INTRODUCTION

Gas is transported by sea in liquefied state in order to reduce its volume and thus make the transportation economical. Due to specific cargo properties special ships called Liquefied Gas Tankers are used which have unique construction features and differ considerably from other classes of ships. Depending on the cargo type, two categories of ships are distinguished, i.e. Liquefied Natural Gas (LNG) and Liquefied Petroleum Gas (LPG) Carriers, [1, 2].

The liquid natural gas is insulated at cryogenic temperature and slightly pressurised above atmospheric pressure. The boil-off gas is used as fuel in the ship boilers. The liquid petroleum gases are transported in one of the following conditions:

- fully refrigerated at slightly above atmospheric pressure,
- refrigerated, semi-pressurised below ambient temperature and over atmospheric pressure,
- fully pressurised at ambient temperature.

In all cases the cargo liquid state is near the boiling temperature at the given pressure. The boil-off petroleum gases are reliquefied and returned to the cargo tank.

Since the transportation of gas is hazardous due to many reasons of potential danger, it is regulated by the International Maritime Organisation (IMO) within IGC Code [3]. Some notes on the practical application of this code are presented in [4]. This document is accepted by the International Association of Classification Societies (IACS) and included in the Classification Rules, for instance [5].

In general, for liquefied gas transportation different cargo tanks are used: integral tanks, membrane tanks, semi-membrane tanks and independent tanks. In the Classification Rules the design features, i.e. tank shape and type of design analysis, and design pressure are used as criteria for tank definition, whereas the grade of refrigerating is of secondary significance. The design vapour pressure for the integral, membrane and semi-membrane tanks is limited at 0.25 bar. However, if the hull scantlings are increased accordingly the pressure may be increased up to 0.7 bar.

The independent cargo tanks are self-supported structures and do not participate in the ship’s strength. They are further subdivided into A, B and C type. The first two tank categories are usually constructed of plane surfaces (gravity tanks) and the design vapour pressure is to be less than 0.7 bar. Type C independent tanks are shell structures (also referred to as pressure vessels) meeting vessel criteria. They operate up to the design vapour pressure of 20 bar.

LNG cargo tanks are usually free standing spherical shell structure, or alternatively may be prismatic of either the free standing, self-supporting type or as a membrane structure. LNG are very large ships with cargo capacity range from 25000 m$^3$ to 145000 m$^3$.

Concerning LPG, fully refrigerated cargo tanks are free standing prismatic type operating at temperatures down to -50°C and limited pressure of 0.7 bar. These ships have cargo capacity from 5000 m$^3$ to 100000 m$^3$.

Refrigerated semi-pressurised tanks are usually of bilobe type. Their operation is limited by pressure of 7 bar and associated boil temperature depending on kind of cargo. The cargo capacity of these ships is up to 15000 m$^3$.

Full-pressurised tanks are spherical, cylindrical or lobed supported by saddles. Maximum value of working pressure is 20 bar. The ships tend to be small with capacity up to 4000 m$^3$.

Pressurised cargo tanks are shell structures and their manufacturing is rather complex due to the curved surface and relatively thick walls. Therefore, they are made of high tensile steel and welded segments with different success of geometrical perfection. Beside the residual stress due to welding, misalignment also causes stress concentration and it must be controlled. An especially difficult problem occurs in the case of misalignment in Y-joint of shells and longitudinal bulkhead of bilobe cargo tanks, [6].

This paper deals with the structure design of the type C independent cargo tanks, also referred to as pressure vessels, as the most interesting task. The tank structure design requires realisation of the following items, [7]:

1. Determination of tank shape and clearances.
2. Selection of higher tensile steel and strength criteria, according to the list of cargos that will be carried.
3. Determination of internal pressure that consists of given design vapour pressure and liquid pressure. The latter is a result of combined gravity and acceleration effects due to ship motion in waves.


5. Calculation of shell thickness using the rather simple formulae for pressure vessels.

6. Strength analysis of stiffening rings which transmit tank load (static + dynamic) to the tank support. The rings are loaded by circumferential forces due to the shear stress determined by the bi-dimensional shear flow theory based on the tank shear forces.

7. Buckling analysis of the tank shell and vacuum rings due to external pressure, i.e. difference between the maximum external pressure and the minimum internal pressure (maximum vacuum).

8. Strength analysis of swash bulkheads due to sloshing pressure.

9. Drawings of tank structure with welding details.

10. List of material and nesting plans.

The design procedure is illustrated for the case of the cylindrical and bilobe cargo tanks of a 6500 m³ LPG Carrier as follows.

2. **SHIP AND TANKS PARTICULARS**

The general arrangement of the considered LPG Carrier with one cylindrical and one bilobe tank of the total capacity of 6500 m³, is shown in Figures 1 and 2. The ship is designed in accordance with the Rules of Germanischer Lloyd (GL) [5] and built in Severnav Shipyard, Turnu-Severin, Romania, for the ship owner Hartmann Reederei, Leer, Germany. The ship main particulars are the following:

- length over all \( L_{\text{oa}} = 114.89 \) m
- length between perpendiculars \( L_{\text{pp}} = 109.211 \) m
- breadth, moulded \( B = 16.80 \) m
- depth to main deck, moulded \( H = 11.825 \) m
- V.C.M. draught \( T_d = 7.60 \) m
- ethylene draught \( T_e = 6.64 \) m
- block coefficient \( C_B = 0.709 \)
- V.C.M. displacement \( D \approx 10716 \) t
- main engine output (MCR) \( P \approx 4400 \) kW
- speed at ethylene draught \( v \approx 16 \) kn

The ship is assigned to transport a list of products from ethylene to vinyl chloride monomer for which the mass density is 0.56 and 0.97 t/m³ respectively. Some ship characteristics for the basic loading conditions are listed in Table 1.

<table>
<thead>
<tr>
<th>Loading condition</th>
<th>Draught, m</th>
<th>Displacement, t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballast departure</td>
<td>4.71</td>
<td>5756</td>
</tr>
<tr>
<td>Ethylene cargo</td>
<td>6.64</td>
<td>8653</td>
</tr>
<tr>
<td>Vinyl chloride monomer</td>
<td>7.60</td>
<td>10176</td>
</tr>
</tbody>
</table>

Tank No. 1 is cylindrical and Tank No. 2 is bilobe type of capacity 1960 m³ and 4485 m³ respectively.

Each tank is placed on two saddle supports covered by wood. One support is fixed while the other one is free in the axial direction, Figures 3 and 4. Cylindrical tank is also secured against rotation, Figure 5. In addition, at the upper part of the stiffening rings antifloating preventions are constructed with a clearance for wood layer, Figure 6.
Thermal insulation is placed on the outer shell side with a thickness of 230 mm. Minimum required values of the clearances between the ship structure (plating and stiffeners) and the insulation are achieved.

Working conditions for the tanks operation are related to the pressure and temperature:

- design vapour pressure, IMO: 4.5 bar
- design vapour pressure, USCG: 3.2 bar
- external pressure: 0.3 bar
- test pressure: 6.75 bar
- working temperature: -104°C +45°C

Since the ship is designated to travel also through the U.S.A. territorial waters, the tanks structure has to meet the requirements of the United States Coast Guard (USCG) [8]. The first issues of the IMCO Code and USCG Code are analysed and discussed in [4].

3. TANKS MATERIAL AND STRENGTH CRITERIA

The GL Rules are followed for the selection of the tanks material based on the design pressure and temperature, and the list of transported products [5]. That is high tensile steel 12Ni19 containing not more than 5% nickel. The material known by the commercial name FAFER 5Ni, produced in accordance with the standard EN 10028-4, is accepted for the tanks structure.

The material mechanical properties are the following:

- yield stress, $R_y = 390$ N/mm²,
- tensile strength, $R_m = 540$ N/mm².

The modulus of elasticity and Poisson's ratio read $E = 2.06 \times 10^5$ kN/m² and $\nu = 0.3$ respectively.

The allowable membrane stress $\sigma_{\text{mem}}$ is the smaller one of the following two values:

- $R_m / A$ and $R_y / B$
where for nickel steels and carbon manganese steels $A = 3$ and $B = 2$. Thus, one finds out
\[
R_m / A = 180 \text{ N/mm}^2 \quad \text{and} \quad R_e / B = 195 \text{ N/mm}^2
\]
finally $\sigma_{am} = 180 \text{ N/mm}^2$.

The allowable total stress $\sigma_{m}$ (membrane + bending) shall not exceed one of the following two values
\[
0.57 R_m = 308 \text{ N/mm}^2, \quad 0.85 R_e = 331.5 \text{ N/mm}^2.
\]
This leads to $\sigma_{m} = 308 \text{ N/mm}^2$.

If membrane stress exists, then for bending stress remains
\[
\sigma_{ab} = 308 - 180 = 128 \text{ N/mm}^2.
\]

This is valid for the circumferential direction. However, in the axial direction the membrane stress is $\sigma_{am} / 2$, and the bending stress may take a higher value, i.e.
\[
\sigma_{ab} = 308 - 90 = 218 \text{ N/mm}^2.
\]

4. INTERNAL PRESSURE

4.1. DESIGN VAPOUR PRESSURE

According to the GL Rules, design vapour pressure is determined by the formula [5]
\[
P_0 = 2 + A \cdot C \cdot \rho_r^{-0.5} \text{ [bar]}
\]
where $A$ depends on the tank material, $C$ is a characteristic of the tank dimensions, and $\rho_r$ is the relative density of the cargo comparing to the fresh water density, at the design temperature.

In the considered case the vapour pressure yields 4.5 bar and 5.5 bar for Tank No. 1 and 2 respectively. The former value is equal to the design vapour pressure and the latter is higher. These values are relevant for the further calculations.

4.2. LIQUID PRESSURE

Liquid pressure is a result of combined effects of gravity and acceleration, excluding sloshing due to fully filled tank. It is calculated by the formula
\[
P_{gd} = \frac{a_{\beta} \cdot z_{\beta} \cdot \rho}{1.02 \cdot 10^4} \text{ [bar]}
\]
where
\[
a_{\beta} \quad \text{relative acceleration due to gravity, resulting from gravitational and dynamical loads, in an arbitrary direction } \beta,
\]
\[
z_{\beta} \quad \text{largest liquid height [m] above the point where the pressure is to be determined, measured from the tank shell in the } \beta \text{ direction}.
\]

To determine $a_{\beta}$ it is necessary to calculate and draw acceleration ellipses for ship’s cross-sections and ship’s longitudinal sections based on the ship acceleration components $a_x$, $a_y$, and $a_z$, as shown in Figure 7 in the former case. In order to avoid ship motion analysis in rough sea, the guidance formulae for acceleration components are given in the Rules, which correspond to the probability of exceedance of $10^{-8}$ in the North Atlantic. Values of $a_x$,

Figure 7. Acceleration ellipses in transverse plane, resulting acceleration $a_{\beta}$ in arbitrary direction $\beta$

Figure 8. Determination of liquid height $z_{\beta}$ in transverse and longitudinal plane
In the considered case the obtained values of the dimensionless acceleration components, i.e. relative acceleration with respect to the gravity constant, for the centre of gravity of each tank are listed in Table 2.

**Table 2 Dimensionless acceleration components**

<table>
<thead>
<tr>
<th></th>
<th>Tank No. 1</th>
<th>Tank No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_x$</td>
<td>0.22968</td>
<td>0.22968</td>
</tr>
<tr>
<td>$a_y$</td>
<td>0.74295</td>
<td>0.70067</td>
</tr>
<tr>
<td>$a_z$</td>
<td>0.88936</td>
<td>0.58043</td>
</tr>
</tbody>
</table>

The values of $\alpha_\beta$ are determined according to Figure 7. For this purpose, the intersection of the acceleration ellipse and a straight line of the liquid level need to be defined. Choosing the origin of the coordinate system $Y_0Z$ in the middle of the ellipse, Figure 7, one finds out

$$a_\beta = \sqrt{[1 + a_{\beta x}^2 + a_{\beta y}^2]^{1/2}}$$

(4)

where

$$a_{\beta x} = k a_{\beta y} - 1, \quad k = \tan(\pi/2 - \beta)$$

(5)

$$a_{\beta y} = \frac{1}{a_{\beta x}}$$

(6)

In a similar way the solution for x-z plane may be obtained.

The values of $z_\beta$ are determined according to Figure 8, i.e. by the formula that gives the distance of the considered point $P(x, z)$ at the tank shell and a straight line tangential to the tank top in the longitudinal section

$$z_\beta = x \cos \alpha + z \sin \alpha - p$$

(7)

where $\alpha = \pi/2 - \beta$ and $p$ is the distance of the straight line from the origin. The solution in the transverse plane is obtained in a similar way.

5. **SHELL THICKNESS**

Structure of the cargo tanks (cylindrical and bilobe) consists of cylindrical shells and spherical dished ends with a torispherical connection for the reduction of stress concentration, or hemispherical shell. Special attention is paid to detail of the shell connection in accordance with the requirement of AD Merkblatt [9]. In the similar way, the tank domes and sumps are constructed, Figures 9 and 10.

![Figure 10. Thickness of forward dished end of Tank No. 2](image)

The thickness of the constitutive shell types is determined by using the GL Rules formulae for pressure vessel and steam boilers, which are based on the membrane theory and allowable stresses, as given below [5].

**Cylindrical shell**

$$t = \frac{D_{at} \cdot p_c}{20 \sigma_{um} \cdot \nu + p_c}$$

(8)

**Spherical shell**

$$t = \frac{D_{at} \cdot p_c}{40 \sigma_{um} \cdot \nu + p_c}$$

(9)

![Figure 9. Shell thickness of Tank No. 2](image)
Toruspherical shell

\[ t = \frac{D_a \cdot p_c \cdot \beta}{40 \sigma_{am} \cdot v} \]  

where

- \( t \) - wall thickness, mm
- \( D_a \) - outside diameter, mm
- \( p_c \) - design pressure, bar
- \( \beta \) - design coefficient for dished ends
- \( v \) - weakening factor
- \( \sigma_{am} \) - allowable membrane stress.

In the considered case the design pressure, \( p_c \), is the internal pressure, \( P_i \).

The shell thickness is calculated in the above way using the GL and USCG design load and strength criteria, where the latter is elaborated in Section 11. The maximum values of the shell thickness are accepted as the final ones of the two requirements and are shown in Figures 9 and 10 for Tank No. 2. Since the shell thickness depends on the liquid height, different thickness values may be noticed for the upper and lower parts of the cylindrical shell. The size of the cylindrical shells is limited by the plate production standard dimensions, while the size of the spherical segments is a result of fabrication conditions for the production of double curved elements.

The thickness of the dome and sump is determined in a similar way according to the GL Rules and AD Merkblatt [9]. Weakening effect in the tank shell due to the dome and sump existence is calculated by taking into account compensation for the open areas.

6. LONGITUDINAL BULKHEAD

The longitudinal bulkhead of the bilobe tanks is watertight for the damage stability reason and is mainly a tension loaded structural element. It compensates the membrane forces induced by the tank shells. Since the circumferential membrane forces, \( N_c \), are double higher than the axial ones, \( N_a \), the former are relevant for the dimensioning of the bulkhead plating.

In that way the membrane force in the bulkhead reads

\[ N_c = 2N_e \cos \alpha \cos \alpha = e/R \]  

where \( R \) is radius of cylinder and \( e \) distance of cylinders centres. Membrane stress \( \sigma_{am} \) in the longitudinal bulkhead should not be larger than that in the tank shell. Taking \( N_e = \sigma_{am} t_b \) and \( N_c = \sigma_{am} t_c \) one obtains a guiding formula for bulkhead thickness \( t_b \) depending on cylinder thickness \( t_c \).

\[ t_b = \frac{2t_e}{R} \]  

Besides the membrane forces, the longitudinal bulkhead has to withstand the difference of liquid pressure in two lobe containments. Filling only one of the containment is unallowable. The tolerable difference between the free surfaces in the two containments is 2 m. However, there are a few worse possible situations:

1. Dynamic pressure in the case of a full tank at the top edge of the longitudinal bulkhead.
2. Difference in the static pressure caused by the ship inclination in the case of equal partial cargo filling in both containments, and dynamic pressure.
3. Single side hydrostatic pressure of the full containment in the case of a malfunction of one cargo pump.

In the considered case the maximum pressure is obtained from the third situation, and its value at the half tank height yields \( p = 46 \) kPa.

The longitudinal bulkhead is reinforced by two supporting rings and vertical girders at the position of the vacuum rings and by longitudinal stiffeners as well, Figure 11. In order to avoid additional stress in the bulkhead plating due to bending, the girders are double sided.

The stiffeners span \( b \) is limited by the allowable bending stress \( \sigma_{ab} \) of the bulkhead plating. For a vertical strip clamped at the stiffeners maximum bending moment and the section modulus yield

![Figure 11. Longitudinal bulkhead](image-url)
\[ M = \frac{pb^2}{12}, \quad W = \frac{t^2}{6(1-v^2)} \]  

Thus, the stress condition
\[ \sigma = \frac{M}{W} \leq \sigma_{ab} \]  
leads to the following formula for the permissible stiffeners span
\[ b = t \left( \frac{2\sigma_{ab}}{(1-v^2)t} \right) = 78.2 \, t \]  
The stiffener cross-section is determined in a simple way by considering a part of the stiffener within one span between the vacuum rings as a clamped girder. To determine the scantlings of the vertical girders a FEM strength analysis of the complete vacuum ring is performed.

7. VACUUM RINGS

The vacuum rings are used to ensure the stability of the tank shell in the event that the internal pressure is reduced below the atmospheric value. The design pressure difference, called external pressure, is \( p_e = 0.3 \) bar.

The cross-section of the vacuum ring (Tank No. 1 230 × 25 mm, Tank No. 2 200 × 25 mm) together with the effective breadth of the cylindrical shell \( b_m = 1.56\sqrt{R} \) forms a T-profile with appropriate moment of inertia of total cross-section, \( I \).

The span between the vacuum rings (\( l = 4 \) m for Tank No. 1, and \( l = 3.157 \) m for Tank No. 2), is determined by the buckling analysis of the cylindrical shell.

The vacuum rings in Tank No. 2 represents an arch of the central angle \( 2\alpha \), where \( \alpha = 124^\circ \). The ring ends at the longitudinal bulkhead are assumed to be clamped and the critical pressure according to \([10]\) reads
\[ p_{cr} = \frac{E \ell}{R^2} \left( k^2 - 1 \right) \]  
For the given \( \alpha \), coefficient \( k = 2.324 \) and the critical pressure takes the value 138 kPa. The safety factor is \( S = p_{cr} / p_e = 4.59 \).

Vacuum rings in Tank No. 1 are closed and \( k = 2 \) in formula (17). Critical pressure is 103 kPa and the safety factor \( S = 3.44 \).

8. SHELL BUCKLING ANALYSIS

8.1. CYLINDRICAL SHELL

In the case of a large external pressure a segment of the cylindrical tank shell between the two vacuum rings may lose stability. According to the GL Rules the critical pressure for elastic buckling of a cylindrical shell is determined by the following formula \([5]\)

\[ p_c = \frac{E}{2(1-\nu^2)} \left[ \frac{t-c}{D_s} \left( \frac{t-c}{D_s} \right) + \frac{(t-c)^3}{3(1-\nu^2)} \right] \]  

where
\[ z = \frac{\pi D_s}{2R}, \quad S_k = 3 + 0.002 \frac{t-c}{R}, \quad R = D_a / 2 \]  

and further
\[ p_c - \text{critical pressure, bar} \]  
\[ t - \text{shell thickness, mm} \]  
\[ D_a - \text{outside diameter, mm} \]  
\[ l - \text{length of shell, mm} \]  
\[ c - \text{allowance for corrosion and wear, mm} \]  
\[ \nu - \text{Poisson's ratio} \]  
\[ S_k - \text{safety factor against elastic buckling} \]  
\[ n - \text{number of buckled folds occurring round the periphery in the event of failure, which gives minimum } p_c \text{ value.} \]

Curves of critical pressure as function of number of buckled folds for Tanks No. 1 and 2 are shown in Figure 12. Minimum value of those curves is real buckling pressure, Table 3.

<table>
<thead>
<tr>
<th>Tank</th>
<th>( l ) [mm]</th>
<th>( t ) [mm]</th>
<th>( S_k )</th>
<th>( n )</th>
<th>( p_c ) [bar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>4005</td>
<td>16.0</td>
<td>3.59</td>
<td>12</td>
<td>0.442</td>
</tr>
<tr>
<td>No. 2</td>
<td>4270</td>
<td>18.0</td>
<td>3.53</td>
<td>11</td>
<td>0.568</td>
</tr>
</tbody>
</table>

8.2. SPHERICAL SHELL

The tank dished ends are designed as segments of the spherical shell. Their stability may be checked by determining the buckling pressure of the complete sphere. According to the GL Rules, buckling pressure is given by the formula \([5]\)
\[ p_c = 3.66 \frac{E}{S_k} \left( 1 - \frac{c}{R} \right)^2 \] (20)

8.3. LONGITUDINAL BULKHEAD

The stability of the plates of the longitudinal bulkhead between the vacuum rings and the longitudinals is checked following the procedure given in [11] for combined axial and transverse compression.

9. STIFFENING RINGS

9.1. SUPPORT REACTIONS

The stiffening rings are structural elements that transfer the tank static and dynamic load to the ship structure by saddle supports. Dimensioning of the rings is a rather complex task since it requires performance of a FEM analysis.

The support reaction consists of a part of the tank and cargo weight, and dynamic load that depends on acceleration. It may be written in the form
\[ F = C \alpha \beta W \] (21)

where C is reaction coefficient as a percentage of weight transferred to the support, \( \alpha \beta \) is dimensionless acceleration including gravity, and \( W \) is total tank weight. The support acceleration \( \alpha \beta \) is determined for the ship in upright position and the biased ship as explained in Section 5. The calculated reactions for all four supports are listed in Table 4.

<table>
<thead>
<tr>
<th>Table 4 Reactions of tank supports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank No.</td>
</tr>
<tr>
<td>Volume, ( V ) [m³]</td>
</tr>
<tr>
<td>Cargo weight, ( W_c ) [kN]</td>
</tr>
<tr>
<td>Steel weight, ( W_s ) [kN]</td>
</tr>
<tr>
<td>Total weight, ( W ) [kN]</td>
</tr>
<tr>
<td>Frame No.</td>
</tr>
<tr>
<td>Reaction coefficient, ( C )</td>
</tr>
<tr>
<td>Acceleration, ( \alpha \beta, \beta = 0° )</td>
</tr>
<tr>
<td>Acceleration, ( \alpha \beta, \beta = 30° )</td>
</tr>
<tr>
<td>Reaction, ( F ) [kN], ( \beta = 0° )</td>
</tr>
<tr>
<td>Reaction, ( F ) [kN], ( \beta = 30° )</td>
</tr>
</tbody>
</table>

9.2. RING LOAD

The stiffening ring is exposed to the action of circumferential shear load due to tank bending between two supports. According to the GL Rules the ring strength has to be considered for the ship in upright and biased positions. For circular stiffening rings of Tank No. 1 the problem may be solved analytically in a rather simple way. However, for bilobe Tank No. 2 numerical procedure has to be applied.

Shear load for the bilobe tank is determined for both vertical and horizontal tank shear force of the unit value. The calculation is performed by the program STIFF [12], based on the theory of the thin-walled girders [13], and the results are shown in Figures 13 and 14 respectively. The resulting shear load for the quarters of the biased tank is obtained as follows, Figure 15:

quarters 1 and 3: \( q = q_c \cos \beta + q_h \sin \beta \)
quarters 2 and 4: \( q = q_c \cos \beta - q_h \sin \beta \)

(22)
where $\beta$ is the inclination angle. The direction of the positive shear load is indicated in Figure 15.

9.3. RING FORCES

The ring sectional forces due to the relative circumferential shear load are determined by the finite element method, using the program SESAM [14]. The model cross-section includes the assumed T-profile of the ring and the effective breadth of the tank shell. The sectional properties are the following:

- cross-section area, $A = 0.057 \text{ m}^2$,
- shear area, $A_s = 0.025 \text{ m}^2$,
- moment of inertia, $I = 0.0098 \text{ m}^4$

The same properties are assumed for the double side girder of the longitudinal bulkhead of the bilobe tank in the first step of the analysis.

The FEM model consists of a number of beam elements and it is placed on elastic springs that simulate behaviour of the wood layer on the tank saddle support. The procedure is illustrated for the case of bilobe ring, Figure 16. The springs are distributed on each tank side within the central angle $-25^\circ$ to $75^\circ$ of the saddle foundation, and they are directed radially. The spring stiffness yields

$$k = E \frac{a b}{h}$$

where
- $E$ - Young's modulus of wood
- $a$ - arc distance between springs
- $b$ - wood breadth
- $h$ - wood thickness

In the considered case, $a = 0.829 \text{ m}$, $b = 0.4 \text{ m}$, $h = 0.2 \text{ m}$, and for the wood material known by the commercial name Lignostone H II/2/30-E5, Rochling Plastics USA, $E = 1.655 \cdot 10^7 \text{ kPa}$.

The calculation is performed for the tank in the upright and biased positions. In the former case all springs are pressed and active, while in the latter case some peripheral springs cause tensile force and are therefore excluded from the analysis.
The obtained results, i.e. the ring deformation, the normal force, the shear force and the bending moment for the biased tank as a worse case are shown in Figs. 17, 18, 19 and 20. The ring is mainly deformed out of the saddle. The normal force is high at the bottom. The shear force is a stepwise function due to the discretised elastic foundation. Thus, an average approximation function is relevant with the maximum value at the ends of the saddle. The bending moment takes the maximum value at the end of the saddle in the lower lobe of the biased tank.

The actual sectional forces for each stiffening ring are obtained by multiplying their relative values calculated for the unit tank shear force with the corresponding value of the support reaction, Table 4.

The stresses caused by the actual sectional forces are calculated at five positions of the ring cross-section in two Gaussian points of each beam element of the ring FEM model. The stress positions are chosen in the symmetry line of the cross-section, at the level of neutral axis, at the ends of the web and at the outer side of the flange and tank shell. Furthermore, the equivalent stresses at the same positions and points are determined using the von Mises formula

\[ \sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \]

(24)

where \( \sigma_x \) and \( \sigma_y \) are normal stresses in the x and y direction respectively, \( \tau_{xy} \) is shear stress in the xy plane, \( \sigma_n \) and \( \sigma_b \) are normal stresses due to the axial force and the bending moment respectively.

The final dimensions of the stiffening rings at different cross sections are determined by varying the initial scantlings until meeting the stress criteria. Based on the difference between equivalent and allowable stresses, the flange and web thickness and the web height are changed. The web height of each stiffening ring at the end of the saddle support is increased due to high values of the sectional forces in the case of the tank biased for 30°, Figure 21.

In a similar way scantlings of Tank No. 1 are determined. Tangential load \( q \) at the angle \( \phi \) of the circular stiffening ring of radius \( r \), due to shear force \( Q \), is presented in the form

\[ q = \frac{Q}{r} \sin(\phi + \beta) \]

(25)

where \( \beta \) is bias angle.

Scantlings of the circular ring are shown in Figure 22. Distribution of the equivalent von Mises stress at five points of the ring cross-section at Fr. 136 in case of biased ship for 30°, is presented in Figure 23. Clockwise numbering of the arch finite elements starts from the tank top. It is evident that maximum stress value is within the allowable value of 308 N/mm².
10. SWASH BULKHEADS

The swash bulkhead is designed as a perforated plate with grillage stiffening, Figure 24. The bulkhead is attached to the vacuum ring and the vertical girder of the longitudinal bulkhead by elastic springs, Figure 25.

The swash bulkhead has to withstand sloshing pressure, which according to the GL recommendation is given by the simple formula

$$ p = \frac{4 - L/150}{l} \rho $$

where $L$ is ship length, $l$ is length of liquid free surface, and $\rho$ is cargo density. Taking into account $l = 12.628$ m, yields $p = 0.4015$ bar.

The scantlings of the bulkhead girders are determined in an ordinary way assuming that they are simply supported at the ends.

The strength of the elastic springs is checked considering one half of the spring as a cantilever clamped at the vacuum ring. This assumption is realistic due to the existence of the inflexion point of spring deflection, Figure 25.
11. USCG REQUIREMENTS

If a Liquefied Gas Tanker intends to enter into the U.S. territorial waters, the ship owner has to apply for a "letter of compliance". In that case the relevant classification society has to certify the compliance with some "special design standards" of the U.S. Coast Guard, which are somewhat stronger than the Classification Rules based on the IMO requirements. When the C type cargo tanks are in question, the most important special design standard is a lower allowable membrane stress for the tank shell. For nickel and carbon-manganese steels factor A is 4 instead of 3, while factor B is the same, i.e. equals 2. In the considered case
\[
\frac{R_m}{A} = 135 \text{ N/mm}^2 \text{ and } \frac{R_m}{B} = 195 \text{ N/mm}^2
\]
i.e. \( \sigma_{am} = 135 \text{ N/mm}^2 \).

Since the USCG use the same formula for the design vapour pressure \( P_0 \), which depends on the allowable membrane stress \( \sigma_{am} \), as a result the value of the USCG vapour pressure is lower than that of the IMO Code. However, the influence of these differences on the tank shell thickness is not linear, and therefore the thickness has to be determined by repeating the classification determination procedure with the new vapour pressure value and the allowable membrane stress.

For the considered tanks, the value of 3.2 bar is found as the USCG design vapour pressure \( P_0 \), which depends on the allowable membrane stress \( \sigma_{am} \), as a result the value of the USCG vapour pressure is lower than that of the IMO Code. However, the influence of these differences on the tank shell thickness is not linear, and therefore the thickness has to be determined by repeating the classification determination procedure with the new vapour pressure value and the allowable membrane stress.

As a result of the repeated calculation, the shell thickness is somewhere increased up to 1 mm.

12. CONCLUSION

Liquefied Gas Carriers are special and sophisticated ships. Due to high pressure and low temperature, the design of their cargo tanks requires special attention. Therefore, the Classification Rules are implementation of the IMO Code requirements. Also, for ships entering the US territorial waters, the US Coast Guard requirements have to be met.

This paper deals with the design of the cylindrical and bilobe C type cargo tanks where the latter is the most interesting problem. This complex task is analysed in details. Determination of scantlings for each structural element is illustrated for the case of an actual LPG Carrier. Thus, the presented procedure may be used as a design standard and applied in similar cases.

Small details as welding joints are not presented. Specification of the material also requires special attention. The size of the tank shell segments depends on standard plate dimensions and possible fabrication in the shipyard, i.e. automatic cutting and bending. For these purpose nesting of structural elements on standard plates has to be done carefully with the required minimum distances since high tensile steel is much more expensive than ordinary steel for ship structures.

REFERENCES


Note: International Maritime Organization (IMO), formerly known as the Inter-Governmental Maritime Consultative Organization (IMCO), was established in 1958 through the United Nations to coordinate international maritime safety and related practices.