S-parameter approximation using rational funtion

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Introduction

- Electrical circuits operating at high frequency
- Component size comparable with wavelengths
- Simulation using lumped elements – netlists with high number of discrete components
- Transmission line behaviour of interconnects
- Skin effect
- Dispersion

SIMULATION CHALLENGE

Simulation using the transfer function

- Defined in the frequency domain
- 1-port networks have impedance or addmitance transfer functions
- N-port networks are represented with impedance or addmitance **matrix**
- Impedance or addmitance measurement are unusable at high frequencies: problem of defining short circuits (parasitic inductance) and open end (parasitic capacitance)
- High frequency characterization are obtained using the S-parameters

Passive devices modelling at high frequencies

- Connectors, interconnects, transmission lines
- Model must retain passive behaviour
- Passive models involve following characteristics:
  1. Causality
  2. Stability
  3. Passivity
- This work considers modelling in the frequency domain

Causality in the frequency domain

- Causal impulse response satisy:

$$h(t) = sign(t)h(t) \qquad sign(t) = \begin{cases} -1 & t < 0 \\ 0 & t = 0 \\ 1 & t > 0 \end{cases} \qquad F\{sign(t)\} = \frac{2}{j\omega}$$

$$F\{h(t)\} = \frac{1}{2\pi} F\{sign(t)\} * F\{h(t)\}$$

$$H(j\omega) = \frac{1}{j\pi} \text{pv} \int_{-\infty}^{+\infty} \frac{H(j\omega')}{\omega - \omega'} d\omega' \qquad H(j\omega) = U(j\omega) + jV(j\omega)$$

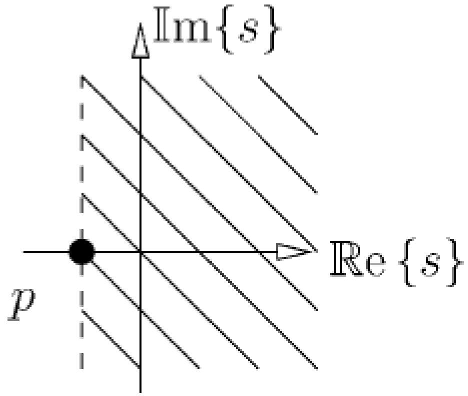
$$U(\omega) = \frac{1}{\pi} \text{pv} \int_{-\infty}^{+\infty} \frac{V(\omega')}{\omega - \omega'} d\omega' \qquad V(\omega) = -\frac{1}{\pi} \text{pv} \int_{-\infty}^{+\infty} \frac{\text{U}(\omega')}{\omega - \omega'} d\omega'$$

Dispersion relations

Stability in the frequency domain

- Impulse responses of unstable systems don't have Fourier transform
- Closely related is the Laplace transform. The region of convergence doesn't include the imaginary axis for unstable systems
- To analyze unstable systems Laplace transform is needed for frequency domain (more precise complex frequency domain)
- Example

$$h_1(t) = e^{pt}S(t) \quad H_1(s) = \frac{1}{s-p} \quad PK\colon Re\{s\} > p$$



Passivity in the frequency domain

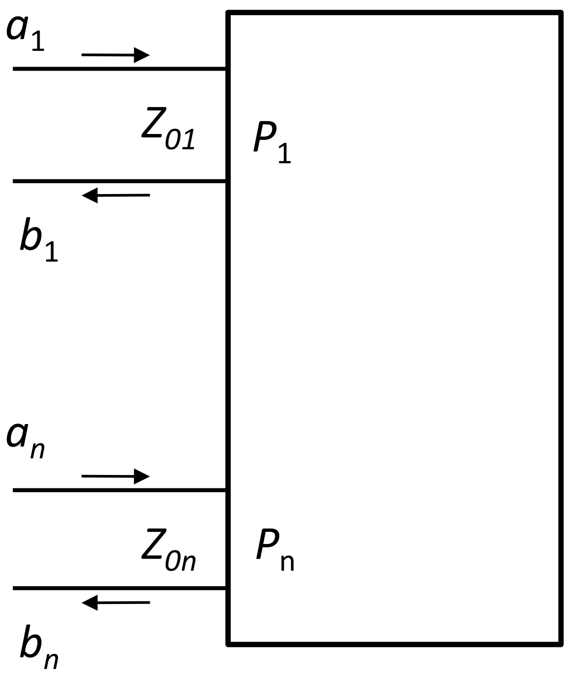
For any frequency the passivity requirment relates theinput and output power

$$\sum_{i=1}^{n}(|a_i|^2 - |b_i|^2) \geq 0$$

$$a^H a - b^H b \geq 0$$

$$a^H a - S^H a^H S a \geq 0$$

$$a^H (I - S^H S) a \geq 0$$



Recapitulation of passivity requirements in the frequency domain

S($j\omega$) matrix is represents a passive network if the following is satisfied:

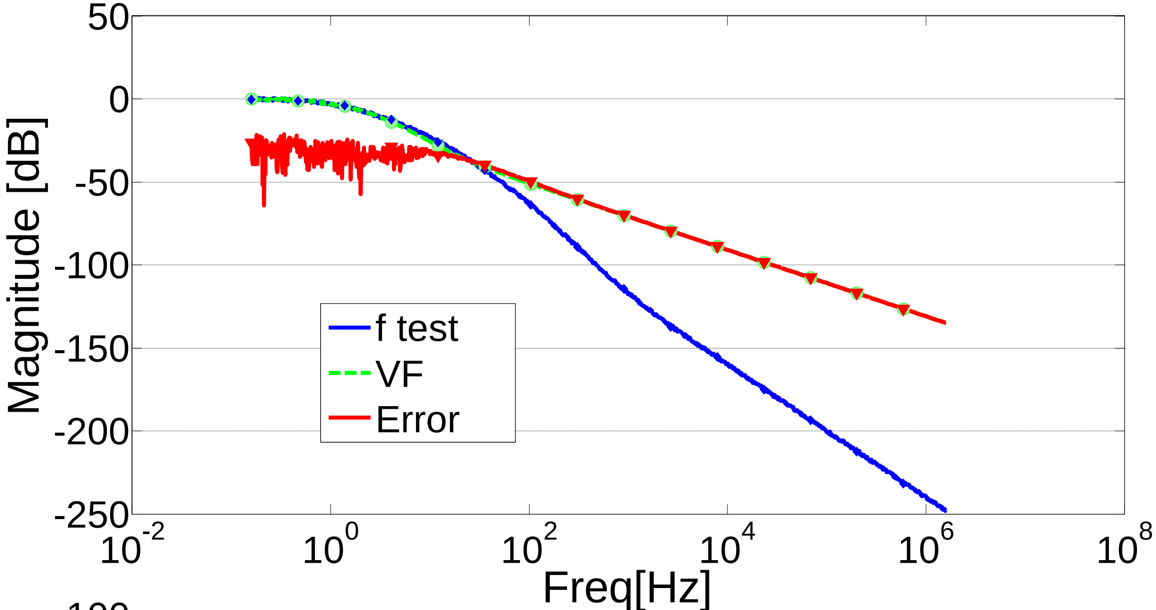
1) Dispersion relation hold
2) $\boldsymbol{I} - \boldsymbol{S}^H(j\omega)\, \boldsymbol{S}(j\omega)$ is positive semidefinite for all ω
3) Fourier transform of the impulse response exist (obviusly satisfied always when S-parameter characteristic is given and the 2rd requirement hold)
4) $\boldsymbol{S}(-j\omega)=\boldsymbol{S}^*(j\omega)$ (time domain impulse response is purely real without imaginary, which is practically always the case)

S-parameter approximated using the rational function

- Linear systems have rational transfer functions
- All of the passivity criteria (EXCEPT one) is already “built in” by choosing the rational basis function for approximation
- Rational function models are represented as: state space, pole-zero, residue-pole, etc.
- Algorithm for rational approximation: VECTOR FITTING (VF)
- Widely used algorithm for rational modelling
- Based on iteratively solving least squares which is a mature and reliable technology
- All poles are fixed in the left complex plane by VF

VF algorithm

- The name comes from the possibility of fitting a number of different functions (vector of functions) with the common set of poles
- Least squares equations are solved using the powerfull Matlab ‘backslash’ operator
- SVD least squares solving can easily be added as the most numerically most reliable method but it is to expensive in term of time with no improvement in accuracy
- Variety of improved basic VF algorithm exist:
- Standard (VF), relaxed (RVF), orthonormal VF (OVF), adding and skimming (VF-AS), recursive VF (RecVF)
- All version implemented in Matlab



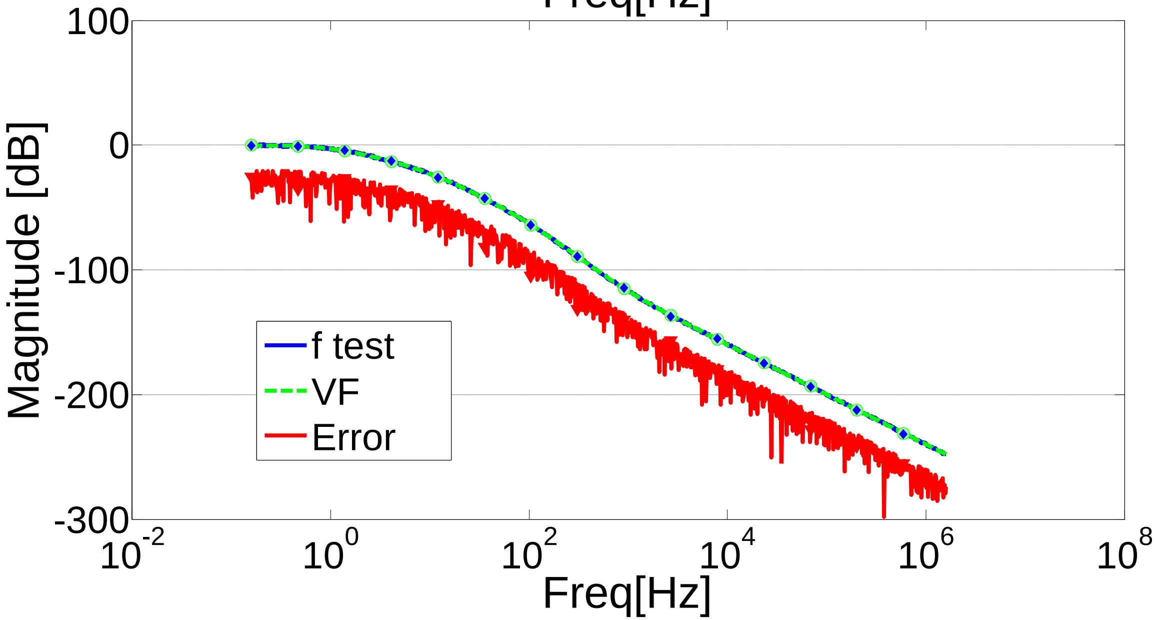
Test function:

-5 poles

-dynamic range 250 db

non weighted VF

-60 poles model results

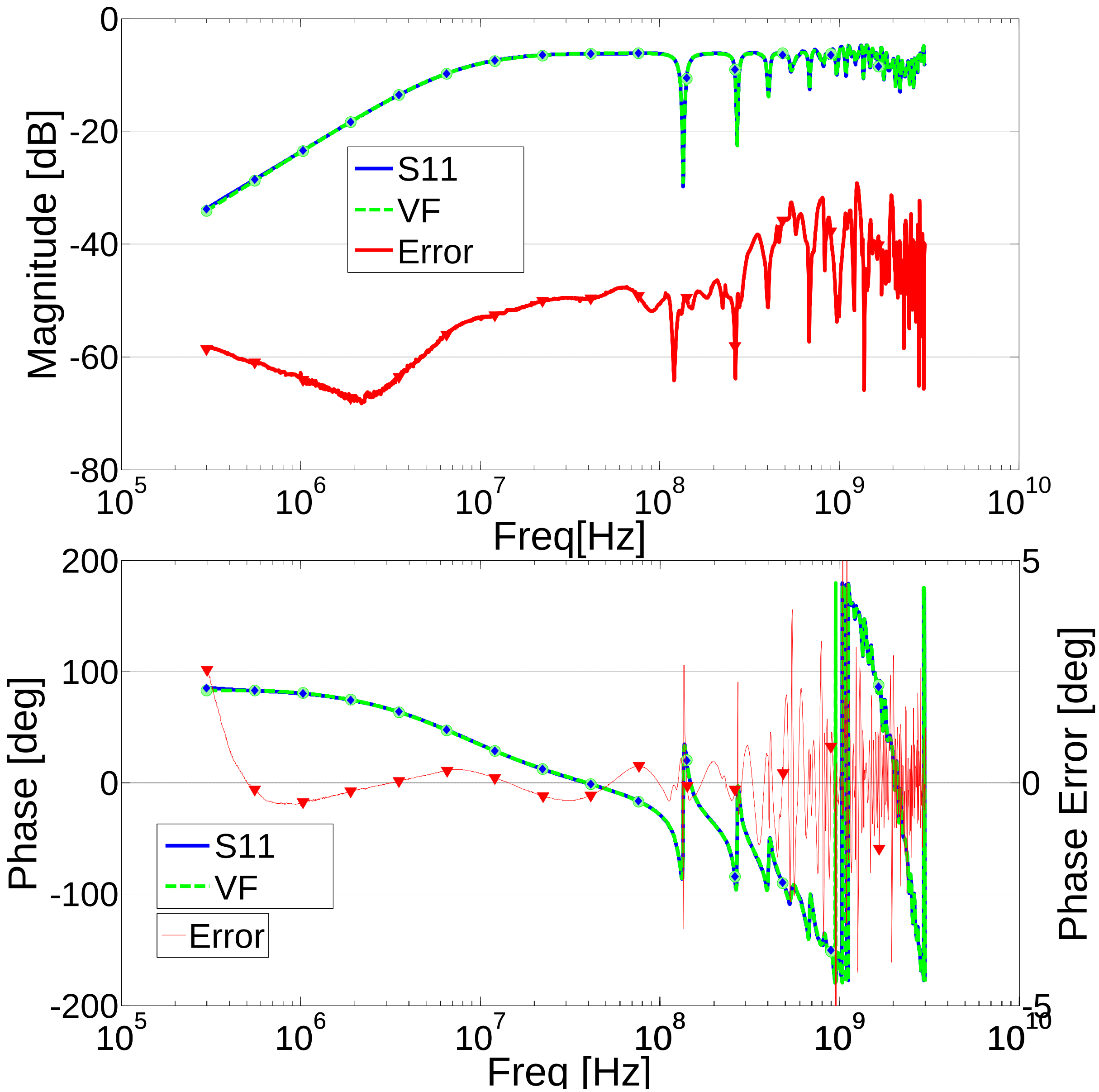


weighted VF

-5 poles results!

-20dB margin in the whole modelling band weighting

S-parameter cable modelling 300kHz-3GHz using 60 poles



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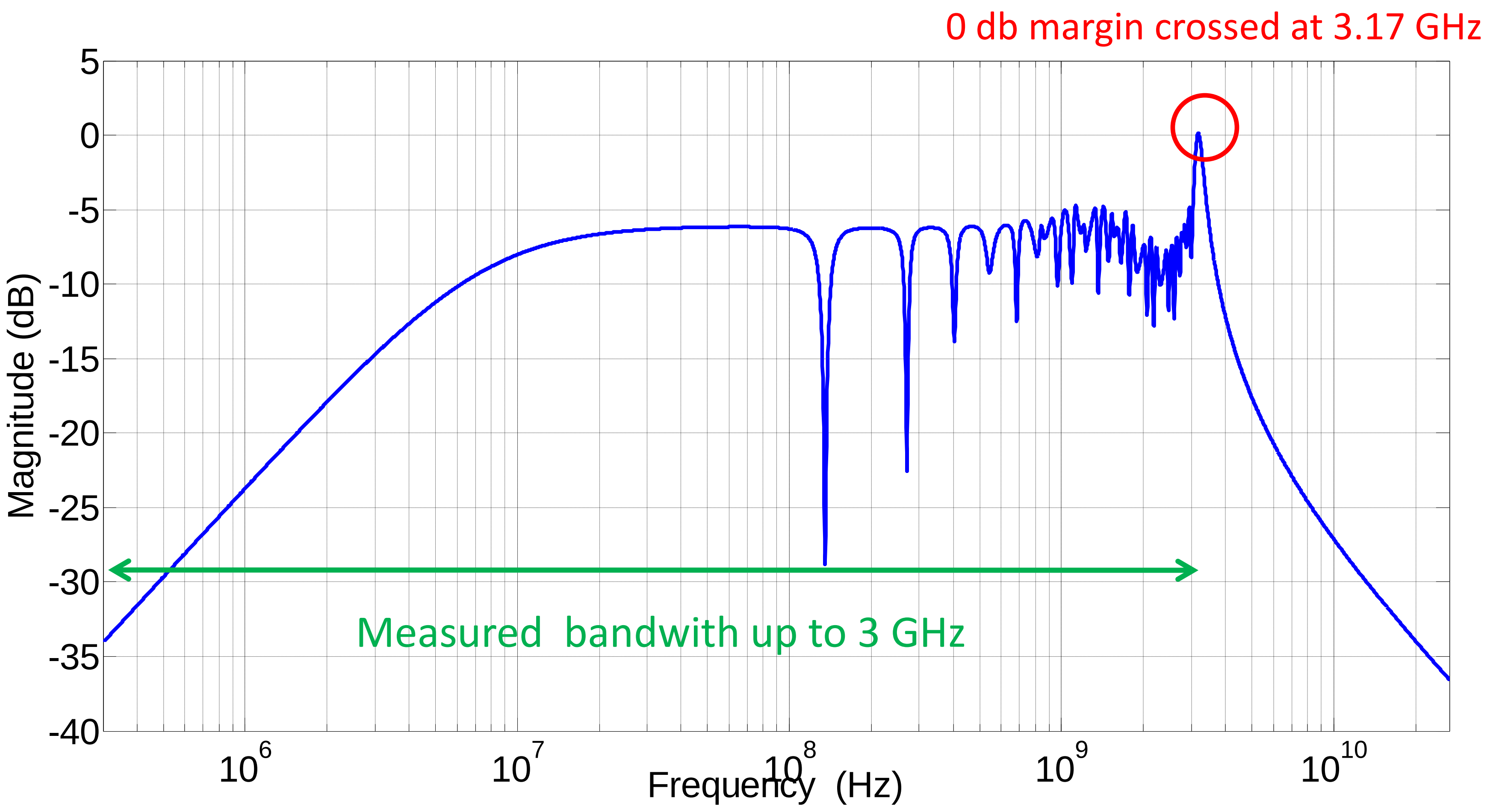

Global passivity of the model

- passivity must be achieved for all frequencies
- $\boldsymbol{I} - \boldsymbol{S}^H(j\omega)\,\boldsymbol{S}(j\omega)$ is psd for all ω
- Rational model achieve zero magnitude at $\omega=\infty$ by construction
- Critical region is the band beyond the modelling bandwidth
- This is the main focus for the future work