COMPUTATION OF CONSTANTS IN MULTIPARAMETRIC ALGEBRAS OF NONCOMMUTATIVE POLYNOMIALS

(Talk)

Milena Sosic
Department of Mathematics, University of Rijeka
msosic@math.uniri.hr

Let $\mathbb{N}_0 = \{0, 1, \dots\}$ be the set of nonnegative integers and let $\mathcal{N} = \{i_1, \dots, i_N\}$ be a fixed subset of $\mathbb{N}_0$. Then we denote by $\mathcal{B} = \mathcal{B}_{\mathcal{N}} = \mathbb{C}\langle e_{i_1}, \dots, e_{i_N}\rangle$ the free unital associative $\mathbb{C}$-algebra with N generators $\{e_i\}_{i\in\mathcal{N}}$, each of degree one. We can think of $\mathcal{B}$ as an algebra of noncommutative polynomials in N noncommuting variables $e_{i_1}, \dots, e_{i_N}$.
We equip $\mathcal{B}$ with a multiparametric q_{ij}-differential structure given by N linear operators $\partial_i \colon \mathcal{B} \to \mathcal{B}$, $i \in \mathcal{N}$ that act as twisted derivations on $\mathcal{B}$:
$\partial_i(1) = 0,\ \partial_i(e_j) = \delta_{ij},\ \partial_i(e_j x) = \delta_{ij} x + q_{ij} e_j \partial_i(x)$ for all $x \in \mathcal{B}$, $i, j \in \mathcal{N}$
(q_{ij} are complex numbers).
The algebra $\mathcal{B}$ is naturally graded by total degree $\mathcal{B} = \bigoplus_{n\geq 0} \mathcal{B}^n$, where $\mathcal{B}^0 = \mathbb{C}$ and $\mathcal{B}^n$ consists of all homogeneous noncommuting polynomials of total degree n in variables $e_{i_1}, \dots, e_{i_N}$. More generally we also have a finer decomposition of $\mathcal{B}$ into multigraded components (= weight subspaces)

$$\mathcal{B} = \bigoplus_{n\geq 0,\, l_1\leq\cdots\leq l_n,\, l_j\in\mathcal{N}} \mathcal{B}_{l_1\dots l_n},$$

where each weight subspace $\mathcal{B}_Q = \mathcal{B}_{l_1\dots l_n}$, corresponds to a multiset $Q = (l_1 \dots l_n)$, is given by

$$\mathcal{B}_Q = span_{\mathbb{C}} \left\{ e_{j_1\dots j_n} := e_{j_1} \cdots e_{j_n} \mid j_1 \dots j_n \in \widehat{Q} \right\}.$$

Here $\widehat{Q} = S_n Q = \{\sigma(l_1 \dots l_n) \mid \sigma \in S_n\}$ denotes the set of all rearrangements of the sequence $l_1, \dots, l_n$ (i.e $\widehat{Q}$ is the set of all distinct permutations of the multiset Q). Thus $\dim \mathcal{B}_Q = \left|\widehat{Q}\right|$.
Of particular interest in algebra $\mathcal{B}$ are elements called *constants* which satisfy $\partial_i C = 0$ for every $i \in \mathcal{N}$. Let $\mathcal{C}$ denotes the space of all constants in algebra $\mathcal{B}$ and similarly let $\mathcal{C}_Q$ denotes the space of all constants in $\mathcal{B}_Q$. Then the main problem of describing the space $\mathcal{C}$ can be reduced to describing the space $\mathcal{C}_Q$. Here we shall give the explicit formulas for nontrivial (basic) constants in $\mathcal{B}_Q$ up to total degree equal to four.

MSC2010: 05Exx.

Keywords: q-algebras, noncommutative polynomial algebras, twisted derivations.

Section: 14.

.

References

[DKKT] G. Duchamp, A. Klyachko, D. Krob, J.-Y. Thibon: Noncommutative symmetric functions III: Deformations of Cauchy and convolution algebras, Discrete Mathematics and Theoretical Computer Science **1** (1997), 159–216.

[F2] Frønsdal C.: On the classification of q-algebras, Lett. Math. Phys. **222** (1999), 708–746 (q-alg/0003146).

[FG] Frønsdal C. and Galindo A.: The ideals of free differential algebras, J. Algebra **222** (1999), 708–746 (q-alg/9806069).

[MPS] Meljanac S., Perica A. and Svrtan D.: The energy operator for a model with a multiparametric infinite statistics, J. Phys. A**36** no. **23** (2003), 6337–6349 (math-ph/0304038).

[MS1] Meljanac S. and Svrtan D.: Determinants and inversion of Gramm matrices in Fock representation of q_{kl}-canonical commutation relations and applications to hyperplane arrangements and quantum groups. Proof of an extension of Zagier's conjecture, arXiv:math-ph/0304040vl, 26 Apr 2003.

[Z] Zagier D.: Realizability of a model in infinite statistics, Commun. Math. Phys. **147** (1992), 199–210.