Analysis of Risk and Non-Linear Optimization - Example of the Croatian Stock Market Index

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Abstract. Amidst the time of crisis, business subjects are risk averse, waiting for better times and for the turmoil to calm down. Theories show that individuals are even ready to reject an investment that has a positive risk premium because the potential gain is insufficient to make up for the risk involved (Bodie, Kane, Marcus, 2008). By doing so, they prolong the crisis even longer. In such situations it is important to protect potential investors against further losses as it is the only thing that can serve as a guarantee of increased optimism that could lead to economic recovery. This paper thus focuses on problem of portfolio optimization of securities that make up the most relevant Croatian stock exchange index, namely Crobex 10. We analyze expected rates of return and portfolio risk and give recommendations to investors on how to choose optimal portfolio and minimize inherent risk.

Keywords: risk, stock market, Croatia.

1. Introduction

For the past few years, the world as well as the Croatian economy, have been shaken by one of the biggest economic crisis in recent history, and according to some authors, the worst crisis of all times. Although the causes and consequences of this crisis have been widely discussed in the professional and scientific community, with many disunited opinions, the facts show that the beginning of the crisis took place in mid 2007ies in the United States of America and, due to the effects of globalization of world economies, have been gradually spilled over to the whole world.

The trigger that induced the crisis was the sudden change in the direction of movement of U.S. real estate market. After the dot com crisis (which was followed by the big drop in share values), the U.S. government authorities decided to stimulate the economy by keeping the interest rates low, that consequently led to an increase of domestic savings which banks invested in, that seemed then, a relatively low-risk financial derivatives insured with real estate mortgages. Even though the government bonds were bearing a relatively low risk, they however carried slim premiums (among other things because China had its export surpluses heavily investing in government bonds of the United States, which lowered their yield) and therefore U.S. banks, looking for relatively lower risk and higher yields, decided to invest considerable amount of their portfolio in the above-mentioned type of financial derivatives. As these financial instruments could be sold on through mechanisms of securitization, soon nearly all the major financial institutions in the world were in possession of these (as will later be determined) high risk assets, which had been evaluated with the highest credit ratings suggesting the lowest risk.

Effects of the crisis in the last five years have been reflected in the large number of failed financial giants, decline in GDP of most countries in the world, rising unemployment, declining consumer wealth measured in trillions of dollars, and the general distrust towards any economic recovery and investment in general. The same sequence of events is happening in Croatia, where all the relevant economic indicators are exacerbating year by year and the „*Leader-Hendal optimism index*“ (an indicator that measures business expectations of people in different industries) points to the stagnation of the economy.

In such situations it is important to protect potential investors against further losses as it is the only thing that can serve as a guarantee of increased optimism that could lead to economic recovery. Therefore, this paper will focus on the problem of optimization of selected securities portfolio, namely Crobex 10.

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2. Modeling the optimization problem

Crobex 10 is the official Zagreb Stock Exchange share index published since 2009[2]. The primary requirement for a certain stock to be considered a component of the Crobex 10 index is that the securities must be traded at least 90% of the trading days in the six-month reference period. Crobex 10 is comprised of 10 top securities included into Crobex index in terms of free-float[3] market capitalization and liquidity.

In the selection process all Crobex securities are ranked by the following two criteria:

1. Free float market capitalization

2. Turnover ratio in preceding six months period.

To each of these criteria, the weight of 50% is attributed and then, the so called weighted market share is calculated. Crobex securities are ranked by the market share and the top ten securities are selected to be included into the Crobex10 index. Securities where 75% of the share capital is owned by a single shareholder are not qualified for Crobex10 selection.

The process of constructing an investor portfolio can be viewed as a sequence of two steps (Bodie, Kane, Marcus, 2008):

1.) Selecting the composition of one's portfolio of risky assets such as stocks and bonds and;

2.) Deciding how much to invest in that risky portfolio versus in a safe asset.

Obviously, since an investor cannot decide how to allocate investment funds without knowing expected return and degree of risk of the stocks preferred, a fundamental part of the asset allocation problem is to characterize the risk-return trade off (Bodie, Kane, Marcus, 2008). There are many definitions of risk as well as methods of its calculation. More general definitions of risk simply state that risk is a likelihood that an undesirable event will occur at a particular time (Merna and Al-Thani, 2008) or an uncertainty concerning the occurrence of a loss (Vaughan and Vaughan, 1998). Since the purpose of this paper is to exclusively focus on the stock market risk, we will define it as volatility of unexpected outcomes, which can represent the value of assets, equity or earnings (Jorion, 2007). As a measure of portfolio risk we will calculate the variance of the observed set of stocks, as it is a standard and widely used measure in this field.

Optimization problem will be reduced to the determination of the investment proportions an investor is willing to put in each share forming the Crobex 10 index, in such a way that the investor minimizes the risk, as well as simultaneously forming a portfolio that would allow for the return on invested assets to be greater than the one that could be achieved by investing in risk free assets. Minimum rate of return on investment had been determined in relation to the benchmark interest rate of the Croatian National Bank, i.e. the interest rate that banks charge to their clients with highest credit ratings. As this rate in the past year ranged between 6% and 6.5% (www.limun.hr), the minimum rate of return that an investor expects was set at the 7% level.

As each optimization problem is based on assumptions (Neralić, 2008), it is important to emphasize the key assumption of our model, which is also the main limitation of it and it implies that, based on historical data, we can extrapolate future trends, which may or may not be the case.

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[2] www.zse.hr

[3] *Free float is the proportion of shares that are freely traded on the stock exchange and, as such, represents a measure of liquidity of individual stocks. Free float is calculated by dividing the total number of issued shares of a company deducting treasury shares in the company (if any), the shares of which are owned by strategic partner companies (those persons or entities that own shares in order to participate in the management decisions of the company and therefore do not plan to sell shares), and all those who have acquired ownership of the shares for long-term investments (e.g. pension funds, board members of companies, etc.) with the total number of issued shares issued. Free float is therefore a percentage which can not be entirely accurately calculated, but only assessed.*

After the previous acquiring of the data on stock returns for all the shares in Crobex 10 index for the past ten years, we calculated the average annual return rate for each share (Table 1). Since only five of the observed ten stocks have been listed on the stock exchange for the past ten years, we took into account only the data for the past five years, in order to standardize the results. Accordingly, the average annual return rate (PGP in table 1) was calculated for the period between the years 2007 and 2011.

Table 1: Annual rates of return of shares comprising the Crobex 10 index[4] per years

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	PGP
ADPL					0,571428	0,569116	0,279419	-0,82165	1,068429	0,507493	-0,14704	0,177331
ADRS							0,007284	-0,62993	0,147664	0,132643	-0,24064	-0,11659
ATGR							-0,01871	-0,47161	0,462281	0,167072	-0,37842	-0,04788
ATPL	1,948056	-0,75008	-0,12695	18,26577	0,744624	0,092897	2,909245	-0,80454	0,312522	-0,24114	-0,55926	0,323366
DLKV	0,127566	0,4	0,315294	0,792253	0,533835	0,990075	1,110287	-0,77466	-0,19987	-0,19798	-0,50184	-0,11281
ERNT	0,519878	0,120225	0,352419	2,344835	0,274743	0,313247	0,364587	-0,64539	0,094014	0,017741	-0,20662	-0,07513
HT							-0,09139	-0,43385	0,343113	0,055438	-0,17041	-0,05942
INGR					0,121002	4,30383	3,227299	-0,99863	-0,49413	-0,57987	-0,5962	0,111694
KOEI	0,615507	0,15378	-0,11323	1,168963	0,800499	1,276949	0,941145	-0,67534	0,082783	0,206328	-0,0315	0,104683
PODR	-0,01384	0,356533	-0,17242	0,356214	0,374829	0,497223	0,053553	-0,489	0,133512	0,064227	-0,25333	-0,09821

Source: Author's own calculations based on the data collected from the Zagreb Stock Exchange, www.zse.hr

Based on the data from Table 1, covariance series of the average annual rates of return have been calculated and shown in Table 2.

Table 2: Covariance matrix

	ADPL	ADRS	ATGR	ATPL	DLKV	ERNT	HT	INGR	KOEI	PODR
ADPL	0,403332	0,176692	0,211813	0,2962	0,161306	0,167721	0,160681	0,177739	0,184568	0,143264
ADRS	0,176692	0,085239	0,090103	0,17305	0,099058	0,086272	0,067403	0,142254	0,108094	0,068074
ATGR	0,211813	0,090103	0,119224	0,143646	0,076399	0,080836	0,084891	0,067526	0,082122	0,074884
ATPL	0,2962	0,17305	0,143646	1,81135	0,860334	0,36841	0,073683	2,066136	0,621168	0,175184
DLKV	0,161306	0,099058	0,076399	0,860334	0,420037	0,188752	0,041414	0,986516	0,317112	0,094121
ERNT	0,167721	0,086272	0,080836	0,36841	0,188752	0,114614	0,058562	0,385907	0,167253	0,072853
HT	0,160681	0,067403	0,084891	0,073683	0,041414	0,058562	0,065753	0,014282	0,05666	0,054125
INGR	0,177739	0,142254	0,067526	2,066136	0,986516	0,385907	0,014282	2,457241	0,702308	0,152766
KOEI	0,184568	0,108094	0,082122	0,621168	0,317112	0,167253	0,05666	0,702308	0,267493	0,092867
PODR	0,143264	0,068074	0,074884	0,175184	0,094121	0,072853	0,054125	0,152766	0,092867	0,055979

Source: Author's own calculations based on the data collected from the Zagreb Stock Exchange, www.zse.hr

By computing the covariance matrix we have obtained all the relevant data necessary to form a goal function which needs to be minimized. In the goal function we try to minimize the variance of portfolio (i.e. portfolio risk), which is given by the following relation:

$$\min V_P = \mathbf{X}^T\mathbf{V}\mathbf{X}$$

where **X** represents a vector of investment proportions in individual shares, while **V** represents the covariance matrix.

[4] Only ticker symbols (and not the full names) of respective companies have been put in the table. Full name behind the ticker as well as all the relevant company details can be checked at the official Zagreb stock exchange pages – www.zse.hr.

Limitations of the general model are contained in the following relations:

$$x1 + x2 + ... + xn = 1$$

$$x1 \geq 0,\ x2 \geq 0,\ ...,\ xn \geq 0;$$

$$x1p1 + x2p2 + ... + xnpn \geq p$$

where x_j represents the proportion of the portfolio invested in investment j, while p_j represents the average rate of return of investment j (in our case, j = 1,2, ..., 10). Finally, p denotes the expected rate of return on selected set of securities.

There were three limitations of the optimization problem. They respectively assumed that: investor is willing to distribute their assets only in ten shares that form Crobex 10 index; investor is willing to bear risk but expects at least 7% of annual rate of return on their investment. The third limitation is the non-negativity of the proportions of portfolio invested in, which is an assumption that has to be added in the mathematical model for computational purposes.

Limitations to the problem written in Lingo 13.0 program software therefore were:

$$x1+x2+x3+x4+x5+x6+x7+x8+x9+x10=1$$

$$x1 \geq 0;\ x2 \geq 0;\ x3 \geq 0;\ x4 \geq 0;\ x5 \geq 0;\ x6 \geq 0;\ x7 \geq 0;\ x8 \geq 0;\ x9 \geq 0;\ x10 \geq 0;$$

$$17.73*x1-11.66*x2-4.79*x3+32.34*x4-11.28*x5-7.5*x6-5.9*x7+11.17*x8+10.47*x9-9.82*x10 \geq 7.$$

After the mathematical formulation of the optimization problem, in order to calculate the optimal solution we used the software package Lingo 13.0.

3. Solution to the optimization problem

Local optimal solution found.	
Objective value:	0.1615909
Infeasibilities:	0.000000
Total solver iterations:	30
Model Class:	NLP
Total variables:	10
Nonlinear variables:	10
Integer variables:	0
Total constraints:	3
Nonlinear constraints:	1
Total nonzeros:	30
Nonlinear nonzeros:	10

Variable	Value	Reduced Cost
X1	0.2273564	0.000000
X2	0.000000	0.1256616
X3	0.000000	0.4514190E-01
X4	0.000000	0.1207215
X5	0.000000	0.2900143
X6	0.000000	0.1198704
X7	0.3128044	0.000000
X8	0.000000	0.3617600

X9	0.4598393	0.000000
X10	0.000000	0.1274855

Row	Slack or Surplus	Dual Price
1	0.1615909	-1.000000
2	0.000000	-0.2380564
3	0.000000	-0.1216077E-01

Based on the solutions obtained, it can be concluded that the minimum variance (as a measure of portfolio risk) of 0.1615909 with an expected rate of return of 7% per year can be achieved by splitting the available assets to just three stocks (variables x1, x7 and x9), namely three companies - AD Plastik, Croatian Telecom and Koncar electrical industry in the ratios of 22.73564%, 31.28044% and 45.98393% respectively. Other stocks from the observed set did not meet the required criteria associated with the expected rate of return, or had excessively fluctuated which affected their variance. *Reduced cost indicator* displays how much it had to reduce the variance of a given data set (i.e. individual stock returns) to be included in the optimal solution, while the *dual price indicator* shows what would the increase of the variance be (0.2380564) for observed portfolio if the expected rate of return of the portfolio increased by 1% (i.e. from 7% to 8%).

4. Conclusion

This paper analyzed the risks and expected returns of shares included in the most relevant index of Zagreb stock exchange, namely Crobex 10. Non-linear risk analysis showed quite odd and unexpected results – the variance and expected rates of return of only three out of ten shares comprising the aforementioned index were satisfactory to invest in. The seven other shares were too volatile and thus bore too high risk or their rates of return have been below those of a risk free asset. Results are even more interesting bearing in mind that only the top and carefully chosen shares can be selected to comprise the Crobex 10 index.

The analysis was conducted in order to protect investors from further losses and help them choose their portfolio wisely. This, in turn could help the economy to overcome the crisis faster. Nevertheless, since the crisis has increased market volatility over the last couple of years, it seems that a significant amount of time still has to pass by in order for the market to stabilize. Until then, risk averse investors should be vigilant and choose only those shares in their portfolio that meet the established risk and rate of return criteria.

5. References

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