Modeling of Worsening

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ABSTRACT

This paper aims to add impetus to understanding of worsening phenomena and prevention of their consequences. The introductory notes outline the uncertainty, the empirical character and the subjective meaning of the term “worsening”. The underlying hypothesis takes up the idea of general cause and effect relations in order to reveal the cause and effect interaction concept of worsening. The concept of worsening is analytically modeled as an accumulation of effects in permanent interaction with causes. The mathematical formulation of the worsening concept is applied to examples of common engineering problems of material plasticity, fatigue and corrosion.

Keywords: causal relations, causality, interaction, system feedback, cumulative causation, fatigue, plasticity, corrosion.

1. INTRODUCTION

Things do not only happen, they also worsen!

Moreover, they frequently worsen faster than it is anticipated or expected in their lifetime. Worsening has direct influences and impact on material objects and live beings - on physical, chemical, biological, technical, and social processes and properties, both natural and those created by people. Worsening also inevitably affects engineering objects, structures and operations, and sometimes very badly. It is an expected phenomenon in the experience of reality; however its resulting consequences are not always predictable with certainty.

Worsening is normally perceived as an accumulation phenomenon of progressive unfavorable but hopefully finite effects due to inescapable causes. The hope that the mechanism by which the cause induces the worsening must be understood is not always sufficient to explain intricate causal relation in which the worsening can retroactively affect the cause.

Sometimes, worsening can be distinguished timely to prevent disadvantageous effects, but more frequently it can be noticed only after the apparent detectable consequences. In many disastrous situations the consequences of worsening cannot be stopped or prevented. For this reason it is vital to have worsening under control, at least until a warning that an urgent preventive action must be taken or considered. Inspections and maintenance actions carried out to reduce the lifetime worsening are usually time consuming and expensive. Repairing of damages can be even more expensive. Any uncontrolled worsening may cause failures, damages, collapses, breaks, devastations and, in the worst case, disasters with possible catastrophic consequences for human life and goods.

2. WORSENING

Worsening has a general connotation of an empirical causality or fatality, sometimes with supernatural and mystical prejudice, since it cannot be always explained and accepted just as a simple causal relation. Worsening involves various factors, sometimes those that cannot be accurately measured or reliably identified; this fact makes it more intricate or fearful and therefore requiring the engagement of intuition, experience and sophisticated rationalization. The experiences in modeling of worsening reveal that cause and effect interactions stay behind earlier concepts such as the “positive feedback” or "cumulative causation" where some effect causes more of itself resulting with the amplification of changes. In contrast, negative feedback and negative accumulative causation opposes changes resulting in attenuation of effects. Positive refers to the direction of change rather than to the consequences of the causation.

The very concept of worsening has important objective or practical character and subjective meaning. The common empirical truth is that the world is constantly changing. Generally, the effects of changes can be described from different viewpoints. One of such viewpoints is the qualitative and quantitative assessment of consequences of changes, which - if they are experienced as bad, harmful, dangerous, undesired or destructive - can be considered as worsening, sometimes subjectively perceived as punishment.

Worsening is also commonly experienced as a relative phenomenon with respect to some predefined constant or uniform reference condition. Such relative changes are sometimes explicated as weakening, yielding, and ageing or fatigue. Yet another observable consequence of worsening is the life shortening with respect to the expected lifetime.

There is no general model of worsening but rather some particular or interdisciplinary views on different problems of worsening. Many processes in sciences and engineering are too complex, and mathematical approximations in practice are often not accurate enough for the appropriate modeling of the multifaceted worsening phenomena.

The above reasons are why this paper focuses on worsening and intends to argue for a concept of worsening as a more general idea within the empirical context of causality. The causal relationship is a widely adopted concept for the understanding of physical reality at micro and macroscopic levels. The paper looks for a more general mental and analytical comprehension of worsening in the context of empirical causality and in the experience of the causal accumulation and positive feedback.

Causality is commonly considered as a relationship between an event (the *cause*) provoked by an outer action and some second event (the *effect*), where the second event is taken as a consequence of the first one - evident to conscious observers.

3. HYPOTHESIS OF WORSENING

The fourth David Hume's [1] statement from 1896 on the judging whether two things are in a cause-effect relation:

> *"The same cause always produces the same effect, and the same effect never arises but from the same cause. This principle we derive from experience, and is the source of most of philosophical reasoning."*

Statements about causality in this study are critically revisited from practicality point of view following the inspiring intuition that causes and effects in reality are of definite and finite character. Once initiated, the cause and the effect can continue to interact with intensity appropriately to their internal causal property not necessarily affected by the external influences. However, the finite cause and effect are substantially dependent on the observable start and on the definite end. The study discusses the thesis that the general causality implies the cause-effect relation and the finite interaction between causes and effects. Internal properties of a relation might affect the dependence of the progression on current and previous initial or input characteristics. Here, the cause-effect interaction concept links to the Wiener's idea of positive feedback from 1948 [2]. Worsening also represents the growth of unfavorable effect and relates to the von Bertalanffy growth theory [3].

Consequently, the paper introduces a more general concept of the finite cause-effect interaction in its preliminary primitive deterministic form. The proposal considers a plausibly practical involvement of the general concept of interactions between causes and effects into the empirical analysis of worsening problems. The mathematical formulation of the interaction concept in the sequel uses the thesis put forward in this study that seemingly complex causal relations are analytically decomposable into simple causal relation and into a cause-effect interaction. The thesis then continues to elaborate that the roots of worsening lie in the realistic cause-effect interactions which are affecting the primarily assumed properties or the expected outcomes of assumingly ideal relations in which the possibilities of cause and effect meddling are not accounted for.

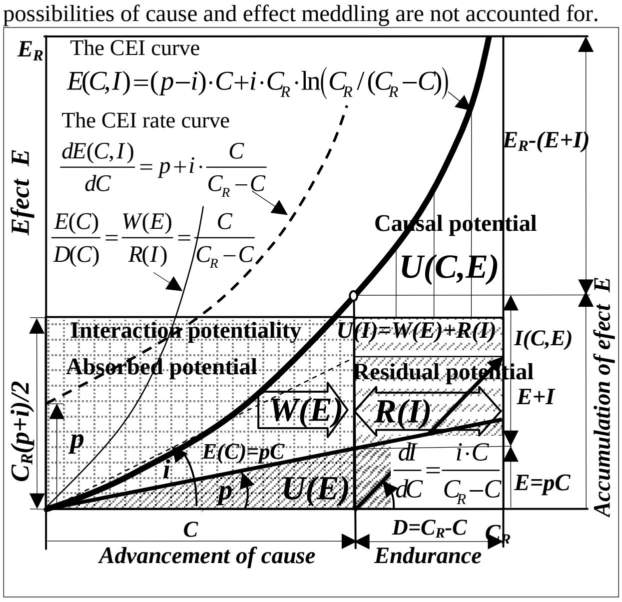

Figure 1. The general concept of cause-effect interactions

The paper, first of all, reminds us how significant the empirical causal relations, explanations, reasoning and inference in sciences and engineering are (e.g. Woodward in 2003 [4] and Pearl in 2009 [5].

4. INTERACTION CONCEPT OF WORSENING

The study at the beginning resumes the Newtonian causation, sometimes denoted as linear causality, between the efficient Cause (C) and the accumulation of a single Effect $E(C)$ at rate p without limits that is analytically presentable by the integral up to the ongoing cause C, as shown:

$$E(C)=\int_0^C pdC = p\cdot C \tag{1}$$

In ideal Cause-Effect (CE) relations with undefined terminal conditions the cause produces the effect $C \Rightarrow E$ or the effect is hold up by the cause $E \Leftarrow C$ but the effect cannot change the cause $E \not\Rightarrow C$ (asymmetry). However, for the finite CE relations in reality the finite cause C_R induces the finite effect E_R as $C_R \Rightarrow E_R$ and interactions between cause and effect can occur due to the influence of the terminal conditions on the causal relationship. The Cause-Effect Interaction (CEI) concept admits that the effect E induced by the cause in turn can affect the subsequent cause C itself $E \Rightarrow C$, where $\Leftrightarrow$ does not stands for symmetry but for interaction $C \Leftrightarrow E$ as $E \Leftrightarrow C$.

The steering thought of the general CEI concept exposed in the study is that the resulting overall effect $E(C,I)=E(C)+I(C,E)$ is decomposable into an observable primary linear CE causal effect $E(C)=p\cdot C$ as in Eq. (1) unaffected by terminal conditions and into less apparent secondary CEI effect $I(C,E)$.

The CEI occurs due to continuous tire out of the residual causal durance D (Fig. 1) at a constant rate $1/i$ Eq. (2), as a consequence of the escalation of the finite cause C_R until reaching the finite effect E_R, defined as follows:

$$D(C)=\frac{1}{i}\int_{C_R-C}^{C_R} dC = C_R - C \tag{2}$$

The interaction rate is then the ratio between the effect E in Eq. (1) and the residual causal effect D (the durance) in Eq. (2) as:

$$\frac{dI(C,E)}{dC}=i\cdot\frac{E(C)}{D(C)}=i\cdot\frac{C}{C_R-C}=i\cdot\frac{c}{1-c} \tag{3}$$

The deterministic parameter i is introduced in Eq. (2) and Eq. (3) to represent the CEI intensity of a CE relation.

The resulting overall rate of change is then the combination of the simple CE rate Eq. (1) and added CEI rate Eq. (3) as shown:

$$E' == p+i\frac{E(C)}{D(C)}=p+i\frac{C}{C_R-C}=p+i\frac{c}{1-c} \tag{4}$$

The deterministic parameter p in Eq. (4) represents the initial propensity to interaction. For $c\to 0$ in (4) it follows: $p=dE/dC$.

The standardized cause $c=C/C_R$ and effect $e=E/C_R$ represent the CE relation with respected to the reference cause C_R and reference effect E_R in the 0-1 space. The interaction rate $E(C)/D(C)$ in Eq. (3) explains the CEI concept: the cause C (in the nominator) upholds the CE relation $E(C)$ as in Eq. (1) but simultaneously reduces the endurance $D=C_R-C$ (in the denominator). Thus, C, D and E jointly induce the CEI. The formulation Eq. (3) enlightens the meaning of the amplifier or final gain in the positive feedback as a consequence of the CEI.

The second derivative of Eq. (4) is the sensitivity of the overall, that is, the rate of the interaction rate, as it is put down below:

$$E''=\frac{dE'}{dC}=\frac{d^2E}{dC^2}=i\cdot\frac{C_R}{(C_R-C)^2}=\frac{i}{C_R}\cdot\frac{1}{(1-c)^2} \tag{5}$$

The overall rate of interaction *E*' Eq. (4) represents the increase of the slope of the tangent on the CE curve (Fig. 1).

The accumulation of CEI effects *I(C,E)* during the *CE* relation is the superposition of the linear and of the logarithmic part after integration of the interaction rate Eq. (4), available from mathematical handbooks, as it follows:

$$E(C,I) = E_o + (p-i)\cdot C + i\cdot C_R \cdot \ln\frac{C_R}{C_R - C} \qquad (6a)$$

Or, rewritten in the standardized c-e (0-1) space (6a) as shown:

$$e(C,I) = e_o + (p-i)\cdot c - i\cdot \ln(1-c) \qquad (6b)$$

The nonlinear part of the CEI interaction rate (4) is at hand in feedback theory [2] as the amplification or the gain factor normally applied in the form of closed loop transfer function $A_f=x/(1+\beta x)$ where the feedback factor (FBF) is preferably $\beta<0$.

The cause *C* is not simply separable from Eq. (6). Due to CE interaction *C* appears on the both sides of the equation:

$$C = C_R \cdot (1 - e^{\frac{1}{iC_R}[(p-i)\cdot C-(E-E_o)]}) \qquad (6c)$$

Only for *p*=*i*, the resulting exponential term in Eq. (6c) (Fig. 1) for pure CEI without the linear part $C = C_R \cdot (1 - e^{-\frac{E-E_o}{i\cdot C_R}})$ represents the exponential low (Fig. 2) that corresponds to the von Bertalanffy growth function (VBGF) [3] where C_R represents the ultimate growth. Thus, the inseparable implicit interaction term (6) generalizes VBGF as a combination of indivisible exponential and linear growth.

The inherent power of a CE relation, its state of being capacitive or resourceful and capable for interaction represents the CEI potentiality *U*. The CEI potentiality is evident from the area under the $E-C$ curves next to the C axis which can be derived by integration of Eq. (6) (Fig. 1), as shown:

$$U(C,E) = \int_0^C E(C,I)dC =$$

$$= \frac{p-i}{2}C^2 + i\cdot C_R \cdot \left[(C_R - C)\cdot\left(\ln\frac{C_R - C}{C_R} - 1\right) + C_R\right] = \qquad (7)$$

$$= C_R^2\left\{(p-i)\cdot c^2/2 + i\cdot\left[(1-c)\cdot\ln(1-c)+c\right]\right\}$$

The average exhaust rate in (7) is $U_{avg}=C_R^2(p+1)/2$ for $c\rightarrow 1$.

The CEI can also be interpreted as the uniform tiring out the residual causal potentiality $R(I)=(1-c)U_{avg}$ of the finite overall causal potentiality *U(C,I)* due to the progressive absorption $W(E)=c\ U_{avg}$ of interactions at average exhaust rate (Fig. 1).

5. INTERACTION PARAMETERS

The parameter *p* is introduced in Eq. (1) to represent the initial propensity to interaction. It is normally evident from the interaction rate *E*' Eq. (4) attainable from the empirically observable CE starting conditions as shown:

For $c\rightarrow 0$ in (4) (the starting condition) it follows: $p=E'$.

The intensity parameter *i* introduced in Eq. (2) can be preliminary assessed from the observed starting rate of the rate of change *E*'' Eq. (5) and from the terminal cause C_R, as:

For $c\rightarrow 0$ in (5) (the starting condition)it follows: $i= C_R E''$.

The checking of the interaction parameter *i* Eq. (6) uses the potentiality *U(C,E)* Eq. (7) (for example by numerical integration of observed test data from CE diagram), as follows:

$$i = \frac{U(C,E)/C_R^2 - pc^2/2}{(1-c)\ln(1-c) + c - c^2/2} \qquad (8)$$

For $c\rightarrow 1$ in (8) (the end condition) it follows $i=2U_{avg}/C_R^2 - p$.

6. EXAMPLES

The first example presents the standardized CEI curves (6b) with different interaction parameters *p* and *i* (Fig. 2) $e=(p-i)\cdot c - i\cdot\ln(1-c)$ indicating levels of worsening. For the reference CE relation is *p*=1 and *i*=0 what implies simple CE relation $E \Leftarrow C$ without worsening due to interaction. Increase of the interaction parameter *i>>0* for any value of propensity parameter *p>0* indicates additional worsening due to the CEI where $E \Leftrightarrow C$ (Fig. 2).

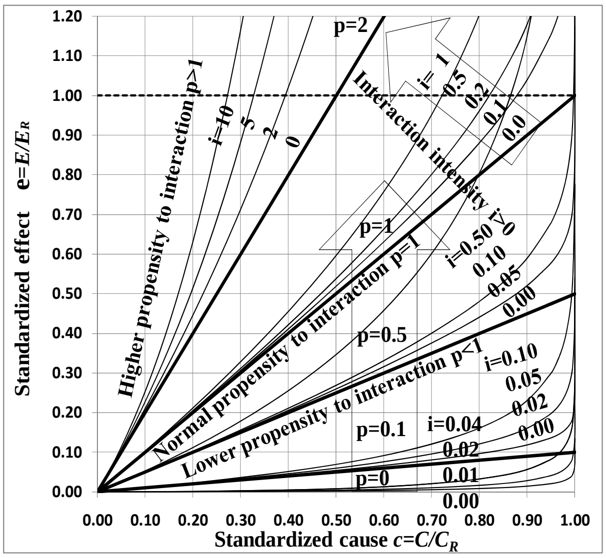

Figure 2. CEI curves $e=(p-i)\cdot c - i\cdot\ln(1-c)$ worsening levels

6.1. Material yielding and plasticity

The following case study considers material plasticity (e.g. Ramberg and Osgood in 1943 [6], Van Vlack in 1985 [7] and Rees in 2006 [8]). The materials under external loads undergo rearrangements of the internal structure within which the particles are being moved to new positions of internal energy equilibrium. The yielding and plasticity imply mobility of particles which occur as a result of dislocation motion in crystalline materials. The consequences of the progression of dislocations in materials at the micro-structural level triggered by sequential internal bond breaking and bond reforming are frequently explicated as interactions. The example applies the CEI concept to investigate the interaction character of plasticity.

The Hook's elastic Stress-Strain (SS) law $\varepsilon(\sigma)=\sigma/E$ is a typical ideal CE relation as in Eq. (1) $\varepsilon \Leftarrow \sigma$.

Experiments confirm that escalation of accumulating material strains ε affects the advancement rate of changes of stresses $\varepsilon \Leftrightarrow \sigma$ in an interactive manner resulting in yielding and plasticity. Following the general CEI concept the study investigates the Stress-Strain Interaction thesis (SSI) of plasticity that the overall strain is decomposable into primary linear strain $\varepsilon_P = \varepsilon_P(\sigma)$ as in Eq. (1) and into accumulation of strains $\varepsilon_I = \varepsilon_I(\sigma,\varepsilon)$ resulting from interactions of strains and stresses (Fig. 1) as in Eq. (6). The overall SSI rate combines the primary strain rate and the interaction rate as in Eq. (4):

$$\varepsilon' = p + i\frac{s}{1-s} \qquad (9)$$

The sensitivity of the interaction rate is as in Eq. (5) (Fig. 4):

$$\varepsilon'' = \frac{d\varepsilon'}{d\sigma} = i \cdot \frac{\sigma_R}{(\sigma_R - \sigma)^2} = i \cdot \frac{1}{\sigma_R} \frac{1}{(1-s)^2} \qquad (10)$$

The resulting overall strain ε after Eq. (6a) is then as follows:

$$\varepsilon = \sigma_R \cdot \left[\frac{1}{E} \cdot s + \frac{1}{Y} \cdot \left(\ln \frac{1}{1-s} - s \right) \right] \qquad (11)$$

The elastic modulus in (11) is $E=1/\varepsilon'=1/p$. The plasticity modulus in (11) is $Y=1/(\sigma_R\,\varepsilon'')=1/i$, (Fig. 3) and (Fig. 4)

Numerical example of plasticity of metallic materials

The numerical example makes use of the tension test data for Steel A36 (e.g. Atlas of Stress-Strain Curves from Tamarin in 2002 [9]). Reference values in the plasticity zone are $\sigma_R = 160\,Mpa$ and $\varepsilon_R = 0.1$ (σ_R/ε_R=1600), (Fig. 4).

The Yield strength and strain are $\sigma_Y = 250\,Mpa$ and $\varepsilon_Y = 0.012$.

The elastic and the plastic modulus are directly attainable from at least three carefully determined test points as close as possible to the start of yielding (Fig. 3).

The elastic modulus amounts to $E=1/\varepsilon'=1/p$=7500 *MPa* .

The plasticity modulus is attainable from test data and amounts to $Y=1/(\sigma_R\,\varepsilon'')=1/i$=11000 *MPa* (Fig. 3).

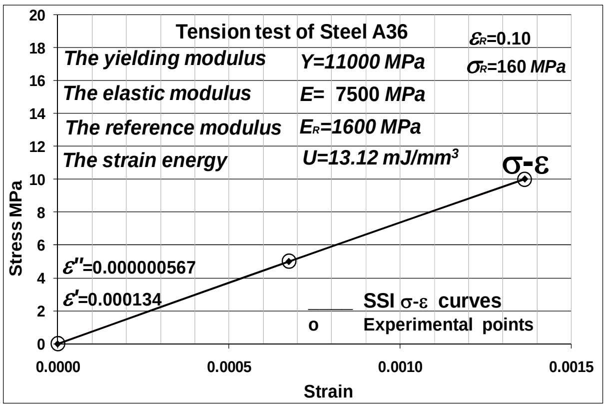

Figure 3. CEI parameters from initial tensile test (Fig. 4) A36

The strain energy is obtained by integration of the experimental σ-ε curve (Fig. 4) and amounts to 13.12 *mJ/mm*^{3}. The causal potentiality is then the complementary strain energy and amounts to U=2.88 *mJ/mm*^{3} . Interaction intensity in now can be checked on the basis of causal potentiality Eq. (7) $i=2U/C_R^2-p=1/Y$=9.17 10^{-5} MPa^{-1} (Fig. 4), same as after Eq. (9) for s=0.

The parameters of the Ramberg Osgud's (RO) [5] power law:

$$\varepsilon = \int_0^{\sigma} \frac{d\varepsilon}{d\sigma} d\sigma = \frac{\sigma}{E} + K \cdot \left(\frac{\sigma}{E} \right)^n \qquad (12)$$

are obtained from tension test data by least squares method: $K=10^5$ and n=3.92 (Fig. 4).

It is commonly recognized that plastic strains in materials have smaller impacts at lower stresses but become greater at higher stress levels, which, in most cases, cannot be described appropriately by the Ramberg-Osgood power law Eq. (12).

The example with steel suggest that the SSI model based on CEI concept using interaction parameters form experimental data can model material yielding properties. Moreover, CEI has potentials to define the full plasticity under higher stresses. The CEI concept applied to material properties indicates how more plasticity adds to the yielding rate and induces further plasticity.

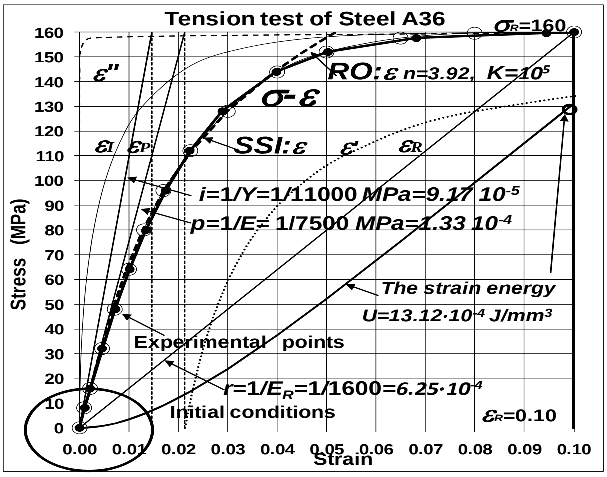

Figure 4. The SSI and RO parameters for steel A36

6.2. Lifetime shortening due to fatigue yield

The next example presents the application of the CEI concept to the Palmgren-Miner's linear fatigue damage rule (LD) (e.g. A Survey Of the State of the Art for Homogeneous Material from Fatemi&Yang in 1998, [10]) on fatigue yield (FY) [11]. The LD fatigue yield model sums up the formerly accumulated fatigue damage fractions $D_{j/i}$ under j^{th} loading block for i^{th} stress amplitude for the measure of fatigue accumulation:

$$W_j = \sum_{i=1}^{j-1} D_{j/i} = D(j-1) \qquad (13)$$

The CEI concept of fatigue worsening admits that the life shortens due to formerly accumulated fatigue W Eq. (13) that affects the residual fatigue strength denoted as the endurance:

$$R_j = 1 - D_i(j-1) \qquad (14)$$

The FY rate relates the fatigue accumulation (worsening) W_j (the cause C) Eq. (13) to the endurance R_j (the effect E) Eq. (14) for each j^{th} loading block patterned after the CEI rate Eq. (4) as:

$$\frac{dY_j}{dD} = \frac{W_j}{R_j} = p + i \cdot \frac{D_i(j-1)}{1 - D_i(j-1)} \qquad (15)$$

Thus for infinitesimally small amounts of damage progressions D, the integral of Eq. (15) represents the CEI relation as in Eq. (5) and indicates the logarithmic and the linear components of the FY denoted $Y(D)$ as it is put down next:

$$Y(D) = \int_0^D \frac{dY}{dD} dD = (p - i) \cdot D - i \cdot \ln(1 - D) \qquad (16)$$

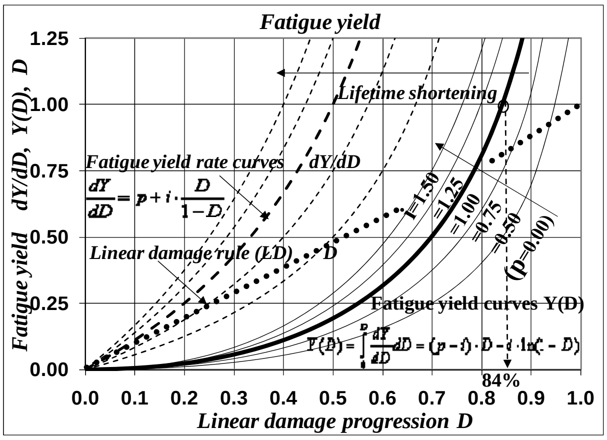

Figure 5. Fatigue lifetime shortening due to fatigue yield

The theoretical fatigue yield curve Eq. (16) can be adjusted to experimental damage progression results [12] by setting the fatigue yield intensity factor i and the initial propensity to yielding p at appropriate values [13] as it is the case with p and i parameters of the general CEI relation Eq. (4-6).
The theoretical fatigue yield rate $D/(1-D)$ demonstrates how the endurance reduction (the effect E) $(1-D)$ influences the damage progression D (the cause C). The study indicates that the fatigue life due to yielding becomes shorter since the formerly accumulated damage reduce endurance, e.g. for i=1 and p=0 the fatigue lifetime shortens at 86% of the expected time (Fig. 5).

6.3. Corrosion wastage

The next example applies the CEI worsening model to the corrosion wastage problem. Corrosion is commonly described as a macro-structural material interaction with the surrounding environment induced by micro-structural electrochemical reactions and interactions, e.g. [14]. The example applies the CEI concept to the theoretical model of time variant corrosion rate *R(t)* (Fig. 6) by Sun & Guedes Soares in 2006 [15]:

$$R(t) = R_s(1 - e^{-\frac{t-T_i}{T_t}}) \qquad (16)$$

where R_s is the steady corrosion rate, T_i is the protective coating lifetime, while T_t is the transition time (Fig. 7). By integration of Eq. (1), the corrosion depth in time *d(t)* is:

$$d(t) = R_s\left[t - (T_i + T_t) + T_t \cdot e^{-\frac{t-T_i}{T_t}}\right] \qquad (17)$$

The models describe corrosion in three phases (Fig. 6):

- The corrosion protection system is effective (up to time T_i)
- The failure of corrosion protection initiation (to time T_t)
- The corrosion rate R_s tends to be constant (after time T_s).

The model assumes constant corrosion rate in the third phase.
The choice of parameters in corrosion model depends on many factors, such as for example coating properties, surrounding properties, temperature and maintenance practice.

The study employs the CEI in the standardized 0-1 space *r-w* (Fig. 6) to explore how the increase of the corrosion wastage $w(r)$ induces higher corrosion rate $r(t)=R/R_s$ due to interaction.
The corrosion wastage rate with respect to the simple CE relation $w(r)=r$ and to what is left over after the wastage denoted as $o(r)=(R_s-R)/R_s=1-r$ Eq. (3), is as follows:

$$w''(r) = p + i \cdot w(r) / o(r) = p + i \cdot r / (1-r) \qquad (18)$$

The corrosion wastage rate *w'(r)* in terms of the CEI model Eq. (4) deepening on the corrosion rate r is the integral of Eq. (18):

$$w'(r) = w'_0 + (p - i) \cdot r - i \cdot \ln(1-r) \qquad (19)$$

The relative corrosion wastage in depth *w(r)* induced by the relative corrosion rate *r(t)* (Fig. 6) in the CEI interpretation Eq. (7) is the integral of Eq. (19) as it follows:

$$w(r) = (w' - w_o) \cdot r - (p-i) \cdot r^2 / 2 - i \cdot r - i \cdot (1-r) \cdot \ln(1-r) \qquad (20)$$

The total corrosion wastage in depth in Eq. (20) (Fig. 7) is:

$$W(r) = T_s \cdot R_s \cdot w(r) \qquad (21)$$

The time to achieve corrosion wastage W at corrosion rate r in Eq. (21) (Fig. 7) is as follows:

$$t(r) = T_s \cdot w'(r) \qquad (22)$$

The CEI parameters according the model Eq. (1) are as shown:

$$p = i = T_t / T_s \qquad (23)$$

Numerical example of corrosion wastage

For mean values of corrosion wastage of main deck plates in cargo tanks of a tanker, [16], reasonable agreement with measurements were achieved with the following parameters of non-linear long-term corrosion propagation model: R_s=0.14 *mm/year*, $T_i = T_t$ =5 *years* and T_s =38 *years* in Eqns. (16-17).
The parameters for the CEI model Eq. (18-23) are as follows:
$p = i = w'_o = T_t / T_s = 5/38 = 0.131$ (Figs. 6 and 7).
These parameters define the corrosion wastage w with respect to the corrosion rate r (Fig. 6) as well as the of corrosion wastage (depth) in time, (Fig. 7).

For the special case when $i=p$ in Eq. (22) the corrosion wastage rate w' in the time domain represents the time t to get the corrosion rate *r(t)* as follows:

$$t(r) = T_s \cdot w'(r) = T_i - T_t \cdot \ln\left[1 - r(t)/r_s\right] \qquad (24)$$

The corrosion rate from equation Eq. (24) is equal to the theoretical model of time variant corrosion rate *R(t)* in Eq. (16):

$$r(t) = (1 - e^{-\frac{t-T_i}{T_t}}) \qquad (25)$$

The Eq. (25) indicates that the theoretical model in Eq. (16) represents pure CEI interaction without the linear part Eq. (6).
Here, the CEI model of corrosion (18-23) generalizes the growth model (16-17) by accounting for a possible linear component of corrosion wastage.

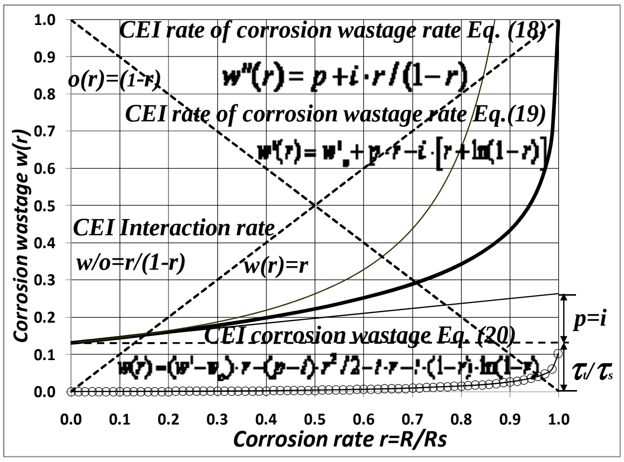

Figure 6. The corrosion wastage

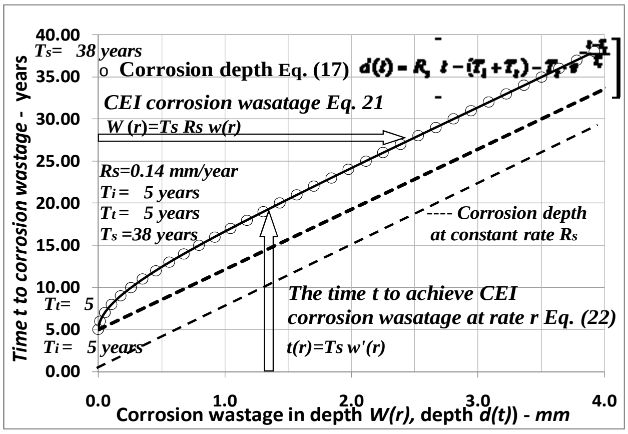

Figure 7. The corrosion depth

7. CONCLUSION

Worsening is a human interpretation of natural and social threatening phenomena whose consequences might be the loss of required or desired existential and operational efficiency. Worsening causes possible breakdown of anticipated individual, personal or general systemic functions and shortening of expected lifetime. Experience of worsening has root in nature but the worsening itself should not be taken as a natural phenomenon but rather as a human interpretation of observable processes. The judgments of the influences of natural processes are beyond human criteria of quality. Worsening is to be accepted as a subjective attribution that expresses human expectations of their safety and wellbeing. It is commonly agreed that the flow of worsening proceeds continuously but irreversibly from the present into the future, rather than into the past insurmountable separated of the future time by the instant of observation. The mental overpass of the instant of observation is the belief in some sort of regularity. The Cause-Effect Interaction (CEI) concept revealed in the paper describes how things worsen with respect to the idealized Cause-Effect relation (CE) and why they in reality worsen faster of anticipated rate due to the negligence or mistreatment of the CEI. The CEI communicates between the elapsed effect and the future cause over the temporal barrier of the instant of observation between past and future time.The CEI concept also enlightens that the cumulative causation and the positive feedback have roots in interactions.

The analytical model takes for granted the encouraging prejudice without proof that there is no endless worsening; worsening must have an observable initiation and an inescapable definite end. Therefore, the concept of CEI is to be accepted rather as another empirical interpretation of observations about worsening phenomena that are valid as long as they can be verified, just as it is the case with the general causality lows. The straightforward mathematical formulation of the cause and effect interaction model indicates that the worsening, although omnipresent and threatening, itself is a simple phenomenon.

When implemented and applied to theoretical or practical engineering models such as the material plasticity, fatigue and corrosion the CEI concept might be a rational approach to an alternative comprehension of worsening, sufficiently straightforward and accurate to tackle various realistic problems of practical importance.

The future research of causal interactions should consider some other forms of non-linear or non-uniform deterministic and probabilistic CE relations as well as the CEI models with correlated multiple causes and effects applied to some other problems, such as for example aging and climate changes.

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