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The $\gamma^\star\gamma \to \pi^0$ transition in a bound-state approach [a]

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We present a recent treatment of the $\pi^0\gamma^\star\gamma$ transition form factor in the coupled Schwinger-Dyson and Bethe-Salpeter approach to stress the desirability of measuring it at CEBAF.

The form factor $T_{\pi^0}(-Q^2,0)$ for the transition $\gamma^\star(k)\gamma(k') \to \pi^0(p)$ (where $k^2 = -Q^2 \neq 0$ is the momentum-squared of the spacelike off-shell photon $\gamma^\star$), is usually thought to be almost completely understood on the basis of the perturbative QCD (pQCD) over the whole range of the momenta-squared covered by the new CLEO data [1] ($1.5\ \mathrm{GeV}^2 \lesssim Q^2 \lesssim 9\ \mathrm{GeV}^2$). However, Radyushkin and Ruskov's [2] QCD sum rule analysis indicates the presence of large nonperturbative contributions even for the largest of these presently accessible momenta. This motivated us to address in Ref. [3] the transition form factor in the coupled Schwinger–Dyson (SD) and Bethe–Salpeter (BS) approach in all spacelike-momentum regimes, from very large Q^2 all the way to $Q^2 = 0$. The $Q^2 = 0$ limit corresponds to the π^0 decay into two real photons ($k^2 = k'^2 = 0$) explained by the Abelian axial *alias* Adler-Bell-Jackiw (ABJ) anomaly, which is usually difficult to incorporate into a bound-state approach [4]. Fortunately, as shown already in Refs. [5,6] and used in the first analysis of $T_{\pi^0}(-Q^2,0)$ in a SD approach [7], such approaches which rely on the chirally well–behaved SD (and BS) equations for the light pseudoscalar mesons ($\pi, K, \eta, ...$) while respecting the Ward-Takahashi identities (WTI) of QED, reproduce (in the chiral and soft limit) the fundamental anomaly result

$$T_{\pi^0}(0,0) = \frac{1}{4\pi^2 f_\pi} \tag{1}$$

naturally and without any fine tuning. This is an advantage with respect to pQCD approaches, which have problems at low Q^2. (*E.g.*, see Ref. [8].)

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In the present coupled SD-BS approach [4,9,10], the anomalous amplitude (1) is also obtained model-independently and without imposing any requirements on the solutions for the π^0 wave function or the quark propagator solutions (*e.g.*, see [5,6]). This is exactly as it should be, since the anomaly – and its result (1) – must not depend on the internal structure of the pion. Subsequently, in Ref. [3], we showed that the SD-BS approach to modeling QCD provides a good description for *both* low and high values of Q^2. We thereby obtained the behavior similar to the Brodsky–Lepage interpolation formula $T_{\pi^0}(-Q^2,0) = (1/4\pi^2 f_\pi) \,/\, (1 + Q^2/8\pi^2 f_\pi^2)$ proposed [11] as a desirable behavior for $T_{\pi^0}(-Q^2,0)$ as it reduces to the ABJ anomaly amplitude (1) at $Q^2 = 0$, while agreeing with the following type of leading behavior for large Q^2:

$$T_{\pi^0}(-Q^2,0) = \mathcal{J}\,\frac{f_\pi}{Q^2} \qquad (\mathcal{J} = \textit{constant} \text{ for large } Q^2), \tag{2}$$

which is favored both experimentally [1] and theoretically (*e.g.*, [2,8,11,12]).

In the coupled SD-BS approach, the BS equation for the pion bound-state $q\bar{q}$ vertex $\Gamma_{\pi^0}(q,p)$ employs the dynamically dressed quark propagator $S(k) = [A(k^2)\not{k} - B(k^2)]^{-1}$, obtained by solving its SD equation. Then, in the case of light pseudoscalars, the $q\bar{q}$ bound states are simultaneously also the (pseudo-)Goldstone bosons of dynamical chiral symmetry breaking (DχSB).

Following Jain and Munczek [13,14], we adopt the ladder-type approximation employing bare quark–gluon–quark vertices but dressed propagators. For the gluon propagator we use an effective, (*partially*) modeled one in Landau-gauge [13,14], given by $G(-l^2)(g^{\mu\nu} - l^\mu l^\nu/l^2)$. (This Ansatz is often called the "Abelian approximation" [15].) The effective propagator function G is the sum of the perturbative contribution G_{UV} and the nonperturbative contribution G_{IR}: $G(Q^2) = G_{UV}(Q^2) + G_{IR}(Q^2)$, $(Q^2 = -l^2)$. The perturbative part G_{UV} is required to reproduce correctly the ultraviolet (UV) asymptotic behavior that unambiguously follows from QCD in its high–energy regime. Therefore, G_{UV} must be given by $\alpha_s(Q^2)/Q^2$, where the running coupling constant $\alpha_s(Q^2)$ is well-known from pQCD, so that G_{UV} is *not* modeled. As in Refs. [4,9,10], we follow Refs. [13,14] and employ the two–loop asymptotic expression for $\alpha_s(Q^2)$. For the modeled, IR part, we adopt from Ref. [14] $G_{IR}(Q^2) = (16\pi^2/3)\, a\, Q^2 e^{-\mu Q^2}$ and $a = (0.387\,\mathrm{GeV})^{-4}$, $\mu = (0.510\,\mathrm{GeV})^{-2}$.

Our calculational procedures are detailed in our Refs. [4,9,10]. We reproduce the solutions of Ref. [14] for the dressed propagator $S(q)$, *i.e.*, $A(q^2)$ and $B(q^2)$, as well as the solutions for the four functions comprising the pion bound-state vertex Γ_{π^0}. Actually, Ref. [14] employs the BS amplitude $\chi_{\pi^0}(q,p) \equiv S(q+p/2)\Gamma_{\pi^0}(q,p)S(q-p/2)$, which is completely equivalent.

We assume that the $\pi^0\gamma^\star\gamma$ transition (and other pseudoscalar meson $\to$

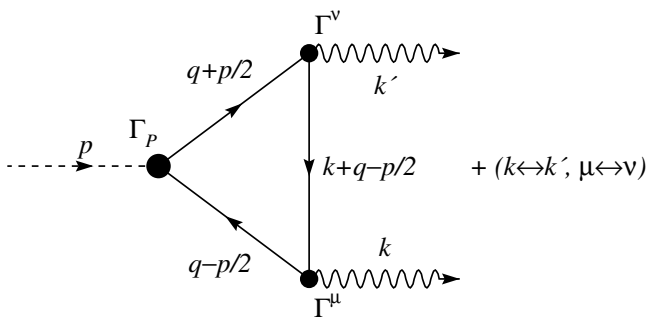

Figure 1: The GIA diagram for $P^0 \to \gamma^{(*)}\gamma^{(*)}$ processes ($P^0 = \pi^0, \eta, \eta', \eta_c, \eta_b$).

two photon transitions) proceeds through the pseudoscalar-vector-vector triangle graph (Fig. 1), and that we calculate the pertinent amplitude $T^{\mu\nu}_{\pi^0}(k,k') = \varepsilon^{\alpha\beta\mu\nu} k_\alpha k'_\beta T_{\pi^0}(k^2, k'^2)$ as in Refs. [4, 9, 10], using the framework advocated by, *e.g.*, Refs. [5–7] in the context of electromagnetic interactions of BS bound states, and often called (*e.g.*, by Ref. [7]) the generalized impulse approximation (GIA). We therefore use the *dressed* quark propagator $S(q)$ and the pseudoscalar BS bound–state vertex $\Gamma_P(q,p)$, as well as consistently dressed *electromagnetic* vertex $\Gamma^\mu(q',q)$, which satisfies the vector Ward–Takahashi identity (WTI) $(q'-q)_\mu \Gamma^\mu(q',q) = S^{-1}(q') - S^{-1}(q)$. Namely, assuming that photons couple to quarks through the bare vertex γ^μ would be inconsistent with our dressed quark propagator $S(q)$, which contains the momentum-dependent functions $A(q^2)$ and $B(q^2)$. The bare vertex γ^μ obviously violates the vector WTI, implying the nonconservation of the electromagnetic vector current and charge. Solving the pertinent SD equation for the dressed quark–photon–quark ($qq\gamma$) vertex Γ^μ is still an unsolved problem, and using the realistic Ansätze for Γ^μ still remains the only practical way to satisfy the WTI. The simplest particular solution of the vector WTI is the Ball–Chiu (BC) [16] vertex

$$\Gamma^\mu_{BC}(q',q) = A_+(q'^2,q^2)\frac{\gamma^\mu}{2} + \frac{(q'+q)^\mu}{(q'^2-q^2)}\{A_-(q'^2,q^2)\frac{(\not{q}'+\not{q})}{2} - B_-(q'^2,q^2)\}\ , \quad (3)$$

where $H_\pm(q'^2,q^2) \equiv [H(q'^2) \pm H(q^2)]$, for $H = A$ or B. It does *not* introduce any new parameters as it is completely determined by the dressed quark propagator $S(q)$. In the SD-BS approach, this minimal WTI-satisfying Ansatz (3) is still the most widely used $qq\gamma$ vertex (*e.g.*, Refs. [6,7,9,10]). A general WTI-

satisfying vertex can be written [16] as $\Gamma^\mu = \Gamma^\mu_{BC} + \Delta\Gamma^\mu$, where the addition $\Delta\Gamma^\mu$ doesn't contribute to the WTI, since it is transverse, $(q'-q)_\mu \Delta\Gamma^\mu(q',q) = 0$. That is, $\Delta\Gamma^\mu(q',q)$ lies in the hyperplane spanned by the vectors $\mathcal{T}^\mu_i(q',q)$ $(i = 1, ..., 8)$ transverse to the photon momentum $k = q' - q$. Curtis and Pennington (CP) [17] advocated an Ansatz for $\Delta\Gamma^\mu(q',q)$ exclusively along the $i = 6$ basis vector: $\Delta\Gamma^\mu(q',q) = \mathcal{T}^\mu_6(q',q)[A_-(q'^2,q^2)/2d(q',q)]$. The coefficient of $\mathcal{T}^\mu_6(q',q)$ can be chosen to ensure renormalizability [17], *e.g.*,

$$d_\pm(q',q) = \frac{1}{q'^2+q^2}\left\{(q'^2 \pm q^2)^2 + \left[M^2(q'^2) + M^2(q^2)\right]^2\right\} \ , \qquad (4)$$

where $M(q^2) \equiv B(q^2)/A(q^2)$ is the DχSB-generated dynamical mass function, which in our case has the large-q^2 dependence [4] in agreement with pQCD.

The choice $d = d_-$ corresponds to the CP vertex Ansatz Γ^μ_{CP} [17]. We use it in analytic calculations of $T_{\pi^0}(-Q^2, 0)$, which are possible for $Q^2 = 0$ and $Q^2 \to \infty$. However, in the numerical calculations, which are necessary for finite values of $Q^2 \neq 0$, we prefer the *modified* CP (mCP) vertex, Γ^μ_{mCP} [3], resulting from the choice $d = d_+$, since it is easier to deal with numerically. Unlike the BC one, the mCP vertex is consistent with renormalizability just like the CP one. In the present context, the important *qualitative* difference between the BC vertex, and the CP as well as the modified, mCP vertex, is that $\Gamma^\mu_{BC}(q',q) \to \gamma^\mu$ when *both* $q'^2, q^2 \to \pm\infty$, whereas $\Gamma^\mu_{CP}(q',q) \to \gamma^\mu$ and $\Gamma^\mu_{mCP}(q',q) \to \gamma^\mu$ as soon as *one* of the squared momenta tends to infinity.

Assuming the isospin symmetry, $\chi \equiv \chi_{u\bar{u}} = \chi_{d\bar{d}}$ is the BS amplitude for both $u\bar{u}$ and $d\bar{d}$ pseudoscalar bound states, and $\chi_{\pi^0}(q,p) \equiv \chi(q,p)\,\lambda^3/\sqrt{2}$. Then, for π^0, GIA yields (see Fig. 1) the tensor amplitude

$$T^{\mu\nu}_{\pi^0}(k,k') = -N_c \frac{1}{3\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\{\Gamma^\mu(q - \frac{p}{2}, k + q - \frac{p}{2}) S(k + q - \frac{p}{2})$$

$$\times \Gamma^\nu(k + q - \frac{p}{2}, q + \frac{p}{2}) \chi(q,p)\} + (k \leftrightarrow k', \mu \leftrightarrow \nu), \qquad (5)$$

leading to the analytical reproduction of the $\pi^0 \to \gamma\gamma$ amplitude (1) with the CP and mCP vertices, in the same way as with the BC vertices.

Let us stress that in the regime of asymptotically large Q^2, we obtained [3] analytical results for the transition form factor in agreement with the behavior (2). Since the pion is light, $k \cdot k' \approx Q^2/2$ for large negative $k^2 = -Q^2$ and $k'^2 = 0$. Taking into account the behavior of the propagator functions $A(q^2)$ and $B(q^2)$ (*e.g.*, see [4]) we can then in Eq. (5) approximate those quark propagators that depend on the photon momenta k and k', by their asymptotic forms: $S(q - p/2 + k) \approx S(k - p/2) \approx -(2/Q^2)(\not{k} - \not{p}/2)$

and analogously for the propagator where k is replaced by k'. Although the relative loop momentum q can be large in the course of integration, its neglecting is justified because the BS amplitude $\chi(q,p)$ decays quickly and thus strongly damps the integrand for the large q's. Ref. [3] then showed that $T^{\mu\nu}_{\pi^0}(k,k') = -(2/N_c\,Q^2)\,\varepsilon^{\mu\lambda\nu\sigma}(k-k')_\lambda p_\sigma\, f_\pi$, finally leading to Eq. (2) with the coefficient $\mathcal{J} = 4/3$. We thus found *model independently* that the asymptotic behavior in the present approach (with $qq\gamma$ vertices such as the bare γ^μ, mCP and CP), agrees exactly with the leading term predicted by OPE [12].

The asymptotic behavior $\propto 1/Q^2$ obtained in the present approach is especially satisfying when compared with the one known to result from the simple constituent quark model (with the constant light–quark mass parameter m_u), where $T_{\pi^0}(-Q^2,0) \propto (m_u^2/Q^2)\ln^2(Q^2/m_u^2)$ as $Q^2 \to \infty$, which overshoots the data considerably because of the additional $\ln^2(Q^2)$-dependence.

It is important to note that this asymptotic behavior, $T_{\pi^0}(-Q^2,0) = (4/3)f_\pi/Q^2$, was obtained for the $qq\gamma$ vertices which reduce to the bare one, γ^μ, as soon as *one* of the squared momenta tends to infinity, such as the renormalizable CP and mCP vertices. Subsequently, Ref. [18] showed that this asymptotic behavior must in fact hold for *any* renormalizable $qq\gamma$ vertex. For the simplest dressed WTI-preserving vertex, the BC one, which reduces to the bare one only when momenta are large in both of its fermion legs, and which is not consistent with multiplicative renormalizability [17], the asymptotic behavior is $(4/3)\widetilde{f}_\pi/Q^2$, where the quantity $\widetilde{f}_\pi$ is given by the same Mandelstam-formalism expression as f_π except that its integrand is modified by the factor $[1+A(q^2)]^2/4$ [3]. For our model solutions [4], $\widetilde{f}_\pi \approx 1.334 f_\pi$, so that the usage of the BC vertices causes the increase of $\mathcal{J}$ in Eq. 2 from $\mathcal{J} = 4/3$ to $\mathcal{J} \approx 1.78$.

Of course, the model-independent asymptotic coefficient $4f_\pi/3$ is the one having the more fundamental meaning, resulting from the renormalizable $qq\gamma$ vertices such as the CP or mCP ones, which have properties closer to the true vertex solution. Also indicative is the asymptotics found [3] for such $qq\gamma$ vertices when both photons are off-shell, $k^2 = -Q^2 << 0$ and $k'^2 = -Q'^2 \leq 0$:

$$T_{\pi^0}(-Q^2,-Q'^2) = \frac{4}{3}\,\frac{f_\pi}{Q^2+Q'^2}\,. \tag{6}$$

The usage of the BC vertex (3) would again modify this result by the substitution $f_\pi \to \widetilde{f}_\pi$. However, Eq. (6) agrees with the leading term of the OPE result of Novikov *et al.* [19] for the special case $Q^2 = Q'^2$. Also, the distribution-amplitude-dependence of the pQCD approach cancels out for that symmetric case, so that $T_{\pi^0}(-Q^2,-Q'^2)$ in this approach (*e.g.*, see [20]), in the limit $Q^2 = Q'^2 \to \infty$, exactly agrees with both our Eq. (6) and Ref. [19]. For that symmetric case, we should thus have even the precise agreement of

the coefficients irrespective of the description of the pion internal structure encoded in the distribution amplitude. Obviously, this favors the renormalizable $qq\gamma$ vertices, such as CP and mCP ones, over the BC vertex. However, the BC vertex may anyway be the one which is more accurate not only for the presently accessible Q^2, but also for larger values before starting to fail.

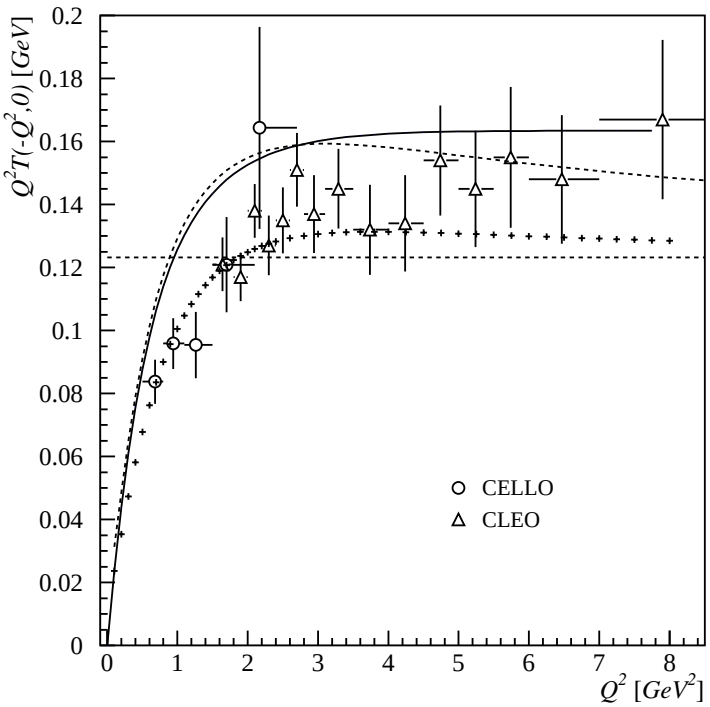

Figure 2: Our numerically obtained $Q^2T_{\pi^0}(-Q^2,0)$ and the data.

For finite Q^2, the transition form factor is evaluated numerically. Fig. 2 compares so obtained $Q^2T_{\pi^0}(-Q^2,0)$ with the CLEO [1] and CELLO [21] data. The solid curve is obtained with the BC vertices, and the dashed curve with the mCP ones. The horizontal dashed line denotes $4f_\pi/3$, the asymptotic value for renormalizable $qq\gamma$ vertices such as the mCP one. It barely touches the experimental error bars from below. Note that also Manohar [12] warned that his OPE approach (also yielding $4f_\pi/3$) indicates the possibility of large corrections to his leading term, but possibly also to the pQCD asymptotic coefficient $2f_\pi$, which is however exact in the *strict* $\ln(Q^2)\to\infty$ limit [11]. We get it lower

by the factor 2/3 because our present approach does not incorporate the effects of the pQCD evolution. However, as Ref. [2] seems to indicate that effects other than the pQCD evolution (including the presently interesting dynamical dressing of quarks) may still play important (and maybe even dominant) role even at Q^2-values larger than the presently accessible ones, one should consider seriously also nonperturbative approaches such as ours. On the other hand, high-precision measurements of $T_{\pi^0}(-Q^2,0)$ can test (and give a hint on how to improve) the SD and BS model solutions which have so far been successful in fitting the low-energy hadron properties such as the meson spectrum. For example, our model choice [14] somewhat overshoots (for both BC and mCP vertices) the present $T_{\pi^0}(-Q^2,0)$ data [1,21] in the region $Q^2 \lesssim 4$ GeV^2.

Once one has a solution for $\chi(q,p)$ that leads to the correct value of f_π, the transition form factor is most sensitive on $A(q^2)$, or, more precisely, on its values at small and intermediate momenta $-q^2$, where $A(q^2)$ is still appreciably different from 1 [3,22]. To illustrate what happens when the $A(q^2)$-profile is decreased, let us enforce by hand the extreme, artificial case $A(q^2) \equiv 1$. (To avoid confusion, we stress it is for illustrative purposes only, as we cannot have such a SD-solution in the adopted approach of Refs. [13,14].) This leads to the curve traced on Fig. 2 by small crosses, pertaining to the usage of both BC and mCP vertices (as well as the CP ones), since $A(q^2) \equiv 1$ makes $\Gamma^\mu_{mCP} \to \Gamma^\mu_{BC}$. This curve reveals how the heights of the curves depicting $Q^2 T_{\pi^0}(-Q^2,0)$ depend on how much the $A(q^2)$-profile exceeds 1. Obviously, for both the solid curve and the dashed one, the agreement with experiment would be improved by lowering them somewhat (at least in the momentum region $Q^2 \lesssim 4$ GeV), which could be achieved by modifying the model [14] and/or its parameters so that such a new solution for $A(q^2)$ is somewhat lowered towards its asymptotic value $A(q^2 \to \infty) \to 1$. (Of course, in order to be significant, this must not be a specialized re-fitting aimed *only* at $A(q^2)$. Lowering of $A(q^2)$ should be a result of a broad fit to many meson properties, comparable to the original fit [14]. This, however, is beyond the scope of this paper.)

By the same token, high precision measurements of $T_{\pi^0}(-Q^2,0)$ can be especially helpful in obtaining information on which solutions for $A(q^2)$ are empirically acceptable and which are not. Of course, measurements of $T_{\pi^0}(-Q^2,0)$ give information on the integrated strength of $A(q^2)$ rather than on $A(q^2)$ itself. However, since it is known that the form of that function must be a rather smooth transition (*e.g.*, see [4]) from $A(q^2) > 1$ for q^2 near 0, to $A(q^2) \to 1$ in the q^2-domain where QCD is perturbative, such measurements [23] would give a useful hint even about $A(q^2)$ itself – namely about what solutions for $A(q^2)$ one may have in sensible descriptions of dynamically dressed quarks and their

bound states. Therefore, the intermediate-momentum ($Q^2 \lesssim 4$ GeV^2) measurements of $T_{\pi^0}(-Q^2, 0)$ at Jefferson Lab, such as those proposed in Ref. [23], would be very desirable. Hopefully, the study of virtual Compton scattering at CEBAF can produce the data on $T_{\pi^0}(-Q^2, 0)$ as envisioned in Ref. [23].

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