The particle’s trajectory - Implementation

DINKA LALE [1], VESNA ŽUPANOVIĆ [2], IVAN GRBAVAC [1]
University of Dubrovnik [1], University of Zagreb [2]
Department of electrical engineering and computing [1], Department of Applied Mathematics [2]
Ćira Carića 4, 20000 Dubrovnik [1], Unska 3, 10000 Zagreb [2]
CROATIA
dinka.lale@unidu.hr http://www.unidu.hr

Abstract: - The paper addresses the application of the mathematical definition of a particle's trajectory in different scientific fields and using various approaches. After the introductory definition of the concept of a particle's trajectory, it go on to provide a definition of the concepts necessary for the understanding of the issues in question: the concept of mathematical chaos (especially the Poincaré section in the context of which the dynamic trajectory is explained) and the Monte Carlo method employed for the estimation of nuclear radiation shields during a nuclear shielding simulation. Finally, an example is given which defines the concept of a wave particle's trajectory and the method of ship manoeuvring along a trajectory.

Key-Words: - the particle, the particle’s trajectory, the interaction, self-similarity, Monte Carlo method, the wave, ship's motion

1 Introduction

The life history of a particle is built up from an acquirement of its trajectory through the particular system of interest. The concept of trajectory has several meanings. It can denote a curve which describes the movement of a planet or comet in space; it can denote a path or progression or the line of a material object movement, a physical body. Therefore, a particle's trajectory describes the path that a certain particle has travelled in the course of its life path.

2 The trajectory

The history of particles is developed from the moment of comprehending their trajectories to the elaboration of a system of particles that in itself is of great significance.

A concept of the path of a particle described its travelling through the medium. Since the typical particle scatters very frequently, the path has „zig-zag“ a form indicated in Fig. 1.

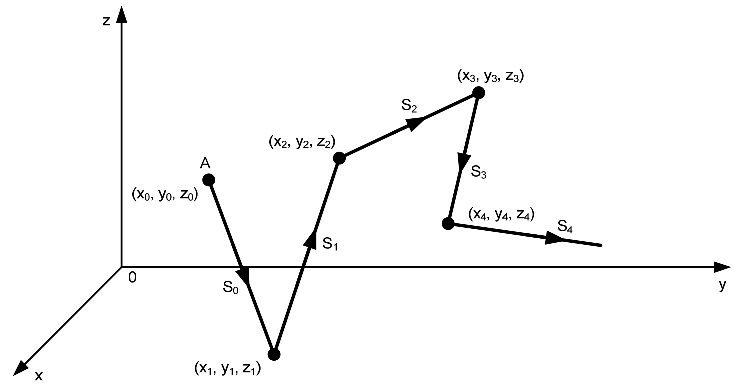

Fig.1. Characteristic “random walk” of a particle through the medium.

Here a particle has a collision with an atom of the medium. That mode of a collision could result in the absorption of the particle and its ending/termination or a particle can continues its travelling through the medium with a new direction and a change of energy or wavelength. Since the change of energy and direction is a statistical process there is not a unique energy or direction after a scattering. In fact, there is a probability distribution for each of these variables. After the first scattering a particle has new direction and energy, it experiences another collision after vice it has new different direction and energy, etc. That process is well described in Fig. 1. In order to track the particle during its journey it requires to know the following quantities: its spatial coordinates (x,y,z), the spherical coordinates of its direction (θ, Φ) and its energy E.

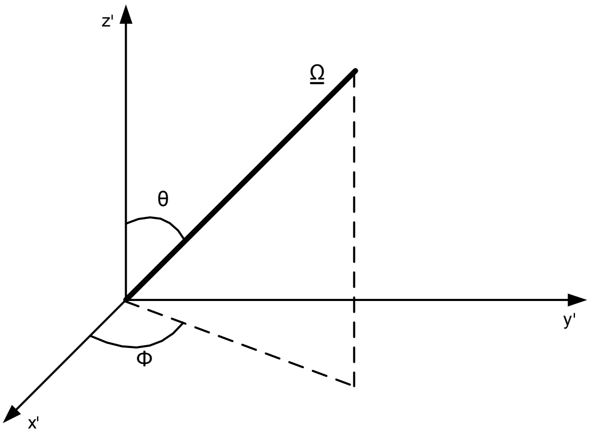

Fig.2. Spherical coordinates of the particle direction (θ, Φ)

The orthogonal coordinate system (x',y',z') is parallel to the basic reference system (x,y,z) shown on Fig. 1.

The quantities above are sufficient to define the state, α, of a particle, where:

$$\alpha \equiv \alpha(x, y, z; E; \theta, \Phi) \qquad (1)$$

A particle's trajectory from collision to collision is depicted like a series of the different states, α_0, α_1, ... α_n. Hence, α_i described i[th] state of a particle and it is defined by the equation:

$$\alpha_i \equiv \alpha_i(x_i, y_i, z_i; E_i; \theta_i, \Phi_i) \ (2)$$

What means in the i[th] state a particle has spatial coordinates of the i[th] collision point, the energy and direction of a particle after the i[th] collision. Each successive state is a function only of the previous state. The exception is the initial state. Conditions which define α_0 are chosen by random sampling from the relevant probability distributions [1].

3 Implementations

3.1 The example of the particle – photon

Use of the mathematical theory is enormous. One of the most interested is in physics and chemistry.The path length of the particle to the next collision point (in this specific case, that particle is photon) is denoted by *s*. The probability of a particle travelling a distance *s* without having an interaction is $e^{-\Sigma s}$. The probability that a particle will have an interaction in the interval *ds* is *Σds*. The conclusion is that the probability that a particle will have an interaction in interval between *s* and *s*+*ds* is

$$\Sigma e^{-\Sigma s} ds \qquad (3)$$

Where Σ is a particle's total cross section (*).

Specific mathematical procedures are required to select the position of the next collision point. If a particle survives that collision, specific mathematical procedures are required for defining its new energy and direction.

A particle, photon, has just undergone its i[th] collision, for photon it is i[th] scattering. After that, the spatial coordinates of its next scattering should be found.

In case of a photon total cross section is denoted as $\mu^{(*)}$. Hence a procedure for picking a random value of *s* from the probability function implied by the expression should be established from equation (3). It is assumed that such a procedure is available and it is selected a particular value of *s* to assign to s_i. Once the value of s_i is determined, the coordinates of the next collision point are ready found from:

$$x_{i+1} = x_i + s_i(\sin\theta_i \cos\Phi_i)$$

$$y_{i+1} = y_i + s_i(\sin\theta_i \cos\Phi_i)$$

$$z_{i+1} = z_i + s_i(\cos\Phi_i) \qquad (4)$$

Next step is to choose the type of interaction, again by sampling from the appropriate probability function. In a case of a photon, the scattering process is Compton scattering, and probability function is given by the Klein-Nishina theory [1].

By sampling from the appropriate scattering law, the new energy, E_i, can be determined. Also, the local angles of the scattering event (Θ_0, Φ) can be determined. That is shown at Fig. 3.

*To describe quantitatively the various interactions, it is necessary to introduce parameter called cross section. The extent to which one particle (for example neutron or photon) interacts with another particle (or nuclei) is described in terms of quantities known as cross-section [17].

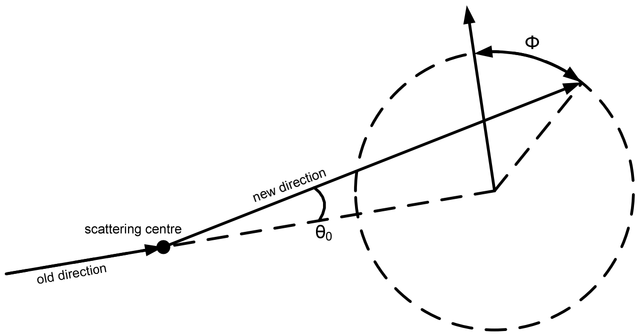

Fig.3. Particle's local angels of scattering

Θ_0 is the defection angle and Φ is the angle of azimuth. Usually the angle Φ can be random assumed and distributed in the range from 0 to 2Π.

3.2 The mathematical chaos

The development of the theory of mathematical chaos it was necessary in physics like in mathematics, especially after developing the quantum theory by Ervin Schrodinger and theory of relativity by Albert Einstein. It is incredibly that lots of figures in the nature also have chaotic shape like, for example, the surface of the sea, the clouds, the snowflakes, the geological shapes, the mesh of neutrons, the architecture of living being etc. The fractal geometry is pursued by those shapes. The fractal geometry is the announcement, in fact the opening of the theory, which is called today theory of chaos. One of the principles of the theory of chaos is opposite of the deterministic principle because it is said the future in random mode depend of the past [3].

When it is talked about the mathematical chaos some names have to be mentioned. First of all, Weierstrass and his function which is continuous, but not differentiable in any point (the length of its diagram is infinite). The next name is Cantor (the Cantor Set). At the end of 19^{th} century Pean defined Pean's curve which fulfilled the space of unit quadrant. At the beginning of 20^{th} century Koch defined Koch's curve which is continuous, has infinite length and it is not differentiable in any point (shown at Fig. 4.) [4].

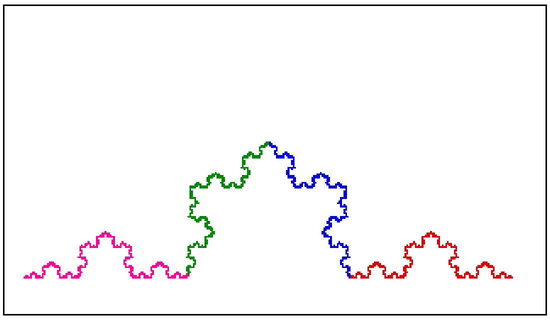

Fig.4. A construction of Koch's curve [14]

After them, the mathematician Henri Poincare, represented the assembly of orbits by the mapping:

$$f : X \to X\,,\ \ \{x, f(x), f^2(x), \ldots | x \in X\} \qquad (5)$$

X is arbitrary m-dimensional area.

The term „dynamic trajectory" describes a trajectory which connects all points of equation (5).

The intersection these trajectory with (m-1)-dimensional area is called Poincare's intersection.

At the beginning of 20^{th} century two other mathematicians appeared, Pierre Fatou and Gaston Maurice Julia. During seventies of 20^{th} theirs discoveries became important because of the fast development of computing, century. In the year 1974 Benoit Mandelbrot got first diagram of Julia's assembly what leads to new term, the fractal (from lat. „fractus"). The fractals have the characteristic of self-similarity shown at Fig. 5. [3].

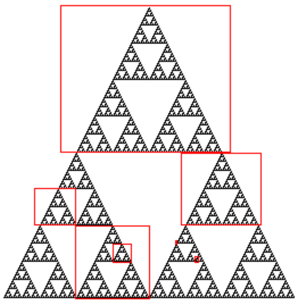

Fig.5. A classical example of fractal shape and its self-similarity [14]

3.3 The particle's trajectory in nuclear physics

The major problem of the theory of nuclear reactor is to determine the distribution of neutrons inside the reactor – to solve neutron transport equation. The neutrons can move around the core of the reactor, can be scattered or absorbed by the reactor or they can escape outside the reactor.

3.3.1 The Monte Carlo method

The Monte Carlo method has become a powerful and versatile tool in solving particle transport problems which are the major problems to reactor and shield designers. In fact, it's very difficult or even impossible to solve adequately by any other mathematical techniques.

The single most important, and also the most distinctive, feature of Monte Carlo is the central role of random sampling methods. In all Monte Carlo calculations it is necessary to draw samples from specified probability distributions [10].

The basic idea of Monte Carlo is to create a series of life histories of the source particles by using random sampling techniques to sample the probability laws that describe the real particle's behaviour. After that, the particle's „random walk“ is traced out step by step until it is information of interest to the problem. The life history is then terminated and a new particle is started from the source. [1]

In considering Monte Carlo, there are two main form of the method

- The basic or analogue, in which the sampling procedures used are straightforward and the schematization followed closely resembles the actual physical processes. In this form there is a strong analogy between the physical particles and the mathematical particles (followed bay computer).
- Non-analogue form is the more sophisticated approach, in which statistical efficiency is improved. Hence, the machine particles are pseudo-particles and no longer the direct counterparts of the real particles.

In Monte Carlo there is no fundamental difference in the treatment of neutrons and photons, there are only differences in the nature of the interaction cross section for the two types of radiation. [1]

In this paper the principle of Monte Carlo method is not described by mathematical equations.

3.3.2 The SCALE

The SCALE (Standardized Computer Analyses for Licensing Evaluation) is computer software system developed at Oak Ridge National Laboratory by requirement of Nuclear Regulatory Commission. It is widely used and accepted around the world for criticality safety analysis. [16] The main object of developing SCALE code is to standard analytical methods. It leads to reducing the costs and increasing the security of the analysis. The SCALE system is very important because it provides the complete analysis of nuclear facility, and also its licensing.

It is include several automatic analytic sequences. Using these different sequences the user can solve problems like the transport equation inside the shield, problems of thermal transport equations, different calculations like the calculation of the criticality etc. Those automatic analytical sequences also called control modules. These control modules use a lot of functional modules. Each of these modules is deterministic to solve some special problem. The sequences SAS2H and SAS4 (Shielding Analytical Sequence) are assigned to calculate the dose of the radiation, and gamma rays and neutrons (in fact, the photons and the neutrons). SAS2H approximates one dimensional transport equation, and SAS4 uses the Monte Carlo method why it is provides radiation dose calculations in positions of detector which are defined in 2 and 3 dimensions.

3.3.3 Example of implementation in physics

There is one more example of implementation the theory of particle's trajectory in physics - in modelling the Earth's atmosphere using Euler's and Lagrange's atmospheric model. The trajectory of the smoke it is used to design mentioned mathematical models.

3.4 The particle’s trajectory in Maritime traffic

The maneuverability of the vessel is a term which analyses her response on sea's waves to estimate capability of the vessel and her safety on rough sea.

The term trajectory has different applications in the theory of maneuverability. One is describing the type of wave and the other one is depicting the driving.

3.4.1 A wave particle's trajectories

If it assumed that the start position of a fluid particle in a wave is the point (x_1,z_1), then $(x-x_1)$ and $(z-z_1)$ are the particle's shifts in relation to that position. Assuming the low wave steepness, these values are low enough for the differences in velocity, which are the consequence of a change in the position, to be neglected, since they are very low and their square is indeed small (negligible) [2].

A new concept is defined - elevation. Elevation or the rising of a free surface is marked with a label ζ. It is described as a harmonic wave of the amplitude ζ_a which is defined as the vertical distance from extreme movements of the free surface (a crest or a trough) to the level of calm water. Further on we define the velocity components that are obtained from the velocity potential. After integration equations of horizontal and vertical velocity components, the variable of time is eliminated. The depth of the sea (water) is marked as *d*, the frequency of the wave is marked as ω and wave's number is marked as *k*. A particle's trajectory has the equation (6).

$$\frac{(x-x_1)^2}{\left(-\zeta_a \frac{\cosh k(d+z_1)}{\sinh kd}\right)^2}+\frac{(z-z_1)^2}{\left(\zeta_a \frac{\sinh k(d+z_1)}{\sinh kd}\right)^2}=(\sin(kx_1-\omega t))^2+(\cos(kx_1-\omega t))^2$$

(6)

3.4.2 Steering a ship along a given trajectory

The development of new technologies connected to the exploitation of the underwater world, the use of the sea and the underwater world for traffic purposes, as well as the progress of mathematics, have led to a new way of steering a ship - sailing along a given trajectory.

Regarding the theory of maneuverability, the vessel is treated as solid particle with six levels of freedom in motion. They are shown at Fig. 6.

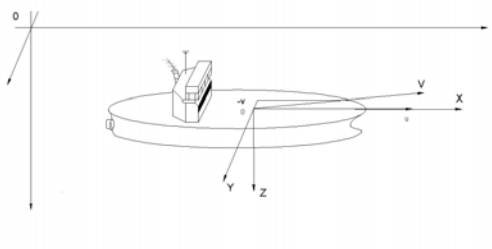

Fig.6. The coordinated system of a ship

0,x,y,z – unfixed coordinated system of a ship

V- the vector of speed of a ship

u - the speed of progress along x-axis

v- the speed of progress along y-axis

While a ship is moving on given course or trajectory, the assignment of control of a ship is to control the moving her centre of mass in horizontal level. Mathematical model of controlling need to be simplified why some things are assumed: the depth of the sea is infinite, there are no other objects near a ship and the velocity of the ship is continuous.

At first glance steering a ship along a course and sailing along a trajectory can be equalized. Namely, in theory, this would be the ideal case, that is, ship manoeuvring without the influence of external disturbances [2].

In the case of a ship's motion this is practically impossible, therefore it is necessary to create a special system for such purposes. An indispensable part of such a system, which is not present in the classical ships of 20 years ago, is a system for the exact determination of the ship's position in the geo reference system. The problem lies in the fact that it is not possible to use the existing navigational systems since the requests for the accurate guidance of a ship along a trajectory allows only for a couple of meters' error. Systems with the necessary preciseness (for example a GPS satellite) are used for such a purpose.

Following the fitting of the entire system, an adequate microprocessor block is installed, which determines the error between the current position of the ship and the given trajectory. The error is then calculated in the adequate steering block into a component of the course error. This course error is processed by the existing autopilot.

The trajectory error is considered to be the shortest distance between the current position of the ship and the given trajectory. Mathematically, this is the distance of a point from the straight line since the trajectory can be divided into a finite number of broken-up linear components. There are lots of ways to calculate the trajectory error, but here it's not necessary to specify it.

4 Conclusion

Scientific discoveries in various fields of science have served scientists in other fields as a basis for their research. Among all of these discoveries, new findings and theorems, those in the field of mathematics are especially outstanding.

It is said that „mathematics is the queen and servant of all the other sciences“.

The discovery and development of the concept of a particle's trajectory, as well as many other discoveries (especially those in the field of the chaos theory), have gained a special, additional, meaning with the development of new technologies, especially computer science. The progress in computers has created the possibility of quickly arriving to the calculation of problems that until then had been almost impossible, if not actually impossible, to solve (calculate). The concept of a particle's trajectory has received the consideration of scientists and engineers dealing with different scientific fields.

The paper cites several of these analyses as examples. Especially prominent is the use of a particle's trajectory theory in the calculations made using a stochastic method known as the Monte Carlo method. The mentioned method is used in the estimation of radiation levels in the calculation of nuclear shields in the way that it observes the transport of neutrons and photons that emit radiation harmful to the living beings and the environment, in case they come into contact with them. It is also used in calculations related to the releasing of harmful substances (effluents) into the Earth's atmosphere. We can especially enjoy the fact that one such discovery, one, at first glance, ordinary mathematical concept, has enabled the projection and construction of safer means of transport, ships. This also allows for the safer navigation of many ships, which today, along with the rough sea and strong sea currents that follow them from the first days of sailing, also face many other problems (wars, pirates).

References:

[1] J. Wood: "Computational Methods in reactor Shielding", Oxford, New York, 1982.

[2] V. Čović: " The maneuverability of the vessel" Rijeka 2006., in Croatian

[3] H.O. Peitgen, H.Juergens, D.Saupe: "Chaos and fractals", Springer, New York

[4] Feynman R.P., Hibbs A.R.: "Quantum Mechanism and Path Integrals", New York 1965.

[5] J.I.Goldstein, D.E.Newbury, P.Echlin, D.C.Joy: "Scanning Electron Microscopy and X-ray Microanalysis, 2nd Edition", New York, 1992

[6] ATLAS Collaboration: "ATLAS Detector and Physics Performance Technical Data Report Volume I", CERN/LHCC

[7] S. Spasić, M. Lj. Kocić: „Application of orthonormal decomposition on fractal interpolating function“, collection of papers, Conference of mathematicians of Macedonia, September 2005, Struga, in Croatian

[8] M. Lj. Kocić: „Fractal history of the chaos“, 2007., in Croatian

[9] Slađana Spasić: „The theory of the chaos and fractal analysis of bio-signal“, dissertation, 2007., The faculty of mathematics, Beograd, in Serbian

[10] Jerome Spanier and Ely M. Gelbard „Monte Carlo Principles and Neutron Transport Problems“, Dover Publications, Inc., Mineola, New York34

[11] Edwin Miles Stoudenmire „Monte Carlo Integration: an Overview with Examples“ MATH 4255, Georgia Tech, 2004.

[12] D. Feretić; Ž. Tomšić; D. Škanata; N. Čavlina; D. Subašić: "The power plants and the environment", Element, Zagreb, 2000., in Croatian

[13] Wilford Zdunkowsky, Andreass Bott: „Dynamics of the Atmosphere A Course in Theoretical Meteorology“, Cambridge University Press

[14] Internet pages:

http://classes.yale.edu/fractals/IntroToFrac/InitGen/KochColors.html, May 2013.

http://sciencepark.etacude.com/simulations/fractal/fractal1.php, May 2013.

http://pages.cs.wisc.edu/~ergreen/honors_thesis/similar.html, May 2013.

[15] Dušan Kalember: „The analysis of stabilisation of the ship on course and trajectory", master thesis, 1991., ETF, Zagreb, in Croatian

[16] The SCALE manual, ORNL/TM-2008/06, September 2008, Oak Ridge National Laboratory

[17] John R. Lamarsh, Anthony J. Baratta: „Introduction to Nuclear Engineering“ Third Edition, Pearson Education International, 2012.