m_3	0.35	0.35	0.35	0.35	0.35
m_4	x	x	0.45	0.45	0.45

4.3.5.2 Algorithm for calculation of tax amounts

The model presented here is made precisely because we *do not know* the amounts of gross incomes. However, we do now, as explained above, the amounts of total net taxable income (N_i^T) and total allowances and deductions (A_i). A procedure had to be developed that calculates hypothetic tax values based on values of net incomes and allowances. We now turn to this procedure, but, before the algorithm itself, we introduce several auxiliary variables.

Note that in the presence of surtax, the marginal PIT rates are larger than the statutory ones. The surtax rate depends on the local unit where a taxpayer lives. Therefore, it is different for each person *i*. To obtain after-surtax marginal rates, the calculations in (4.3) through (4.6) are done.

$$t_{1,i} = (1 + r_i) \times m_1 \qquad (4.3)$$

$$t_{2,i} = (1 + r_i) \times m_2 \qquad (4.4)$$

$$t_{3,i} = (1 + r_i) \times m_3 \qquad (4.5)$$

$$t_{4,i} = (1 + r_i) \times m_4 \qquad (4.6)$$

For convenience, it is helpful to obtain some transformations.

$$T_{1,i} = t_{1,i} \times P_1 \tag{4.7}$$

$$T_{2,i} = t_{2,i} \times (P_2 - P_1) \tag{4.8}$$

$$T_{3,i} = t_{3,i} \times (P_3 - P_2) \tag{4.9}$$

$$N_{1,i} = A_i + P_1 - T_{1,i} \tag{4.10}$$

$$N_{2,i} = A_i + P_2 - (T_{1,i} + T_{2,i}) \tag{4.11}$$

$$N_{3,i} = A_i + P_3 - (T_{1,i} + T_{2,i} + T_{3,i}) \tag{4.12}$$

And now, to the algorithm. Post-tax income of a person i is compared to certain pre-determined values to decide which *one* of the five conditions listed in (4.13) is satisfied. Based on the decision, imputed PIT amount (T_i^T) is obtained. Now, it is easy to obtain the amount of imputed pre-tax income (X_i^T), as shown in (4.14).

$$N_i^T \le A_i \qquad \Rightarrow \qquad T_i^T = 0 \tag{4.13a}$$

$$A_i < N_i^T \le N_{1,i} \qquad \Rightarrow \qquad T_i^T = \frac{t_{1,i}}{1 - t_{1,i}}(N_i^T - A_i) \tag{4.13b}$$

$$N_{1,i} < N_i^T \le N_{2,i} \qquad \Rightarrow \qquad T_i^T = T_{1,i} + \frac{t_{2,i}}{1 - t_{2,i}}(N_i^T - N_{1,i}) \tag{4.13c}$$

$$N_{2,i} < N_i^T \le N_{3,i} \qquad \Rightarrow \qquad T_i^T = T_{1,i} + T_{2,i} + \frac{t_{3,i}}{1 - t_{3,i}}(N_i^T - N_{2,i}) \tag{4.13d}$$

$$N_i^T > N_{3,i} \qquad \Rightarrow \qquad T_i^T = T_{1,i} + T_{2,i} + T_{3,i} + \frac{t_{4,i}}{1 - t_{4,i}}(N_i^T - N_{3,i}) \tag{4.13e}$$

$$X_i^T = N_i^T + T_i^T \tag{4.14}$$

4.3.6 Calculation of SSC amounts

Modelling social security contributions is much easier because SSC are single rate taxes applied to the same tax base, and there are no allowances and deductions. They are applied on several income sources, or in other words, paid by several groups of income earners: (a) workers who earn wages and salaries, (b) firms employing these workers, (c) people earning income from contractual work, (d) employers with the latter and (e) self-employed entrepreneurs and professionals.[20] For each group, the tax base is defined differently.

For employees and their employers the tax base is gross wage. As can be seen in Table 4.15 employers pay three kinds of SSCs, each calculated as a percentage of a gross wage: SSC for the unemployment protection system, primary SSC for the health insurance and special SSC for the health insurance. Employees pay SSC for the pension insurance, in two separate pillars, also as a percentage of a gross wage.[21]

SSC tax base for the self-employed is a lump sum, i.e. it is independent of a taxpayer's income. Three different groups of the self-employed are recognized: (a) entrepreneurs running business accounts, (b) professionals and (c) producers in agriculture. Their tax bases are calculated as products of the average national gross wage in the preceding year (ANGW) and a coefficient (k), which varies according to the group of the self-employed. Thus, for groups (a), (b) and (c), the value of the coefficient is equal to 0.65, 1.1 and 0.4, respectively. The rates are shown in Table 4.15.

For income from contractual work, the tax base for SSC is a gross revenue (gross receipt), which is equivalent to gross wage of the worker. Similarly as employee, contractual worker pays SSC to the pension system, while employer pays SSC to primary health insurance, using the rates presented in Table 4.15.

Table 4.15: Tax base and rates for SSC

Taxpayer	**Tax base**	**Tax rate (%)**				
		Pension system– 1st pillar	**Pension system– 2nd pillar**	**Health insurance**		**Unemploy-ment insurance**
				Primary	**Special**	
Employer of EE	gross wage	-	-	15	0.5	1.7
Employee	gross wage	15	5	-	-	-
Employer of CW	gross receipt	-	-	15	-	-
Contractual worker	gross receipt	15	5	-	-	-
Self-employed	ANGW * *k*	15	5	15	0.5	-

[20] In fact, this is a simplified view of the SSC system in Croatia. Besides these groups, the law defines a myriad of other groups of SSC payers, each with its own tax bases and rates. However, the bulk of SSC revenue arrives from the five groups mentioned above. Also, they are easily recognizable and possible to model.
[21] When employee's SSC are deducted from gross wage, we obtain pre-PIT income from wages. Thus, SSC are not taxed by PIT.

Notes (Table 4.15): ANGW = average national gross wage in the preceding year; EE = employee; CW = contractual worker, k = coefficient (0.65 for entrepreneurs running business accounts; 1.1 for professionals and 0.4 for producers in agriculture)

Table 4.16: Variables for calculation of SSCs

X_i^T	imputed pre-PIT income
X_i^W	imputed pre-PIT income from wages and salaries
X_i^{GW}	imputed gross wage
X_i^C	imputed pre-PIT income from contractual work
X_i^{GC}	imputed gross income from contractual work
s^{P1}	tax rate: SSC for pension system – 1st pillar
s^{P2}	tax rate: SSC for pension system – 2nd pillar
s^{H1}	tax rate: SSC for health insurance – primary
s^{H2}	tax rate: SSC for health insurance – special
s^U	tax rate: SSC for unemployment insurance
B_i^{EE}	tax base: SSCs paid by employers and employees
B_i^{SE}	tax base: SSCs paid by self-employed
B_i^C	tax base: SSCs paid for income from contractual work
T_i^{P1}	imputed SSC for pension system – the 1st pillar
T_i^{P2}	imputed SSC for pension system – the 2nd pillar
T_i^H	imputed SSC for health insurance – primary and special
T_i^U	imputed SSC for unemployment insurance
X^{ANGW}	average national gross wage in the preceding year
k	coefficient

Table 4.16 presents the variables related to SSC. Pre-PIT incomes from wages and salaries, X_i^W, and contractual work, X_i^C, were obtained multiplying total imputed pre-PIT income, X_i^T, with the shares of respective income sources in total net taxable income (these equations are not shown here). Then, gross incomes from wages and contractual work are calculated as in (4.15) and (4.17). Tax bases for SSC on these sources of income are equal to these gross incomes, as seen in (4.16) and (4.18). For self-employed people, tax base is obtained somewhat differently, as in (4.19), as already explained above.

$$X_i^{GW} = X_i^W /(1 - s^{P1} - s^{P2}) \quad (4.15)$$

$$B_i^{EE} = X_i^{GW} \quad (4.16)$$

$$X_i^{GC} = X_i^C /(1 - s^{P1} - s^{P2}) \quad (4.17)$$

$$B_i^C = X_i^{GC} \tag{4.18}$$

$$B_i^{SE} = k \times X^{ANGW} \tag{4.19}$$

$$T_i^{P1} = s^{P1} \times (B_i^{EE} + B_i^{SE} + B_i^C) \tag{4.20}$$

$$T_i^{P2} = s^{P2} \times (B_i^{EE} + B_i^{SE} + B_i^C) \tag{4.21}$$

$$T_i^H = s^{H1} \times (B_i^{EE} + B_i^{SE} + B_i^C) + s^{H2} \times (B_i^{EE} + B_i^{SE}) \tag{4.22}$$

$$T_i^U = s^U \times B_i^{EE} \tag{4.23}$$

Finally, amounts of different SSC are obtained as products of the corresponding tax rates and tax bases, as shown in (4.20) through (4.23).

4.4 Relevance of the data

4.4.1 Taxable income and taxes

The empirical study whose results will be presented in the next chapter, analyses social security contributions and personal income tax. Since APK does not collect data on these taxes, the amounts were imputed using the microsimulation model described above. Assuming that this model is correct, the representativeness of the data will depend on the quality of original data. We decided to compare the data on taxable incomes and PIT from two sources: APK and official Tax Administration data.

Tax Administration data come from another study of the redistributive effects (Urban 2006), and relate to large 5% samples of Croatian taxpayers. The most recent available database is from 2005, and these data are compared with 2005 APK data. Since both sets contain data on individuals, it is possible to compare both the aggregate values of incomes and taxes, and look more deeply into their distributions.

Table 4.17 contains the comparison of aggregate values for pre-PIT and post-PIT income and PIT, and the average tax rates. The columns respectively show values obtained for Tax Administration databases, APK dataset[22] and the ratio between the former two values. This

[22] APK data were aggregated using the household sampling weights.

ratio is calculated to illustrate which part of taxable income and PIT is covered by APK. Pensioners are separated from other taxpayers to identify some peculiarities.

Table 4.17: Comparison of official and survey based aggregate figures: PIT 2005

	Tax Administration	**APK**	**Ratio (%)**
	1	2	3(=2/1)
Non-pensioners			
Post-PIT income	67.7	57.8	85
PIT and surtax	8.8	5.7	64
Pre-PIT income	76.5	63.5	83
Average tax rate (%)	11.5	9.0	
Pensioners			
Post-PIT income	24.4	26.0	107
PIT and surtax	0.5	0.5	97
Pre-PIT income	24.9	26.5	106
Average tax rate (%)	2.0	1.8	
All taxpayers			
Post-PIT income	92.1	83.8	91
PIT and surtax	9.3	6.2	66
Pre-PIT income	101.4	90.0	88
Average tax rate (%)	9.2	6.9	

Notes: income and taxes in billions HRK; average tax rate = (PIT and surtax) ÷ (Pre-PIT income) x 100; Ratio in column 3 = (APK) ÷ (Tax Admin.) x 100

Source: *author's calculations*

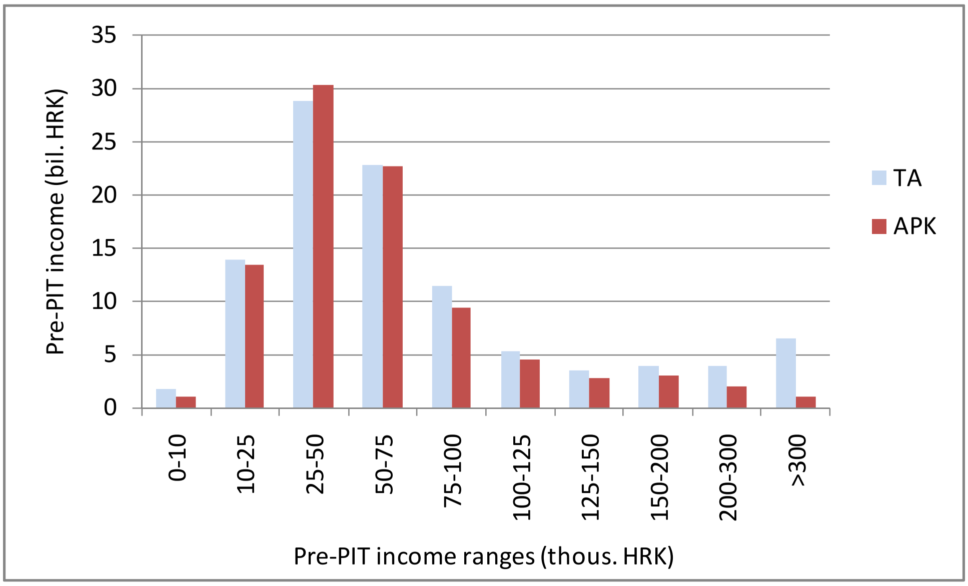
APK understates total post-PIT income by 15% for non-pensioners. However, for pensioners the APK total value is 7% larger. Where these differences come from? Checking APK incomes by source, it was revealed that pensioners make additional self-employment income, primarily from agriculture. By rule, the microsimulation model treats this income as taxable, which probably opposes the real practice. Namely, from tax administrative data we learn that only a small portion of non-pension income of pensioners is taxable.[23]

[23] Pensioners have "relaxed" PIT treatment, with relatively high personal allowance, which is manifested in low average tax rate of about 2%, and the deviation explained above will only have small affect on PIT variable. Therefore, we have decided not to adapt microsimulation model to exclude self-employment income from pensioners' taxable income.

Much greater discrepancy in administrative and survey data is found for PIT and surtax. For non-pensioners, APK covers less than 2/3 of actual revenue. This is 20 percentage points lower than the corresponding coverage for post-PIT income. For non-pensioners, the average tax rate obtained from administrative data is 11.5%, and from survey data only 9%. This difference may arise from two reasons: (a) microsimulation model is downward biased in imputation of PIT amounts (for example, through inclusion of too much allowances and deductions) and/or (b) the distribution of survey taxable income does not reflect actual income distribution.

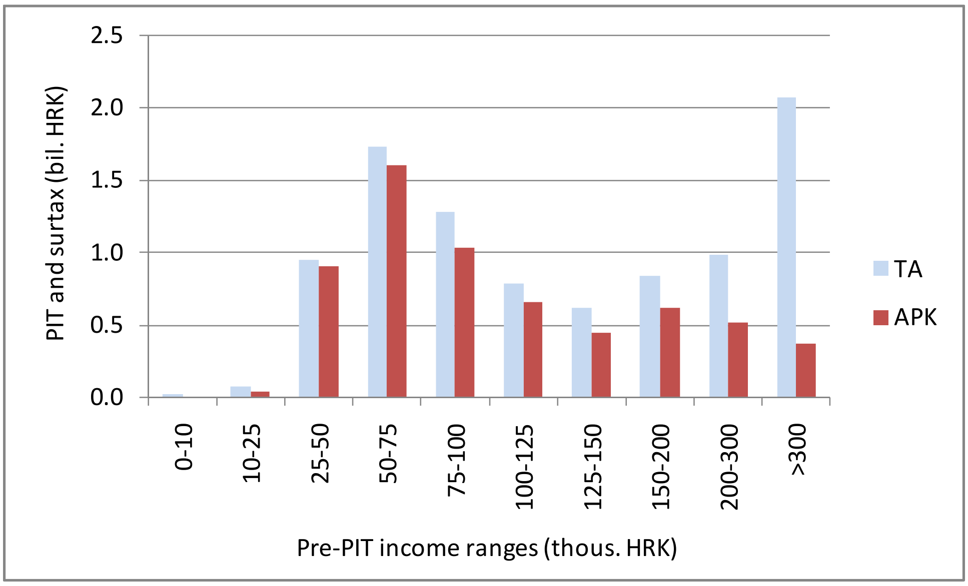
Figure 4.2 shows comparison of distributions of aggregated pre-PIT income for all taxpayers in 2005 from tax administration data (TA) and survey data (APK). Taxpayers are sorted in ten income classes according to their pre-PIT income. For the first for classes (up to the yearly income of 75,000 HRK or 10,300 EUR), magnitudes of total earned income are quite similar. However, for higher income groups (incomes greater than 75,000 HRK), total administrative incomes are higher than survey everywhere, with relative difference that is increasing in income. Thus, for the highest income class (incomes greater than 300,000 HRK or 41,100 EUR), TA data measure six times higher total income than APK data.

Figure 4.2: Distribution of taxable income from administrative and survey sources, 2005



Source: author's calculations

Figure 4.3: Distribution of PIT and surtax from official and survey sources, 2005



Source: *author's calculations*

Since PIT is in nature progressive, higher income classes contribute relatively more to total tax revenue. Figure 4.3 helps to reveal where the "missing" PIT could be. Looking at official data, over 2 billion HRK or 22% of total PIT revenue arrives from the top income class (incomes greater than 300,000 HRK or 41,100 EUR). Now, since the survey data under represent these top incomes, they also hide a large portion of PIT revenue.

Table 4.18 offers a closer look at the upper tail of income distribution. According to TA data, there are 26,000 taxpayers (1% of all taxpayers) with income higher than 200,000 HRK (27,400 EUR); their income share is 10.2% and they provide 32.7% of PIT and surtax, about 3 billion HRK. By APK data, there are only 12,000 such taxpayers (0.5% of all taxpayers); they earn 3.4% of income and pay less than 1 billion HRK of tax, providing 14.3% of total PIT and surtax. If we concentrate only on the top incomes (higher than 300,000 HRK or 41,100 EUR), the relative differences in the number of taxpayers and amounts of income and tax become even more diverging for two data sources.

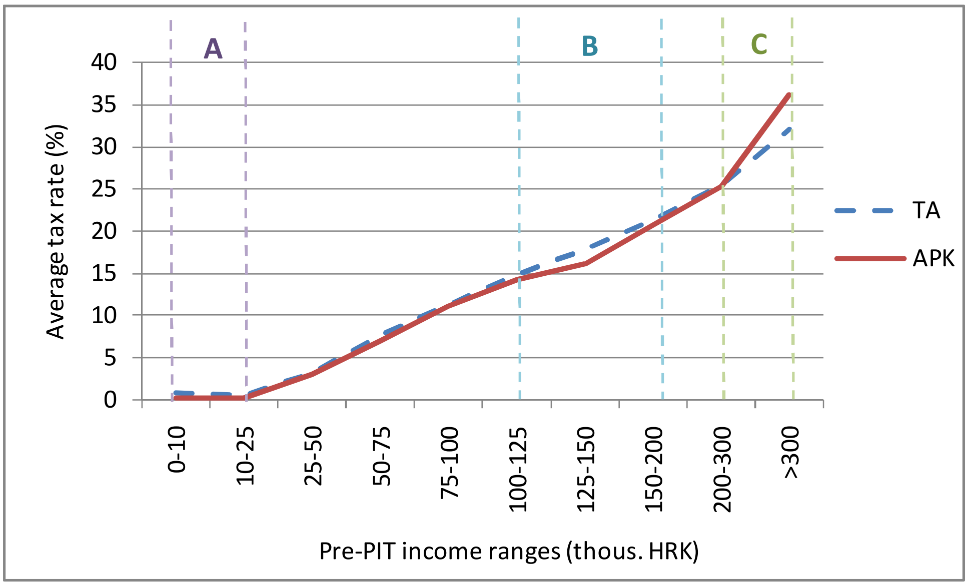
Table 4.18: Comparison of official and survey based aggregate figures: Top incomes 2005

Income class (thousands HRK)	Taxpayers (thou.)	% of total	Pre-PIT income (bill. HRK)	% of total	PIT and surtax (bill. HRK)	% of total
Tax Administration data						
200-300	16	0.6	3.889	3.8	0.988	10.6
>300	10	0.4	6.461	6.4	2.067	22.2
>200	26	1.0	10.350	10.2	3.055	32.7
Survey data						
200-300	9	0.4	2.052	2.3	0.519	8.4
>300	3	0.1	1.013	1.1	0.366	5.9
>200	12	0.5	3.065	3.4	0.885	14.3

Source: *author's calculations*

Thus, we have discovered one of the reasons why overall average PIT rate for survey data (9%) is smaller than the one for official data (11.5%): incomplete coverage of high incomes. What about validity of microsimulation model? According to Figure 4.4 it seems that the model simulates PIT and surtax rather well, as evidenced by the fact that full and dotted (respectively representing Tax Administration data and data simulated from APK) fit into each other over the whole income range. Three intervals on which there are divergences are designated by A, B and C on the figure 4.3.

Figure 4.4: Average PIT rate obtained for official and survey data, 2005



Source: *author's calculations*

The interval A ranges from zero to incomes of 25,000 HRK (3,400 EUR). The lowest incomes are tax-free through the use of personal allowances. However, some sources of incomes are taxed by withholding, without accounting for personal allowance (income from contractual work, dividends, rental activity). If taxpayers earn income only from one or more of these sources, they are not obliged to submit a tax return, but they can do that if they want to use personal allowances and other deductions. In reality, these taxpayers sometimes abstain from submitting tax returns, and therefore we have a situation in which the average tax rate for the lowest income interval is greater than zero. On the other hand, the microsimulation model assumes that all taxpayers submit tax returns. Therefore, we notice a slight difference in the average tax rate between official and survey data on interval A.

On interval B (pre-PIT incomes from 100,000 to 200,000 HRK or 13,700 to 27,400 EUR), the simulated data show the average tax rate that is smaller by few percentage points than for the official data. The reason may be that the microsimulation model ascribes more allowances and deductions to the upper-middle income class than they really use. It is the opposite for interval C (pre-PIT incomes larger 200,000 HRK or 27,400 EUR); here we have many small entrepreneurs that use deductions connected with incentives to employ new workers, which are not built into the model. These deductions may significantly lower the average tax rate for the top incomes. There is another explanation for the divergence on intervals B and C, and actually, it is the same as for interval A: tax withholding without submission of tax return; in this case, income is taxed by flat rate and not by general schedule; the presence of such cases will push the average tax rate up (down) below (above) certain income thresholds.

We will briefly turn to comparison of different data sources regarding SSCs. In microsimulation model, SSCs are imputed on pre-PIT incomes; thus, the reliability of SSCs variables depends on the quality of microsimulation model in imputation of PIT and surtax, but ultimately, on the appropriateness of income data. As Table 4.19 shows, the coverage of SSCs by the imputed data ranges from 72% to 82%, as compared with the central government budget data. This is significantly higher than for PIT data (see Table 4.17). The reason may be the following: in the upper tail of income distribution (underrepresented by APK data, as we have seen), we have many self-employed people. They pay lump sum SSCs, unlike the employed, which pay in proportion with their wage income. Now, take the example of two persons with income of 250,000 HRK (34,200 EUR) that both escape APK: one is employed and the other self-employed; the amount of hidden PIT is relatively higher than the “loss” of

SSCs, because two persons should pay similar amounts of PIT and quite different amounts of SSCs (recall Table 4.15).

Table 4.19: Comparison of budget data and APK aggregate figures: SSCs, 2005

	Budget	**APK data**	**Ratio (%)**
	1	2	3(=2/1)
SSC for the health system	14,165	11,597	82
SSC for the pension system (1st pillar)	15,714	11,231	72
SSC for the unemployment protection system	1,422	1,112	78

Note: taxes in billions HRK

Source: author's calculations

4.4.2 Social benefits

In this section, we continue with comparisons of APK and TA data, first concentrating on cash social benefits other than public pensions. Since the individual administrative data for cash benefits are not available, we can only compare the aggregate amounts from the central government budget and APK. This analysis will remain incomplete for another reason as well, and that is a shortage of data on social benefits paid by the local governments. Despite these limitations, the following comparison will be illustrative of the fact that APK data are far from perfect in suiting the needs of investigation into the distribution of income.

Table 4.20 collects the information from Table 4.3 and Table 4.5, adding the aggregate figures from APK for various social benefit variables. First of all, APK contains only nine variables covering cash social benefits (items 6.1 to 6.9 in Table 4.5), while we know that the number of benefits is much larger. However, we also mentioned that many of these benefits are insignificant in their amount. Now, we try to establish the connections between the instruments covered by APK variables with those existing in the budget.

Thus, the "subsistence income allowance" (A1) and "one-time money allowance" (A2) correspond nicely with the variable "basic support allowances" (i-*6.1*) in APK. The same can be said for "unemployment benefit" (C1; i-*6.3*), "sick-leave benefit" (C2; i-*6.7*) and "child allowance" (D1; i-*6.2*), where one-to-one relationship exists. Also, "maternity allowance and layette supplement" (D2) corresponds to two APK variables, "maternity allowance" (i-*6.5*) and "layette supplement" (i-*6.6*).

On the other hand, there are several instruments for which we cannot find the correspondents in APK. However, we must recall that "support for military and civil people with disabilities" (B1) and "support for HW veterans with disabilities" (B2) are in nature similar to the benefits from group A, with the difference that group B relates to people related to war. Therefore, it is very likely that items i-*6.1*, i-*6.4* and i-*6.8* from APK also cover benefits B1 and B2. Similarly, item i-*6.7* might have covered C3 and C4, and item i-*6.2* also benefit D3.

Again, we use the ratio between APK and budget figures to reveal how much of the actual amount is covered by the survey data. The benefits from group A, or "basic support", seem to be very well represented in APK: for i-*6.1* the ratio is 100%, and 87% for items i-*6.4* and i-*6.8*. However, if we assume that benefits from group B were also presented by i-*6.1*, i-*6.4* and i-*6.8*, then the combined ratio falls to only 44% (see the 3rd row from below of Table 4.20). Unemployment benefit is relatively well presented in APK, as manifested by the ratio of 71%, and similar is true for "child allowance" with ratio of 73%. The worst situation appears for sick-leave benefit, where less than one third of total amount can be found in APK.

Overall, only about half of cash social benefits can be "found" in APK.

Table 4.20: Comparison of official and APK aggregate figures: social benefits, 2005

<table>
<tr><th>Budget</th><th>Amount</th><th>APK</th><th>Amount</th><th>Ratio (%)</th></tr>
<tr><td>A1: Subsistence income allowance</td><td>499</td><td rowspan="2">i-6.1: Basic support allowances</td><td rowspan="2">557</td><td rowspan="2">100</td></tr>
<tr><td>A2: One-time money allowance</td><td>58</td></tr>
<tr><td>A3: Personal invalidity allowance</td><td>169</td><td rowspan="2">i-6.4: Supplement for the injured
i-6.8: Support for rehab. and employment of people with disabilities</td><td rowspan="2">412</td><td rowspan="2">87</td></tr>
<tr><td>A4: Allowance for help and care</td><td>307</td></tr>
<tr><td>B1: Support for military and civil people with disabilities</td><td>386</td><td></td><td></td><td></td></tr>
<tr><td>B2: Support for HW veterans with disabilities</td><td>774</td><td></td><td></td><td></td></tr>
<tr><td>C1: Unemployment benefit</td><td>889</td><td>i-6.3: Unemployment benefit</td><td>634</td><td>71</td></tr>
<tr><td>C2: Sick-leave benefit</td><td>1,000</td><td>i-6.7: Sick-leave benefit</td><td>318</td><td>32</td></tr>
<tr><td>C3: Benefits for HW veterans</td><td>58</td><td></td><td></td><td></td></tr>
</table>

Budget	Amount	APK	Amount	Ratio (%)
C4: Other payments to the health insured	189			
D1: Child allowance	1,435	i-*6.2*: Child allowance	1,047	73
D2: Maternity allowance and layette supplement	718	i-*6.5*: Maternity allowance and i-*6.6*: Layette supplement	453	63
D3: Allowance for parental care	114			
Total	6,596	**Total**	3,433	52
A1+A2+A3+A4+B1+B2	2,193	i-*6.1*+i-*6.4*+i-*6.8*	969	44
C2+C3+C4	1,247	i-*6.7*	318	26
D1+D2+D3	2,267	i-*6.2*+i-*6.5*+i-*6.6*	1,500	66
Total – B1 – B2	5,436	Total	3,421	63

Note: amounts in millions HRK

Source: *author's calculations; Executive's budget proposal for 2007, Ministry of Finance, Croatia*

4.4.3 Public pensions

For the end of the analysis, we have left the largest single fiscal instrument under investigation – public pensions. Table 4.21 reveals many differences between the data on pensions in the central government budget and APK in 2005. An amount of old-age pensions in the budget is 11.9 billion compared to 16.3 in APK. Why such differences? Firstly, APK devotes only three variables to pensions (old-age, invalidity, family). The supplements to pensions, paid by the central and local government budgets are not distinguishable from the basic pension. Furthermore, HW related pensions cannot be separately identified and this too relates to pensions of other special groups of pensioners (items B1, B2, etc.).

Therefore, we have not compared the budget and APK items inside the same rows, as in the previous table, but created "adjusted" items for the three major kinds of pensions. "Adjusted old-age" comprises A1 and all items from B2 to C5. "Adjusted invalidity" is a sum of A2 and 50% of B1, while "adjusted family" contains A3 and other 50% of B1. All three adjusted items are augmented by their aliquot proportion in A4. We obtained somewhat more comparable items through this procedure. Still it remains a fact that the amount of old-age pensions is exaggerated in APK, compared to family and invalidity pensions.

Still, if we do not mind the structure and concentrate only on the total amount of pensions, this fiscal instrument is well represented, as evidenced by the ratio of 93% in the bottom row of Table 4.21.

Table 4.21: Comparison of budget and APK aggregate figures: public pensions, 2005

Budget	Amount	APK	Amount	Ratio (%)
A1: Old-age	11,924			
A2: Invalidity	4,238			
A3: Family	3,167			
A4: Supplement to pensions	1,487			
B1: Invalidity and family pensions for HWV and HWVD	2,943			
B2: Croatian army servants	355			
B3: Ministry of interior	328			
B4: Ex-JNA servants	396			
B5: Pensions for NOR	666			
B6: Pensions for HDV	168			
C1: Members of parliament	39			
C2: Public officials and servants from ex-Yugoslavia	17			
C3: Political ex-prisoners	121			
C4: Croatian academy of sciences and arts	13			
C5: Other	32			
Adjusted old-age	14,916	i-*5.1*: Public pensions – Old-age	16,271	109
Adjusted invalidity	6,057	i-*5.3*: Public pensions – Invalidity	3,863	64
Adjusted family	4,921	i-*5.2*: Public pensions – Family	3,505	71
Total	**25,894**		**23,980**	93

Note: amounts in millions HRK

Source: *author's calculations; Executive's budget proposal for 2007, Ministry of Finance, Croatia*

4.5 Measures of living standard

4.5.1 Defining pre-fiscal and post-fiscal income

The task of research in fiscal incidence is to measure the difference between the living standards of different individuals in the pre-fiscal and post-fiscal situation, i.e. the situations before and after the government intervention takes place. The key question is what we mean by "fiscal"? That depends on the choice of the fiscal instruments that will be covered by a research. This coverage may span from a single tax or benefit instrument to the whole fiscal system. The choice for this research is to analyze fiscal subsystem consisting of most important personal taxes and cash social benefits in Croatia.

Therefore, in this research, the term *pre-fiscal* relates to incomes *before* personal taxes and cash benefits, and hence *pre-tax-and-benefit* (pre-TB) income. On the other hand, the term post-fiscal relates to incomes after direct taxes and cash benefits, hence *post-tax-and-benefit* (post-TB) income. Usually, pre-TB income includes market income together with the value of production for own use and non-government transfers. Post-TB income relates to the disposable income of households, and is equal to pre-TB income *minus* personal taxes *plus* cash benefits.

4.5.2 Overview of income, tax and benefit variables

Table 4.22 overviews all the different variables that will be used in this research. At the top part of the table, we find the main variables, discussed a moment earlier. Shortly later in this section, we will see that neither of these variables (pre-TB income, post-TB income, taxes, benefits) is uniquely defined; an analyst combines different income, tax and benefit variables presented in Table 4.22 to arrive at desired definition(s) of them.

Three types of market income are recognized: market income taxable by PIT (*xtmi*), market income not taxable by PIT (*nntx*) and obligatory contributions to the private pension fund (*pfcp*). The latter item is actually "SSC for pension system – the 2nd pillar"; inclusion of this item into market incomes means that contributions to the 2nd pillar of the pension system will always be treated as private outlay of an individual, and not as a tax.

Notice that public pensions are presented by four different variables. The first division is inspired by Immervoll et al (2005), who introduced separate treatment of two groups of pensioners: those aged less than 65, and those aged 65 and more. The second division is between the pre-PIT and post-PIT pension income. The use of pre-PIT pensions (*xpyo* and *xpol*), when public pensions are not part of pre-TB income, creates an anomaly that would prevent the proper estimation of redistributive effect and reranking of this fiscal instrument.[24] Therefore, the variables for post-PIT pensions (*npyo* and *npol*) are also included.

[24] See Urban (2008) for explanation of this anomaly.

Taxes are presented by six variables. SSCs for the pension system (*sscp*) relate only to contributions to the 1^{st} pillar. Because the distinction between pre-PIT and post-PIT pensions has to be made, as explained above, we also needed to create separate PIT variables (*pito* and *pitp*), besides the main PIT variable that presents total PIT and surtax (*pitt*).

Finally, the six benefit items are defined according to the previous discussions. Note only that maternity allowance (i-*6.5*) and layette supplement (i-*6.6*) are joined into one variable (*mata*), while support for rehabilitation and employment of people with disabilities (i-*6.8*) is merged with supplement for the injured (i-*6.4*) to create another new variable (*rehs*).

Table 4.22: Variables of income, taxes and benefits

Notation	**Description**
General	
X	Pre-TB income
N	Post-TB income
T	Total individual taxes
B	Total cash benefits
Market incomes	
xtmi	Market income taxable by PIT: wages and salaries, self-employment income, income from part-time and contractual work, rental income and income from property rights
nntx	Non-taxable market income (interest on private saving and investment)
pfcp	Obligatory contributions to the private pension fund (i.e. SSC for pension system – the 2^{nd} pillar)
Non-market-non-fiscal incomes	
ownu	Value of production for own use
trnk	Periodic transfers from private persons: gifts, alimonies
Public pensions	
xpyo	Public pensions to persons aged less than 65, before PIT
npyo	Public pensions to persons aged less than 65, after PIT (hence also „Pensions (<65)“)
xpol	Public pensions to persons aged 65 and more, before PIT
npol	Public pensions to persons aged 65 and more, after PIT (hence also „Pensions (65&>)“)
Taxes	
sscp	SSC for the pension system - the 1^{nd} pillar
ssch	SSC for the health system
sscu	SSC for the unemployment protection system
pitt	PIT and surtax, total
pito	PIT and surtax, on *xtmi*
pitp	PIT and surtax, on *xpyo* and *xpol*
Benefits	
bspa	Basic support allowances
uneb	Unemployment benefit

Notation	Description
chla	Child allowance
sicb	Sick-leave benefit
mata	Maternity allowance and layette supplement
rehs	Supplement for the injured and support for rehabilitation and employment of people with disabilities

Notes: (a) The most important of these variables are derived in section 4.3.6. Let us mention some of the identities: $xtmi = X_i^T$; $pfcp = T_i^{P2}$; $sscp = T_i^{P1}$; $ssch = T_i^H$; $sscu = T_i^U$; $pitt = T_i^T$. (b) Observe the following relationships: (*xpyo*+*xpol*) – *pitp* = *npyo*+*npol*; *pitp* = *pitt* – *pito*; *xpyo* + *xpol* – *pitt* = *npyo* + *npol* – *pito*

4.5.3 Income, tax and benefit definitions

As we have already indicated, the analysts use different definitions of pre-TB income, taxes and benefits, which depend on various assumptions concerning the economic role and incidence of taxes and benefits. In this research, we will search for answers on three important questions:

(a) Can employers shift the burden of SSCs on employees? In other words, who bears a burden of SSCs for which the legal taxpayers are employers?
(b) Should the public pensions be treated as social benefits or as a market income?
(c) Should SSC to the 1st pillar of the pension system be treated as personal saving or as a tax?

Instead of attempting to provide definitive answers to these questions, in the empirical part of the research, we are going to employ six scenarios or income-tax-benefit definitions (henceforth ITBD) reflecting different choices.

Remember that in Croatia, employers are legal taxpayers of SSC for the health system (*ssch*) and SSC for the unemployment protection system (*sscu*), while pension contributions are paid by employees. The ITBDs 1, 2 and 3 assume that SSCs paid by employers (*ssch* and *sscu*) are fully shifted on employees, which conforms to a classical view of tax incidence theory. Therefore, *ssch* and *sscu* are included in pre-TB income; see (4.26), (4.30) and (4.34). Consequently, they are also included into taxes; see (4.27), (4.31) and (4.35). On the other hand, ITBDs 4, 5 and 6 assume that SSCs paid by employers are also economically borne by employers; therefore, they are neither income nor taxes for employees; observe that they are not mentioned in income definitions (4.38), (4.42) and (4.46), and tax definitions (4.39), (4.43) and (4.47).

Then we have the issue of public pensions. We have two extreme choices: to treat public pensions as social benefits, or to treat them as market income. In ITBDs 1 and 4, public pensions (*npyo* and *npol*) are regarded as benefits, as can be seen in (4.28) and (4.40). In all other ITBDs, public pensions (*xpyo* and *xpol*) are a part of pre-TB income, as defined in (4.30), (4.34), (4.42) and (4.46).

Finally, the issue whether SSCs to the 1st pillar of the pension system are taxes or savings is solved in the following way. ITBDs 3 and 6 do not treat these contributions (*sscp*) as taxes, but as a form of saving; see (4.35) and (4.47). Therefore, they add to disposable income; see (4.37) and (4.49). All other ITBDs (1, 2, 4 and 5) treat SSCs to the 1st pillar as taxes.

Table 4.23: Definitions of incomes, taxes and benefits (ITBDs)

$f = xtmi + pfcp + nntx + ownu + trnk$	(4.24)
$g = uneb + sicb + chla + bspa + mata + rehs$	(4.25)
ITBD 1	
$X_1 = f + (sscp + ssch + sscu)$	(4.26)
$T_1 = (sscp + ssch + sscu) + pito$	(4.27)
$B_1 = g + (npyo + npol)$	(4.28)
$N_1 = X_1 - T_1 + B_1 =$	
$= (f + sscp + ssch + sscu) - (sscp + ssch + sscu + pito) + (g + npyo + npol) =$	
$= f + g + (npyo + npol) - pito$	(4.29)
ITBD 2	
$X_2 = f + (sscp + ssch + sscu) + (xpyo + xpol)$	(4.30)
$T_2 = (sscp + ssch + sscu) + pitt$	(4.31)
$B_2 = g$	(4.32)
$N_2 = X_2 - T_2 + B_2 =$	
$= (f + sscp + ssch + sscu + xpyo + xpol) - (sscp + ssch + sscu + pitt) + g =$	
$= f + g + (xpyo + xpol) - pitt$	(4.33)
ITBD 3	

$$X_3 = f + (sscp + ssch + sscu) + (xpyo + xpol) \quad (4.34)$$

$$T_3 = ssch + sscu + pitt \quad (4.35)$$

$$B_3 = g \quad (4.36)$$

$$N_3 = X_3 - T_3 + B_3 = \quad (4.37)$$

$$= (f + sscp + ssch + sscu + xpyo + xpol) - (ssch + sscu + pitt) + g =$$

$$= f + g + sscp + (xpyo + xpol) - pitt \quad (4.38)$$

ITBD 4

$$X_4 = f + sscp \quad (4.39)$$

$$T_4 = sscp + pito \quad (4.40)$$

$$B_4 = g + (npyo + npol) \quad (4.41)$$

$$N_4 = X_4 - T_4 + B_4 =$$

$$= (f + sscp) - (sscp + pito) + (g + npyo + npol) =$$

$$= f + g + (npyo + npol) - pito \quad (4.42)$$

ITBD 5

$$X_5 = f + sscp + (xpyo + xpol) \quad (4.43)$$

$$T_5 = sscp + pitt \quad (4.44)$$

$$B_5 = g \quad (4.45)$$

$$N_5 = X_5 - T_5 + B_5 =$$

$$= (f + sscp + xpyo + xpol) - (sscp + pitt) + g =$$

$$= f + g + (xpyo + xpol) - pitt \quad (4.46)$$

ITBD 6

$$X_6 = f + sscp + (xpyo + xpol) \quad (4.47)$$

$$T_6 = pitt \quad (4.48)$$

$$B_6 = g \quad (4.49)$$

$$N_6 = X_6 - T_6 + B_6 =$$

$$= (f + sscp + xpyo + xpol) - pitt + g =$$

$$= f + g + sscp + (xpyo + xpol) - pitt \quad (4.50)$$

The following relationships may be useful.

$$X_4 = X_1 - ssch - sscu \quad (4.51)$$

$$X_{23} = X_2 = X_3 \quad (4.52)$$

$$X_{23} = X_1 + xpyo + xpol \quad (4.53)$$

$$X_{56} = X_5 = X_6 \quad (4.54)$$

$$X_{56} = X_{23} - ssch - sscu \quad (4.55)$$

$$N_{1245} = N_1 = N_2 = N_4 = N_5 \quad (4.56)$$

$$N_{36} = N_3 = N_6 \quad (4.57)$$

$$N_{36} = N_{1245} + sscp \quad (4.58)$$

Figure 4.5 shows graphically the ITBDs described above as combinations of six main elements: benefits other than pensions (g), public pensions, market income plus non-market-non-fiscal incomes (f), SSCs to the 1st pillar of the pension system ($sscp$), employers' SSCs ($ssch$ & $sscu$) and PIT. The dark cell indicates that the element is present in the definition of pre-TB income (*X*), taxes (*T*) or benefits (*B*).

Thus, g is always included in benefits, and PIT is always included in taxes. f and $sscp$ are always a part of pre-TB income. Inclusion of other elements varies.

Figure 4.5: Definitions of incomes, taxes and benefits

	ITBD 1			**ITBD 2**			**ITBD 3**			**ITBD 4**			**ITBD 5**			**ITBD 6**		
	X	*T*	*B*	*X*	*T*	*B*	*X*	*T*	*B*	*X*	*T*	*B*	*X*	*T*	*B*	*X*	*T*	*B*
g			■			■			■			■			■			■
public pensions			■	■			■					■	■			■		
f	■			■			■			■			■			■		
sscp	■	■		■	■		■			■	■		■	■		■		
ssch & *sscu*	■	■		■	■		■	■										
PIT		■			■			■			■			■			■	

5 RESULTS

5.1 Introduction

In this chapter, we present the estimations of the redistributive and reranking effects for Croatia in the period from 2001 to 2006. The results are obtained using methodological tools developed in *chapter 3*. Basic indicators are calculated for six different income-tax-benefit definitions (ITBD) defined in section 4.5 and six income equivalence scales (IES), mentioned in section 2.4.2.1. This gives us overall 36 "scenarios" for the basic indicators. For decompositions of redistributive effect to reveal contributions of taxes and benefits, 12 scenarios were run (6 ITBDs times 2 IESs), and the results for one IES are presented.

The chapter is organized as follows. Section 5.2 is divided into two major parts: the first presents the indicators of inequality, redistributive effect and reranking. We have chosen 2006 as the basic year and present results for all 36 scenarios. For other years, we choose one IES and compare results across years and ITBDs. Section 5.3 presents further decompositions of redistributive effect and reranking to reveal roles of individual taxes and benefits.

5.2 Redistributive effect and reranking in Croatia

5.2.1 Income definitions and equivalence scales

Different ITBDs reflect contrasting assumptions about (a) the incidence of taxes – who bears the economic burden of SSC paid by employers (*ssch* and *sscu*), (b) the role of public pensions – whether they are social benefits or just another form of market income, and (c) the role of SSC to the 1st pillar of pension system – whether they are taxes or should be treated as means of private saving. Thus, we designed ITBDs 1, 2 and 3 which regard that SSCs paid by employers are fully shifted on employees, and ITBDs 4, 5, 6 which exclude these SSCs from pre-TB incomes. ITBDs 1 and 4 treat public pensions as social benefits, unlike four other ITBDs, which include them into pre-TB income. Finally, ITBDs 3 and 6 assume that SSCs to 1st pillar are not taxes, but an equivalent to voluntary saving; for other definitions, these SSCs are taxes. For a thorough discussion on ITBDs design, see section 4.5.

The analysis also takes into account three types of most exploited equivalence scales in the research of redistributive effects, as evidenced by section 2.4.2.1: (a) "OECD" scale (E_3), (b) Cutler and Katz scale (E_2) and (c) "power" scale (E_1). Each type of scale appears with two different sets of parameters: (a) the so-called "modified OECD scale" ($E_3(.5,.3)$) and the "original OECD scale" ($E_3(.7,.5)$); (b) "Cutler and Katz scale" features in common configuration of $E_2(.5,.5)$, and the alternative one, $E_2(.7,.6)$; (c) "power scale" appears with typical square-root set of $E_1(.5)$, and also $E_1(1)$, which is actually a setup which ignores both economies of scale and the differences in age of household members.

5.2.2 Basic indicators of the redistributive effect

5.2.2.1 Estimates for 2006

Table 5.1 summarizes the indices of income inequality and redistribution in Croatia 2006. Gini coefficients of pre-TB income (G_X) and post-TB income (G_N) and their differences (RE), are calculated for 36 different scenarios explained above.

The scales $E_2(.5,.5)$ and $E_1(.5)$ show the largest Gini coefficients for all ITBDs, to the opposite of $E_3(.7,.5)$ and $E_1(1)$, which result in the smallest inequalities, whether for G_X or G_N. Looking at the redistributive effect as a percentage of pre-TB Gini coefficient (rows $RE\,(\%G_X)$), we observe very small difference between the results obtained for different equivalence scales, with exception of $E_1(1)$ in case of ITBDs 1, 2 and 4.

It seems that different equivalence scales do not lead to significant differences in results. Therefore, in further analysis, we may focus on one or two most interesting equivalence scales. We proceed with comparison of results across ITBDs.

Recall that ITBDs 1, 2, 4 and 5, on one side, and ITBDs 3 and 6 on the other, have identical post-TB income. For the former group, post-TB income is denoted as N_{1245}, and N_{36} for the latter, as explained in section 4.5. For all equivalence scales, N_{1245} shows smaller inequality than N_{36}, which is expected, because N_{36} contains *sscp*, unlike N_{1245}. The reason is that *sscp* are part of the income earned only by the working people who are, on average, richer than pensioners and non-working individuals.

The differences between Gini coefficients of pre-TB income for different ITBDs are much larger. First, recall that ITBDs 2 and 3 on one side, and 5 and 6 on the other, have identical pre-TB incomes, denoted as X_{23} and X_{56}, respectively. Next, remember that X_{23} and X_{56} differ in SSCs paid by employers (see (4.54); $X_{56} = X_{23} - ssch - sscu$). This results in 1.5 percentage higher Gini coefficients for X_{23}, for the same reason as in case of *sscp*, in the previous paragraph. Interestingly, X_1 and X_4 differ in identical respect (see (4.50); $X_4 = X_1 - ssch - sscu$), but the difference between corresponding Gini coefficients is almost negligible. This suggests that the effect on system's inequality of one instrument depends on the structure and size of the whole system: recall that ITBDs 1 and 4 include public pensions into benefits, while ITBDs consider it as part of pre-TB income.

Already a comparison of pre-TB income Gini coefficients for different ITBDs reveals the fact that public pensions are the major factor influencing inequality in Croatia. Observe that in comparisons between X_1 and X_2, and between X_4 and X_5, which differ in public pensions, substantial differences in G_X exist, equal to 15 percentage points.

Further analysis of the results in Table 5.1 confirms a following thesis: there is a large discrepancy in estimates of RE, and they crucially depend on how the fiscal system is defined. The redistributive effect of systems which exclude public pensions from pre-TB income and treat them as benefits (ITBD 1 and 4) amounts to more than 40% of G_X. All other systems show at most half of this figure (ITBD 2), down to modest 10-11% of G_X, when only PIT and cash benefits are involved (ITBD 6). We will turn to these issues again later.

Table 5.1: Redistributive effects of taxes and benefits in Croatia, 2006

	$E_3(.5,.3)$	$E_3(.7,.5)$	$E_2(.5,.5)$	$E_2(.7,.6)$	$E_1(.5)$	$E_1(1)$
ITBD 1						
G_X	0.5061	0.4950	0.5220	0.5106	0.5180	0.4862
G_N	0.2901	0.2849	0.3034	0.2932	0.3012	0.2913
RE	0.2160	0.2101	0.2186	0.2174	0.2168	0.1949
RE (%G_X)	42.7	42.4	41.9	42.6	41.9	40.1

	E_3(.5,.3)	E_3(.7,.5)	E_2(.5,.5)	E_2(.7,.6)	E_1(.5)	E_1(1)
ITBD 2						
G_X	0.3622	0.3546	0.3760	0.3657	0.3735	0.3549
G_N	0.2901	0.2849	0.3034	0.2932	0.3012	0.2913
RE	0.0721	0.0697	0.0726	0.0725	0.0724	0.0636
RE (%G_X)	19.9	19.6	19.3	19.8	19.4	17.9
ITBD 3						
G_X	0.3622	0.3546	0.3760	0.3657	0.3735	0.3549
G_N	0.3088	0.3016	0.3235	0.3125	0.3207	0.3043
RE	0.0534	0.0530	0.0524	0.0533	0.0528	0.0505
RE (%G_X)	14.7	14.9	13.9	14.6	14.1	14.2
ITBD 4						
G_X	0.4998	0.4887	0.5157	0.5043	0.5118	0.4802
G_N	0.2901	0.2849	0.3034	0.2932	0.3012	0.2913
RE	0.2097	0.2038	0.2123	0.2111	0.2106	0.1889
RE (%G_X)	42.0	41.7	41.2	41.9	41.2	39.3
ITBD 5						
G_X	0.3466	0.3404	0.3591	0.3496	0.3572	0.3436
G_N	0.2901	0.2849	0.3034	0.2932	0.3012	0.2913
RE	0.0564	0.0555	0.0557	0.0564	0.0561	0.0523
RE (%G_X)	16.3	16.3	15.5	16.1	15.7	15.2
ITBD 6						
G_X	0.3466	0.3404	0.3591	0.3496	0.3572	0.3436
G_N	0.3088	0.3016	0.3235	0.3125	0.3207	0.3043
RE	0.0378	0.0389	0.0356	0.0372	0.0366	0.0392
RE (%G_X)	10.9	11.4	9.9	10.6	10.2	11.4

Source: *author's calculations*

5.2.2.2 Estimates for the whole period: 2001-2006

In this section, we compare the results for 2006 with those for earlier years. Figure 5.1 presents Gini coefficients of pre-TB and post-TB income and the redistributive effect as a percentage of pre-TB inequality, for six ITBDs in the period 2001-2006. The similar trend in inequality exists for both pre-TB and post-TB income and all ITBDs: inequality is lowest in 2001; thereafter it increases, reaching a peak in 2004; then it falls, and in 2006 it is slightly higher than in 2001. The differences in inequality within the period are not small: they range from 2 to 3 percentage points.

For ITBDs 1 and 4, we can observe a significant rise of the redistributive effect, expressed in terms of pre-TB income inequality ('$RE(\%G_X)$'), after initial decline in 2002. For other definitions, the picture is less conclusive. These trends are also present for other equivalence scales.

Figure 5.1: Redistributive effects of taxes and benefits in Croatia, $E_3(.5,.3)$, 2001-2006

Source: *author's calculations*

5.2.3 Basic indicators of reranking effect

5.2.3.1 Estimates for 2006

Table 5.2 shows values of the Atkinson-Plotnick and Lerman-Yitzhaki indices of reranking for 36 chosen scenarios. The systems in which public pensions are not considered as benefits (ITBD 2, 3, 5 and 6) show a relatively small amount of reranking compared to systems in which public pensions are benefits (ITBD 1 and 4). Thus, for the low-reranking systems, the Lerman-Yitzhaki index of reranking ranges from less than 1% of G_X and 6% of RE (ITBD

6) to 3% of G_X and 16% of RE (ITBD 2). On the other hand, high-reranking systems (ITBDs 1 and 4) have R^{LY} of up to 20% of G_X and 50% of RE.

Looking within ITBDs, for these four low-reranking systems, all equivalence scales reveal similar magnitudes of reranking index, as expressed in terms of G_X and RE. On the other hand, reranking indices for ITBDs 1 and 4 show high divergence when compared across the equivalent scales. Thus, for ITBD 1, the Lerman-Yitzhaki index of reranking ranges from 11.9% of G_X and 28.2% of RE for $E_2(.5,.5)$, to 20.9% of G_X and 51.6% of G_X for $E_1(1)$. Also, the variation of values 'R^{LY} (%G_X)' and 'R^{LY} (%RE)' within the definitions ITBDs 1 and 4 is much higher than it was for the 'RE (%G_X)' in Table 5.1.

The difficult question posed several times in this study is: should public pensions be regarded as social benefits or as a form of market income, such as wages? If analytical decision is to regard public pensions as social benefits, the basic results of this research have shown that: (a) they are major redistributive factor, (b) they probably introduce large quantity of reranking in the system, and (c) their relative importance is sensitive to the choice of equivalence scale. These pre-findings will be challenged in further analysis.

Table 5.2: Reranking effects of taxes and benefits in Croatia, 2006

	E_3(.5,.3)	E_3(.7,.5)	E_2(.5,.5)	E_2(.7,.6)	E_1(.5)	E_1(1)
ITBD 1						
R^{AP}	0.0511	0.0595	0.0444	0.0481	0.0467	0.0755
R^{LY}	0.0737	0.0852	0.0620	0.0693	0.0658	0.1009
R^{LY} (%G_X)	14.6	17.2	11.9	13.6	12.7	20.8
R^{LY} (%RE)	34.1	40.6	28.4	31.9	30.4	51.8
ITBD 2						
R^{AP}	0.0072	0.0080	0.0067	0.0070	0.0068	0.0090
R^{LY}	0.0085	0.0092	0.0078	0.0082	0.0080	0.0101
R^{LY} (%G_X)	2.3	2.6	2.1	2.3	2.1	2.8
R^{LY} (%RE)	11.7	13.2	10.8	11.4	11.0	15.8
ITBD 3						
R^{AP}	0.0029	0.0031	0.0029	0.0029	0.0028	0.0034
R^{LY}	0.0032	0.0034	0.0032	0.0032	0.0031	0.0038
R^{LY} (%G_X)	0.9	1.0	0.8	0.9	0.8	1.1
R^{LY} (%RE)	6.1	6.5	6.1	6.0	5.9	7.5

	$E_3(.5,.3)$	$E_3(.7,.5)$	$E_2(.5,.5)$	$E_2(.7,.6)$	$E_1(.5)$	$E_1(1)$
ITBD 4						
R^{AP}	0.0497	0.0578	0.0432	0.0469	0.0454	0.0733
R^{LY}	0.0714	0.0825	0.0601	0.0671	0.0638	0.0976
R^{LY} (%G_X)	14.3	16.9	11.7	13.3	12.5	20.3
R^{LY} (%RE)	34.1	40.5	28.3	31.8	30.3	51.6
ITBD 5						
R^{AP}	0.0041	0.0043	0.0041	0.0041	0.0040	0.0047
R^{LY}	0.0047	0.0049	0.0045	0.0046	0.0045	0.0052
R^{LY} (%G_X)	1.3	1.4	1.3	1.3	1.3	1.5
R^{LY} (%RE)	8.3	8.8	8.1	8.2	8.0	10.0
ITBD 6						
R^{AP}	0.0023	0.0022	0.0024	0.0023	0.0023	0.0021
R^{LY}	0.0024	0.0024	0.0025	0.0024	0.0024	0.0023
R^{LY} (%G_X)	0.7	0.7	0.7	0.7	0.7	0.7
R^{LY} (%RE)	6.3	6.1	7.1	6.5	6.6	5.8

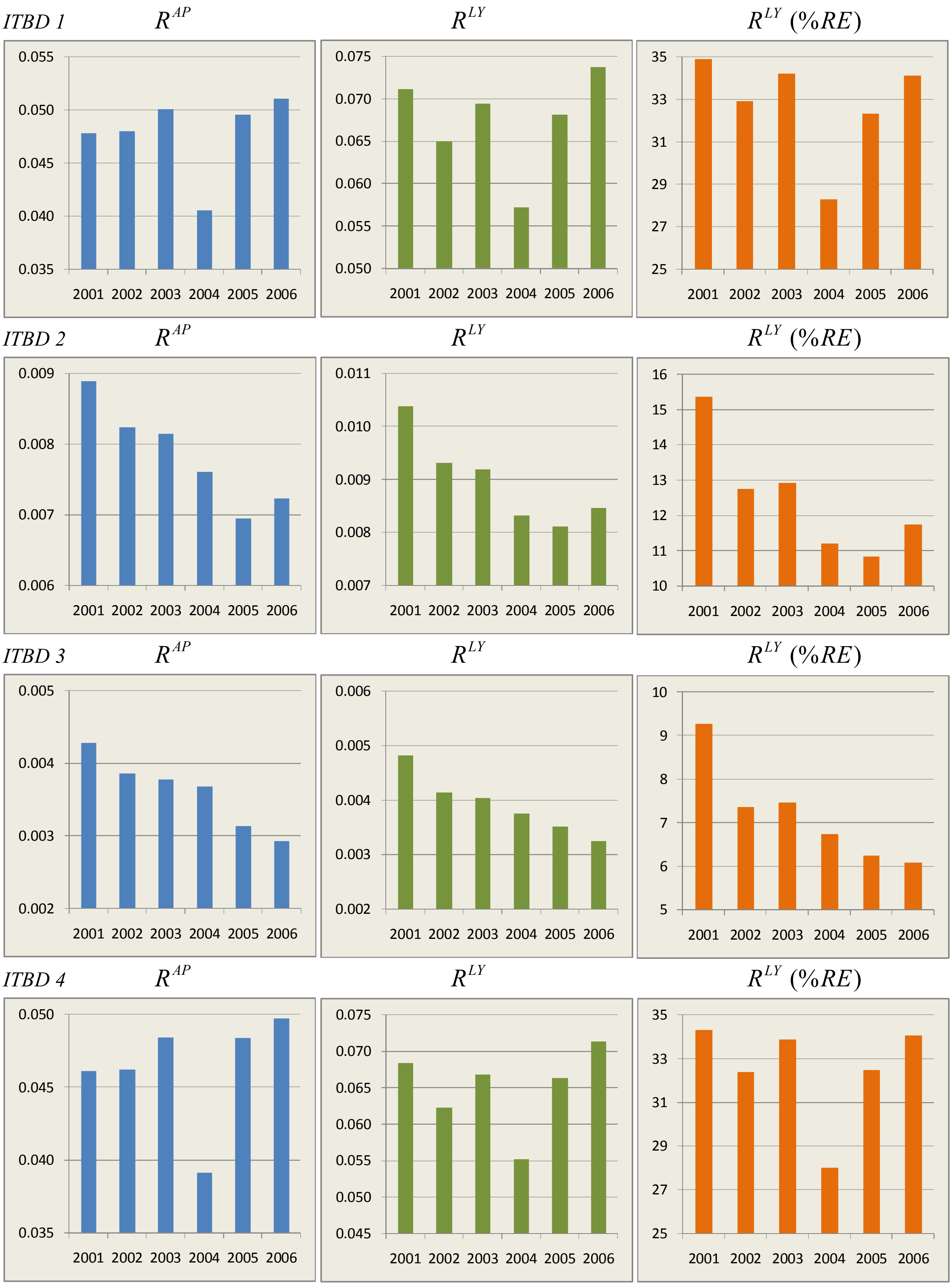
Source: *author's calculations*

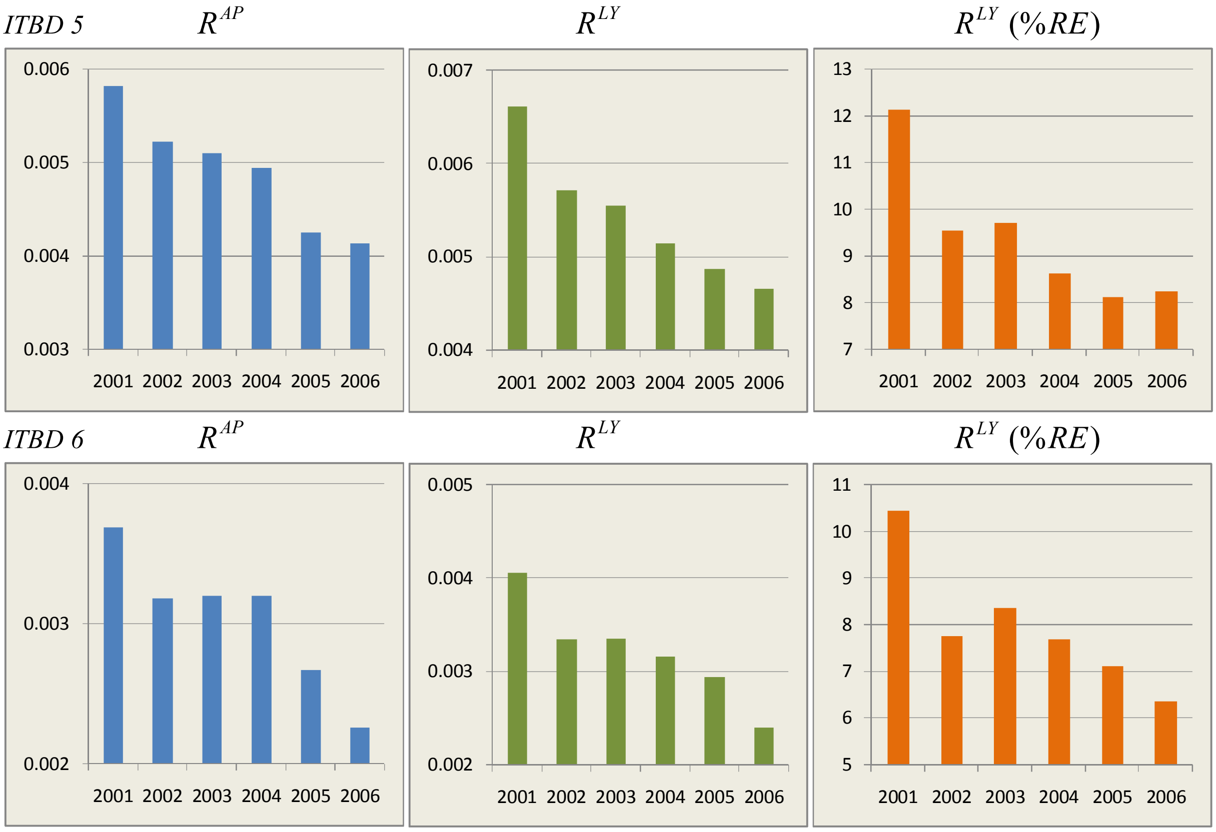
5.2.3.2 Estimates for the whole period: 2001-2006

Figure 5.2 presents Atkinson-Plotnick (R^{AP}) and Lerman-Yitzhaki (R^{LY}) indices of reranking and also the latter index expressed in terms of redistributive effect ('R^{LY} (%RE)'), for all six ITBDs in the period from 2001 to 2006. The first interesting thing, easy to observe owing to the graphical presentation, is that two reranking indices have very similar patterns, but they differ in amounts, whereby R^{LY} is higher. Considering ITBDs 1 and 4, the second interesting issue is that reranking in 2004 is significantly lower than in other years. Year 2004 behaves here as an outlier, but this is not the case for other ITBDs.

For ITBDs 1 and 4, reranking sustains at the high level during the period, but for all other ITBDs, we notice a decreasing trend in all indicators shown in Figure 5.2., R^{AP}, R^{LY} and 'R^{LY} (%RE)'. The decrease of R^{LY} as a share in RE is significant, ranging from 3 percentage points for ITBDs 2 and 3, and 4 percentage points for ITBDs 5 and 6.

Figure 5.2: Reranking effects of taxes and benefits in Croatia, $E_3(.5,.3)$, 2001-2006





Source: *author's calculations*

5.2.4 Contributions of taxes and benefits to redistributive effect

5.2.4.1 Estimates for 2006

We proceed with decompositions of reranking and the redistributive effect to reveal the roles of various taxes and benefits (henceforth, TB). The methodology section presented two approaches to further analyze redistributive and reranking effects. One approach is based on absolute differences between amounts of TB (section 3.3.2), and the other, on deviations of TB from proportionality (section 3.3.3). Furthermore, two different basic decompositions of redistributive effect were used as starting points. The first is Kakwani (1984) decomposition into vertical and reranking effect (see equation (2.8)), and the other is Lerman-Yitzhaki (1995) decomposition (from equation (2.23)).

Thus, we have four distinct decompositions of redistributive effect into contributions of TB, which were presented in sections 3.3.2 and 3.3.3. Since each of them decomposes three indicators (redistributive effect, vertical effect and reranking) to reveal contributions of 12 different fiscal instruments, we obtain a lot of information. This calls for a systematic presentation. Only the results for $E_3(.5,.3)$, are presented since the underlying empirical work has proved that there are no significant differences between the results for different IESs.

Table 5.3 shows the results for Lerman-Yitzhaki (1995) decompositions and Table 5.4 is devoted to Kakwani (1984) decompositions. All the results are obtained for six ITBDs. Values of indicators are skipped for convenience and, instead, percentage contributions are given (denoted as %*RE* or %R^{LY}). Subtotals for all taxes and all benefits are also shown in rows '*taxes'* and '*benefits'*.

Major instruments like public pensions will have many times larger contributions to overall reranking and redistributive effect, than, for example, sickness benefits. Naturally, we will also be interested in 'normalized' contributions, defined in the following way. First, we obtain a share of instrument in total sum of TB, as shown in column '*% T&B'* of Table 5.3 and Table 5.4. Now, we can compare each percentage contribution of an instrument to RE, R^{LY} or some other indicator with the instrument's share in total amount of TB. For easier assessment, columns 'RE (÷)' are calculated, dividing 'RE (%)' by '% *T&B*'. Numbers in "RE (÷)" greater than 1 mean that the tax or benefit contributes to the redistributive effect over proportionately.

We will simultaneously analyze the results from Table 5.3 and Table 5.4, tracking the similarities and differences between them. The former shows decompositions of Lerman-Yitzhaki indicators, V^{LY}, R^{LY} and their sum, RE (recall that $RE = V^{LY} + R^{LY}$). The latter decomposes indicators from Kakwani decomposition, V^K, R^{AP} and their difference, RE (remember that $RE = V^K - R^{AP}$). Bellow the head '*Amounts*' of both tables, there are results obtained for "amounts of TB" approach explained in section 3.3.2.3, and below *'Deviations'* title, there are results for "deviation of TB from proportionality" approach, based on section 3.3.3.3.

We have seen that ITBDs 1 and 4 are characterized by large amount of distance narrowing, but that was also followed by huge quantity of reranking. Now we can confirm our expectations that pensions are the principal cause of reranking. Public pensions (*pyo* and *pol*) create about 77-95% of total reranking, as shown in column 'R^{LY} (%R^{LY})' and 'R^{AP} (%R^{AP})', which is almost twice their share in total amount of TB in these ITBDs.

On the other pole, there is PIT which creates relatively small amounts of reranking. For ITBDs 1 and 4, PIT contributes with only 2% to R^{LY} and less than 1% to R^{AP}. For other ITBDs, which exclude pensions from the fiscal system, the contribution of PIT to reranking is relatively larger, but still several times lower than the share of PIT in total amount of TB.

SSCs also contribute under-proportionately to reranking, but their overall size is high, and together they make the second contributor to reranking, after public pensions. For ITBD 1, they create about 20% of R^{LY} and 12% of R^{AP}. In ITBD 2, they make two-thirds of R^{LY} and R^{AP}, which is roughly their proportion in overall amount of TB for this ITBD. For other ITBDs, namely 3, 5 and 6, they are the largest contributor, but still their share in reranking is not higher than their share in overall TB.

The role of cash benefits (*unem*, *sick*, *chbn*, *bspa*, *matr*, *rehb*) in reranking crucially depends on the choice of the scenario. For ITBDs 1 and 4, they even reduce reranking or contribute significantly below their proportion in overall TB. For other definitions, it is quite the opposite. The extreme example is ITBD 6, where cash benefits contribute with 31.5% to total amount of TB, but create 90% of reranking.

Columns 2, 3 and 4 ('V^{LY} ($\%RE$)', 'R^{LY} ($\%RE$)' and 'RE ($\%RE$)') of Table 5.3, and columns 2, 3 and 4 ('V^{K} ($\%RE$)', 'R^{AP} ($\%RE$)' and 'RE ($\%RE$)') of Table 5.4, present full decomposition of redistributive effect (RE) that use "amounts of TB" approach in calculation. Observe that the sum of vertical (columns 2) and reranking (columns 3) shares for all instruments amounts to 100, signifying 100% of RE. Thus, RE is decomposed in two "dimensions" – into vertical/reranking effects and across fiscal instruments. For Lerman-Yitzhaki decomposition ($RE = V^{LY} + R^{LY}$), contributions naturally add to 100, implying that reranking is a positive contributor. On the other hand, for Kakwani decomposition ($RE = V^{K} - R^{AP}$), in order to get a total sum of 100, we had to multiply each contribution to reranking by -1.

Summing the values of 'V^{LY} ($\%RE$)' and 'R^{LY} ($\%RE$)' for each row, we obtain 'RE ($\%RE$)' – a contribution of each fiscal instrument to the redistributive effect. The same is done with 'V^{K} ($\%RE$)' and 'R^{AP} ($\%RE$)'. Comparing column 4 ('RE ($\%RE$)') of Table

5.3 and Table 5.4, we notice that they are equal! This interesting property is not a coincidence – it occurs in all data exercises, as we noted in 3.3.4.4.

Which instruments contribute most to the redistributive effect? The taxes are the dominant contributor for all ITBDs. Even for ITBD 4, where taxes make less than 40% of total TB, they contribute with 63.3% to RE. For all ITBDs, the contribution of taxes is at least proportional to their share in total taxes benefits, while for benefits the ratio "RE (÷)" in column 5 is only as high as 0.6. This is the picture we get when "amounts of TB" approach is used. What does the other approach say?

For "deviation of TB from proportionality" approach, the results are quite reverse. For Lerman-Yitzhaki decomposition, taxes contribute 40% of RE for ITBD 1 and only 30% of RE for ITBD 4. The situation for Kakwani decomposition is even worse for taxes: they bring about less than 12% of RE for these two ITBDs. The "deviation of TB from proportionality" approach presents benefits as much more important contributors. Why such discrepancy in the results obtained for two approaches?

The reasons are already explained in section 3.3.4.2, and they lie in the choice of benchmark used to establish the notions of inequality decrease and inequality increase. In the "deviations of TB from proportionality" approach, the conditions to declare the tax instrument as inequality reducing are much stronger than for the benefit instrument, while in the "amounts" approach, it is the opposite. It is similar when magnitudes of contributions are determined: the "amounts" approach favours taxes, while the "deviations" approach favours benefits.

This discrepancy can be further illustrated by calculation of the simple correlation coefficient of the column 5 ("RE (÷)" obtained for "amounts" approach) and column 10 ("RE (÷)" for "deviations" approach), capturing the data for all ITBDs together. This coefficient ranges from 0.17 in 2006 to 0.40 in 2004. On the other hand, if we do the same exercise for reranking, the correlation coefficient for "R^{LY} (÷)" of two approaches (not shown in Table 5.3 and Table 5.4), is almost 1, evidencing that the two approaches decompose reranking almost identically, which may be also proved by comparing the columns 3 and 8 ("R^{LY} (% RE)"). In section 3.3.4.3 we recommended the decomposition of reranking based on the "amounts" approach, but from this empirical evidence we conclude that "deviations" approach is similarly relevant.

Table 5.3: Decompositions of V^{LY}, R^{LY} and RE, $E_3(.5,.3)$, 2006

		"Amounts"					*"Deviations"*			
	% T&B	V^{LY} (%*RE*)	R^{LY} (%*RE*)	*RE* (%*RE*)	*RE* (÷)	R^{LY} (%R^{LY})	V^{LY} (%*RE*)	R^{LY} (%*RE*)	*RE* (%*RE*)	*RE* (÷)
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
ITBD 1										
sscp	19.7	22.1	3.2	25.3	*1.3*	9.7	8.6	4.1	12.8	*0.6*
ssch	20.4	22.9	3.3	26.2	*1.3*	10.1	8.9	4.3	13.2	*0.6*
sscu	2.0	2.3	0.3	2.6	*1.3*	0.9	1.0	0.4	1.4	*0.7*
pit	10.4	18.6	0.6	19.2	*1.8*	2.0	11.9	1.2	12.8	*1.2*
unem	0.9	0.2	-0.1	0.1	*0.1*	-0.2	0.9	-0.1	0.8	*0.8*
sick	0.5	-0.1	0.1	0.1	*0.1*	0.4	0.3	0.1	0.5	*0.8*
chbn	1.3	1.1	-0.3	0.8	*0.6*	-0.9	2.2	-0.3	1.8	*1.3*
bspa	0.9	1.2	0.0	1.1	*1.3*	-0.1	1.9	-0.1	1.8	*2.0*
matr	0.8	-0.4	0.1	-0.3	*-0.4*	0.2	0.2	0.0	0.3	*0.3*
rehb	0.5	0.2	0.2	0.4	*0.7*	0.5	0.6	0.1	0.7	*1.4*
pyo	16.0	-2.3	8.5	6.2	*0.4*	26.0	9.5	7.7	17.3	*1.1*
pol	26.4	1.4	16.8	18.2	*0.7*	51.3	21.2	15.4	36.8	*1.4*
taxes	52.6	65.9	7.5	73.3	*1.4*	22.7	30.4	10.0	40.2	*0.8*
benefits	47.4	1.3	25.4	26.7	*0.6*	77.3	36.8	22.8	59.8	*1.3*
ITBD 2										
sscp	34.0	30.0	2.1	32.1	*0.9*	32.8	21.8	2.1	24.5	*0.7*
ssch	35.1	31.0	2.1	33.2	*0.9*	33.9	22.6	2.2	25.3	*0.7*
sscu	3.4	3.1	0.2	3.3	*1.0*	3.2	2.4	0.2	2.7	*0.8*
pit	18.9	26.5	0.5	27.0	*1.4*	7.5	31.4	0.5	30.6	*1.6*
unem	1.6	0.2	0.2	0.4	*0.3*	2.9	2.2	0.2	2.4	*1.5*
sick	0.9	-0.1	0.4	0.3	*0.3*	6.3	0.9	0.4	1.5	*1.6*
chbn	2.3	1.5	0.1	1.6	*0.7*	1.9	5.4	0.1	5.3	*2.3*
bspa	1.5	1.6	0.2	1.8	*1.2*	3.1	4.8	0.2	4.8	*3.2*
matr	1.4	-0.5	0.3	-0.2	*-0.1*	4.9	0.6	0.3	1.1	*0.8*
rehb	0.9	0.3	0.2	0.5	*0.5*	3.5	1.5	0.2	1.8	*2.0*
pyo										
pol										
taxes	91.3	90.7	4.9	95.6	*1.0*	77.5	78.2	5.0	83.1	*0.9*
benefits	8.7	3.0	1.4	4.4	*0.5*	22.5	15.4	1.3	16.9	*2.0*
ITBD 3										
sscp										
ssch	53.1	48.2	1.2	49.4	*0.9*	33.5	31.8	1.2	33.1	*0.6*
sscu	5.2	4.9	0.1	5.0	*1.0*	3.1	3.4	0.1	3.5	*0.7*
pit	28.6	38.9	0.3	39.3	*1.4*	9.7	40.1	0.4	39.7	*1.4*
unem	2.4	0.3	0.3	0.6	*0.3*	7.6	3.0	0.3	3.3	*1.4*
sick	1.4	0.0	0.5	0.5	*0.3*	14.4	1.4	0.5	2.2	*1.5*
chbn	3.5	2.0	0.2	2.3	*0.6*	6.1	7.1	0.2	7.3	*2.1*
bspa	2.3	2.4	0.2	2.6	*1.1*	6.7	6.5	0.2	6.7	*2.9*
matr	2.2	-0.7	0.4	-0.3	*-0.1*	11.9	1.0	0.4	1.6	*0.8*
rehb	1.4	0.5	0.2	0.7	*0.5*	7.0	2.2	0.2	2.6	*1.9*
pyo										
pol										
taxes	86.9	92.0	1.6	93.6	*1.1*	46.3	75.3	1.7	76.3	*0.9*
benefits	13.1	4.5	1.9	6.4	*0.5*	53.7	21.2	1.8	23.7	*1.8*

		"Amounts"					"Deviations"			
	% T&B	V^{LY} (%RE)	R^{LY} (%RE)	RE (%RE)	RE (÷)	R^{LY} (%R^{LY})	V^{LY} (%RE)	R^{LY} (%RE)	RE (%RE)	RE (÷)
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
ITBD 4										
sscp	25.4	31.6	4.2	35.8	*1.4*	10.4	9.1	5.6	14.7	*0.6*
ssch										
sscu										
pit	13.4	26.6	0.9	27.5	*2.0*	2.3	12.4	1.6	15.3	*1.1*
unem	1.2	0.3	-0.1	0.2	*0.2*	-0.2	0.9	-0.1	0.9	*0.8*
sick	0.7	-0.1	0.2	0.1	*0.2*	0.5	0.4	0.2	0.5	*0.8*
chbn	1.7	1.6	-0.4	1.2	*0.7*	-1.0	2.3	-0.5	2.1	*1.2*
bspa	1.1	1.7	-0.1	1.6	*1.4*	-0.1	2.0	-0.1	2.1	*1.9*
matr	1.1	-0.5	0.1	-0.4	*-0.4*	0.2	0.3	0.0	0.3	*0.3*
rehb	0.7	0.3	0.2	0.5	*0.8*	0.6	0.6	0.2	0.9	*1.3*
pyo	20.6	-3.3	11.7	8.4	*0.4*	29.3	9.9	11.0	20.1	*1.0*
pol	34.0	2.0	23.2	25.2	*0.7*	58.1	22.2	22.1	43.0	*1.3*
taxes	38.9	58.2	5.1	63.3	*1.6*	12.7	21.5	7.2	30.0	*0.8*
benefits	61.1	1.9	34.9	36.7	*0.6*	87.3	38.6	32.7	70.0	*1.1*
ITBD 5										
sscp	55.2	47.9	1.9	49.8	*0.9*	38.2	30.2	2.0	32.4	*0.6*
ssch										
sscu										
pit	30.7	42.3	0.6	42.9	*1.4*	13.0	43.5	0.7	43.1	*1.4*
unem	2.5	0.4	0.3	0.7	*0.3*	6.8	3.0	0.3	3.5	*1.4*
sick	1.5	-0.1	0.7	0.5	*0.3*	13.2	1.2	0.6	2.2	*1.4*
chbn	3.7	2.4	0.3	2.6	*0.7*	5.6	7.5	0.3	7.7	*2.1*
bspa	2.5	2.5	0.3	2.8	*1.1*	6.0	6.6	0.3	6.9	*2.7*
matr	2.3	-0.8	0.5	-0.2	*-0.1*	10.7	0.9	0.5	1.7	*0.7*
rehb	1.5	0.4	0.3	0.8	*0.5*	6.6	2.1	0.3	2.5	*1.7*
pyo										
pol										
taxes	85.9	90.2	2.6	92.7	*1.1*	51.1	73.6	2.7	75.6	*0.9*
benefits	14.1	4.8	2.4	7.3	*0.5*	48.9	21.3	2.3	24.4	*1.7*
ITBD 6										
sscp										
ssch										
sscu										
pit	68.6	85.0	0.6	85.7	*1.2*	12.5	62.1	0.7	62.2	*0.9*
unem	5.6	0.7	0.7	1.4	*0.2*	13.2	4.6	0.7	5.3	*0.9*
sick	3.4	-0.1	1.2	1.1	*0.3*	22.7	2.1	1.1	3.5	*1.0*
chbn	8.4	4.5	0.7	5.1	*0.6*	12.7	11.0	0.6	11.7	*1.4*
bspa	5.6	5.1	0.5	5.6	*1.0*	9.6	10.1	0.5	10.5	*1.9*
matr	5.2	-1.5	1.0	-0.5	*-0.1*	19.3	1.5	1.0	2.7	*0.5*
rehb	3.3	1.1	0.5	1.6	*0.5*	10.0	3.5	0.5	4.1	*1.2*
pyo										
pol										
taxes	68.6	85.0	0.6	85.7	*1.2*	12.5	62.1	0.7	62.2	*0.9*
benefits	31.4	9.8	4.5	14.3	*0.5*	87.5	32.8	4.4	37.8	*1.2*

Source: *author's calculations*

Table 5.4: Decompositions of V^K, R^{AP} and RE, E_3(.5,.3), 2006

		"Amounts"					"Deviations"			
	% T&B	V^K (%RE)	R^{AP} (%RE)	RE (%RE)	RE (÷)	R^{AP} (%R^{AP})	V^K (%RE)	R^{AP} (%RE)	RE (%RE)	RE (÷)
	1	2	3	4	5	6	7	8	9	10
ITBD 1										
sscp	19.7	26.6	-1.3	25.3	1.3	5.9	2.4	-0.4	2.1	0.1
ssch	20.4	27.5	-1.3	26.2	1.3	6.1	2.5	-0.4	2.1	0.1
sscu	2.0	2.7	-0.1	2.6	1.3	0.5	0.3	0.0	0.3	0.1
pit	10.4	19.3	-0.1	19.2	1.8	0.3	6.8	0.4	7.4	0.7
unem	0.9	0.1	0.1	0.1	0.1	-0.3	1.2	0.0	1.3	1.4
sick	0.5	0.2	-0.1	0.1	0.1	0.5	0.9	-0.1	0.8	1.4
chbn	1.3	0.5	0.3	0.8	0.6	-1.2	2.3	0.2	2.6	1.9
bspa	0.9	0.9	0.2	1.1	1.3	-0.9	2.1	0.2	2.3	2.6
matr	0.8	-0.2	-0.1	-0.3	-0.4	0.3	0.8	-0.1	0.7	0.9
rehb	0.5	0.4	-0.1	0.4	0.7	0.4	1.2	-0.1	1.1	2.0
pyo	16.0	12.9	-6.7	6.2	0.4	30.3	34.4	-7.5	26.8	1.7
pol	26.4	31.1	-12.9	18.2	0.7	58.2	67.1	-14.3	52.7	2.0
taxes	52.6	76.2	-2.8	73.3	1.4	12.8	12.0	-0.3	11.8	0.2
benefits	47.4	45.9	-19.3	26.7	0.6	87.2	110.1	-21.8	88.2	1.9
ITBD 2										
sscp	34.0	33.5	-1.4	32.1	0.9	32.6	22.0	-1.4	20.0	0.6
ssch	35.1	34.6	-1.4	33.2	0.9	33.7	22.7	-1.4	20.7	0.6
sscu	3.4	3.5	-0.1	3.3	1.0	3.1	2.4	-0.1	2.3	0.7
pit	18.9	27.2	-0.2	27.0	1.4	5.2	31.4	-0.2	32.7	1.7
unem	1.6	0.6	-0.1	0.4	0.3	3.3	3.7	-0.2	3.5	2.3
sick	0.9	0.6	-0.3	0.3	0.3	7.4	2.9	-0.3	2.2	2.4
chbn	2.3	1.7	-0.1	1.6	0.7	1.7	7.3	-0.1	7.5	3.3
bspa	1.5	1.9	-0.1	1.8	1.2	3.2	6.6	-0.2	6.6	4.3
matr	1.4	0.1	-0.2	-0.2	-0.1	5.8	2.4	-0.3	1.9	1.3
rehb	0.9	0.7	-0.2	0.5	0.5	3.9	2.9	-0.2	2.6	2.9
pyo										
pol										
taxes	91.3	98.7	-3.2	95.6	1.0	74.7	78.5	-3.0	75.7	0.8
benefits	8.7	5.5	-1.1	4.4	0.5	25.3	25.8	-1.2	24.3	2.8
ITBD 3										
sscp										
ssch	53.1	50.2	-0.9	49.4	0.9	31.1	28.4	-0.8	27.5	0.5
sscu	5.2	5.0	-0.1	5.0	1.0	2.9	3.1	-0.1	3.0	0.6
pit	28.6	39.5	-0.2	39.3	1.4	7.8	39.2	-0.2	39.9	1.4
unem	2.4	0.8	-0.2	0.6	0.3	8.2	4.6	-0.2	4.2	1.8
sick	1.4	0.9	-0.5	0.5	0.3	16.4	3.6	-0.5	2.7	1.9
chbn	3.5	2.4	-0.2	2.3	0.6	6.1	9.1	-0.2	9.0	2.6
bspa	2.3	2.8	-0.2	2.6	1.1	6.6	8.3	-0.2	8.1	3.5
matr	2.2	0.1	-0.4	-0.3	-0.1	13.1	3.0	-0.4	2.3	1.1
rehb	1.4	1.0	-0.2	0.7	0.5	7.8	3.6	-0.2	3.2	2.4
pyo										
pol										
taxes	86.9	94.8	-1.2	93.6	1.1	41.8	70.6	-1.1	70.4	0.8
benefits	13.1	8.0	-1.6	6.4	0.5	58.2	32.2	-1.7	29.6	2.3

		"Amounts"					*"Deviations"*			
	% T&B	V^K (%*RE*)	R^{AP} (%*RE*)	*RE* (%*RE*)	*RE* (÷)	R^{AP} (%R^{AP})	V^K (%*RE*)	R^{AP} (%*RE*)	*RE* (%*RE*)	*RE* (÷)
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
ITBD 4										
sscp	25.4	37.6	-1.8	35.8	*1.4*	6.0	2.6	-0.4	2.2	*0.1*
ssch										
sscu										
pit	13.4	27.6	-0.1	27.5	*2.0*	0.5	7.7	0.6	7.9	*0.6*
unem	1.2	0.1	0.1	0.2	*0.2*	-0.3	1.4	0.0	1.3	*1.1*
sick	0.7	0.3	-0.2	0.1	*0.2*	0.5	1.0	-0.2	0.8	*1.1*
chbn	1.7	0.8	0.4	1.2	*0.7*	-1.2	2.5	0.3	2.7	*1.6*
bspa	1.1	1.3	0.3	1.6	*1.4*	-1.0	2.3	0.2	2.4	*2.1*
matr	1.1	-0.3	-0.1	-0.4	*-0.4*	0.3	0.9	-0.1	0.7	*0.7*
rehb	0.7	0.6	-0.1	0.5	*0.8*	0.4	1.3	-0.2	1.1	*1.6*
pyo	20.6	18.4	-10.0	8.4	*0.4*	32.5	37.6	-10.7	27.2	*1.3*
pol	34.0	44.3	-19.1	25.2	*0.7*	62.3	73.4	-20.2	53.6	*1.6*
taxes	38.9	65.3	-2.0	63.3	*1.6*	6.4	10.3	0.2	10.1	*0.3*
benefits	61.1	65.5	-28.8	36.7	*0.6*	93.6	120.4	-30.9	89.9	*1.5*
ITBD 5										
sscp	55.2	51.2	-1.4	49.8	*0.9*	36.1	27.6	-1.3	26.0	*0.5*
ssch										
sscu										
pit	30.7	43.3	-0.4	42.9	*1.4*	10.7	42.5	-0.4	43.2	*1.4*
unem	2.5	1.0	-0.3	0.7	*0.3*	7.4	4.9	-0.3	4.5	*1.8*
sick	1.5	1.1	-0.6	0.5	*0.3*	14.7	3.8	-0.6	2.8	*1.9*
chbn	3.7	2.9	-0.2	2.6	*0.7*	5.7	9.7	-0.2	9.6	*2.6*
bspa	2.5	3.0	-0.2	2.8	*1.1*	5.9	8.5	-0.2	8.3	*3.3*
matr	2.3	0.2	-0.5	-0.2	*-0.1*	12.1	3.2	-0.5	2.4	*1.0*
rehb	1.5	1.0	-0.3	0.8	*0.5*	7.3	3.7	-0.3	3.2	*2.2*
pyo										
pol										
taxes	85.9	94.6	-1.8	92.7	*1.1*	46.8	70.1	-1.7	69.2	*0.8*
benefits	14.1	9.3	-2.1	7.3	*0.5*	53.2	33.8	-2.2	30.8	*2.2*
ITBD 6										
sscp										
ssch										
sscu										
pit	68.6	86.1	-0.5	85.7	*1.2*	10.2	58.3	-0.4	58.5	*0.9*
unem	5.6	2.0	-0.6	1.4	*0.2*	13.6	6.7	-0.7	6.0	*1.1*
sick	3.4	2.2	-1.1	1.1	*0.3*	24.1	5.3	-1.1	3.9	*1.1*
chbn	8.4	5.7	-0.6	5.1	*0.6*	12.4	13.3	-0.6	12.7	*1.5*
bspa	5.6	6.0	-0.4	5.6	*1.0*	8.6	11.7	-0.4	11.3	*2.0*
matr	5.2	0.5	-1.0	-0.5	*-0.1*	20.5	4.4	-1.0	3.2	*0.6*
rehb	3.3	2.1	-0.5	1.6	*0.5*	10.6	5.0	-0.5	4.4	*1.4*
pyo										
pol										
taxes	68.6	86.1	-0.5	85.7	*1.2*	10.2	58.3	-0.4	58.5	*0.9*
benefits	31.4	18.5	-4.2	14.3	*0.5*	89.8	46.4	-4.3	41.5	*1.3*

Source: *author's calculations*

5.2.4.2 Estimates for the whole period: 2001-2006

In this section, we continue analysis from the previous section, extending it to the whole period. For presentation of results in Table 5.5 and Table 5.6 we choose three years: 2001 and 2006, as a start and the end of the period, and also 2004 as a middle year, especially interesting because in this year some of the previously shown indicators were "irregularly" different than in other years. Table 5.5 shows the results for "amounts of TB" approach, and Table 5.6 for "deviations of TB from proportionality" approach.

We observe steady contributions over the period for all instruments and ITBDs. Small exception is public pensions. In 2004 their contribution to R^{LY} is lower than in other two years; this is not fully compensated by their higher contribution to V^{LY}, and contribution of public pensions to RE is lower. One explanation is that the share of public pensions (*pyo* and *pol*) in overall TB was 40.7% in 2001 and 42.4% in 2006, while only 37.9% in 2004.

Table 5.5: Decompositions of V^{LY}, R^{LY} and RE, "amounts", $E_3(.5,.3)$, 2001-2006

	% T&B			V^{LY} (%*RE*)			R^{LY} (%*RE*)			*RE* (÷)		
	2001	2004	2006	2001	2004	2006	2001	2004	2006	2001	2004	2006
ITBD 1												
sscp	19.3	20.7	19.7	21.3	22.8	22.1	3.3	2.6	3.2	*1.3*	*1.2*	*1.3*
ssch	20.0	21.3	20.4	22.0	23.6	22.9	3.4	2.6	3.3	*1.3*	*1.2*	*1.3*
sscu	1.8	2.1	2.0	2.2	2.4	2.3	0.3	0.2	0.3	*1.4*	*1.3*	*1.3*
pit	11.7	12.4	10.4	21.0	21.3	18.6	0.9	0.6	0.6	*1.9*	*1.8*	*1.8*
unem	1.0	1.0	0.9	-0.3	0.3	0.2	0.1	0.0	-0.1	*-0.2*	*0.3*	*0.1*
sick	0.8	0.7	0.5	-0.6	-0.7	-0.1	0.4	0.3	0.1	*-0.3*	*-0.6*	*0.1*
chbn	2.0	1.6	1.3	0.7	1.0	1.1	-0.3	-0.3	-0.3	*0.2*	*0.4*	*0.6*
bspa	0.9	0.9	0.9	1.1	1.3	1.2	0.1	0.0	0.0	*1.4*	*1.4*	*1.3*
matr	1.3	1.0	0.8	-0.5	-0.5	-0.4	0.1	0.1	0.1	*-0.3*	*-0.5*	*-0.4*
rehb	0.4	0.6	0.5	-0.1	0.2	0.2	0.2	0.1	0.2	*0.1*	*0.6*	*0.7*
pyo	18.6	15.6	16.0	-3.5	-2.4	-2.3	10.7	7.1	8.5	*0.4*	*0.3*	*0.4*
pol	22.1	22.3	26.4	3.1	5.1	1.4	14.3	12.3	16.8	*0.8*	*0.8*	*0.7*
taxes	52.9	56.5	52.6	66.6	70.1	65.9	7.9	6.1	7.5	*1.4*	*1.3*	*1.4*
benefits	47.1	43.5	47.4	-0.1	4.3	1.3	25.6	19.5	25.4	*0.5*	*0.5*	*0.6*
ITBD 2												
sscp	32.2	33.0	34.0	28.9	29.7	30.0	2.3	1.7	2.1	*1.0*	*1.0*	*0.9*
ssch	33.3	34.1	35.1	29.8	30.7	31.0	2.3	1.8	2.1	*1.0*	*1.0*	*0.9*
sscu	3.1	3.3	3.4	3.0	3.1	3.1	0.2	0.2	0.2	*1.0*	*1.0*	*1.0*
pit	20.7	20.7	18.9	30.0	28.7	26.5	0.7	0.5	0.5	*1.5*	*1.4*	*1.4*
unem	1.7	1.5	1.6	-0.4	0.4	0.2	0.4	0.2	0.2	*0.0*	*0.4*	*0.3*
sick	1.3	1.1	0.9	-0.8	-0.9	-0.1	0.7	0.5	0.4	*-0.1*	*-0.3*	*0.3*
chbn	3.3	2.6	2.3	1.0	1.3	1.5	0.3	0.2	0.1	*0.4*	*0.6*	*0.7*
bspa	1.5	1.4	1.5	1.5	1.7	1.6	0.4	0.2	0.2	*1.3*	*1.3*	*1.2*
matr	2.2	1.5	1.4	-0.7	-0.7	-0.5	0.6	0.3	0.3	*-0.1*	*-0.3*	*-0.1*
rehb	0.7	0.9	0.9	-0.2	0.3	0.3	0.3	0.2	0.2	*0.1*	*0.6*	*0.5*
pyo												
pol												
taxes	89.3	91.0	91.3	91.6	92.2	90.7	5.5	4.1	4.9	*1.1*	*1.1*	*1.0*
benefits	10.7	9.0	8.7	0.3	2.0	3.0	2.6	1.7	1.4	*0.3*	*0.4*	*0.5*
ITBD 3												
sscp												
ssch	49.1	50.9	53.1	45.9	46.8	48.2	1.3	1.0	1.2	*1.0*	*0.9*	*0.9*
sscu	4.5	4.9	5.2	4.6	4.7	4.9	0.1	0.1	0.1	*1.0*	*1.0*	*1.0*
pit	30.5	30.8	28.6	43.5	41.8	38.9	0.5	0.4	0.3	*1.4*	*1.4*	*1.4*
unem	2.5	2.3	2.4	-0.5	0.5	0.3	0.5	0.3	0.3	*0.0*	*0.4*	*0.3*
sick	2.0	1.6	1.4	-1.0	-1.2	0.0	0.9	0.7	0.5	*0.0*	*-0.3*	*0.3*
chbn	4.9	3.8	3.5	1.2	1.8	2.0	0.5	0.3	0.2	*0.3*	*0.5*	*0.6*
bspa	2.2	2.1	2.3	2.2	2.4	2.4	0.4	0.3	0.2	*1.2*	*1.3*	*1.1*
matr	3.2	2.3	2.2	-1.1	-1.0	-0.7	0.8	0.4	0.4	*-0.1*	*-0.3*	*-0.1*
rehb	1.1	1.3	1.4	-0.2	0.5	0.5	0.3	0.3	0.2	*0.2*	*0.6*	*0.5*
pyo												
pol												
taxes	84.2	86.6	86.9	94.0	93.3	92.0	1.9	1.5	1.6	*1.1*	*1.1*	*1.1*
benefits	15.8	13.4	13.1	0.7	3.0	4.5	3.5	2.3	1.9	*0.3*	*0.4*	*0.5*

	% T&B			V^{LY} (%*RE*)			R^{LY} (%*RE*)			*RE* (÷)		
	2001	2004	2006	2001	2004	2006	2001	2004	2006	2001	2004	2006
ITBD 4												
sscp	24.7	27.0	25.4	30.2	32.5	31.6	4.2	3.3	4.2	*1.4*	*1.3*	*1.4*
ssch												
sscu												
pit	15.0	16.2	13.4	29.8	30.4	26.6	1.2	0.9	0.9	*2.1*	*1.9*	*2.0*
unem	1.3	1.2	1.2	-0.4	0.4	0.3	0.1	0.0	-0.1	*-0.2*	*0.3*	*0.2*
sick	1.0	0.9	0.7	-0.9	-1.0	-0.1	0.5	0.4	0.2	*-0.4*	*-0.7*	*0.2*
chbn	2.5	2.1	1.7	1.0	1.5	1.6	-0.4	-0.4	-0.4	*0.2*	*0.5*	*0.7*
bspa	1.1	1.2	1.1	1.5	1.8	1.7	0.1	0.0	-0.1	*1.5*	*1.5*	*1.4*
matr	1.7	1.2	1.1	-0.8	-0.8	-0.5	0.2	0.1	0.1	*-0.4*	*-0.5*	*-0.4*
rehb	0.6	0.7	0.7	-0.2	0.3	0.3	0.2	0.2	0.2	*0.1*	*0.7*	*0.8*
pyo	23.8	20.4	20.6	-4.9	-3.4	-3.3	14.7	9.7	11.7	*0.4*	*0.3*	*0.4*
pol	28.3	29.1	34.0	4.4	7.3	2.0	19.5	16.9	23.2	*0.8*	*0.8*	*0.7*
taxes	39.7	43.2	38.9	60.0	62.9	58.2	5.4	4.1	5.1	*1.6*	*1.6*	*1.6*
benefits	60.3	56.8	61.1	-0.2	6.2	1.9	34.9	26.8	34.9	*0.6*	*0.6*	*0.6*
ITBD 5												
sscp	50.7	52.7	55.2	45.3	46.7	47.9	2.1	1.5	1.9	*0.9*	*0.9*	*0.9*
ssch												
sscu												
pit	32.5	33.0	30.7	47.0	45.1	42.3	0.9	0.7	0.6	*1.5*	*1.4*	*1.4*
unem	2.7	2.4	2.5	-0.6	0.6	0.4	0.6	0.4	0.3	*0.0*	*0.4*	*0.3*
sick	2.1	1.7	1.5	-1.3	-1.4	-0.1	1.2	0.8	0.7	*-0.1*	*-0.4*	*0.3*
chbn	5.2	4.1	3.7	1.5	2.1	2.4	0.6	0.4	0.3	*0.4*	*0.6*	*0.7*
bspa	2.3	2.3	2.5	2.3	2.6	2.5	0.6	0.3	0.3	*1.3*	*1.3*	*1.1*
matr	3.4	2.4	2.3	-1.2	-1.1	-0.8	1.0	0.6	0.5	*-0.1*	*-0.2*	*-0.1*
rehb	1.2	1.4	1.5	-0.3	0.4	0.4	0.4	0.3	0.3	*0.1*	*0.6*	*0.5*
pyo												
pol												
taxes	83.2	85.7	85.9	92.2	91.7	90.2	3.0	2.2	2.6	*1.1*	*1.1*	*1.1*
benefits	16.8	14.3	14.1	0.5	3.2	4.8	4.3	2.8	2.4	*0.3*	*0.4*	*0.5*
ITBD 6												
sscp												
ssch												
sscu												
pit	65.9	69.7	68.6	90.2	87.8	85.0	0.8	0.8	0.6	*1.4*	*1.3*	*1.2*
unem	5.5	5.1	5.6	-1.0	1.1	0.7	0.9	0.8	0.7	*0.0*	*0.4*	*0.2*
sick	4.2	3.6	3.4	-2.2	-2.5	-0.1	2.0	1.4	1.2	*0.0*	*-0.3*	*0.3*
chbn	10.5	8.7	8.4	2.5	3.7	4.5	1.4	0.8	0.7	*0.4*	*0.5*	*0.6*
bspa	4.6	4.9	5.6	4.6	5.1	5.1	0.9	0.6	0.5	*1.2*	*1.2*	*1.0*
matr	6.9	5.1	5.2	-2.2	-2.2	-1.5	1.8	1.0	1.0	*-0.1*	*-0.2*	*-0.1*
rehb	2.3	3.0	3.3	-0.3	1.0	1.1	0.6	0.5	0.5	*0.1*	*0.5*	*0.5*
pyo												
pol												
taxes	65.9	69.7	68.6	90.2	87.8	85.0	0.8	0.8	0.6	*1.4*	*1.3*	*1.2*
benefits	34.1	30.3	31.4	1.3	6.2	9.8	7.6	5.2	4.5	*0.3*	*0.4*	*0.5*

Source: *author's calculations*

Table 5.6: Decompositions of V^{LY}, R^{LY} and RE, "deviations", $E_3(.5,.3)$, 2001-2006

	% T&B			V^{LY} (%*RE*)			R^{LY} (%*RE*)			*RE* (÷)		
	2001	2004	2006	2001	2004	2006	2001	2004	2006	2001	2004	2006
ITBD 1												
sscp	19.3	20.7	19.7	7.4	9.1	8.6	4.3	3.3	4.1	0.6	0.6	0.6
ssch	20.0	21.3	20.4	7.7	9.4	8.9	4.4	3.4	4.3	0.6	0.6	0.6
sscu	1.8	2.1	2.0	0.9	1.0	1.0	0.4	0.3	0.4	0.7	0.7	0.7
pit	11.7	12.4	10.4	13.1	14.2	11.9	1.4	1.1	1.2	1.2	1.2	1.2
unem	1.0	1.0	0.9	0.5	1.1	0.9	0.1	0.0	-0.1	0.5	1.1	0.8
sick	0.8	0.7	0.5	0.0	-0.2	0.3	0.3	0.2	0.1	0.4	0.1	0.8
chbn	2.0	1.6	1.3	2.3	2.5	2.2	-0.4	-0.4	-0.3	0.9	1.2	1.3
bspa	0.9	0.9	0.9	1.9	2.2	1.9	0.1	0.0	-0.1	2.2	2.3	2.0
matr	1.3	1.0	0.8	0.4	0.2	0.2	0.0	0.1	0.0	0.4	0.2	0.3
rehb	0.4	0.6	0.5	0.2	0.7	0.6	0.1	0.1	0.1	0.8	1.4	1.4
pyo	18.6	15.6	16.0	11.0	10.1	9.5	9.7	6.3	7.7	1.1	1.1	1.1
pol	22.1	22.3	26.4	20.9	24.1	21.2	13.1	11.2	15.4	1.5	1.6	1.4
taxes	52.9	56.5	52.6	29.2	33.8	30.4	10.5	8.0	10.0	0.7	0.7	0.8
benefits	47.1	43.5	47.4	37.3	40.6	36.8	23.0	17.5	22.8	1.3	1.3	1.3
ITBD 2												
sscp	32.2	33.0	34.0	19.4	21.2	21.8	2.3	1.7	2.1	0.7	0.7	0.7
ssch	33.3	34.1	35.1	20.1	21.9	22.6	2.4	1.8	2.2	0.7	0.7	0.7
sscu	3.1	3.3	3.4	2.4	2.4	2.4	0.2	0.2	0.2	0.8	0.8	0.8
pit	20.7	20.7	18.9	36.1	33.9	31.4	0.8	0.6	0.5	1.7	1.6	1.6
unem	1.7	1.5	1.6	1.3	2.6	2.2	0.3	0.2	0.2	1.1	1.9	1.5
sick	1.3	1.1	0.9	-0.1	-0.6	0.9	0.7	0.5	0.4	0.9	0.4	1.6
chbn	3.3	2.6	2.3	6.1	5.8	5.4	0.2	0.1	0.1	1.9	2.2	2.3
bspa	1.5	1.4	1.5	4.9	5.1	4.8	0.3	0.2	0.2	3.5	3.6	3.2
matr	2.2	1.5	1.4	1.1	0.4	0.6	0.5	0.3	0.3	0.9	0.6	0.8
rehb	0.7	0.9	0.9	0.6	1.6	1.5	0.2	0.2	0.2	1.3	2.2	2.0
pyo												
pol												
taxes	89.3	91.0	91.3	78.0	79.3	78.2	5.7	4.3	5.0	0.9	0.9	0.9
benefits	10.7	9.0	8.7	14.0	14.9	15.4	2.4	1.5	1.3	1.6	1.9	2.0
ITBD 3												
sscp												
ssch	49.1	50.9	53.1	28.3	29.9	31.8	1.4	1.0	1.2	0.6	0.6	0.6
sscu	4.5	4.9	5.2	3.2	3.3	3.4	0.1	0.1	0.1	0.7	0.7	0.7
pit	30.5	30.8	28.6	44.4	42.8	40.1	0.6	0.5	0.4	1.4	1.4	1.4
unem	2.5	2.3	2.4	1.9	3.5	3.0	0.4	0.3	0.3	1.0	1.7	1.4
sick	2.0	1.6	1.4	0.3	-0.4	1.4	0.9	0.7	0.5	0.9	0.5	1.5
chbn	4.9	3.8	3.5	7.5	7.5	7.1	0.5	0.3	0.2	1.7	2.0	2.1
bspa	2.2	2.1	2.3	6.4	6.8	6.5	0.4	0.3	0.2	3.2	3.3	2.9
matr	3.2	2.3	2.2	1.6	0.6	1.0	0.7	0.4	0.4	0.9	0.6	0.8
rehb	1.1	1.3	1.4	0.9	2.3	2.2	0.3	0.2	0.2	1.3	2.0	1.9
pyo												
pol												
taxes	84.2	86.6	86.9	76.0	76.0	75.3	2.1	1.6	1.7	0.9	0.9	0.9
benefits	15.8	13.4	13.1	18.7	20.2	21.2	3.3	2.2	1.8	1.5	1.7	1.8

	% T&B			V^{LY} (%RE)			R^{LY} (%RE)			RE (÷)		
	2001	2004	2006	2001	2004	2006	2001	2004	2006	2001	2004	2006
ITBD 4												
sscp	24.7	27.0	25.4	7.7	9.9	9.1	5.6	4.3	5.6	0.5	0.5	0.6
ssch												
sscu												
pit	15.0	16.2	13.4	13.6	15.4	12.4	2.0	1.5	1.6	1.1	1.1	1.1
unem	1.3	1.2	1.2	0.5	1.2	0.9	0.1	-0.1	-0.1	0.5	1.0	0.8
sick	1.0	0.9	0.7	0.0	-0.3	0.4	0.5	0.3	0.2	0.3	0.0	0.8
chbn	2.5	2.1	1.7	2.4	2.7	2.3	-0.6	-0.5	-0.5	0.9	1.1	1.2
bspa	1.1	1.2	1.1	1.9	2.4	2.0	0.1	-0.1	-0.1	2.0	2.1	1.9
matr	1.7	1.2	1.1	0.5	0.2	0.3	0.1	0.1	0.0	0.3	0.2	0.3
rehb	0.6	0.7	0.7	0.2	0.8	0.6	0.2	0.1	0.2	0.7	1.3	1.3
pyo	23.8	20.4	20.6	11.4	10.9	9.9	13.8	9.1	11.0	1.0	1.0	1.0
pol	28.3	29.1	34.0	21.6	26.1	22.2	18.5	16.0	22.1	1.4	1.4	1.3
taxes	39.7	43.2	38.9	21.3	25.2	21.5	7.6	5.8	7.2	0.8	0.7	0.8
benefits	60.3	56.8	61.1	38.5	43.9	38.6	32.6	25.1	32.7	1.2	1.2	1.1
ITBD 5												
sscp	50.7	52.7	55.2	25.9	28.8	30.2	2.2	1.6	2.0	0.6	0.6	0.6
ssch												
sscu												
pit	32.5	33.0	30.7	48.1	46.0	43.5	1.0	0.8	0.7	1.5	1.4	1.4
unem	2.7	2.4	2.5	1.7	3.5	3.0	0.5	0.4	0.3	0.9	1.6	1.4
sick	2.1	1.7	1.5	-0.1	-0.8	1.2	1.1	0.8	0.6	0.8	0.3	1.4
chbn	5.2	4.1	3.7	8.2	7.8	7.5	0.6	0.3	0.3	1.7	2.0	2.1
bspa	2.3	2.3	2.5	6.5	6.9	6.6	0.5	0.3	0.3	3.1	3.1	2.7
matr	3.4	2.4	2.3	1.5	0.5	0.9	0.9	0.5	0.5	0.9	0.6	0.7
rehb	1.2	1.4	1.5	0.8	2.2	2.1	0.4	0.3	0.3	1.2	1.9	1.7
pyo												
pol												
taxes	83.2	85.7	85.9	74.1	74.8	73.6	3.2	2.4	2.7	0.9	0.9	0.9
benefits	16.8	14.3	14.1	18.6	20.2	21.3	4.1	2.7	2.3	1.5	1.7	1.7
ITBD 6												
sscp												
ssch												
sscu												
pit	65.9	69.7	68.6	64.5	63.9	62.1	1.0	0.9	0.7	1.0	0.9	0.9
unem	5.5	5.1	5.6	2.8	5.2	4.6	0.9	0.7	0.7	0.7	1.2	0.9
sick	4.2	3.6	3.4	0.4	-0.7	2.1	1.9	1.4	1.1	0.7	0.3	1.0
chbn	10.5	8.7	8.4	10.9	11.1	11.0	1.4	0.8	0.6	1.2	1.4	1.4
bspa	4.6	4.9	5.6	9.3	10.1	10.1	0.9	0.6	0.5	2.2	2.2	1.9
matr	6.9	5.1	5.2	2.3	0.9	1.5	1.8	1.0	1.0	0.6	0.4	0.5
rehb	2.3	3.0	3.3	1.3	3.5	3.5	0.6	0.5	0.5	0.9	1.4	1.2
pyo												
pol												
taxes	65.9	69.7	68.6	64.5	63.9	62.1	1.0	0.9	0.7	1.0	0.9	0.9
benefits	34.1	30.3	31.4	27.1	30.1	32.8	7.5	5.1	4.4	1.0	1.2	1.2

Source: *author's calculations*

6 CONCLUSION AND IMPLICATIONS

6.1 Implications of the research

This dissertation analysed the concepts in measurement of income redistribution existing in the literature for the last thirty years, including the Atkinson (1980) and Plotnick (1981) index of reranking and Kakwani (1984) decomposition of redistributive effect into vertical and reranking terms. These concepts received a great amount of attention among the scholars and researchers. Their methodologies were updated and extended, and used in a variety of empirical studies of income redistribution.

Algebraic validity of the well-known decompositions is not questionable: the decomposed index *is* a sum (or difference, or some other combination) of its components. Thus, in case of Kakwani (1984) decomposition, we have that $RE = V^K - R^{AP}$. Each index (R^{AP}, V^K and RE) can be independently derived and the above identity can be proved. What is doubtful is whether these indices and their interactions are properly interpreted. Are the indices R^{AP} and V^K separate and independent "beings"? Can we increase RE by decreasing R^{AP} leaving V^K unchanged?

These are the questions to which the major part of methodological section of this dissertation is devoted. The whole investigation was motivated by antagonism between the inventors of reranking effect, Atkinson and Plotnick, and originator of the progressivity index, Kakwani. The former two scholars have claimed that reranking does not influence the redistributive effect, and also suggested that reranking index should not be involved into a more comprehensive framework, together with the progressivity index. The latter author has neglected both advices and Kakwani (1984) decomposition appeared, becoming one of the most widely used tools in analysis of income redistribution, progressivity and reranking.

The *first hypothesis* of this dissertation is that reranking cannot influence the redistributive effect, in a manner proposed by researchers using Kakwani decomposition. They claim that the vertical effect (V^K) has a meaning of potential redistributive effect, which would be achieved if reranking (presented by R^{AP}) would be 'somehow eliminated'. However, this work has proved that such task is incomprehensible: 'elimination of reranking' always leads to decrease of vertical effect leaving the redistributive effect unchanged. The implication for researchers is that they should avoid interpretation mentioned above. Moreover, they must be cautious when interpreting results of any decomposition.

This analysis also shows that the Kakwani index of vertical effect (V^K) has major weaknesses as a measure of progressivity, exactly because it is not independent of reranking; actually, it contains reranking in itself. Some of these notions were proposed already by Lerman and Yitzhaki (1995), but in this work they are extended and analytically proved. Thus, another implication of this research would be to avoid using V^K as an index of progressivity.

Ruling out some interpretations and uses of indices created a hole which had to be filled. Therefore, new indices – or rather say – new interpretations of existing indices are proposed: indices of fiscal deprivation, fiscal domination, distance narrowing, and others are invented. The implication is that the well-known indices do not have to be forgotten; instead, they are given new life, and should be used and developed further in the new contexts.

Another methodological innovation is presented in this work: decomposition of reranking effect to reveal contributions of individual taxes and benefits. This was not the first attempt in this task; Jenkins (1988) and Duclos (1993) proposed their models, but each has certain weaknesses. A new model of decomposition fully and straightforwardly decomposes Atkinson-Plotnick (R^{AP}) and Lerman-Yitzhaki (R^{LY}) reranking effects. This also enables us to determine contributions of different fiscal instruments to the redistributive effect (RE). The implication of this advancement is self-evident: we can learn from the data how much each tax and benefit contributes to distance narrowing and reranking.

These new indices and decompositions are then applied to check the *second hypothesis*, which is empirical and relates to Croatian system of individual taxes and social transfers. In this case we have a group of hypotheses, concerning the role of fiscal instruments on income inequality. They are proved to be correct and empirical analysis has discovered other important findings.

This study has analysed a section of total fiscal system in Croatia, consisting of social security contributions, personal income tax, public pensions and cash social benefits. The data are obtained from the household budget survey and tax variables are imputed using a microsimulation model designed purposefully for this study.

Income, tax and social transfer variables are combined to create six different definitions of the analysed fiscal subsystem, respecting various assumptions on tax and benefit incidence. For example, two definitions treat public pensions as social benefits, while other definitions consider them as market income. Some definitions regard that social security contributions paid by employers are fully shifted to employees, while other definitions assume the opposite. Also, social security contributions to the pension fund are treated as tax in one set of definitions, while they are individual saving in other definitions.

Reduction of income inequality caused by the fiscal system is estimated using a measure of redistributive effect(RE), which is a difference between Gini coefficients of pre-fiscal (G_X) and post-fiscal income (G_N). Here, pre-fiscal income denotes income before taxes and benefits, while post-fiscal income represents income after taxes and benefits. Pre-fiscal income, post-fiscal income, taxes and benefits are separately defined for each of the six definitions of the fiscal system.

The empirical analysis has confirmed the *hypothesis* that the fiscal system is one of the prime determinants of disposable income inequality in Croatia. Reduction of income inequality caused by the fiscal system ranges from 10% for the least comprehensive definition of the fiscal system, including only personal income tax and cash social benefits, to 40% for the most inclusive definition of the fiscal system, capturing public pensions and social security contributions, together with personal income tax and cash social benefits.

If we assume the latter, all-inclusive definition of the fiscal system, the largest part of income redistribution is achieved by public pensions, social security contributions, and personal income tax. However, this study does not offer the definite conclusion about the order of importance of these three groups of fiscal instruments in achieving inequality reduction. One of the *hypotheses* claimed that public pensions are the main contributor to the redistributive effect. Indeed, one set of decompositions – those based on “deviations of taxes and benefits from proportionality" – provided evidence for that conclusion. On the other hand, decompositions based on "amounts of taxes and benefits" have shown that social security contributions and personal income tax are far more important than public pensions.

The two approaches mentioned above significantly differ in criterions by which progressivity and regressivity are defined. Therefore, it is not surprising that results notably disagree. "Amounts" approach is inclined to stress the primary role of taxes, and "deviations" approach favours benefits. The implication of this finding is interesting for researchers as it may offer a new perspective on a role of taxes and benefits in inequality reduction.

Whatever the order of importance of major fiscal instruments in fiscal distribution, the important finding of this research is that they all decrease income inequality.

Further findings of the empirical section are concerned with reranking of income units. How much reranking is introduced by Croatian fiscal system? The estimates of reranking are largely dependent on how we define the fiscal system, and the measures range from modest 1% of G_X (6% of RE) for the narrowly defined system, containing only personal income tax and cash social benefits, to vary large amounts of over 15% of G_X (34% of RE) when the fiscal system is widely defined, involving public pensions and social security contributions, together with personal income tax and cash social benefits.

Again, if we assume the widest definition of the fiscal system, which includes public pensions as benefits and all social security contributions as taxes, then public pensions are undoubtedly the largest contributor to reranking, with a share of more than 75%. They are followed by social security contributions, whose share is about 20%. Personal income tax, on the other hand, contributes only mildly to reranking.

Reranking is generally considered as inequitable, and implication of these findings is that the policy makers could reduce inequity felt by the public by redesigning some parts of the fiscal system. However, they have to be very cautious in interpretation of the results, as will be noted in the next section.

6.2 Limits of the research

The first set of limits of this research lies in the indicators used in measurement of the redistributive effect. Each indicator has its own limitations, and a researcher must be aware of them when interpreting the results. A lot of attention in this work is devoted exactly to arguments that the results of Kakwani decomposition were misapprehended in the literature for a long time; the indicators of reranking and vertical effect were given interpretations that were above their limits.

New indices and new interpretations of the existing indices are offered in this paper, each with its own limit. One of the limits of this research is in the fact that – due to limited time for writing of this thesis – not all properties and nuisances of the new indices could be tested and interpreted. This is left for further research.

The limits of every empirical research lie in the quality of data. In this case, data issues pose a great problem. It is typical for household budget data surveys that they underrepresent incomes at the upper tail of income distribution. The comparative analysis presented in this work provides such evidence for Croatia. It also seems that amounts of certain social benefits are seriously underreported. We may expect that these data deficiencies have a grave impact on the results of empirical study.

Although substantial fraction of Croatian fiscal system is covered by this research, a large part is not analyzed; for example, health and education system, and indirect taxes. However, this is not a shortage of this particular study. It has pursued an approach that is often in research of income redistribution: to analyze fiscal subsystem consisting of personal taxes and cash social benefits. One of the reasons is that for these fiscal instruments determination of amounts for each individual is relatively easy.

Another problem was how to define the fiscal subsystem that is object of analysis. Should public pensions be considered as social benefits? Are social security contributions taxes? Definite answers were not provided here, and therefore, different scenarios or definitions were used, as already explained in this chapter. However, neither of these scenarios provide true picture – it lies somewhere between. Therefore, we have to be very cautious when interpreting the results and deriving conclusions.

6.3 Recommendations for further research

Suggestions for further research are already announced in the previous section on the limits of this research. For a more convincing analysis of redistributive effects in Croatia, better databases must be formed. This can be done by merging datasets from various official sources; for example, population survey, tax administration, pension funds and relevant welfare state ministries and agencies.

A special analysis of the pension system should be undertaken to estimate its redistributive properties. It would be ideal if we could divide each pension into two parts: one that is "earned" and the other that is a benefit from the government (or tax, if the pension is lower than it should be, based on contributions). The former part would always be included into pre-fiscal incomes, the latter would always be benefit (or tax). Something similar could be done for social security contributions. The data needed for such investigation are perhaps obtainable from the Croatian Pension Institute.

This research opens a path to further investigation of fiscal incidence and income redistribution in Croatia. Further research would dwell with other fiscal instruments, such as indirect taxes and benefits in-kind, leading to the coverage of the whole fiscal system.

7 LITERATURE

Ankrom, J. (1993). An analysis of horizontal and vertical equity in Sweden, the United States and the UK. *Scandinavian Journal of Economics*, 95(1), 119-124.

Aronson, J.R., Johnson, P. & Lambert, P.J. (1994). Redistributive effect and unequal income tax treatment. *The Economic Journal*, 104(423), 262-270.

Aronson, R., Lambert, P.J. & Trippeer, D. (1999). Estimates of the Changing Equity Characteristics of the U.S. Income Tax with International Conjectures. *Public Finance Review*, 27, 138–59.

Atkinson, A.B. (1980). Horizontal Equity and the Distribution of the Tax Burden. H.J. Aaron & M.J. Boskins (eds), *The Economics of Taxation* (pp. 3-18). Washington D.C.: Brookings.

Baum, S. (1987). On the Measurement of Tax Progressivity: Relative Share Adjustment. *Public Finance Review*, 15(2), 166-187

Berliant, M.C. & Strauss, R.P. (1985). The Vertical and Horizontal Equity Characteristics of the Federal Individual Income Tax. M. David & T. Smeeding (eds), *Horizontal Equity, Uncertainty, and Economic Well-being* (pp. 179-211). University of Chicago Press.

Bishop, J., Chow, V. & Formby, J. (1995). The Redistributive Effect of Direct Taxes: a Comparison of Six Luxembourg Income Study-Countries. *Journal of Income Distribution*, 5, 65–90.

Cissé, B., S. Luchini & Moatti, J. P. (2007). Progressivity and horizontal Equity in Health Care Finance and Delivery: What about Africa? *Health Policy*, 80, 51-68.

Čok, M. et al. (2008). Use of Simulation Models for the Tax Reform in Slovenia. Financial Theory and Practice, 32 (1), 29-43.

Čok, M. & Urban, I. (2007). Distribution of income and taxes in Slovenia and Croatia. *Post-Communist Economies*, 19(3), 299-316.

Creedy, J. (2002). The GST and Vertical, Horizontal and Reranking Effects of Indirect Taxation In Australia, *Australian Economic Review*. 35, 380–90.

Creedy, J., et al. (2008). Equity and Efficiency Measures of Tax-Transfer Systems: Some Evidence for New Zealand. New Zealand Treasury Working Paper 08/04.

Creedy, J. & van de Ven, J. (2001). Decomposing Redistributive Effects of Taxes and Transfers in Australia: Annual and Lifetime Measures, *Australian Economic Papers*. 40, 185–98.

Cutler, D.M. & Katz, L. (1992). Rising inequality? Changes in the distribution of income and consumption in the 1980's. *American Economic Review*, 82, 546-551.

Dardanoni, V. & Lambert, P.J. (2001). Horizontal Inequity Comparisons, *Social Choice and Welfare*. 18, 799–816.

Dardanoni, V. & Lambert, P.J. (2002). Progressivity Comparisons. *Journal of Public Economics*, 86, 99–122.

Decoster, A. & Van Camp, G. (2001). Redistributive Effects of the Shift From Personal Income Taxes to Indirect Taxes: Belgium 1988-1993. *Fiscal Studies*, 22, 79–106.

Dilnot, A., J. Kay & Norris, C. (1984). The UK Tax System, Structure and Progressivity, 1948-1982. *Scandinavian Journal Of Economics*, 86, 150–65.

Duclos, J.-Y. (1993). Progressivity, redistribution and equity, with application to the British tax and benefit system. *Public Finance / Finances Publiques*, 48(3), 350-365.

Duclos, J.-Y. (2000). Gini Indices and the Redistribution of Income. *International Tax and Public Finance*, 7, 141–162.

Duclos, J.-Y. & Lambert, P.J. (2000). A Normative and Statistical Approach to Measuring Classical Horizontal Inequity. *Canadian Journal of Economics*, 33(1), 87–113.

Duclos, J.-Y., Jalbert, V. & Araar, A. (2003). Classical horizontal inequity and reranking: an integrated approach. *Research on Economic Inequality*, 10, 65–100.

Dyck, D. (2005). Fiscal Redistribution in Canada, 1994-2000. *Canadian Tax Journal / Revue Fiscale Canadienne*, 53(4), 974-1006.

Ervik, R. (1998). The Redistributive Aim of Social Policy: A Comparative Analysis of Taxes, Tax Expenditure Transfers and Direct Transfers in Eight Countries. Luxembourg Income Studies Working Paper 184.

Fellman, J., Jäntti, M. & Lambert, P.J. (1999). Optimal Tax-Transfer Systems and Redistributive Policy. *Scandinavian Journal of Economics*, 101, 115–26.

Förster, M.F. (2000). Trends and Driving Factors in Income Distribution and Poverty in the OECD Area. OECD Labour Market and Social Policy Occasional Paper 42.

Heady, C., Mitrakos, T. & Tsakloglou, P. (2001). The Distributional impact of social transfers in the EU: Evidence from the ECHP. *Fiscal Studies*, 22 (4), 547-565.

Hyun, J.K. & Lim, B.I. (2005). Redistributive effect of Korea's income tax: equity decomposition. *Applied Economics Letters*, 12, 195-198.

Immervoll, H., et al. (2005). Household incomes and redistribution in the European Union: quantifying the equalising properties of taxes and benefits. EUROMOD Working Paper EM9/05.

Iyer, G. S. & Seetharaman, A. (2000). An Evaluation of Alternative Procedures for Measuring Horizontal Inequity. *Journal of the American Taxation Association*, 22, 89–110

Jäntti, M. (1997). Inequality in Five Countries in the 1980s: The Role of Demographic Shifts, Markets and Government Policies. *Economica*, 64, 415–40.

Jakobsson, U. (1976). On the measurement of the degree of progression. *Journal of Public Economics*, 5, 161-168.

Jenkins, S. (1988). Reranking and the analysis of income redistribution. *Scottish Journal of Political Economy*, 35(1), 65-76.

Johannes, T.A., Akwi, T. & Anzah, P.E. (2006). The Distributive Impact of Fiscal Policy in Cameroon: Tax and Benefit Incidence. PMMA Working Paper 2006-16.

Kakwani, N.C. (1977a). Applications of Lorenz curves in economic analysis. *Econometrica*, 45, 719-727.

Kakwani, N.C. (1977b). Measurement of Progressivity: An International Comparison. *The Economic Journal*, 87, 71-80.

Kakwani, N.C., 1984, On the measurement of tax progressivity and redistributive effect of taxes with applications to horizontal and vertical equity. *Advances in Econometrics*, 3, 149-168.

Kakwani, N.C. (1986). Analyzing redistribution policies: A study using Australian data. Cambridge: Cambridge University Press.

Kakwani, N.C. & Lambert, P.J. (1998). On Measuring Inequity in Taxation: A New Approach. *European Journal of Political Economy*, 14, 369–80.

Kaplow, L. (1989). Horizontal Equity: Measures in Search of a Principle. *National Tax Journal*, 42(2), 139-154.

Kaplow, L. (2000). Horizontal Equity: New Measures, Unclear Principles. National Bureau of Economic Research Working Paper 7649.

Kim, K. & Lambert, P.J. (2009). Redistributive Effect of US Taxes and Public Transfers, 1994-2004. *Public Finance Review*, 37 (1), 3-26.

Lambert, P.J. (1985). On the redistributive effect of taxes and benefits. *Scottish Journal of Political Economy*, 32(1), 39-54.

Lambert, P.J. (1988). Net fiscal incidence progressivity: some approaches to measurement. W. Eichhorn (ed.), *Measurement in Economics: Theory and Applications of Economic Indices* (pp. 519-532). Heidelberg: Physica-Verlag.

Lambert, P.J. (2001). The Distribution and Redistribution of Income. (3rd ed.) Manchester and New York: Manchester University Press.

Lerman, R. I. & Yitzhaki, S. (1995). Changing ranks and the inequality impacts of taxes and transfers. *National Tax Journal*, 48(1), 45-59.

Lerman, R. & Yitzhaki, S. (1994). Effect of Marginal Changes in Income Sources On U.S. Income Inequality. *Public Finance Review*, 22, 403-417.

Mahler, V.A. & Jesuit, D.K. (2006). Fiscal redistribution in the developed countries: new insights from the Luxembourg Income Study. *Socio-Economic Review*, 4, 483-511.

Musgrave, R. A. & Thin, T. (1948). Income tax progression, 1929-48. *The Journal of Political Economy*, 56(6), 498-514.

Nolan, B. (1987). Direct Taxation, Transfers and Reranking: Some Empirical Results for the UK. *Oxford Bulletin of Economics and Statistics*, 49, 273 –290.

Norregaard, J. (1990). Progressivity of Income Tax Systems. *OECD Economic Studies*, 15, 83-110.

Plotnick, R. (1981). A Measure of Horizontal Equity. *The Review of Economics and Statistics*, 63(2), 283-288.

Plotnick, R. (1985). A Comparison of Measures of Horizontal Inequity. M. David & T. Smeeding (eds), *Horizontal Equity, Uncertainty, and Economic Well-being* (pp. 239-268). Chicago: University of Chicago Press.

Pyatt, G., Chen, C. & Fei, J. (1980). The Distribution of Income by Factor Components. *The Quarterly Journal of Economics*, 95(3), 451-73.

Reynolds, M. & Smolensky, E. (1977). Public Expenditure, Taxes and the Distribution of Income: The United States, 1950, 1961, 1970. New York: Academic Press.

Shorrocks, A. (1982). Inequality Decomposition by Factor Components. *Econometrica*, 50, 193–211.

Smith, J.P. (2001). Progressivity of the Commonwealth Personal Income Tax, 1917-1997. *The Australian Economic Review*, 34(3), 263-278.

Thoresen, T.O. (2004). Reduced Tax Progressivity in Norway: The Effect from Tax Changes. *International Tax and Public Finance*, 11, 487–506.

Urban, I. (2006). Progressivity of personal income tax in Croatia: decomposition of tax base and rate effects. *Financial Theory and Practice*, 30(3), 207-231

Urban, I. (2008). Income Redistribution in Croatia: The Role of Individual Taxes and Social Transfers. *Financial Theory and Practice*, 32 (3), 387-403.

Urban, I. & Lambert, P.J. (2005). Redistribution, horizontal inequity and reranking: how to measure them properly. University of Oregon Economics Department Working Papers 2005-12.

Urban, I. & Lambert, P.J. (2008). Redistribution, horizontal inequity and reranking: how to measure them properly. *Public Finance Review*, 36 (5), 563-587.

van de Ven, J., Creedy, J. & Lambert, P.J. (2001). Close equals and calculation of the vertical, horizontal and reranking effects of taxation. *Oxford Bulletin of Economics and Statistics*, 63, 381-394.

van Doorslaer, E. et al. (1999). The redistributive effect of health care finance in twelve OECD countries. *Journal of Health Economics*, 18, 291-313.

Verbist G. (2004). Redistributive Effect and Progressivity of Income Taxes: An International Comparison across the EU using EUROMOD. EUROMOD Working Paper EM5/04.

Wagstaff, A. et al. (1999a). Redistributive effect, progressivity and differential tax treatment: personal income taxes in twelve OECD countries. *Journal of Public Economics*, 72, 73-98.

Wagstaff, A., et al. (1999b). The redistributive effect of health care finance in twelve OECD countries. *Journal of Health Economics*, 18, 291-313.

Wagstaff, A. & van Doorslaer, E. (2001). What Makes The Personal Income Tax Progressive? A Comparative Analysis of Fifteen OECD Countries. *International Tax and Public Finance*, 8, 299–315.

Zaidi, S. (2009). Main Drivers of Income Inequality in Central European and Baltic Countries. The World Bank: Policy Research Working Paper 4815.

REDISTRIBUTIVNI UČINKI NEPOSREDNIH DAVKOV IN SOCIALNIH PREJEMKOV NA HRVAŠKEM

Ivica Urban

POVZETEK

Predmet in cilji analize

V zadnjih treh desetletjih se je na globalni ravni močno povečalo zanimanje za merjenje redistributivnih učinkov znotraj fiskalnih sistemov. Raziskave na tem področju so podkrepljene s splošnim prepričanjem, da igra država glavno vlogo pri določanju ekonomske neenakosti v družbi. Ta trditev je nesporno dokazana z različnimi empiričnimi študijami. Mnogi raziskovalci s področja prerazdelitve dohodkov so si zastavili naslednje vprašanje: kaj pomenijo posamezni davki in prejemki za redistributivni učinek?

Literatura na temo merjenja redistributivnega učinka je pričela izhajati sredi sedemdesetih let, pol desetletja po začetnih delih s tesno povezanega področja merjenja ekonomske neenakosti. Čeprav je bila prva znana raziskava o redistributivnem učinku avtorjev Musgrave in Thin (1948) napisana pred šestdesetimi leti, sta deli Jakobssona (1976) in Kakwanija (1977a, 1977b) oblikovali temelje za vse nadaljnje raziskave. Kot poudarja Lambert (2001): "Njuna osrednja dognanja so znana […] kot *Jakobsson/Kakwani* teoremi in izpostavljajo povezave med progresivnim obdavčenjem dohodkov in lastnosti krivulje koncentracije pred in po obdavčitvi".

Progresivni davek potisne Lorenzovo krivuljo proti liniji enakosti. Obseg tega premika, izmerjen kot razlika med Ginijevim koeficientom dohodkov pred in po obdavčitvi, je danes znan kot *redistributivni učinek,* ki je tem večji, čim večja sta progresivnost davka in povprečna davčna stopnja. Vsa ta stališča je razložil Kakwani (1977b). Vendar pa je eden od pojavov ostal prikrit kar nekaj časa, dokler ga nista odkrila Atkinson (1980) in Plotnick (1981). Opazila sta, da obdavčitev poleg zmanjšanja dohodkovnih razlik sproža dodaten proces - prerangiranje dohodkovnih enot. Izmeri se kot razlika med Ginijevim koeficientom in koeficientom koncentracije po obdavčitvi.

Znano sintezo vseh teh konceptov poda Kakwani (1984) kot razčlenitev redistributivnega učinka v vertikalni ali progresivni učinek in učinek prerangiranja. Ta je v literaturi na temo prerazdelitve dohodkov postala in ostala eno najpomembnejših orodij. Popularnost te razčlembe temelji na njeni obsežnosti (zajema različne vidike pravičnosti prerazdelitve), enostavnosti in preprosti izračunljivosti ter razpoložljivosti za neposredno interpretacijo politik (učinek prerazdelitve je večji, če se zmanjša horizontalna neenakost).

Kot prvi je metodološki instrumentarij za razčlenitev redistributivnega učinka z namenom razkritja relativnih prispevkov individualnih davkov in prejemkov predstavil Lambert (1985). Njegov model je pripomogel k razkritju pomembne interakcije med davki in prejemki: tudi v primeru, ko je davčni sistem regresiven, lahko davke še vedno štejemo kot dejavnik krepitve redistributivnega učinka fiskalnega sistema. Vendar pa ta model ni vključeval prerangiranja in je pravzaprav razčlenil Kakwanijev (1984) vertikalni učinek fiskalnega sistema, ne pa redistributivnega učinka ali učinka prerangiranja.

Več študij je potrdilo, da je dohodkovna neenakost na Hrvaškem v mednarodni primerjavi zmerna. Je relativno majhna dohodkovna neenakost naravni sestavni del hrvaškega gospodarstva ali pa predstavlja posledico fiskalnih aktivnosti? Glede na izkušnje iz drugih držav in dejstvo, da je delež javnih izdatkov v BDP visok, lahko predvidevamo, da ima vlada pomemben vpliv na razdelitev dohodkov na Hrvaškem. Ker pa je razdelitev davkov in prejemkov na Hrvaškem doslej le delno raziskana, nimamo zanesljivega dokaza za to domnevo.

Primarni cilj moje raziskave je odgovoriti na naslednja vprašanja: (1) Kakšen je učinek fiskalnega sistema na dohodkovno neenakost na Hrvaškem?, in (2) Kako različni fiskalni instrumenti vplivajo na ta učinek? Cilj raziskave je analiza redistributivnih učinkov različnih davkov in prejemkov na Hrvaškem: prispevkov za socialno varnost, dohodnine, javnih pokojnin ter denarnih prejemkov, ki so oz. niso osnovani na dohodkovnem preizkusu.

V začetku dela na disertaciji so bili v ospredju empirični vidiki. Kot model merjenja sem izbral najbolj razširjene razčlenitve redistributivnega učinka, ki sta jih predlagala Kakwani (1984) in

Lambert (1985). Vendar pa sta na določeni točki stopila v ospredje dva metodološka problema, in sicer: (a) interpretacija mer vertikalnega učinka in učinka prerangiranja, (b) razčlenitev učinka prerangiranja. Pred uporabo za empirične namene je bilo treba razrešiti ti metodološki nasprotji.

Prvi problem je povezan z vlogo prerangiranja v redistribucijskem procesu. Ob ponovnem pregledu literature sem ugotovil, da so ta problem nekateri avtorji upoštevali, drugi pa zanemarili. Na eni strani sta bila Atkinson (1980) in Plotnick (1981), avtorja učinka prerangiranja, ki sta trdila, da prerangiranje enot ne vpliva na razdelitev dohodka po obdavčitvi. Opozorila sta na to, da merjenje učinka prerangiranja ne bi smeli vključevati v obširnejši sistem, s katerim bi poskušali izmeriti tudi progresivnost. Po drugi strani pa Kakwani (1984) ni sledil temu nasvetu temveč izdelal model, ki zajema tako prerangiranje kot progresivnost.

Drugi problem zadeva razčlenitev prerangiranja z namenom razkritja doprinosov različnih davkov in prejemkov. Kot sem že omenil, Lambertova razčlenitev (1985) razčlenjuje samo vertikalni učinek. Obstajata pa dva poskusa razčlenitve prerangiranja, ki sta ju opisala avtorja Jenkins (1988) in Duclos (1993), vendar imata oba določene omejitve.

Glede na to, da sta oba zgoraj omenjena problema ključnega pomena, sem se odločil razširiti raziskave z namenom, da zanju poiščem rešitve. Rezultat so novi kazalci in ponovne interpretacije obstoječih kazalcev ter več novih razčlenitev. Kljub temu pa nisem zanemaril empiričnega cilja. Vsi ti različni kazalci in razčlenitve so tudi uporabljeni za analizo podatkov za Hrvaško za obdobje 2001 do 2006.

Za samo pripravo podatkov in v oceno njihove relevantnosti je potrebnega veliko dela, saj podatki o anketi o porabi gospodinjstev, ki so osnova empiričnemu delu disertacije ne vsebujejo podatkov o osebnih davkih. Le-te je bilo treba zato izračunati s pomočjo mikrosimulacijskega modela, ki sem ga razvil v ta namen.

Hipoteza

Na osnovi zgoraj navedenega so v disertaciji tako postavljene metodološka hipoteza (1) in več empiričnih hipotez (2).

(1) Kakwani (1984) nudi *napačno interpretacijo* vloge prerangiranja v redistribucijskem procesu. Stališče, ki ga zastopam v disertaciji pa pravi, da prerangiranje dohodkovnih enot ne more vplivati na redistributivni učinek RE.

(2) Fiskalni sistem je eden od najpomembnejših dejavnikov, ki vplivajo na dohodkovno neenakost na Hrvaškem. Največji delež dohodkovne prerazdelitve je dosežen z javnimi pokojninami. Pomembno vlogo igrajo tudi dohodnina in prispevki za socialno varnost.

Metodologija

Večji del disertacije je posvečen sami metodologiji. Osnovno orodje celotne analize je Ginijev koeficient. Za njegovo izračunavanje obstaja več različnih pristopov, od katerih so trije uporabljeni v tej raziskavi. Druga uporabljena metoda pa je analiza različnih prehodov med dohodkovnimi vektorji.

Eden od razlogov za priljubljenost Ginijevega koeficienta je ta, da ga je mogoče izpeljati in interpretirati na veliko različnih načinov. S tem koeficientom je tesno povezana Lorenzova krivulja in povezava med njima je kar najbolj naravna in intuitivna, vendar, ko nas zanima, kaj se zgodi s kazalci, ki temeljijo na Ginijevem koeficientu, *ko se pojavi majhna sprememba*, je primernejša uporaba drugih analitičnih orodij. Formule, ki temeljijo na rangiranju dohodka in dohodkovnih razlikah, omogočajo prikaz, kako majhen transfer iz ene dohodkovne enote v drugo vpliva na mero. Ta metodološki pristop je pogosto uporabljen za dokazovanje, da Ginijev koeficient izpolnjuje "načelo transferjev", vendar pa ne izpolnjuje "načela padajočih transferjev" (npr. v Lambert, 2001:35).

Redistribucijski proces si je mogoče predstavljati tudi kot vrsto majhnih transferjev med enotami opazovanja in kazalce redistributivnega učinka in prerangiranja je mogoče rekonstruirati s temeljito preiskavo sprememb, ki jih povzročajo ti majhni transferji. Takšen pristop predvsem pripomore k spoznanju, zakaj prerangiranje ne more vplivati na zmanjšanje dohodkovnih razlik. Zgolj z uporabo "agregatnih" kazalce lahko namreč pomembni problemi pri interpretaciji ostanejo prikriti. Zato so vsi kazalci redistributivnega učinka, poleg pristopa, ki temelji Lorenzovi krivulji, izraženi še s pomočjo *oddaljenosti od povprečja* (distance from mean), *razdalje med enotami opazovanja* (distance between units), ter *Ginijeve funkcije blaginje* (Gini welfare function).

Naslednjo metodo, uporabljeno v tej disertaciji bi lahko poimenovali *analiza vektorjev dohodkovnih prehodov* (analysis of income vectors' transitions). Podatki o dohodkih opazovanih enot so zapisani v ločenih vektorjih dohodkov pred in po obdavčitvi. Dohodki enot so lahko razvrščeni na različne načine, nas pa zanimata predvsem dve razvrstitvi: po dohodku pred obdavčitvijo in po dohodku po obdavčitvi. Tako lahko dohodke pred in dohodke po obdavčitvi razvrstimo glede na velikost dohodkov pred obdavčitvijo ali glede na velikost dohodkov po obdavčitvi, iz česar dobimo štiri vektorje. Na tej podlagi nato analiziramo različne prehode med temi vektorji in razkrijemo njihove lastnosti. Ti prehodi so v osnovi glavnih kazalcev redistributivnega učinka ter prerangiranja in s poznavanjem prehodov lahko ocenimo lastnosti kazalcev, ki na njih temeljijo.

Tretja metoda za razumevanje narave in pomena kazalcev redistributivnega učinka in prerangiranja se prav tako spusti na raven individualnih enot. Pri njej se vprašamo: Kakšen je "občutek" dohodkovne enote A o njenem aktualnem dohodkovnem statusu v odnosu do druge enote B? Enota A lahko občuti "prednost" ali "superiornost" nasproti enoti B, če je njen dohodek višji, vendar pa se lahko to zaradi fiskalnega procesa spremeni: razlika se lahko zmanjša, "kar prizadene občutke enote A" in B lahko celo preseže A z višjim dohodkom po obdavčitvi ter tako še poglobi "prikrajšanost" enote A. Tovrstni koncepti olajšajo razumevanje pomena različnih numeričnih kazalcev, ki merijo redistributivni učinek.

Vse tri zgoraj navedene metode so opisane v literaturi na temo prerazdelitve dohodkov. Vendar pa obstaja še četrta, ki se zdi nova. Ta izvira iz pristopa "razdalje med enotami opazovanja" za izračun Ginijevega koeficienta in se uporablja za razčlenitev redistributivnih učinkov z namenom ocene posameznih davkov in prejemkov. Ta pristop uporablja posebej izdelane matrike, ki vsebujejo razlike (razdalje) med vrednostmi izbrane spremenljivke za vsak par dohodkovnih enot. Spremenljivke predstavljajo dohodki pred in po obdavčitvi ter posamezni davki in prejemki. Če kombiniramo in primerjamo te matrike na različne načine in zberemo pare enot, dobimo glavne kazalce in njihove razčlenitve.

Struktura

Disertacija je sestavljena iz šestih poglavij. *Prvo poglavje* pojasnjuje predmet, cilje, hipoteze in metodologijo. *Drugo poglavje* vsebuje pregled literature o merjenju redistributivnih učinkov. Poglobi se v Kakwanijeve kazalce in razčlenitve, ki so v literaturi deležni največje pozornosti. Pregled ni le povzetek metod, ki se uporabljajo na tem področju, temveč kritični prikaz razvoja metodološkega instrumentarija, ki razkriva določene probleme pri aktualnih pristopih do merjenja redistributivnega učinka. V tem smislu služi kot uvod v metodološki del disertacije.

Tretje poglavje vsebuje metodologijo in predstavlja jedro disertacije. V njegovem *prvem* delu sem razvil osnovne koncepte merjenja, ki temeljijo na pristopu "oddaljenost od povprečja" in "razdalje med enotami opazovanja" za izračunavanje Ginijevega koeficienta. Izpeljani so novi kazalci redistributivnih učinkov, kazalci zmanjševanja dohodkovnih razlik in prerangiranja, opredeljena je njihova povezanost z obstoječimi kazalci. Ti novi koncepti se nato uporabijo za interpretacijo Kakwanijeve razčlenitve redistributivnega učinka. V *drugem* delu poglavja je predstavljena metodologija, ki te kazalce razčlenjuje, da bi razkrila doprinos posameznih fiskalnih instrumentov k redistributivnemu učinku in prerangiranju.

Naslednji dve poglavji sta namenjeni uporabi novo razvitih metodologij za analizo hrvaškega fiskalnega sistema. *Četrto poglavje* obravnava podatke, uporabljene v empirični analizi. Kot uvod je podan kratek opis obstoječih fiskalnih instrumentov. Temu sledi opis mikrosimulacijskega modela, uporabljenega za izračun davkov. Poglavje se konča z oceno kakovosti uporabljenih podatkov v primerjavi s podatki iz drugih, tj. upravnih virov. *Peto poglavje* analizira empirične rezultate za Hrvaško v obdobju od 2001 do 2006.

Šesto poglavje je zaključno poglavje disertacije, v katerem so navedene omejitve aktualne raziskave in priporočila za nadaljnje raziskovanje.

Pomen raziskav

Disertacija predstavlja analizo konceptov merjenja prerazdelitve dohodkov v obstoječi literaturi v zadnjih tridesetih letih, vključno z Atkinsonovim (1980) in Plotnickovim (1981) kazalcem prerangiranja ter Kakwanijevo (1984) razčlenitvijo redistributivnega učinka v vertikalne učinke in učinke prerangiranja. Njihove metodologije so bile posodobljene in razširjene ter uporabljene v različnih empiričnih študijah prerazdelitve dohodkov.

Algebraična veljavnost znanih razčlenitev je nesporna: razčlenjeni kazalec *je* vsota (ali razlika ali kaka druga kombinacija) njegovih komponent. Tako v primeru Kakwanijeve razčlenitve (1984) dobimo $RE = V^K - R^{AP}$. Vsak kazalec (R^{AP}, V^K in RE) se lahko izpelje neodvisno in zgornja enakost je dokazljiva. Vprašljivo pa je, ali so bili ti kazalci in njihove interakcije pravilno interpretirani. Sta na primer kazalca R^{AP} in V^K ločeni in neodvisni "entiteti"? Ali je mogoče povečati RE z zmanjšanjem R^{AP} ob tem, da je V^K nespremenjen? Tem vprašanjem je namenjen večji del metodološkega dela disertacije. Celotno raziskavo je motiviral antagonizem med iznajditeljema učinka prerangiranja, Atkinsonom in Plotnickom, ter očetom kazalca progresivnosti, Kakwanijem. Prva avtorja sta trdila, da prerangiranje ne vpliva na redistributivni učinek in še predlagala, da kazalec prerangiranja naj ne bi bil vključen v obširnejši okvir skupaj s kazalcem progresivnosti. Kakwani tega ni upošteval in pojavila se je njegova razčlenitev (Kakwani, 1984), ki je postala eno najbolj razširjenih orodij za analizo redistributivnega učinka, progresivnosti in prerangiranja.

Prva hipoteza v disertaciji je, da prerangiranje ne more vplivati na redistributivni učinek na način, kot ga navajajo raziskovalci, ki so uporabili Kakwanijevo razčlenitev. Ti trdijo da vsebuje vertikalni učinek (V^K) potencialni redistributivni učinek, ki bi bil dosežen, če bi bilo prerangiranje (prikazano z R^{AP}) nekako eliminirano. V disertaciji je dokazano, da bi bila to nemogoče: izključitev prerangiranja vedno vodi k zmanjšanju vertikalnega učinka in redistributivni učinek ostane nespremenjen. Iz tega sledi, da se je treba prej omenjeni interpetaciji izogniti. Še več, previdni moramo biti pri interpretaciji katerekoli razčlenitve.

Disertacija prav tako pokaže, da ima Kakwanijev kazalec vertikalnega učinka (V^K) kot merilo progresivnosti pomembne pomanjkljivosti prav zato, ker ni neodvisen od prerangiranja in ga pravzaprav vsebuje. Nekatera od teh stališč sta predstavila že Lerman in Yitzhaki (1995), vendar so le-ta v disertaciji razširjena in analitično dokazana. Iz tega sledi drugi sklep disertacije - izognitev uporabi V^K kot kazalca progresivnosti.

Izključevanje nekaterih interpretacij in načinov uporabe kazalcev je ustvarilo praznino, ki jo je bilo treba zapolniti. To je narejeno z novimi kazalci, ali raje z novimi interpretacijami obstoječih kazalcev: kazalcev fiskalne deprivacije, fiskalne dominacije, zmanjševanja razlik in drugih, ki jih je treba uporabljati in razvijati naprej v okviru novih kontekstov.

V disertaciji je predstavljena še ena metodološka inovacija: razčlenitev učinka prerangiranja za oceno doprinosa posameznih davkov in prejemkov. Vendar to ni bil prvi poskus v tej smeri. Jenkins (1988) in Duclos (1993) sta predlagala svoja modela, a imata oba določene pomanjkljivosti. Model razčlenitve, ki je predstavljen v disertaciji, popolnoma in neposredno razčleni učinke prerangiranja Atkinson-Plotnicka (R^{AP}) in Lerman-Yitzhakija (R^{LY}). To tudi omogoča oceno, v kakšni meri različni fiskalni instrumenti doprinašajo k redistributivnemu učinku (RE). Posledica tega pristopa je očitna: iz podatkov lahko izračunamo, v kakšni meri vsak davek in prejemek prispeva k zmanjšanju dohodkovnih razlik in prerangiranju.

Kazalci in razčlenitve se nato uporabijo za preverjanje druge hipoteze, ki je empirična in povezana s hrvaškim fiskalnim sistemom. Analiziran je del fiskalnega sistema, ki sestoji iz prispevkov za socialno varnost, dohodnine, javnih pokojnin in denarnih socialnih prejemkov. Podatki so pridobljeni iz anket o porabi gospodinjstev, davčne spremenljivke pa so izračunane z mikrosimulacijskim modelom, izdelanim za namene disertacije.

Dohodki, davki in socialni prejemki so med seboj kombinirane tako, da ustvarijo šest različnih definicij analiziranega fiskalnega sistema, upoštevaje različne predpostavke glede davkov in prejemkov. Tako na primer dve definiciji obravnavata javne pokojnine kot socialne prejemke, medtem ko jih druge definicije upoštevajo kot tržni dohodek.

Zmanjšanje dohodkovne neenakosti, ki ga povzroča fiskalni sistem, je ocenjeno na podlagi redistributivnega učinka (RE), ki je razlika med Ginijevima koeficientoma dohodka pred obdavčitvijo (G_X) in dohodka po obdavčitvi (G_N). Pri tem termin dohodek pred obdavčitvijo predstavlja dohodek pred odtegljajem davkov in prispevkov, medtem ko termin dohodek po obdavčitvi pomeni dohodek po odtegljaju davkov in prispevkov. Dohodek pred obdavčitvijo, dohodek po obdavčitvi, davki in prejemki so ločeno definirani za vsako od šestih definicij fiskalnega sistema.

Empirična analiza je potrdila hipotezo, da je fiskalni sistem ena od primarnih determinant dohodkovne neenakosti na Hrvaškem. Razpon zmanjšanja dohodkovne neenakosti, ki ga povzroča fiskalni sistem, sega od 10% za najmanj obširno definicijo fiskalnega sistema, ki vključuje samo dohodnino in denarne socialne prejemke, do 40% za najobširnejšo definicijo fiskalnega sistema, ki zajema javne pokojnine in prispevke za socialno varnost, skupaj z dohodnino in denarnimi socialnimi prejemki.

Če upoštevamo slednjo, najobširnejšo definicijo fiskalnega sistema, je največji del prerazdelitve dohodkov dosežen z javnimi pokojninami, prispevki za socialno varnost in dohodnino. Vendar pa raziskava ne nudi dokončnega zaključka glede zaporedja teh treh skupin fiskalnih instrumentov glede pomembnosti za zmanjšanje dohodkovne neenakosti. Ena od hipotez predpostavlja, da javne pokojnine v največji meri prispevajo k redistributivnemu učinku. Skupina razčlenitev, ki temelji na "odstopanju davkov in prejemkov od proporcionalnosti", jo je vsekakor dokazala. Na drugi strani pa so razčlenitve, ki temeljijo na "zneskih davkov in prejemkov", pokazale, da so prispevki za socialno varnost in dohodnina veliko pomembnejši od javnih pokojnin. Razlika v rezultatih je posledica kriterijev za definiranje progresivnosti in regresivnosti. Pristop "zneskov" teži k poudarku primarne vloge davkov, pristop "odstopanj" pa podpira prejemke. Posledica tega dognanja je zanimiva za raziskovalce, saj utegne nuditi novo perspektivo glede vloge davkov in prejemkov pri zmanjšanju dohodkovne neenakosti.

Ne glede na zaporedje po pomembnosti glavnih fiskalnih instrumentov, pa je pomembna ugotovitev, da vsi skupaj zmanjšujejo dohodkovno neenakost.

Nadaljnji rezultati empiričnega dela zadevajo prerangiranje dohodkovnih enot. V kakšni meri je prerangiranje vpeljano v hrvaški fiskalni sistem? Ocene prerangiranja so v veliki meri odvisne od načina definiranja fiskalnega sistema, in razpon meril sega od skromnega 1% od G_X (6% od RE) za ozko definiran sistem, ki vključuje samo dohodnino in denarne socialne prejemke, do vrednosti nad 15% od G_X (34% od RE), ko fiskalni sistem vključuje javne pokojnine in prispevke za socialno varnost, skupaj z dohodnino in denarnimi socialnimi prejemki.

Če torej vzamemo najširšo definicijo fiskalnega sistema, ki vključuje javne pokojnine kot prejemke in prispevke za socialno varnost kot davke, so javne pokojnine nedvomno tiste, ki z več kot 75% deležem največ doprinesejo k prerangiranju. Sledijo jim prispevki za socialno varnost z 20% deležem. Na drugi strani pa dohodnina na prerangiranje vpliva le malo.

Prerangiranje splošno velja za nepravično in posledica teh ugotovitev je, da bi nosilci politik lahko zmanjšali neenakost, ki jo občuti javnost, s preoblikovanjem nekaterih delov fiskalnega sistema.

Meje raziskave

Prva skupina omejitev se nanaša na kazalce, uporabljene pri merjenju redistributivnega učinka. Vsak kazalec ima lastne omejitve, ki jih je treba upoštevati pri interpretaciji rezultatov. Enako velja za kazalce razvite v disertaciji in nove interpretacije obstoječih kazalcev saj niso bile testirane in interpretirane vse njihove lastnosti in pomanjkljivosti. Na tem področju bodo potrebne nadaljnje raziskave.

Vsaka empirična raziskava je odvisna tudi od kakovosti podatkov. Za podatke iz anket o porabi gospodinjstev je značilno, da podcenjujejo dohodke na zgornjem delu (krivulje) razdelitve dohodkov. Komparativna analiza, predstavljena v disertacij, potrjuje da to velja tudi za Hrvaško. Zdi se tudi, da so poročila o zneskih določenih socialnih prejemkov zelo pomanjkljiva. Pričakujemo lahko, da so te podatkovne pomanjkljivosti močno vplivale na rezultate empirične analize.

Nadaljnji problem je način definiranja fiskalnega sistema, ki je predmet analize. Ali naj se javne pokojnine obravnavajo kot socialni prejemki? Ali so prispevki za socialno varnost davki? V pričujoči disertaciji nisem predstavil definitivnih odgovorov, zato sem uporabil različne scenarije ali definicije. Vendar pa nobeden od teh scenarijev ne predstavlja realne podobe - ta obstaja nekje vmes. Zato je pri interpretaciji in izpeljavi zaključkov potrebna previdnost.

Priporočila za nadaljnje raziskave

Priporočila za nadaljnje raziskave so že nakazana pri omejitvah te disertacije. Za prepričljivejšo analizo redistributivnih učinkov na Hrvaškem bi bilo treba izdelati boljše podatkovne baze. To je mogoče storiti z združevanjem podatkov iz različnih uradnih virov, kot so na primer podatki iz popisa prebivalstva, podatki davčne uprave, pokojninskih skladov in relevantnih ministrstev in agencij.

Za oceno redistributivnih lastnosti pokojninskega sistema bi bila potrebna posebna analiza. Idealno bi bilo, če bi vsako pokojnino lahko razdelili na dva dela: na del, ki je "zaslužen" in del ki je prejemek od vlade (ali davek, če je pokojnina nižja kot bi morala biti, glede na vplačane prispevke). Prvi del bi bil vedno vključen v dohodke pred obdavčitvijo, drugi del bi se vedno obravnaval kot prejemek (davek). Nekaj podobnega bi lahko storili tudi s prispevki za socialno varnost.

Disertacija tako odpira pot do nadaljnjih raziskav fiskalne incidence in prerazdelitve dohodkov na Hrvaškem. Nadaljnje raziskave pa bi morale obravnavati tudi druge fiskalne instrumente, kot so posredni davki in prejemki v naravi, s čimer bi bil pokrit celoten fiskalni sistem.

Literatura

Atkinson, A.B. (1980). Horizontal Equity and the Distribution of the Tax Burden. H.J. Aaron & M.J. Boskins (eds), *The Economics of Taxation* (pp. 3-18). Washington D.C.: Brookings.

Duclos, J.-Y. (1993). Progressivity, redistribution and equity, with application to the British tax and benefit system. *Public Finance / Finances Publiques*, 48(3), 350-365.

Jakobsson, U. (1976). On the measurement of the degree of progression. *Journal of Public Economics*, 5, 161-168.

Jenkins, S. (1988). Reranking and the analysis of income redistribution. *Scottish Journal of Political Economy*, 35(1), 65-76.

Kakwani, N.C. (1977a). Applications of Lorenz curves in economic analysis. *Econometrica*, 45, 719-727.

Kakwani, N.C. (1977b). Measurement of Progressivity: An International Comparison. *The Economic Journal*, 87, 71-80.

Kakwani, N.C. (1984). On the measurement of tax progressivity and redistributive effect of taxes with applications to horizontal and vertical equity. *Advances in Econometrics*, 3, 149-168.

Lambert, P.J. (1985). On the redistributive effect of taxes and benefits. *Scottish Journal of Political Economy*, 32(1), 39-54.

Lambert, P.J. (2001). The Distribution and Redistribution of Income. (3rd ed.) Manchester and New York: Manchester University Press.

Lerman, R. I. & Yitzhaki, S. (1995). Changing ranks and the inequality impacts of taxes and transfers. *National Tax Journal*, 48(1), 45-59.

Musgrave, R. A. & Thin, T. (1948). Income tax progression, 1929-48. *The Journal of Political Economy*, 56(6), 498-514.

Plotnick, R. (1981). A Measure of Horizontal Equity. *The Review of Economics and Statistics*, 63(2), 283-288.