Theories within Theories

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Overview

- In this talk I will try to sketch the grounds for repositioning and redefining contemporary metaphysics, in the view of:
  1. the failure of metaphysics conceived as an isolated theory,
  2. the failure of metaphysics conceived as a repository of necessary truths,
  3. the failure of metaphysics conceived as a theory on nature of things.

  Notions of physical time will be used as a critical example.[1]
- An appraisal of Tim Maudlin's approach to fundamental ontology will be attempted in the end, in the view of its characteristics as:
  - a theory within a theory,
  - a revisable theory,
  - a theory on the fundamental structures of the world.

[1] Notion of time in the realm of intentionality will not be discussed.

Part I

Metaphysics as an isolated theory?

Time as a priori form of representation

Immanuel Kant, Critique of Pure Reason, 2.rev.ed 1787

Time is not an empirical concept that is somehow drawn from an experience. For simultaneity or succession would not themselves come into perception if the representation of time did not ground them a priori. Only under its presupposition can one represent that several things exist at one and the same time (simultaneously) or in different times (successively) .

- In contrast to Kant, in STR (special theory of relativity) simultaneity is not a primitive notion. It is not the intuition of time but the notion of (constancy of) speed of light that grounds STR simultaneity.

> *Two events e_1 and e_2, occurring at points p_1 and p_2 of an inertial frame F respectively, are simultaneous in F if and only if light emitted at e_1 meets light emitted at e_2 at the midpoint m of the segment p_1 p_2 in F.*

N. Rakić. 1997. Common Sense Time and Special Relativity,
Institute for Logic, Language and Computation, Amsterdam.

Metaphysics as experimental philosophy

- The issue of primitive or derivative position of time ought to be decided on empirical grounds.
- In 1928. Albert Einstein suggested to Jean Piaget (one of fathers of experimental philosophy) to investigate the origins in children of notions of time and in particular of notions of simultaneity, and later was amused by Piaget's findings.
- Piaget's (anti-Kantian) hypothesis that the notion of speed is more fundamental than the notion of time was consistent with observations made in his experiments.

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No primitive intuition of simultaneity

Let us look at the development of the notion of simultaneity. . .
(*) In one of our experiments the experimenter has two little dolls, one in each hand, that walk along the table side by side . . . The child says go; the two dolls start off at exactly the same time and the same speed. The child says stop, and the two dolls stop, once again side by side having gone exactly the same distance. In this situation children have no problem in admitting that the dolls started at the same time and stopped at the same time.
(**) But if we change the situation slightly, so that one of the dolls has a slightly longer hop each time than the other, then, when the child says stop, one doll will be farther along than the other. In this situation the child will agree that the dolls started at the same time, but he will deny that they stopped at the same time. He will say that one stopped first; it did not go as far. We can then ask him, “When it stopped, was the other one still going?” And he will say no. Then we will ask him, “When the other one stopped, was this one still going?” And he will say no again. This is not, then, a question of a perceptual illusion. Finally, we will ask again, “Then did they stop at the same time?” The child will still say, ”No, they did not stop at the same time because this one did not get as far.”
The notion of simultaneity—two things happening at the same time—simply does not make sense for these children when it refers to two qualitatively different motions. It makes sense for two qualitatively similar motions taking place at the same speed, as in the first situation described, but when two different kinds of motions are involved it simply makes no sense. *There is no primitive intuition of simultaneity. . . This is going to require an intellectual construction.*
Jean Piaget. Genetic Epistemology, 1971.

Coming out from self-imposed isolation

- Kant's grounding of metaphysics of time on the psychology of perception was wrong.
  1. The notion of time in physics is radically different from the one postulated in Kant's theory.
     - E.g. not all of the events need be ordered by simultaneity or succession. If light emitted at e_1 cannot reach e_2, then e_1 and e_2 are not ordered: they are neither simultaneous nor successive.
  2. The experiments in genetic epistemology have falsified Kant's hypothesis on existence of time as an immutable form of perception.
- The history of the notion of time shows how mistaken were the expectations that a privileged immutable part of our knowledge could be found and how mistaken was the methodology that assumed that a fundamental theory can be built in isolation from other theories.

Part II

Metaphysics as repository of necessary truths?

Theories in motion

- In AGM theory two types of theory dynamics are studied:

  expansion addition of a new sentence x to an existing theory A and resulting expanded theory A' is deductive closure of the union $A' = Cn(A \cup \{x\})$;

  revision complex and underdetermined theoretical change occurring when a new sentence x cannot be consistently added to a theory A which therefore undergoes

  contraction an underdetermined change of A to contracted theory A^* that enables consistent addition of x,

  expansion and expansion by x

Alchourrón, Carlos, Peter Gärdenfors, and David Makinson. 1985.
On the Logic of Theory Change: Partial Meet Contraction and Revision Functions.
Journal of Symbolic Logic 50:510–530.

- Like in other theories of theoretical dynamics (cf Piaget, Quine) in AGM theory the dynamics is understood as a process of establishing and preserving logical equilibrium (or consistency) under the need of accepting new items of knowledge.

Principles of contraction

- In AGM theory the operation of contraction of set A by a sentence x, $A \div x$ results in maximal subset of A that does not entail x.
- In general there will be more than one maximal subset of A of the kind, and the set of these is called the remainder set of A by x, $A \perp x$. The remainder set $A \perp x$ contains all and only those sets B such that
  1. $B \subseteq A$,
  2. $x \notin Cn(B)$, and
  3. there is no B' such that $B \subset B' \subseteq A$ and $x \notin Cn(B')$.
- One of the ways to define contraction $A \div x$ is to say that it is a choice operation γ picking a member of the remainder set: $A \div x \in A \perp x$ or $A \div x = \gamma(A \perp x)$. This function is called maxichoice contraction. The definition of the contraction operation is given in syntactic terms and it has three elements:
  1. preservation condition — contracted set is a subset of the original set,
  2. not-entailment condition — contracted set does not entail contracted sentence,
  3. maximality condition — contracted set retains the maximal number of sentences from the original set.

AGM revision: leaving a logic intact

- Revision is needed when consistent expansion is not possible, i.e. when $A \dot{+} p = Cn(\bot)$.

Definition (Revision)

$$A \dot{+} x = Cn\big((A \dot{-} \neg x) \cup \{x\}\big)$$

- It is presupposed in AGM theory of theory revision that the logic of the theory A is invariant, that it has classical negation, and, therefore, that the consistency of A can be defined by not having both x and $\neg x$ as a consequence.
- The invariant logic restriction can be removed and revision of logic allowed.
- Let us make explicit the logic defining particular consequence relation by introducing a place for it within the binary function $Cn(L, A)$ and denoting by $InCon(L, A)$ the relation of a set A being inconsistent in logic L. The notion of inconsistency differs from logic to logic since not all of them ought to have classical negation, explosive element and the like.

Generalized revision

- The generalized notion of revision cannot be defined using negation.

Definition (Free revision)

Free revision of a theory $A \subseteq \Phi$ founded on a logic $L \subseteq \wp\Phi \times \wp\Phi$ by a sentence x is a theory A^* founded on a logic L^* iff for some $B \subseteq A$

1. $A^* = Cn\big(L^*, B \cup \{x\}\big)$,
2. $\neg InCon\big(L^*, B \cup \{x\}\big)$,
3. there is no B' such that $B \subset B' \subseteq A$ and $\neg InCon\big(L^*, B' \cup \{x\}\big)$.

- The three conditions above correspond to preservation, non-entailment and maximality of contraction, and therefore, by inheritance, of revision. To non-entailment here corresponds consistency condition since in original notion which presupposes classical negation, non-entailment of $\neg x$ serves to unsure the possibility of consistent addition of x.
- AGM revision is a special case of the free revision with invariant logic, i.e., $L = L^*$.
- Let us take a look at Quine's sketch of theoretical dynamics, understood as a kind of free revision.

A sketch of free revision in Quine's *Two Dogmas of Empiricsm*

(Semantic holism). . . total science is like a field of force whose boundary conditions are experience. A conflict with experience at the periphery occasions readjustments in the interior of the field. Truth values have to be redistributed over some of our statements. Re-evaluation of some statements entails re-evaluation of others, because of their logical interconnections—the logical laws being in turn simply certain further statements of the system, certain further elements of the field.

(Underdetermination; equilibrium) . . . the total field is so undetermined by its boundary conditions, experience, that there is much latitude of choice as to what statements to re-evaluate in the light of any single contrary experience. No particular experiences are linked with any particular statements in the interior of the field, except indirectly through considerations of equilibrium affecting the field as a whole.

(No privileged parts) Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system. . . . no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanic. . .

(Revision preferences) . . . our natural tendency [is] to disturb the total system as little as possible. . . [leaving intact, B. Ž] . . . highly theoretical statements of physics or logic or ontology. . . [which]. . . may be thought of as relatively centrally located within the total network. . .

Free revision, Quine and metaphysics

- Main elements of free revision are present in Quine's metaphor of science:
  1. "underdeterminacy"—typically there are many alternative paths of revision,
  2. "equilibrium"—revision preserves a kind of consistency,
  3. "natural tendency to disturb the total system as little as possible"—revision is parsimonious.
- Quine's metaphor of periphery and center serves to indicate both
  - how closely or how loosely a statement is related to sense data,
  - the complexity of he possible revision, where proximal position implies more complex revision.
- Although less likely to be revised due to the large amount of changes required, metaphysics is open to revision as any other fragment in the web of science.

Part III

Metaphysics as science on nature of things?

Expressive power

- A language $\mathcal{L}$ *can distinguish* between structures $\mathfrak{M}$ and $\mathfrak{N}$ if there is a sentence $\varphi \in \mathcal{L}$ such that $\mathfrak{M} \models \varphi$ but $\mathfrak{N} \not\models \varphi$.
- First order structures $\mathfrak{M}$ and $\mathfrak{N}$ *are isomorphic* iff
  - there is a bijection $f : \mathfrak{M}(\forall) \to \mathfrak{N}(\forall)$ such that
  - for any n-place predicate P

$$\text{if } \langle d_1, \ldots, d_n \rangle \in \mathfrak{M}(P), \text{ then } \langle f(d_1), \ldots, f(d_n) \rangle \in \mathfrak{N}(P)$$

- If two structures are isomorphic, then no first-order sentence can be true in one structure and false in the other. Therefore, there is *fixed limit in (im)precision* for theories formulated within first-order language.

Indiscernibility of lexicographic and naturalistic structures

Example

Let $\mathcal{T}$ be a satisfiable first-order theory. Let us call naturalistic the structure $\mathfrak{N}_{Nat}$ whose domain is a collection of clusters of sense data and which satisfies $\mathcal{T}$. Let us call Henkin's structure—the structure $\mathfrak{M}_H$ constructed over $\mathcal{L}$ and truth assignment h such that

$$h(\varphi) = \text{true} \;\text{ iff }\; \mathfrak{N}_{Nat} \models \varphi$$

In Henkin structure $\mathfrak{M}_H$ objects are sets of individual constants (in a metaphor: "books of names")

$$[\![t]\!]_H = \{x \mid h(t = x) = \text{true}\}$$

while the relations between "Henkin objects" are defined by truth assignment h that satisfies $\mathcal{T}$ as follows

$$\mathfrak{M}_H(P) = \{\langle [\![t_1]\!]_H, \ldots, [\![t_n]\!]_H\rangle \mid h(P(t_1, \ldots, t_n) = \text{true}\} \,.$$

First-order language can not make a distinction between "lexicographic structure" $\mathfrak{M}_H$ and naturalistic structure $\mathfrak{N}_{Nat}$ because they are isomorphic.

Futile expansion and Pythagoreanism

- It is futile to expand a theory with non-structural metaphysics.

Definition

An expansion of a first-order theory T by a theory M is futile if for any structure $\mathfrak{M}$ it holds that

$$\mathfrak{M} \models \varphi \text{ for all } \varphi \in T \text{ just in case } \mathfrak{M} \models \psi \text{ for all } \psi \in T \cup M$$

- Therefore, it is a limitation of the language that makes it impossible to decide what the world is "made of": whether everything is a mathematical object as Pythagoras claimed, or everything is a book of names like in a Henkin-structure, a cluster of sense data or whatever you like.

Another inessential difference

- The case in which a theory M that can be replaced by its logical duplicate T' gives another example of ineffective difference introduction.

Definition

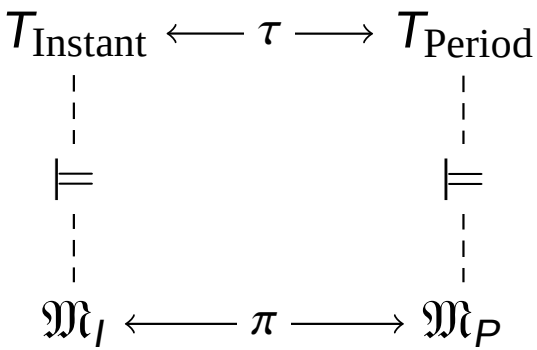
The difference between theories M and M' is unessential iff

- there is a translation τ taking sentences from M and delivering sentences expressed in the vocabulary of M' et vice versa,
- and it is theoremhood preserving: if $M \vdash P$, then $\tau(M) \vdash \tau(P)$, and if $M' \vdash P'$, then $\tau(M') \vdash \tau(P)$ (where $\tau(X) = \{\tau(P) \mid P \in X\}$).

- If theories M and M' are unessentially different and if expansion of a theory T by M is conservative over translation, i.e. $T \cup M \vdash P \Leftrightarrow T \cup M' \vdash \tau(P)$, then the difference between the two theories $T \cup M$ and $T \cup M'$ is inessential itself.
- If the proposition above holds, and the theories M and M' have different ontology, i.e. if they disagree on the question of what there is, then this ontological difference is theoretically ineffective.

Time ontology

- There has been a debate about what the time is made of: instants or periods?
- Johan van Benthem (in late 1980ies) has investigated the conditions of theoretical interchangeability of point structures and period structures.
- Applying this approach, M. Arsenijević (2003) has proved for the two plausible axiomatic systems, one of time instants T_{Instant} and the other of time periods T_{Period}, that their difference is unessential (trivial).
- This example shows that even in the case when two theories with ontological implications do not have isomorphic models, their addition to a richer theory can be theoretically inert, it need not make any theoretical difference.



Part IV

Contemporary metaphysics

Weakening and restitution

Changed status—revisability Metaphysics is not a theory consisting of necessary or a priori statements as experimental philosophy and epistemology show. It is a theory within another theory, and it shares its fate. It an indispensable part of more global and open project of knowledge acquisition.

Remaining in central position Metaphysics retains its central position in the web of science, due to the fact that its revision is a complex process with the far reaching consequences.

Redefinition—science of patterns Metaphysics cannot be a science on “the nature of things”, on “the substance of the world”, on things of which the world is made of,. . . because of limitations imposed on the expressive power of language and because of possible theoretical ineffectiveness of ontological assumptions. Rather, it is **a theory on fundamental structures of the world**, a theory informed by particular sciences, but not formed solely by them.

Contemporary metaphysics

- The philosophical investigation represented by Tim Maudlin's *formal fundamental ontology* defines the main qualities of contemporary metaphysics:

  Revisability it offers a theory that is not conceived as collection of purportedly necessary truth, rather, the value of the theory offered is to be judged by conceptual distinctions it introduces, by its ability to assimilate diverse answers to open questions in science,. . . ,

  Central position it discusses fundamental issues, it is a theory within a theory;

  Science of patterns it is a formal discipline.

. . .

The Theory of Linear Structures offers an alternative conceptual tool, built on the notion of a line or directed line. It allows for a different and more detailed account of the sub-metrical structure of a space. It has useful application to discrete spaces as well as continua. And most important, it articulates the central role that time plays in unifying space-time into a structured whole. That role emerges clearly only in Relativistic space-times, opening up possibilities that do not exist in the classical account. But in order to appreciate this role of time, we must reconstruct the mathematics used in the most basic geometrical notions from a new foundation.

Tim Maudlin. 2010. Time, topology and physical geometry.

Proceedings of the Aristotelian Society Supplementary Volume LXXXIV, 63–78