Use of technology for enhancement of teaching mathematics: perspective, problems, and criteria and selection method

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Abstract. Already in the eighties of the last century it has been recognized that *we are on the threshold of a revolution in mathematics how we think about it, how we practice it, and how we learn it. The revolution centers on the computer as a mathematical tool. As in every revolution, people are taking sides some would like to see a complete exploration of the computer s role in human mathematics, while others feel that using a computer in math is cheating [12]. Despite the availability of hardware and software, mathematics staff in a technology-rich secondary school rarely used computers in their teaching* [15]. We are presenting some of reasons for this phenomenon. We are also aware that *mathematical developments lay at the heart of recent advances in engineering, biomedical science, commerce, and information technology. Students in these areas need mathematical education but their backgrounds, abilities and attitudes vary widely* [8]. We are introducing some resources that are available on the Internet and could enhance teaching mathematics. *Since the World Wide Web is becoming wider at an increasing rate, it is virtually impossible to take any kind of accurate snapshot of the state of its development* [7]. We are presenting methodology for selection by defining criteria and selection method to choose the best appropriate. Those criteria ensure better accomplishment of learning objectives and avoiding risks of giving up from using resources because they are not appropriate for teachers by being either too complicated or time consuming to use.

Keywords: teaching mathematics, teaching with technology, decision making, mathematics

Introduction

Internet and technology have offered new teaching materials, software and other resources that can be implemented in schools to enhance students learning

and performance. It has been debated if technology, in general, is enhancing educational process in sense of ensuring better accomplishment of learning objectives. We are presenting just some of argument for and against using the technology. Some of arguments for usage are better engagement of student in learning, stimulated cognitive grow and performance, better identification of student s weaknesses in content area, computers and internet allow teaching at the moment with many information than adhere prepared lesson. Arguments against usage are need of change of teaching style to fit some technological device and open question about methodological approach that can insure improved students achievement.

The role of technology can be summarized National Research Council s statement The process of using technology to improve learning is never solely a technical matter, concerned only with the properties of educational hardware and software. Like a text book or any other cultural object, technology resources for education function in a social environment, mediated by learning conversations with peers and teachers [2].

In professional literature has been recognized that use of technology doesn t increase success of teaching process by itself [1]. Preparation with using technology consumes much more effort and time then classical preparation. Benefits of involving such effort are not obvious, and rarely adequately recognized in working or wider community. Mostly, motive to give such effort is teachers inner motivation driven by enthusiasm of preparing for better future or just looking for self-fulfillment.

Educational science and practice, e.g. [9],[11],[12],[15] recognized great technological change of the world. With that change of world they recognize urgent need of change of methodological approach in teaching mathematics. Educational process should be more focused on acquiring permanent and useful knowledge, skills and abilities. Involving technology can make process of teaching mathematics modern. Using technology can save time on operational part of teaching process. Saved time can be spent on explaining basic ideas in mathematics (basic knowledge), discussions and creative mathematical thinking (reflective knowledge).

It has also been recognized that technology doesn t necessary mean better education. Some possible negative effect on students, could be fall of persistence, patience, accuracy and concentration - what teachers usually believe that mathematics give to their students. Technology and on-line teaching applications can also have great problem: be very interesting and time consuming but with small educational benefit.

By being aware and giving relatively small effort probability that those problems happen can be significantly reduced. There is something new that arise with use of technology. It can t be called problem, maybe possibility or question: What is main question for our mathematical subjects: why or what if . Answering this question can give us should we or maybe must we use technology in education.

In our paper we present method for selection software in course Selected chapters of mathematics. We present decision criteria and selection method.

Decision theory

Decision about using technology can have large influence on teaching and learning process. Using new technology, software, hardware or just available on-line materials introduce threats and opportunities, has their own strengths and weaknesses. In this part we are introducing method for recognizing those aspects in software selection process.

Teaching process is dynamic process. Before taking teaching actions, teacher should think if action will produce desired goal. In the decision process he or she should take new information if available and be open for feedback. Teachers decide about those actions generally based on their creativity, experience or intuition. Some actions influence achieving course objectives and can be called strategic. Decisions about those actions shouldn t be made only on those principles. Systematic analytical approach should also be used. There are different approaches and systematic methods of mathematical models that characterize the problem and argue the decision. In the systematic approach lies the difference between good and bad decisions. In fact, a good decision will be one that uses a quantitative approach, based on logic, taking into account all the available input data and possible alternatives. The omission of some alternatives, which may even be very insignificant, can lead to the wrong choice. It is known that sometimes a good decision can result in unexpected outcomes but in general analytical approach in decision making is better than one done just on intuition. One of strategic decisions is decision on need of use teaching software and selection the best appropriate if needed.

Decision theory studies decision-making processes in a systematic and analytical way. The environment in which decision can be taken may be more or less deterministic and uncertain, and therefore the decisions we make in terms of certainty, or conditions of risk or uncertainty. Although, we can t be totally sure what will be results of our actions our decision making problem can be characterized as one in terms of certainty. Decision maker should also be aware that there are times when a decision an isolated one-time decision, but rather as the first in a series of sequential decisions that are interconnected in some future time intervals.

In this paper we decided to use the Analytic Hierarchy Process (AHP) technique for multi criteria decision making. The procedure for using the AHP can be summarized as:

1. Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives.
2. Establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements.
3. Synthesize these judgments to yield a set of overall priorities for the hierarchy.
4. Check the consistency of the judgments.
5. Come to a final decision based on the results of this process [16].

There are different commercially available or free software tools that were developed for the analysis of multi criteria decision making using pairwise comparisons such as Expert Choice, Decision Lab, D-Sight, ERGO, MakeItRational and various other tools. Our calculations were made in Expert Choice.

Educational software

Educational software is software whose primary purpose is in teaching and learning. Four different software packages were in final consideration for use in course Selected Chapters of Mathematics. We describe some of properties of the software.

GeoGebra is teaching and learning tool that integrates geometry, algebra and calculus. It is a cross-platform application written in Java and can serve for development of instructional materials in mathematics in many different forms, types and styles, and for all levels of mathematical education. *GeoGebra* has a built-in Cartesian coordinate system and can accept geometric commands (drawing points, lines, vectors, perpendicular line, angle bisector,...) and algebraic commands (pairs of coordinates, equation of a curve, function,...). This double representation, the geometric and the algebraic, is one of the greatest advantages of *GeoGebra*. Moving the objects in the Geometry window changes the expressions in the Algebra window accordingly and vice-versa, editing the expressions in the Algebra window results in the respective change in the Geometry window. Moreover, with a Spreadsheet window, which is also dynamically connected with Geometry and Algebra window, *GeoGebra* is ready for statistics commands and charts [10].

Mathematica is most widely used, complete and currently the most powerful advanced computer algebra system. *Mathematica* computations can be divided into three main classes: numerical, symbolic and graphical. Unlike the usual programming languages such as C and C++, it is not restricted to a small number of data types. It uses symbolic expressions to provide a very general representation of mathematical and other structures. *Mathematica* has a large number of built-in functions, but also includes own powerful programming language which supports several programming styles including:

- Procedural programming with block structure, conditionals, iterations and recursion
- Functional programming with pure functions and functional operators
- Rule-based programming with pattern matching and object orientation

The biggest disadvantage of *Mathematica* is price because it is to expensive for average users [18].

Maxima is a powerful computer algebra system written in Lisp programming language which combines symbolic, numerical and graphical abilities. It is a free and open source program which is being continuously improved by a team of volunteers. Compared with *Mathematica*, *Maxima* has more basic and simpler interface, but has big advantage in price. Currently, two most popular interfaces for *Maxima* are *wxMaxima* and *XMaxima*. For new users is the best to start with *wxMaxima* interface because it has convenient icons which help locate *Maxima* functions for common tasks. Through menus and buttons in *wxMaxima* new users gradually learn basic *Maxima* syntax. Also, *wxMaxima* implements its own math display engine to nicely display *Maxima* output. *XMaxima* is more lightweight interface with very small number of menu commands than *wxMaxima*, but it is more stable. Therefore, experienced users rather use *XMaxima* interface because they already know *Maxima* syntax and it is faster to them just type the name of command than

to search particular *Maxima* command in menus (if it is in menu at all). *XMaxima* is also a faster environment for testing and playing with code ideas than *wxMaxima* [13], [14].

SAGE is an open source mathematics computing environment written in very powerful and popular Python programming language. Most mathematics computing environments contain some kind of mathematics- oriented high-level programming language. Some of these mathematics-oriented programming languages were created specifically for the environment they work in while *SAGE* is built around an existing Python programming language. This means that expert Python programmers are also expert *SAGE* users. Beginners must first learn Python to being able solving problems with *SAGE* because *SAGE* is just powerful mathematics extension of Python. While most mathematics computing environments are self-contained entities, *SAGE* takes the umbrella-like approach of providing some algorithms itself and some by wrapping around other mathematics computing environments (*Maxima*, *GAP*, *BLAS*, *LAPACK*, *mathematics Python packages*, etc.). *SAGE* is built out of nearly 100 open source programs and can be used to study elementary and advanced, pure and applied mathematics like basic and advanced algebra, calculus, elementary and advanced number theory, cryptography, numerical computation, group theory, combinatorics, graph theory, linear algebra, etc. *SAGE* interface is a notebook in a web browser or the command line. With notebook interface, *SAGE* can connect to locally installation of *SAGE* on hard disc or to a *SAGE* server on the network. *SAGE* server on network is great tool for windows users because *SAGE* native port is *Linux*. Windows users must install virtual machine if they want run *Linux* and *SAGE* locally on Windows. *SAGE* currently works best with the Firefox web browser [17].

Case study: selection of educational software

In this part we present our decision making process of selection educational software.

A. Decision goal, the alternatives and the criteria

Course Selected chapters of mathematics is course in second year of pregraduate study of informatics. Using technology is expected for all courses so there is now need of decision if it s needed or not. Decision goal is to select best appropriate software to accomplish course objectives. In section Educational software we presented four alternatives that are considerable for use. Criteria for selection are Mastering Software by Teacher, Mastering Software by Students, Price, Covering Syllabus, Available Materials and User Help.

B. Establishing priorities among the elements

Figure 1. Hierarchical structure of decision problem shows hierarchical structure of our problem.

Figure 2. shows pairwise comparisons of importance of each of criteria. Pairwise comparisons are done by 1-9 scale recommended by AHP methodology. Final

rating of our criteria are: Price $(34,8\%)$, Covering Syllabus $(32,5\%)$, Mastering Software by Teacher $(12,9\%)$, Mastering Software by Students $(9,0\%)$, Available Materials $(6,5\%)$, User Help $(4,2\%)$.

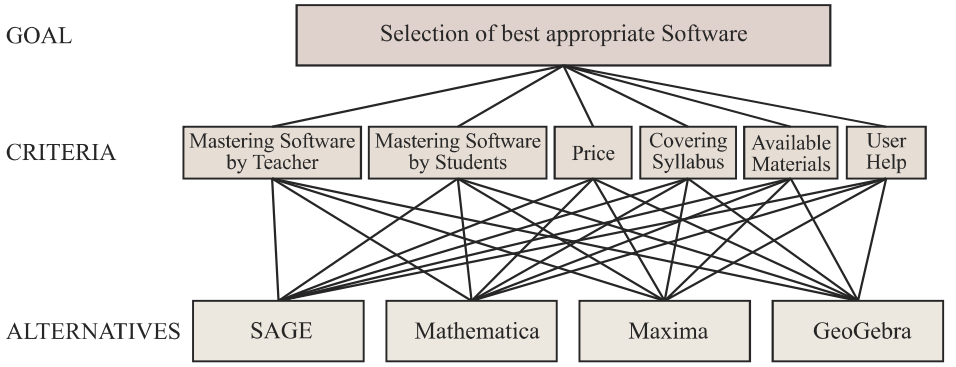

Figure 1. Hierarchical structure of decision problem.

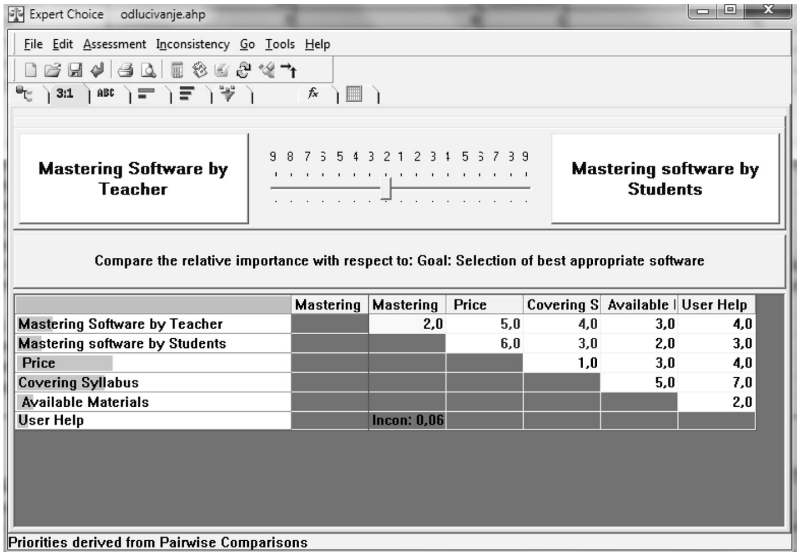

Figure 2. Pairwise comperisons of criteria .

Similar method was used to pairwise comparison of alternatives according to every criteria.

C. Overall priorities and consistency

Gathering all comparisons we can conclude that *SAGE* is best appropriate software for use in course Selected chapters of mathematics. In Table 1. Overall priorities in decision process is shown rating of every alternative according to every criteria. All comparisons were checked on consistency. Consistency ration on every comparison table were less then 0,10 what is acceptable by AHP methodology.

	Mastering Software by Teacher	Mastering Software by Students	Price	Covering Syllabus	Available Materials	User Help	Overall points
Weights	**12,9%**	**9,0%**	**34,8%**	**32,5%**	**6,5%**	**4,2%**	
SAGE	0,124	0,218	0,320	0,293	0,362	0,236	**0,275**
Mathematica	0,319	0,078	0,040	0,477	0,375	0,472	**0,265**
Maxima	0,074	0,149	0,320	0,186	0,165	0,106	**0,207**
GeoGebra	0,484	0,555	0,320	0,044	0,098	0,186	**0,253**

Tablica 1. Overall priorities in decision process.

D. Final decision

Using AHP methodology for multi criteria decision making we can conclude that *SAGE* is the most appropriate software for use in course Selected Chapters of Mathematics. Some of top advantages of *SAGE* are price, available materials and coverage of syllabus. Although, comparing to other software *SAGE* is just best in price, and in every other criteria there is one or even two alternatives better. For example, *Mathematica* is better in Covering syllabus, Available Materials, User Help but it is commercial software relatively difficult to earn for students. Teachers are more familiar with *Mathematica* while they graduated mathematics, but that isn t reason good enough to make it obligatory for students of informatics. *GeoGebra*, even free and easy to learn is not covering course program good enough to be the best choice.

Conslusion

Technology has enlarged opportunities to enhance mathematics teaching. There are some positive and negative aspects of using technology. Preparation for use of technology in teaching consumes more time. This effort doesn t produce tangible results in short time period so it s not recognized in community as it should be. Usage of technology opens new perspective on mathematics teaching methodology. Because of change of world, in some aspects, it becomes necessary to accomplish learning objectives. In our paper we have shown application of AHP, multi criteria decision methodology, as possibility for best appropriate software selection methodology. In decision process we used six criteria and four alternatives.

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