BLUR IDENTIFICATION USING AVERAGED SPECTRA OF DEGRADED IMAGE SINGULAR VECTORS

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ABSTRACT

In this paper we propose new blur identification algorithm based on singular value decomposition (SVD) of degraded image. An unknown space-invariant point-spread function (PSF) is also decomposed using SVD. Magnitude functions of PSF singular vectors (left and right) are identified using averaged spectra of corresponding singular vectors of degraded image. Phase functions of PSF singular vectors are supposed to be zero, except for the case when zero crossings can be detected from corresponding magnitude functions. In the proposed method, two dimensional PSF estimation procedure is decomposed into several one-dimensional estimation procedures. PSF estimation algorithm does not require numerical optimization, what implicates fast and straightforward procedure.

1. INTRODUCTION

Image degradation manifests itself as a blurring of image and adding of noise to the image. It is caused by imperfection of imaging systems and inappropriate imaging conditions. Image blurring is modeled as a 2-D convolution of image with non-causal point-spread function (PSF). Blur identification is determination of unknown PSF, and is the most important step towards successful digital image restoration.

Early approaches to blur identification take advantage of 2-D power cepstrum of degraded image [1]. Periodicity in logarithm of spectrum enables identification of blurs that have zeros on the unit bicircle (motion blur, out-of-focus blur). More recent approaches are based on the 2-D autoregressive moving average (ARMA) model of degraded image. AR part of the model represents original image model, and MA part represents non-causal PSF [2]. Estimation of unknown model parameters is performed using maximization of likelihood function. This method is extended to more general case, using spectrally equivalent minimum phase (SEMP) system representation [3]. In [4] 2-D ARMA model identification is decomposed into parallel 1-D complex ARMA model identification.

Iterative algorithms, where identification and restoration steps are performed one after another, are described in [5] and [6]. In [7] the PSF is chosen from a collection of a candidate PSFs to provide the best match between the restoration residual power spectrum and its expected value. Unknown PSF parameter estimation based on continuous blur model is described in [8].

Our blur identification algorithm operates on singular vectors of degraded image. We identify components of SVD decomposition of the unknown PSF. First (left and right) singular vectors of PSF can be estimated using corresponding first few singular vectors of degraded image. When S/N ratio is high, it is possible to estimate second singular vectors of PSF.

This paper is organized as follows. In Section 2 we discuss changes in singular vectors of original image due to degradation. In Section 3 we describe our blur identification algorithm. Section 4 contains identification results and discussion, while in Section 5 we summarize proposed algorithm.

2. DEGRADATION OF SINGULAR VECTORS

Blur identification is possible from physical modeling and measurement of imaging system (test images), but it is frequently necessary to identify blur from degraded image only. Standard linear discrete model of image degradation is described using following equation:

$$g(i,j) = \sum_{k=-K}^{K} \sum_{l=-L}^{L} h(k,l) f(i-k, j-l) + n(i,j), \quad (1)$$

where i and k represent discrete variables in vertical direction, j and l represent discrete variables in horizontal direction, $f(i,j)$ is original image, $g(i,j)$ is degraded image, $n(i,j)$ is Gaussian white noise field, and $h(k,l)$ is space-invariant PSF of size $(2K+1)(2L+1)$. If we look at the images in the equation (1) as $N \times N$ matrices, than we can analyze changes in SVD representation of original image matrix $\mathbf{F}$ due to degradation. Original image is blurred in such a way that original image is displaced by (k,l), and linear combination using weighting factors $h(k,l)$ is computed:

$$\sum_{r=1}^{R_G} s_{rG} \mathbf{u}_{rG} \mathbf{v}_{rG}^T = \\ = \sum_{k=-K}^{K} \sum_{l=-L}^{L} h(k,l) \sum_{r=1}^{R_F} s_{rFkl} \mathbf{u}_{rFkl} \mathbf{v}_{rFkl}^T + \mathbf{N}, \quad (2)$$

where $\mathbf{N}$ is $N \times N$ white Gaussian noise field, R_G and R_F are the ranks, s_{rG} and s_{fRkl} are singular values, $\mathbf{u}_{rG}$ and $\mathbf{u}_{rFkl}$ are left singular vectors, $\mathbf{v}_{rG}$ and $\mathbf{v}_{rFkl}$ are right singular vectors of degraded image and original image displaced for (k, l), respectively. Important question is what happens with image SVD when image is displaced by k rows and l columns. Providing that displacement satisfies $k, l \ll N$, the first R_1 singular vectors will be shifted versions (except border) of singular vectors of image at position $(0, 0)$, while first R_1 singular values will change insignificant. Shifts in vertical direction will be visible on left singular vectors, while shifts in horizontal direction will be visible on right singular vectors:

$$\mathbf{u}_{rFk} \equiv \mathbf{u}_{rFkl}, \quad \mathbf{v}_{rFl} \equiv \mathbf{v}_{rFkl}, \quad r \leq R_1, \quad k, l \ll N. \quad (3)$$

As we move towards smaller singular values, corresponding singular vectors will be more and more perturbed and equivalence in (3) isn't valid any more. It is supposed that $h(k, l)$ is also decomposed using SVD:

$$\mathbf{h} = \sum_{p=1}^{R_h} s_{ph} \mathbf{u}_{ph} \mathbf{v}_{ph}^T, \quad (4)$$

where $\mathbf{h}$ denotes PSF matrix, R_h is the rank, s_{ph} are singular values, $\mathbf{u}_{ph}$ and $\mathbf{v}_{ph}$ are left and right singular vectors of PSF matrix, respectively. We will approximate $\mathbf{h}$ using only first singular vectors. Vector $\mathbf{u}_{1h}$ operates in vertical direction, and $\mathbf{v}_{1h}$ operates in horizontal direction. Changing the order of summation, and grouping the members that operate separate in horizontal and vertical direction, (2) is transformed into:

$$\sum_{r=1}^{R_1} s_{rG} \mathbf{u}_{rG} \mathbf{v}_{rG}^T \approx \sum_{r=1}^{R_1} \left[s_{rFkl} s_{1h} (\mathbf{u}_{1h} * \mathbf{u}_{rFk}) \cdot \right. \left. \cdot (\mathbf{v}_{1h} * \mathbf{v}_{rFl})^T \right] + \mathbf{N}_{R1}, \quad (5)$$

where (*) denotes convolution operation, and N_{R1} represents a sum of first R_1 base images of N. It is evident from (5) and confirmed experimentally on various PSFs and various types of images, that blurring effect of $\mathbf{u}_{1h}$ is present in the first left singular vectors of degraded image, while blurring effect of $\mathbf{v}_{1h}$ is present in the first right singular vectors of degraded image. We found experimentally that activity of $\mathbf{u}_{2h}$ and $\mathbf{v}_{2h}$ is present in left and right degraded image singular vectors in the vicinity of $r = N/2$.

3. PSF ESTIMATION ALGORITHM

In order to perform estimation procedure we need a model of singular vectors of original image. Covariance function of singular vectors is modeled as a first-order stationary Markov sequence. Effect of noise also have to be incorporated into estimation procedure. Effect of additive noise and model of singular vectors of non-degraded image are analyzed experimentally. Characteristics depicted in Fig. 1(a) and 1(b) are result of averaging on a number of images of various scenes. Fig. 1(a) depicts value of one-step correlation for non-degraded images, $N = 256$. As r moves from

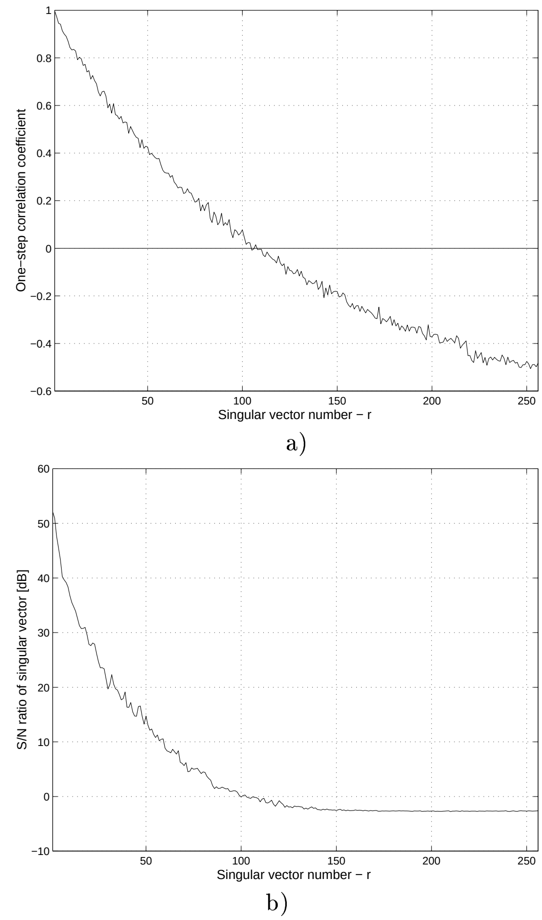

Figure 1: Experimental results: a) one step correlation coefficient as a function of r, b) S/N ratio of singular vectors as a function of r, for images with S/N ratio = 35 dB.

1 to N, singular vectors contain more higher frequency components. We are also interested how additive noise is distributed over singular vectors of blurred image. Images are blurred, and SVD of blurred image was computed. Then, noise is added to blurred image, and SVD of that image was computed. Change in singular vectors due to added noise was analyzed, and expressed as dependence of S/N ratio on singular vector number r. Functional dependence depicted in Fig. 1(b) is a result of averaging over various PSFs. Shape of characteristic does not depend significantly on S/N ratio.

Estimation procedure for $\mathbf{u}_{1h}$ and $\mathbf{v}_{1h}$ will be described. It is evident from Fig. 1(b) that first few singular vectors will have S/N ratio significantly better than image S/N ratio. The idea is to average spectra of first R_1 singular vectors, in order to further eliminate effect of noise. Estimates of averaged spectra of original image singular vectors, $\hat{S}_{\mathbf{u}F}$ for left and $\hat{S}_{\mathbf{v}F}$ for right vectors, are constructed on the base of exponential model of covariance function:

$$\hat{S}_{\mathbf{u}F} = \hat{S}_{\mathbf{v}F} = DFT(\rho_{R1}^{|n|}), \quad |n| \leq \frac{N}{2}, \quad (6)$$

where ρ_{R1} is averaged value of first R_1 one-step correlation coefficients, according to characteristic depicted in Fig. 1(a). For simplicity, the same model is used for left and right singular vectors, and no additional information about original image is extracted from degraded image. Now, when we have referent models, magnitude functions of $\mathbf{u}_{1h}$ and $\mathbf{v}_{1h}$ are estimated as follows:

$$\hat{U}_G = \sqrt{\frac{\sum_{r=1}^{R1} S_r \mathbf{u}_G}{R_1 \hat{S}\mathbf{u}_F}}, \quad \hat{V}_G = \sqrt{\frac{\sum_{r=1}^{R1} S_r \mathbf{v}_G}{R_1 \hat{S}\mathbf{v}_F}}, \qquad (7)$$

where $\hat{U}_G$ is estimate of the magnitude function of $\mathbf{u}_{1h}$, $\hat{V}_G$ is estimate of magnitude function of $\mathbf{v}_{1h}$, $S_r\mathbf{u}_G$ is the spectrum of r-th left singular vector and $S_r\mathbf{v}_G$ is the spectrum of r-th right singular vector of degraded image. It is expected that averaging will somewhat eliminate negative effects due to simple model of non-degraded image singular vectors.

The magnitude function has to be coupled with the corresponding phase function. We take the phase function to be zero, except for the case when zero-crossings can be detected from the magnitude function. In that case π radian jumps take place at the frequencies where zero crossings are detected [3]. Finally, the inverse Fourier transform is applied, and real part of the resulting sequences is analyzed. Positive values in the vicinity of the maximum are of interest. Appearance of very small or negative values indicate that we have reached boundaries of the PSF. This way, support of PSF is determined directly from the sequence where first PSF singular vectors are estimated. Described procedure is carried out separately for left and right singular vectors. Vector $\hat{\mathbf{u}}_{1h}$ is estimate of $\mathbf{u}_{1h}$, and $\hat{\mathbf{v}}_{1h}$ is estimate of $\mathbf{v}_{1h}$. Finally, estimated PSF ,

$$\hat{\mathbf{h}} = \hat{\mathbf{u}}_{1h}\hat{\mathbf{v}}_{1h}^T \qquad (8)$$

is normalized to have the sum of the matrix elements equal to one. When S/N ratio is better than 45 dB, $\hat{\mathbf{u}}_{2h}$ and $\hat{\mathbf{v}}_{2h}$ can be estimated using similar procedure from singular vectors in the vicinity of the $r = N/2$.

4. RESULTS AND DISCUSSION

Important information extracted from Fig. 1 is: first singular vectors contain small amounts of noise, but problem is in the fact that they are highly correlated, and contain less high frequency components. Obviously, some trade-off must be done in determination of R_1. We choose R_1=20, the number of singular vectors of degraded image that will be used to determine first singular vectors of PSF. For the original singular vectors model in (6) we used $\rho_{R1} = 0.84$. This value was determined experimentally, averaging first R_1 values of function depicted in Fig. 1(a). The same model is used for left and right singular vectors.

We tested described algorithm for blur identification on simulated and real world degraded images. Table 1. contains results from one identification example, for different S/N ratios. Image of town, N=256, was blurred using 3×3 truncated Gaussian PSF. Then certain amount of noise was added to produce degraded image. Estimated PSF is good

0.07511	0.12384	0.07511
0.12384	0.20420	0.12384
0.07511	0.12384	0.07511

a)

0.07187	0.12583	0.07187
0.12286	0.21510	0.12286
0.07187	0.12583	0.07187

b)

0.07185	0.12603	0.07185
0.12266	0.21517	0.12266
0.07185	0.12603	0.07185

c)

0.07266	0.12616	0.07266
0.12231	0.21237	0.12231
0.07266	0.12616	0.07266

d)

Table 1: a) True PSF, b) PSF identified at S/N ratio=50 dB, c) PSF identified at S/N ratio=40 dB d) PSF identified at S/N ratio=30 dB.

description of true PSF, and is almost constant in a wide range of S/N ratios of interest. This behaviour is result of cumulative effect of characteristic noise distribution (depicted in Fig. 1(b)) and averaging of singular vectors. Magnitude functions (7) were also used to calculate real cepstrum. In the case of motion blur, spikes in cepstrum [1] were visible up to S/N ratios of 20 dB. Proposed algorithm is an addition to methods described in [9]. For uniform PSFs we estimated all values, not just PSF support. Some ringing was present in the restored images. When PSF is identified from degraded image only, than unknown original image is test signal for identification procedure. Lower power in high-frequency components of test signal implicate less accurate identification of degradation system transfer function in that frequency range.

Important point is determination of PSF support. As we already said, PSF support is determined directly from the sequences where PSF components are estimated. Appearance of very small or negative values is a signal that PSF boundaries are reached. In the cases when PSF support was not properly estimated, restoration results were also good.

Using described procedure we can accurately estimate $\hat{\mathbf{u}}_{1h}$ and $\hat{\mathbf{v}}_{1h}$. That is a separable approximation of the true PSF. In fact, among all separable approximations of the true PSF, this approximation has the smallest mean square error. But, this is not valid image restoration criterion. Nevertheless, from experiments we can conclude that as S/N ratio goes down, difference between true and separable approximation of PSF becomes less important. The S/N ratio is the parameter which ultimately limits the restoration quality and need for an accurate PSF model [10]. We can write:

$$S/N \quad ratio \downarrow \ \Rightarrow \sum_{i,j} [\hat{F}_{h(k,l)} - \hat{F}_{h(k)h(l)}]^2 \quad \downarrow \quad , \qquad (9)$$

where $\hat{F}_{h(k,l)}$ is image restored using the true PSF, and $\hat{F}_{h(k)h(l)}$ is image restored using the separable SVD approximation of the true PSF. When S/N ratio is better than 45 dB, $\hat{\mathbf{u}}_{2h}$ and $\hat{\mathbf{v}}_{2h}$ can be estimated from singular vectors in the vicinity of the $r = N/2$. In that region singular vectors contain more higher frequency components, but S/N ratio is low. In estimation procedure we change value of ρ_{R1} in (6) to -0.15. In this region spectrum of singular vectors has band-pass characteristic.

Example of real world identification for image restoration is presented in Fig. 2. Image from low-quality airborne TV sensor was restored using PSF that was estimated using proposed procedure. There is a significant improvement in the image resolution. Except for computation of degraded image SVD , estimation procedure is very fast.

a)

b)

Figure 2: a) Image from low-quality airborne TV sensor, b) restored image, PSF is estimated using proposed algorithm.

5. CONCLUSIONS

In the proposed blur identification algorithm components of SVD decomposition of the unknown PSF are estimated. First singular vectors (left and right) of the unknown PSF are estimated using corresponding singular vectors of degraded image. Estimated PSF is almost constant in a wide range of S/N ratios of interest. Main part of the error in PSF estimation is due to simplified model of non-degraded image singular vectors. Estimation algorithm does not require numerical optimization, what implicates fast operation. Proposed procedure is especially applicable at lower S/N ratios. Restoration results on simulated and real world images prove the validity of the approach.

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