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EFFECTIVE STIFFNESS FOR STRUCTURAL ANALYSIS OF BUILDINGS IN EARTHQUAKE

Summary

The latest trend in seismic design of structures according to Eurocode 8 implies taking into account the effect of cracking while evaluating the stiffness of reinforced concrete elements – the stiffness affects on size of seismic forces and lateral displacements. In this paper an analysis of effective stiffness of cracked reinforced concrete elements depending on the size of axial force and the amount of longitudinal reinforcement was conducted, taking into account the tension stiffening effect. The results obtained through this analysis were then applied to a seven – storey residential building.

Key words

cracked reinforced concrete elements, effective stiffness, seismic forces

DJELOTVORNE KRUTOSTI ZA PRORAČUN KONSTRUKCIJA ZGRADA NA POTRESNO DJELOVANJE

Sažetak

Suvremeni pristup proračuna konstrukcija na potres prema Eurokodu 8 podrazumijeva uzimanje u obzir utjecaj raspucavanja na krutost armiranobetonskih elemenata. O krutosti ovise veličine potresnih sila i bočnih pomaka. Provedena je analiza djelotvorne krutosti raspucanih armiranobetonskih elemenata ovisno o veličini uzdužne sile i količini uzdužne armature, uzimajući u obzir sudjelovanje betona u nošenju na vlak između pukotina. Dobiveni rezultati primijenjeni su na proračunu sedmerokatne stambene zgrade.

Ključne riječi

raspucani armiranobetonski elemenati, djelotvorna krutost, potresne sile

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1. INTRODUCTION

Ductile reinforced concrete (RC) structures experience large plastic deformations during an earthquake. In critical regions of structures plastic hinges open up, plastic deformations occur in steel reinforcement while concrete often maintains its integrity due to confining reinforcement. Cracks in concrete, which reduce the stiffness of RC elements, occur at loads much smaller than those that correspond to yielding of reinforcement and bearing capacity of elements. In order to describe this process of how structures behave in earthquake as accurately as possible a non-linear dynamic analysis should be performed. However, today for most of the structures that are expected to be exposed to seismic actions a linear analysis based on a response spectrum is used. According to EN-1998-1 [1] linear analysis of structures in earthquake should be performed using a design spectrum which is reduced with respect to the elastic one by using the behaviour factor q. The elastic stiffness used in analysis should correspond to the stiffness of the elastic branch of such a bilinear global force-deformation response. This means that the use of the full elastic stiffness of uncracked concrete in the analysis is inapropriate [2]. By performing a linear analysis, under seismic actions, it is important that the distribution of member forces is based on realistic stiffness values (including cracks) applying at close to member yield forces. This will ensure that member ductilities are reasonably uniformly distributed [3]. An accurate assessment of member stiffness is also required in order to obtain the vibration periods of structure and seismic forces. Addition to the above, horizontal displacements also depend on member stiffness. In order to meet the criteria of serviceability it is important to take into account the effect of cracking while evaluating member stiffness. Excessive lateral dispalcements of structure can lead to unfavorable second order effects (P-Δ effects). In that case, by using stiffness of uncracked elements, lateral displacements and second order effects would be underestimated.

According to Eurocode 8, in concrete buildings, in composite steel – concrete buildings and in masonry buildings, the stiffness of the load bearing elements should be evaluated taking into account the effect of cracking. Such stiffness should correspond to the initiation of yielding of reinforcement. Unless a more accurate analysis is performed, the flexural and shear stiffness of RC or masonry elements may be taken as one half of the corresponding stiffness of the uncracked element. ACI 318-08 [4] and Paulay [3] provide recommendations for stiffness reduction depending on the type of RC elements (columns, beams or walls).

In this paper an analysis of ratio of effective stiffness and stiffness of uncracked element (initial stiffness), corresponding to the initiation of yielding of reinforcement, was conducted, taking into account the tension stiffening effect. The analysis was performed for different strength classes, reinforcement ratios and normalised axial forces. The results obtained through this analysis were then applied to a seven – storey residential building. Structural analysis for Model 1 was performed according to EN-1998-1 [1]. The ratio of effective stiffness and stiffness of uncracked element was obtained based on the amount of longitudinal reinforcement and size of normalised axial forces from Model 1. In this way, more accurate, obtained stiffness for walls was then applied in Model 2. In Model 3 initial stifness of all structural elements was used (the effect of cracking was neglected).

2. EFFECTIVE STIFFNESS OF RC WALLS

An analysis of stiffness of slender RC walls with rectangular cross-section (with height to length ratio $h_w/l_w \geq 2$), corresponding to the initiation of yielding of reinforcement, was carried out. Tensile reinforcement yields at stress equal to the characteristic yield strength f_{yk}. The wall (which is symmetrically reinforced) is in that case loaded by bending moment M_y. Only the reinforcement located at the ends of the wall (in boundary elements) was taken into account; the web reinforcement was neglected. Stress – strain relationship for concrete in compression was taken as non – linear according to EN 1992-1-1 [5]. A bilinear stress – strain relationship with a horizontal top branch for reinforcing steel B500B (f_{yk}=500 MPa) was assumed. For structural analysis of buildings it is practical to know the ratio of effective stiffness and stiffness of uncracked element (ie initial stiffness) $(EI)_{eff}/(E_{cm}I_c)$, where I_c is the second moment of area of concrete section, and E_{cm} mean value of modulus of elasticity of concrete. The effective stiffness can be determined using the following expression:

$$(EI)_{eff} = M_y / (1/r) \tag{1}$$

where M_y is the yielding moment and $1/r$ the mean curvature.

The mean curvature in any section of an element according to CEB-FIP Model Code 1990 [6] is given by the relationship:

$$\frac{1}{r} = \frac{\varepsilon_{sm} - \varepsilon_{cm}}{d} \tag{2}$$

where

ε_{sm} is the mean steel strain, ε_{cm} is the mean concrete strain and d is the effective depth of a cross section.

The mean curvature $(1/r)$ in case when bending moment M is equal to the yielding moment M_y [6] is:

$$\frac{1}{r} = \frac{1}{r_y} - \left(\frac{1}{r_{2r}} - \frac{1}{r_{1r}}\right) \cdot \beta_1 \cdot \beta_2 \cdot \left(\frac{M_r}{M_y}\right) \tag{3}$$

where

$1/r_y$ is the curvature corresponding to M_y and N,

$1/r_{2r}$ is the curvature in state II-naked corresponding to the action of M_r and N,

$1/r_{1r}$ is the curvature in state I corresponding to the action of M_r and N,

M_r is the cracking moment,

N is the applied normal force,

β_1 is the coefficient characterizing the bond quality of the reinforcing bars (β_1=1,0 for high bond bars and 0,5 for smooth bars), and

β_2 is the coefficient representing the influence of the duration of loading (β_2=0,5 for long – term loading or for a large number of load cycles and 0,8 at first loading).

Bending moment M_y was determined using equilibrium equations for axial forces and bending moments acting in the cross – section. Bending moment M_r was determined for the corresponding cross – section dimensions, axial force and mean value of tensile strength of concrete f_{ctm}. On the basis of the analysis in this research, it was found out that the ratio $(EI)_{\text{eff}}/(E_{cm}I_c)$ depend on concrete class, symetric tension and compression reinforcement ratio (ρ_1=ρ_2) and normalised axial compression force ν_d given by :

$$\nu_d = \frac{N_{Ed}}{A_c \cdot f_{cd}} \tag{4}$$

where N_{Ed} is the axial force, A_c area of section of concrete member and f_{cd} design value of concrete compressive strength.

Ratios of effective stiffness and initial stiffness $(EI)_{\text{eff}}/(E_{cm}I_c)$ for slender walls for concrete class C25/30 and different ν_d and ρ are illustrated in Figure 1. These ratios can as well be used for columns.

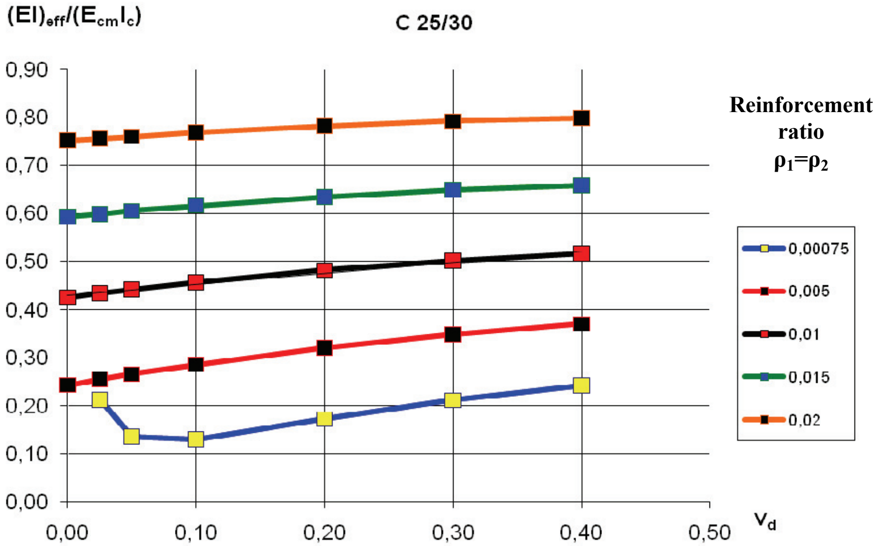

Figure 1. Ratio $(EI)_{eff}/(E_{cm}I_c)$ for different ν_d and ρ (concrete class C25/30, steel B500 B)

3. STRUCTURAL MODEL

The building consists of a basement and 6 floors (Figure 2). The structural grid of columns and walls is regular - the spacing of grid lines equals 6,0 m in both directions, as shown in Figure 3. In each direction the building is braced by two walls and colums that run without interruption from their foundations to the top of the building. Along the perimeter of the building columns are connected with beams, while the rest of the columns are connected with floor slabs, thus making a frame system. The storey height is 3,4 m. The cross – section dimensions of structural elements are: columns 50/50 cm, perimeter beams 22/40 cm, plate thickness 20 cm, wall length 6,0 m and thickness 22 cm.

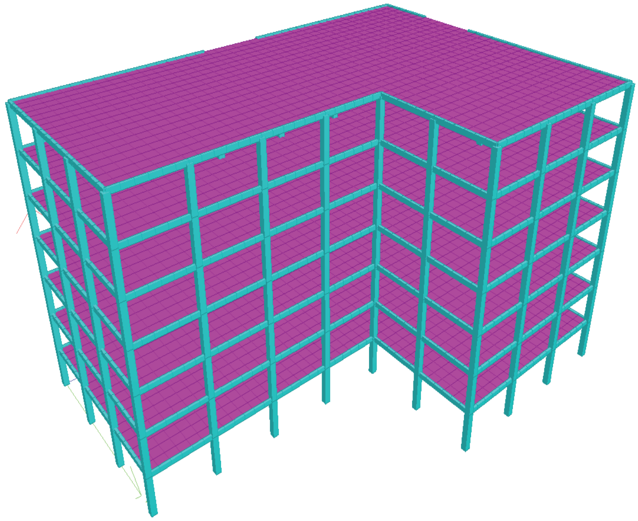

Figure 2. Residential building – 3D model

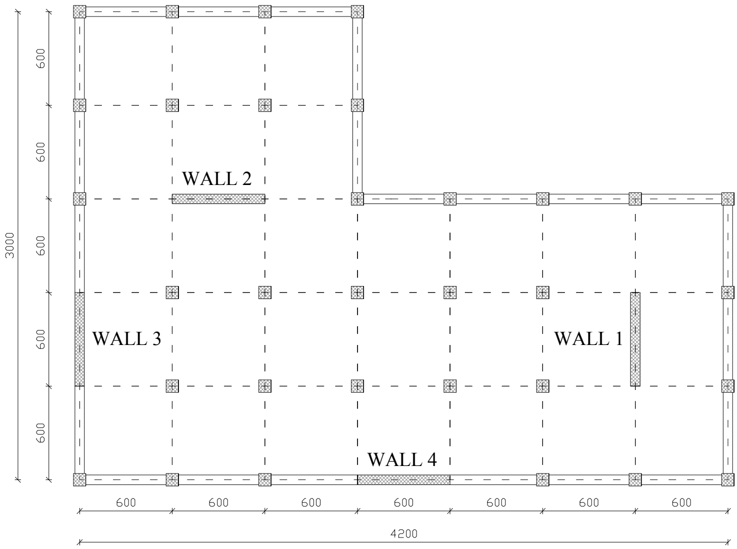

Figure 3. Floor plan

3.1. ACTIONS ON STRUCTURE

Structural analysis of the building was performed using STAAD.Pro 2007 [7]. The dead load and load of the partition walls was assumed as g=2,0 kN/m^2. The purpose of the building is for residential activities (category A); according to EN 1991-1 [8] the live load was taken equal to q_k=2 kN/m^2. Seismic design was performed using modal response spectrum analysis which is applicable to all types of buildings, ie to buildings whose response is significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction. The disign spectrum (Figure 4) was determined according to [1] with reference peak ground acceleration a_{gR}=0,2g. The structure is a wall-equivalent dual system and the behaviour factor used was q=3,0.

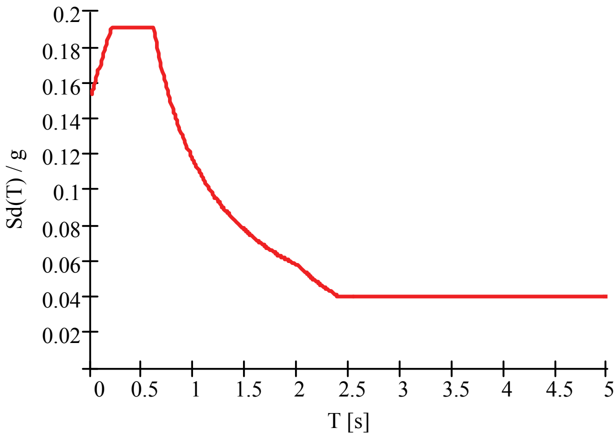

Figure 4. Design spectrum for ground type C, q=3,0 and a_g=0,2 g

3.2. MODEL 1

The stiffness of all columns, beams and walls was taken as one half of the corresponding stiffness of the uncracked element. The stiffness of floor slabs was not reduced due to the estimation that they will not experience large deformations, that could cause significant cracking.

3.3. MODEL 2

The stiffness of the elements was chosen according to the provided amount of reinforcement from the analysis in Model 1. The stiffness of floor slabs was not reduced. Ratio of effective and initial stiffness of RC walls $(EI)_{\mathrm{eff}}/(E_{cm}I_c)$ depends on ρ_1 and v_d, and has a value from 0,33 to 0,39 in areas from the bottom of the walls till a height of 10,2 m, while on the remaining height of the walls the ratio of effective and initial stiffness is approximately 0,25. For the analysis the adopted values were: 0,35 (from the bottom of the wall to 10,2 m) and 0,25 on the remaining height. The ratio of effective and initial stiffness of the columns was not further analyzed because of their small contribution to the overall stiffness of the building; so a value of $(EI)_{\mathrm{eff}}/(E_{cm}I_c)$=0,50 was adopted as in Model 1. For the same reason, ratio of effective and initial stiffness for all the beams was taken as $(EI)_{\mathrm{eff}}/(E_{cm}I_c)$=0,50, as in Model 1. According to [4] the stiffness of cracked walls should be taken as 0,35I_g, while according to [3] the stiffness of cracked walls should be chosen from 0,25I_g to 0,45I_g, depending on the size of axial force, where I_g is second moment of area of uncracked section.

3.4. MODEL 3

The stiffness of elements was chosen as there will be no cracking. Ratio of effective and initial stiffness (slab, columns, walls and beams) was set as $(EI)_{\mathrm{eff}}/(E_{cm}I_c)$=1,0.

4. RESULT COMPARISON

Structural analysis of the building considered, with different stiffness assigned in Models 1, 2 and 3 gives different values of fundamental periods, base shear for each direction of seismic action, bending moments and shear forces at the bottom of the walls (given in Table 1).

Table 1. Comparison of periods, base shear, bending moments and shear forces for Models 1, 2 and 3 and their relations

Value	**Model 1 $(EI)_{eff}/(E_{cm}I_c)$=0,5**	**Model 2 (more accurate stiffness)**	**Model 3 $(EI)_{eff}/(E_{cm}I_c)$=1,0**	**Model 1/ Model 2**	**Model 3/ Model 2**
1. mode period	1,091 s	1,219 s	0,861 s	0,89	0,71
2. mode period	1,085 s	1,208 s	0,857 s	0,90	0,71
3. mode period	0,855 s	0,971 s	0,655 s	0,88	0,67
Base shear x	4737,83 kN	4372,07 kN	5726,42 kN	1,08	1,31
Base shear y	4886,69 kN	4433,58 kN	5885,21 kN	1,10	1,33
Bending moment wall 1	25670 kNm	20140 kNm	38270 kNm	1,27	1,90
Bending moment wall 2	24180 kNm	19680 kNm	34870 kNm	1,23	1,77
Bending moment wall 3	27030 kNm	22330 kNm	37990 kNm	1,21	1,70
Bending moment wall 4	20380 kNm	16880 kNm	28580 kNm	1,21	1,69
Shear force wall 1	2484 kN	2130 kN	3102 kN	1,17	1,46
Shear force wall 2	2363 kN	2077 kN	2900 kN	1,14	1,40
Shear force wall 3	2477 kN	2169 kN	3010 kN	1,14	1,39
Shear force wall 4	1866 kN	1672 kN	2284 kN	1,12	1,37

As a consequence of different stiffness different storey and interstorey drifts are obtained under seismic actions. Storey drifts, shown in Figure 5, were determined for no-collapse requirement according to EN 1998-1 [1] by means of the following expression:

$$d_s = q_d \cdot d_e \tag{5}$$

where q_d is displacement behaviour factor (assumed equal to behaviour factor q), and d_e is the displacement of the same point of the structural system, as determined by a linear analysis based on the design response spectrum.

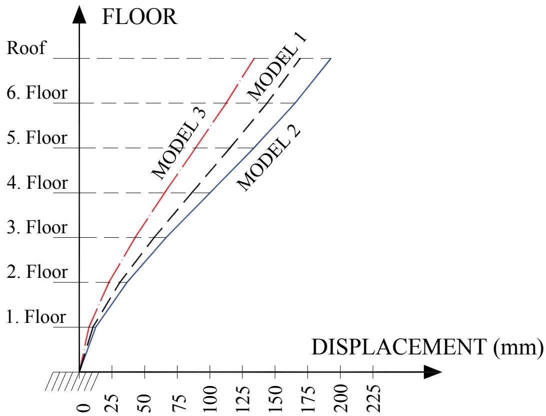

Figure 5. Comparison of storey drifts

5. CONCLUSIONS

The resulting bending moments at the bottom of the walls for Model 1 are about 1,2 times greater than the moments calculated in Model 2 (model where the effective stiffness of walls was more accurate). In Model 3, with full element stiffness, bending moments at the bottom of the wall are about 1,7 times greater than in Model 2. Horizontal displacements of the structure in Model 2 are 1,1 times greater than those in Model 1, while horizontal displacements in Model 2 are 1,3 times greater than those in Model 3. According to this ratios, it can be concluded that horizontal displacements vary less than internal forces with the change of stiffness. It can be generally concluded, that the more accurate effective stiffness will give smaller internal forces in RC wall and frame buildings in earthquake, which can affect a more economical structural design.

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