EFFECTIVE STIFFNESS FOR STRUCTURAL ANALYSIS OF BUILDINGS IN EARTHQUAKE

Summary
The latest trend in seismic design of structures according to Eurocode 8 implies taking into account the effect of cracking while evaluating the stiffness of reinforced concrete elements – the stiffness affects on size of seismic forces and lateral displacements. In this paper an analysis of effective stiffness of cracked reinforced concrete elements depending on the size of axial force and the amount of longitudinal reinforcement was conducted, taking into account the tension stiffening effect. The results obtained through this analysis were then applied to a seven – storey residential building.

Key words
cracked reinforced concrete elements, effective stiffness, seismic forces

DJELOTVORNE KRUTOSTI ZA PRORAČUN KONSTRUKCIJA ZGRADA NA POTRESNO DJELOVANJE

Sažetak
Suvremeni pristup proračuna konstrukcija na potres prema Eurokodu 8 podrazumijeva uzimanje u obzir utjecaj raspucavanja na krutost armiranobetonskih elemenata. O krutosti ovise veličine potresnih sila i bočnih pomaka. Provedena je analiza djelotvorne krutosti raspučanih armiranobetonskih elemenata ovisno o veličini uzdužne sile i količini uzdužne armature, uzimajući u obzir sudjelovanje betona u nošenju na vlak između pukotina. Dobiveni rezultati primijenjeni su na proračunu sedmerokatne stambene zgrade.

Ključne riječi
raspučani armiranobetonski elemenati, djelotvorna krutost, potresne sile

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1. INTRODUCTION

Ductile reinforced concrete (RC) structures experience large plastic deformations during an earthquake. In critical regions of structures plastic hinges open up, plastic deformations occur in steel reinforcement while concrete often maintains its integrity due to confining reinforcement. Cracks in concrete, which reduce the stiffness of RC elements, occur at loads much smaller than those that correspond to yielding of reinforcement and bearing capacity of elements. In order to describe this process of how structures behave in earthquake as accurately as possible a non-linear dynamic analysis should be performed. However, today for most of the structures that are expected to be exposed to seismic actions a linear analysis based on a response spectrum is used. According to EN-1998-1 [1] linear analysis of structures in earthquake should be performed using a design spectrum which is reduced with respect to the elastic one by using the behaviour factor $q$. The elastic stiffness used in analysis should correspond to the stiffness of the elastic branch of such a bilinear global force-deformation response. This means that the use of the full elastic stiffness of uncracked concrete in the analysis is inappropriate [2]. By performing a linear analysis, under seismic actions, it is important that the distribution of member forces is based on realistic stiffness values (including cracks) applying at close to member yield forces. This will ensure that member ductilities are reasonably uniformly distributed [3].

An accurate assessment of member stiffness is also required in order to obtain the vibration periods of structure and seismic forces. Addition to the above, horizontal displacements also depend on member stiffness. In order to meet the criteria of serviceability it is important to take into account the effect of cracking while evaluating member stiffness. Excessive lateral displacements of structure can lead to unfavorable second order effects (P-Δ effects). In that case, by using stiffness of uncracked elements, lateral displacements and second order effects would be underestimated.

According to Eurocode 8, in concrete buildings, in composite steel – concrete buildings and in masonry buildings, the stiffness of the load bearing elements should be evaluated taking into account the effect of cracking. Such stiffness should correspond to the initiation of yielding of reinforcement. Unless a more accurate analysis is performed, the flexural and shear stiffness of RC or masonry elements may be taken as one half of the corresponding stiffness of the uncracked element. ACI 318-08 [4] and Paulay [3] provide recommendations for stiffness reduction depending on the type of RC elements (columns, beams or walls).

In this paper an analysis of ratio of effective stiffness and stiffness of uncracked element (initial stiffness), corresponding to the initiation of yielding of reinforcement, was conducted, taking into account the tension stiffening effect. The analysis was performed for different strength classes, reinforcement ratios and normalised axial forces. The results obtained through this analysis were then applied to a seven – storey residential building. Structural analysis for Model 1 was performed according to EN-1998-1 [1]. The ratio of effective stiffness and stiffness of uncracked element was obtained based on the amount of longitudinal reinforcement and size of normalised axial forces from Model 1. In this way, more accurate, obtained stiffness for walls was then applied in Model 2. In Model 3 initial stiffness of all structural elements was used (the effect of cracking was neglected).
2. EFFECTIVE STIFFNESS OF RC WALLS

An analysis of stiffness of slender RC walls with rectangular cross-section (with height to length ratio \( h_w/l_w \geq 2 \)), corresponding to the initiation of yielding of reinforcement, was carried out. Tensile reinforcement yields at stress equal to the characteristic yield strength \( f_{yk} \). The wall (which is symmetrically reinforced) is in that case loaded by bending moment \( M_y \). Only the reinforcement located at the ends of the wall (in boundary elements) was taken into account; the web reinforcement was neglected. Stress – strain relationship for concrete in compression was taken as non – linear according to EN 1992-1-1 [5]. A bilinear stress – strain relationship with a horizontal top branch for reinforcing steel B500B \((f_{yk}=500 \text{ MPa})\) was assumed. For structural analysis of buildings it is practical to know the ratio of effective stiffness and stiffness of uncracked element (ie initial stiffness) \((EI)_{eff}/(EcmIc)\), where \( Ic \) is the second moment of area of concrete section, and \( Ecm \) mean value of modulus of elasticity of concrete. The effective stiffness can be determined using the following expression:

\[
(EI)_{eff} = M_y/(1/r)
\]

where \( M_y \) is the yielding moment and \( 1/r \) the mean curvature.

The mean curvature in any section of an element according to CEB-FIP Model Code 1990 [6] is given by the relationship:

\[
\frac{1}{r} = \frac{\varepsilon_{cm} - \varepsilon_{sm}}{d}
\]

where

\( \varepsilon_{sm} \) is the mean steel strain, \( \varepsilon_{cm} \) is the mean concrete strain and \( d \) is the effective depth of a cross section.

The mean curvature \((1/r) \) in case when bending moment \( M \) is equal to the yielding moment \( M_y \) [6] is:

\[
\frac{1}{r} = \frac{1}{r_y} - \left( \frac{1}{r_{y_2}} - \frac{1}{r_{y_1}} \right) \cdot \beta_1 \cdot \beta_2 \cdot \frac{(M_y - Mr)}{M_y}
\]

where

\( 1/r_y \) is the curvature corresponding to \( M_y \) and \( N \),

\( 1/r_{y_2} \) is the curvature in state II-naked corresponding to the action of \( M_r \) and \( N \),

\( 1/r_{y_1} \) is the curvature in state I corresponding to the action of \( M_r \) and \( N \),

\( M_r \) is the cracking moment,

\( N \) is the applied normal force,

\( \beta_1 \) is the coefficient characterizing the bond quality of the reinforcing bars (\( \beta_1 = 1,0 \) for high bond bars and 0,5 for smooth bars), and

\( \beta_2 \) is the coefficient representing the influence of the duration of loading (\( \beta_2 = 0,5 \) for long term loading or for a large number of load cycles and 0,8 at first loading).
Bending moment $M_y$ was determined using equilibrium equations for axial forces and bending moments acting in the cross-section. Bending moment $M_y$ was determined for the corresponding cross-section dimensions, axial force and mean value of tensile strength of concrete $f_{ctm}$. On the basis of the analysis in this research, it was found out that the ratio $(EI)_{eff}/(EcmIc)$ depend on concrete class, symmetric tension and compression reinforcement ratio ($\rho_1=\rho_2$) and normalised axial compression force $\nu_d$ given by:

$$\nu_d = \frac{N_{Ed}}{A_c \cdot f_{cd}}$$

(4)

where $N_{Ed}$ is the axial force, $A_c$ area of section of concrete member and $f_{cd}$ design value of concrete compressive strength.

Ratios of effective stiffness and initial stiffness $(EI)_{eff}/(EcmIc)$ for slender walls for concrete class C25/30 and different $\nu_d$ and $\rho$ are illustrated in Figure 1. These ratios can as well be used for columns.

Figure 1. Ratio $(EI)_{eff}/(EcmIc)$ for different $\nu_d$ and $\rho$ (concrete class C25/30, steel B500 B)

3. **STRUCTURAL MODEL**

The building consists of a basement and 6 floors (Figure 2). The structural grid of columns and walls is regular - the spacing of grid lines equals 6.0 m in both directions, as shown in Figure 3. In each direction the building is braced by two walls and columns that run without interruption from their foundations to the top of the building. Along the perimeter of the building columns are connected with beams, while the rest of the columns are connected with floor slabs, thus making a frame system. The storey height is 3.4 m. The cross-section dimensions of structural elements are: columns 50/50 cm, perimeter beams 22/40 cm, plate thickness 20 cm, wall length 6.0 m and thickness 22 cm.
3.1. ACTIONS ON STRUCTURE

Structural analysis of the building was performed using STAAD.Pro 2007 [7]. The dead load and load of the partition walls was assumed as $g=2\,\text{kN/m}^2$. The purpose of the building is for residential activities (category A); according to EN 1991-1 [8] the live load was taken equal to $q_e=2\,\text{kN/m}^2$. Seismic design was performed using modal response spectrum analysis which is applicable to all types of buildings, i.e. to buildings whose response is significantly affected by contributions from modes of vibration higher than the
fundamental mode in each principal direction. The design spectrum (Figure 4) was determined according to [1] with reference peak ground acceleration $a_g^R = 0.2 g$. The structure is a wall-equivalent dual system and the behaviour factor used was $q = 3.0$.

![Figure 4. Design spectrum for ground type C, $q=3.0$ and $a_g=0.2$ g](image)

### 3.2. MODEL 1

The stiffness of all columns, beams and walls was taken as one half of the corresponding stiffness of the uncracked element. The stiffness of floor slabs was not reduced due to the estimation that they will not experience large deformations, that could cause significant cracking.

### 3.3. MODEL 2

The stiffness of the elements was chosen according to the provided amount of reinforcement from the analysis in Model 1. The stiffness of floor slabs was not reduced. Ratio of effective and initial stiffness of RC walls $(EI)_{eff}/(EcmI_c)$ depends on $\rho_1$ and $\nu_d$, and has a value from 0.33 to 0.39 in areas from the bottom of the walls till a height of 10.2 m, while on the remaining height of the walls the ratio of effective and initial stiffness is approximately 0.25. For the analysis the adopted values were: 0.35 (from the bottom of the wall to 10.2 m) and 0.25 on the remaining height. The ratio of effective and initial stiffness of the columns was not further analyzed because of their small contribution to the overall stiffness of the building; so a value of $(EI)_{eff}/(EcmI_c) = 0.50$ was adopted as in Model 1. For the same reason, ratio of effective and initial stiffness for all the beams was taken as $(EI)_{eff}/(EcmI_c) = 0.50$, as in Model 1. According to [4] the stiffness of cracked walls should be taken as $0.35I_g$, while according to [3] the stiffness of cracked walls should be chosen from $0.25I_g$ to $0.45I_g$, depending on the size of axial force, where $I_g$ is second moment of area of uncracked section.
3.4. MODEL 3

The stiffness of elements was chosen as there will be no cracking. Ratio of effective and initial stiffness (slab, columns, walls and beams) was set as \((EI)_{\text{eff}}/(E_{\text{cm}}I_c)=1,0\).

4. RESULT COMPARISON

Structural analysis of the building considered, with different stiffness assigned in Models 1, 2 and 3 gives different values of fundamental periods, base shear for each direction of seismic action, bending moments and shear forces at the bottom of the walls (given in Table 1).

<table>
<thead>
<tr>
<th>Value</th>
<th>Model 1 (\frac{(EI)<em>{\text{eff}}}{(E</em>{\text{cm}}I_c)=0.5})</th>
<th>Model 2 (\text{(more accurate stiffness)})</th>
<th>Model 3 (\frac{(EI)<em>{\text{eff}}}{(E</em>{\text{cm}}I_c)=1.0})</th>
<th>Model 1/Model 2</th>
<th>Model 3/Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. mode period</td>
<td>1,091 s</td>
<td>1,219 s</td>
<td>0,861 s</td>
<td>0,89</td>
<td>0,71</td>
</tr>
<tr>
<td>2. mode period</td>
<td>1,085 s</td>
<td>1,208 s</td>
<td>0,857 s</td>
<td>0,90</td>
<td>0,71</td>
</tr>
<tr>
<td>3. mode period</td>
<td>0,855 s</td>
<td>0,971 s</td>
<td>0,655 s</td>
<td>0,88</td>
<td>0,67</td>
</tr>
<tr>
<td>Base shear x</td>
<td>4737,83 kN</td>
<td>4372,07 kN</td>
<td>5726,42 kN</td>
<td>1,08</td>
<td>1,31</td>
</tr>
<tr>
<td>Base shear y</td>
<td>4886,69 kN</td>
<td>4433,58 kN</td>
<td>5885,21 kN</td>
<td>1,10</td>
<td>1,33</td>
</tr>
<tr>
<td>Bending moment wall 1</td>
<td>25670 kNm</td>
<td>20140 kNm</td>
<td>38270 kNm</td>
<td>1,27</td>
<td>1,90</td>
</tr>
<tr>
<td>Bending moment wall 2</td>
<td>24180 kNm</td>
<td>19680 kNm</td>
<td>34870 kNm</td>
<td>1,23</td>
<td>1,77</td>
</tr>
<tr>
<td>Bending moment wall 3</td>
<td>27030 kNm</td>
<td>22330 kNm</td>
<td>37990 kNm</td>
<td>1,21</td>
<td>1,70</td>
</tr>
<tr>
<td>Bending moment wall 4</td>
<td>20380 kNm</td>
<td>16880 kNm</td>
<td>28580 kNm</td>
<td>1,21</td>
<td>1,69</td>
</tr>
<tr>
<td>Shear force wall 1</td>
<td>2484 kN</td>
<td>2130 kN</td>
<td>3102 kN</td>
<td>1,17</td>
<td>1,46</td>
</tr>
<tr>
<td>Shear force wall 2</td>
<td>2363 kN</td>
<td>2077 kN</td>
<td>2900 kN</td>
<td>1,14</td>
<td>1,40</td>
</tr>
<tr>
<td>Shear force wall 3</td>
<td>2477 kN</td>
<td>2169 kN</td>
<td>3010 kN</td>
<td>1,14</td>
<td>1,39</td>
</tr>
<tr>
<td>Shear force wall 4</td>
<td>1866 kN</td>
<td>1672 kN</td>
<td>2284 kN</td>
<td>1,12</td>
<td>1,37</td>
</tr>
</tbody>
</table>

As a consequence of different stiffness different storey and interstorey drifts are obtained under seismic actions. Storey drifts, shown in Figure 5, were determined for no-collapse requirement according to EN 1998-1 [1] by means of the following expression:

\[
d_x = q_d \cdot d_c\]

where \(q_d\) is displacement behaviour factor (assumed equal to behaviour factor \(q\)), and \(d_c\) is the displacement of the same point of the structural system, as determined by a linear analysis based on the design response spectrum.
5. CONCLUSIONS

The resulting bending moments at the bottom of the walls for Model 1 are about 1.2 times greater than the moments calculated in Model 2 (model where the effective stiffness of walls was more accurate). In Model 3, with full element stiffness, bending moments at the bottom of the wall are about 1.7 times greater than in Model 2. Horizontal displacements of the structure in Model 2 are 1.1 times greater than those in Model 1, while horizontal displacements in Model 2 are 1.3 times greater than those in Model 3. According to this ratios, it can be concluded that horizontal displacements vary less than internal forces with the change of stiffness. It can be generally concluded, that the more accurate effective stiffness will give smaller internal forces in RC wall and frame buildings in earthquake, which can affect a more economical structural design.

LITERATURE

[4] Building Code Requirements for Structural Concrete (ACI 318M-08) and Commentary, American Concrete Institute, Farmington Hills, 2008.