Correcting RMS value of a sine waveform using limited number of periods during generation and determinate aperture time on DMM

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Sampling of a sine waveform with high-resolution integrating DMM with the input parameters, such as aperture, sampling and measurement time, number of bursts, number of harmonics, expected RMS and measured frequency of a signal can be accurate to 5 ppm. Precision of a measured frequency can highly influence on sampling parameters. The amplitude of expected RMS behaves as sinc function due change of an aperture time. Still the error exists. The alternative method for correction of a root-mean-square value of a measured sine signal due impact of aperture time is presented.

t_a = aperture time
$t_a\ [T]$ = aperture time in percentage of a period
F_s = sampling frequency
n = number of samples
ppm = part per million

1. Introduction

A 10ppm accurate digital AC measurement algorithm was introduced in 1991 and since then exhaustively tested on DMM 3458A instrument. Since the accuracy of DMM is higher on DC range then on AC range (see appendix), Swerlein algorithm samples AC input on DC range. The A/D's error relative to perfect sampling can be expressed by *sinc* function.

2. Correcting root mean square – simulation part

2.1. Sine waveform

Sine waveform can be generated according to the following equation: $y[i] = amp \times sin(initial\_phase + frequency \times 360.0 \times i/F_s)$, for $i = 0, 1, 2, \ldots, n - 1$. To become composite signal, LabVIEW is collecting mean values. From collected mean values, LabVIEW calculates root mean square value (*RMS*) of a signal waveform (equation 1):

$$RMS = \sqrt{\frac{1}{n}\sum_{i=0}^{n-1}|x_i|^2} \qquad (1)$$

The first method for correcting root mean square value, because of limited aperture time (which is affecting sampling info – systematic error), is*:*

$$\mathbf{RMS' = RMS + amp \times (1 - sinc\ (\pi \times t_a\ [T]))} \qquad (2)$$

And it is valid: $t_a[T] = \frac{t_a}{T}$.

The corresponding measurement uncertainty is:

$$Error' = (RMS' - amp) \cdot 10^6 \text{ [ppm]}. \qquad (3)$$

The normalized *sinc* function is shown on figure 1. This function is fundamental in the concept of reconstructing the original continuous band limited signal from uniformly spaced samples of that signal.

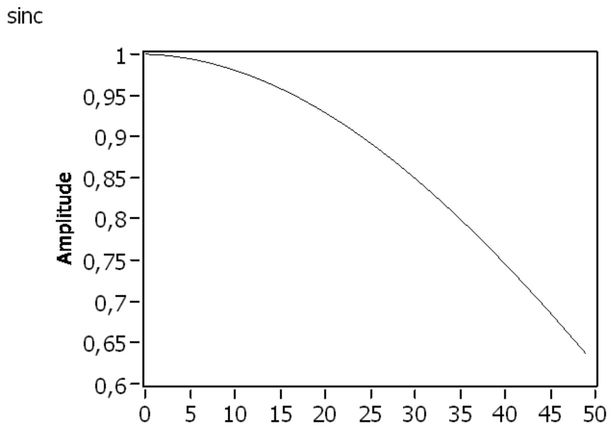

Figure 1. ***sinc*** **function**

The second method for correcting root mean square value, which is used in Swerlein's algorithm[1], is:

$$RMS'' = \frac{RMS}{sinc(\pi * t_a[T])} \quad (4)$$

The corresponding measurement uncertainty is:

$$Error'' = (RMS'' - amp) \cdot 10^6 \text{ [ppm]}. \quad (5)$$

The standard uncertainty associated with the RMS estimate depends on the waveform stability, harmonic content, and noise variance, was evaluated to be less than $\mathbf{5 \cdot 10^{-6}}$ in the 1-1000 V and 1-100 Hz ranges[2].

If the aperture time is increased until 10 % of a period and the **number of samples per period** are changed from 60 to 200, it can be seen that the difference between *Error''* and *Error'* is fluctuating between positive and negative values (depending on aperture time) and the parabolic curve in the graph representing *(RMS''-RMS')/RMS'* in a relation to $t_a[T]$ is negative for particular number of samples per period. Thus, from the results of simulation, a more effective formula for correcting the root mean square value, because of influence of aperture time, is equation (2) (figures 2 and 3).

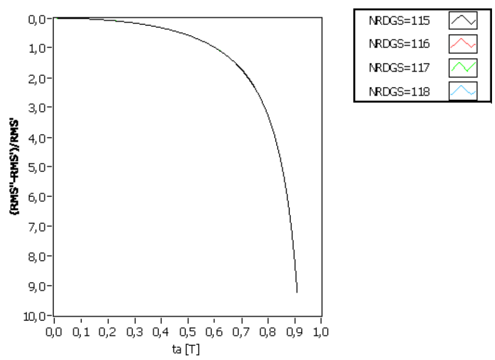

Figure 2. Negative parabolic trend in correcting root mean square value depending on equation (2) and (4)

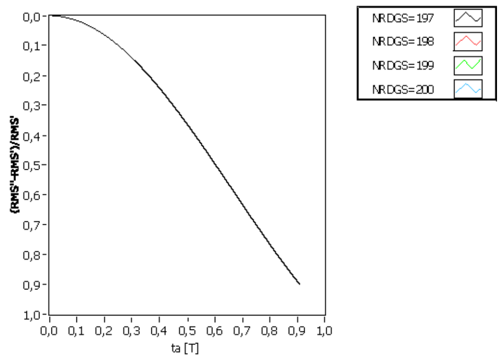

Figure 3. Negative linear trend in correcting root mean square value depending on equation (2) and (4)

2.2. Behavior of A/D converter

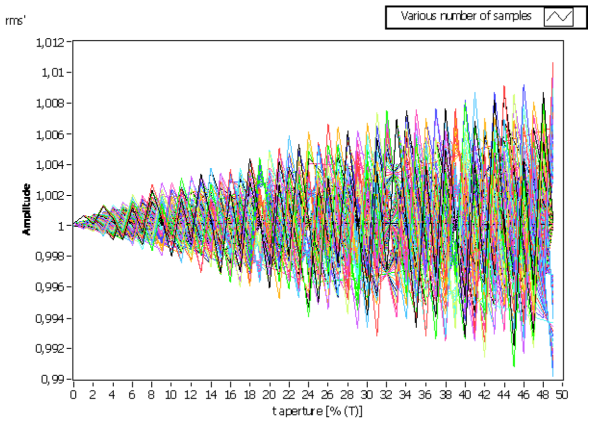

Figure 4. Impact of number of samples and aperture time on amplitude

3. Correcting root mean square – real data

In Table 1 are the measured values of voltage source FLUKE at ranges 10 V and 90 V sampled on DMM 3458A instrument[3].

Table 1.Measured voltages depending on chosen aperture time

t_a[T]	Uncorrected results		Corrected results	
	U [V]	U [V]	RMS'[V]	RMS'[V]
0.05	10.69855	92.62285	10.7425	93.0033
0.1	10.70412	91.60841	10.8792	93.1245
0.15	10.29746	89.05499	10.6891	92.4452
0.2	9.93074	86.18991	10.6209	92.1651
0.25	9.69551	83.13903	10.762	92.372
0.3	9.01402	78.01701	10.529	91.133
0.35	8.68937	75.0851	10.7185	92.6527
0.4	8.0608	69.26894	10.6624	91.7923
0.45	7.3792	64.12156	10.6032	92.0338
0.5	6.77459	59.51871	10.6622	93.176
0.55	6.08706	52.51602	10.6701	92.1938
0.6	5.38479	46.49261	10.6854	92.3825
0.65	4.65388	40.07212	10.6843	92.2806
0.7	3.95416	34.1617	10.7169	92.7102
0.75	3.24063	28.16249	10.7285	92.9887
0.8	2.53086	21.8883	10.7273	92.8492
0.85	1.85894	16.04848	10.7386	92.9244
0.9	1.22049	10.53116	10.7498	93.031
0.95	0.11504	0.6267	10.2528	88.3947

Figure 5 presents the relative error of a corrected result on the real data.

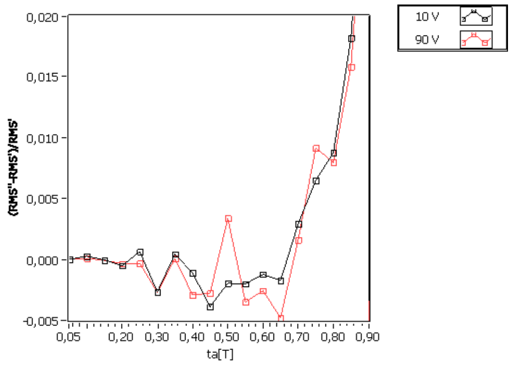

Figure 5. Relative error

3.1. Temperature independence

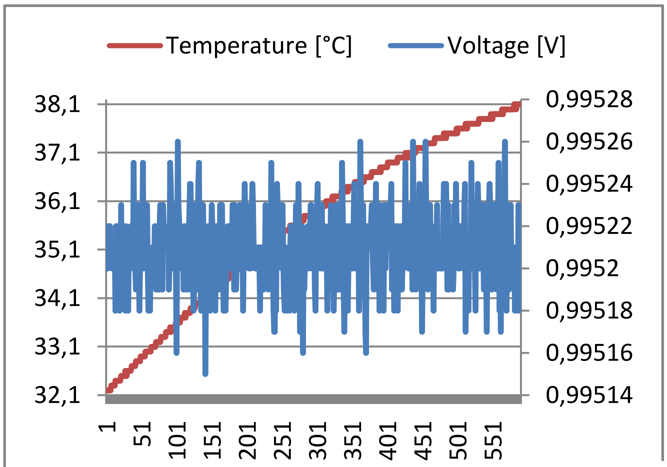

Figure 6. Temperature independence of Swerlein algorithm

4. Conclusions

Because of the diversity in results in using equation (2) or equation (4), Swerlein algorithm could use additional case for correcting the accuracy of the DMM 3458A depending on selected aperture time and range of measured voltage.

REFERENCES

1. R. L. Swerlein, “A 10 ppm accurate digital AC measurement algorithm”, *in Proc. NCSL Workshop Symp.*, pp. 17-36, 1991.
2. Kyriazis, G.A.; Swerlein, R., “Evaluation of uncertainty in AC voltage measurement using a digital voltmeter and Swerlein’s algorithm”, Precision Electromagnetic Measurements, pp. 24-25, 2002.
3. Agilent Technologies 3458A Multimeter, “User's guide”, *Agilent Technologies*, Manual Part Number: 03458-90014.

APPENDIX

Figure 7. Keithley 199 and Digital Multimeter 3458A

The multimeter measures DC voltage on any of five ranges. Table 2 shows each DC voltage range and its full scale reading (which also shows the maximum number of digits for the range). Table 2 also shows the maximum resolution for each range.

Table 2. DC Voltage Ranges

DCV range	Full Scale Reading	Maximum Resolution
100 mV	120.00000 mV	10 nV
1 V	1.20000000 V	10 nV
10 V	12.0000000 V	100 nV
100 V	120.00000 V	1 µV
1000 V	1050.00000 V	10 µV

The A/D converter's configuration determines the measurement speed, resolution, accuracy, and normal mode rejection for DC or ohms measurements. The factors that affect the A/D converter's configuration are the reference frequency, the specified integration time and the specified resolution. The multimeter's frequency and period counter accepts AC voltage or AC current inputs. The maximum resolution is 7 digits for both frequency and period measurements. Before the multimeter will take readings, three separate events must occur in the proper order: the trigger arm event, the trigger event and the sample event. For most applications, you will need to use only one or two of these events and leave the other event(s) set to AUTO. Readings can be stored in one of five formats: ASCII (16 bytes), single integer (2 bytes), double integer (4 bytes), single real (4 bytes), or double real (8 bytes).

The 3458A accuracy is specified as shown in table 3 for DCV measurements.

Table 3. DCV accuracy (ppm of Reading + ppm of Range)

Range	24 Hour	90 Day	1 year	2 year
100 mV	2,5+3	5,0+3	9+3	14+3
1 V	1,5+0,3	4,6+0,3	8+0,3	14+0,3
10 V	0,5+0,05	4,1+0,05	8+0,05	14+0,05
100 V	2,5+0,3	6,0+0,3	10+0,3	14+0,3
1000 V	2,5+0,1	6,0+0,1	10+0,1	14+0,1

In ACV measurements the best accuracy is 100 ppm for synchronous subsampled technique (table 4).

Table 4. ACV accuracy

Technique	Frequency range	Best Accuracy
Synch. subsampled	1 Hz – 10 MHz	0,010 %
Analog	10 Hz – 2 MHz	0,03 %
Random sampled	20 Hz – 10 MHz	0,1 %