Three-Phase Inverters Supplying Non-Symmetrical, Non-Linear and Single Phase Loads

Igor Musulin
Power Electronics and Control Department
Končar – Electrical Engineering Institute, Inc.
Zagreb, Croatia
igor.musulin@koncar-institut.hr

Željko Jakopović
Department of Electric Machines, Drives and Automation
Faculty of Electrical Engineering and Computing
Zagreb, Croatia
zeljko.jakopovic@fer.hr

Abstract— **In this paper, an overview of three-phase inverter topologies for supplying non-symmetrical, non-linear and single phase loads is presented. Today non-symmetrical and non-linear loads dominate in electric power systems. This is even more evident in the isolated systems like uninterruptible power supply, renewable energy sources operating in island mode or auxiliary power supply in railway vehicles. Three-phase inverters therefore must be able to supply three-phase as well as single-phase loads and at the same time not affect the power quality. There are currently three inverter topologies capable of doing this. A special emphasis is given on a three-phase four-leg inverter topology. In addition, two of the most used modulation techniques for three-phase four-leg inverter are described. Mathematical model in abc and dq0 coordinate system is also derived. At the end, the paper outlines some of the existing control design challenges and their solutions.**

Keywords— three-phase four-leg inverter; 3D space vector modulation; carrier based PWM; proportional − resonant controller

I. INTRODUCTION (*Heading 1*)

Today power quality is one of the most important research topics in the field of electrical engineering. It is estimated that between 75 − 90 percent of all loads is in fact nonlinear loads [1]. By definition, a non-linear load is a load where the wave shapes of the steady-state current does not follow the wave shape of the applied voltage. Power losses in the low voltage distribution network due to the non-linear and non-symmetric loads are more than 10 percent of the average transmitted energy [2]. Thus, it is extremely important to limit these power quality deteriorating factors. Various standards and laws concerning power quality are published imposing limitations on current harmonics and voltage non-symmetry [3] – [7]. Problems with non-linear and non-symmetric loads are the main concern in isolated systems like UPS, renewable energy sources operating in island mode, and auxiliary power supply for ships, airplanes and railway coaches. Large numbers of non-linear and/or non-symmetric loads generate harmonics and cause unbalance, neutral current, etc.

Various commercial solutions are already successfully used for many years in dealings with current harmonics (active filters) and decreased power factor (capacitor banks). Only recently, mainly due to the high increase of renewable energy sources in a supply system, particularly operating in island mode, non-symmetry is also being recognized as a problem. From this point forward, non-symmetry will be regarded as a simultaneous supply of three-phase and single-phase loads. For this, our wires are needed, i.e. three-phase wires and a neutral wire.

Chapter II gives an overview of three-phase four-wire topologies used for supplying non-linear and non-symmetric loads. Due to numerous advantages, three-phase four-leg inverter topology is chosen for further consideration. In Chapter III two of the most frequently used modulation techniques are presented. Chapters IV and V give a mathematical model of three-phase four-leg inverter and a short overview of control strategies, respectively. The paper ends with the conclusion.

II. TOPOLOGY

A three-phase inverter bridge consisting of six power semiconductors arranged in three half-bridges, each including two power semiconductors (Fig. 1), generates a three-phase power supply for a consumer from the voltage provided by the DC link. The output voltage pulses are controlled, so that when averaged in time a three-phase voltage is generated, the rms and frequency of which are optionally variable. LC filter is used to attenuate PWM modulation frequency. This type of inverter is used largely for supplying balanced loads, powers ranging from few watts to more than megawatts. Load imbalance generates electric current flow in the soil because the source (distribution system) and load neutral are grounded at different potential. In that case, any unbalanced current that may flow in the neutral will partly return through the earth. This current is referred as stray current and is usually harmless if the system is properly designed. However, higher values of stray current can cause DC voltage oscillation and unbalance in the DC capacitor voltage. Resistors for symmetrical distribution, high capacitor values or additional voltage control circuits are then used to solve this problem. Also, this topology is unable to supply single-phase loads since there is no neutral wire available. Therefore, one of the next three topologies is generally used.

The first one consists of an inverter and an isolated transformer connected to the inverter output terminals, (Fig. 2). Due to its many advantages like galvanic isolation of primary

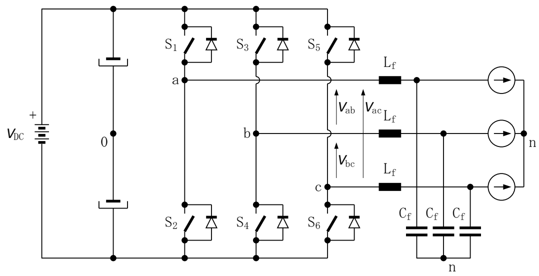

Fig 1. Three-phase inverter

and secondary voltage level, elimination of neutral current, overvoltage protection in certain cases and transformer inductance can be used in filter impedance, this configuration is often used. Also, it is possible to supply single-phase loads. Transformer price and weight can be regarded as disadvantages, mainly in mobile applications like railways.

It is already known that in a symmetrical system only two variables are independent, and as a result, can be separately controlled ($X_a + X_b + X_c = 0$). On the other hand, non-symmetrical system has three independent variables ($X_a + X_b + X_c \neq 0$). Separate control of three variables in three-phase inverter is therefore possible by using:

- Split DC-linked capacitors and tying the neutral point to the midpoint of the DC-linked capacitors.
- A four-leg inverter topology and tying the neutral point to the midpoint of the fourth neutral leg.

The first one, three-phase inverter with split DC bus capacitors (Fig. 3) is actually consisting of a three single-phase half bridge inverters. This enables the independent control of the each leg, i.e. phase. With unbalanced loads, neutral current returns through DC bus capacitors. Expressions for capacitor current and voltages are in (1) and (2), respectively.

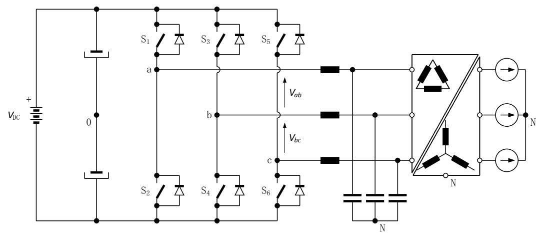

Fig 2. Three-phase inverter with isolated transformer

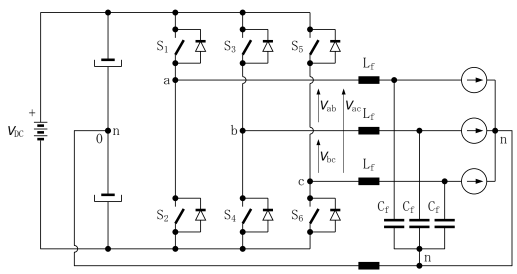

Fig 3. Three-phase inverter with split DC bus capacitors

$$i_{C_1} = \frac{1+s_a}{2} i_a + \frac{1+s_b}{2} i_b + \frac{1+s_c}{2} i_c = \frac{s_a i_a + s_b i_b + s_c i_c + 3i_0}{2} \quad (1)$$

$$i_{C_2} = \frac{1-s_a}{2} i_a + \frac{1-s_b}{2} i_b + \frac{1-s_c}{2} i_c = \frac{s_a i_a + s_b i_b + s_c i_c - 3i_0}{2} \quad (-)$$

$$v_{C_1} = \frac{V_{DC} - R_{DC} i_{C_1}}{s C_1 R_{DC} + 2} \quad (2)$$

$$v_{C_2} = \frac{V_{DC} - R_{DC1} i_{C_2}}{s C_2 R_{DC} + 2} \quad (-)$$

where R_{DC} is DC source resistance, s Laplace operator, s_a, s_b and s_c switching states.

From there it can be seen that capacitor voltage is directly influenced by zero current i_0 [8]. For voltage balancing high value capacitors and additional balancing circuits must be used. [4]. Single-phase voltage supply is also possible with this topology.

With the continual cost reduction of semiconductors and the development of faster microprocessors, a three-phase four-leg inverter topology (Fig. 4) has lately become more interesting. Although it was first mentioned in the literature in the early 90is, up until now it has not been extensively used in commercial applications. The neutral wire is connected to the neutral point of the added fourth leg. The advantages are: higher utilization of DC voltage (more than 15 % compared to conventional one from Fig. 1), current does not flow through DC capacitors, DC voltage oscillation are minimized and capacitor values can be smaller. Complex modulation scheme and large number of semiconductor devices can be sometimes considered a disadvantage. Nevertheless, this type of inverter is ideal solution in four-wire systems for supplying non-symmetrical and single-phase loads.

III. MODULATION

Sinusoidal Pulse Width Modulation (SPWM) is today frequently used in three-phase inverters, mainly due to its simplicity and easy implementation. With the development of advanced and faster microprocessors and DSPs, Space Vector Pulse Width Modulation (SV PWM) emerges as first choice for induction machine control applications. Advantages of space vector modulation are reduction in commutation losses and

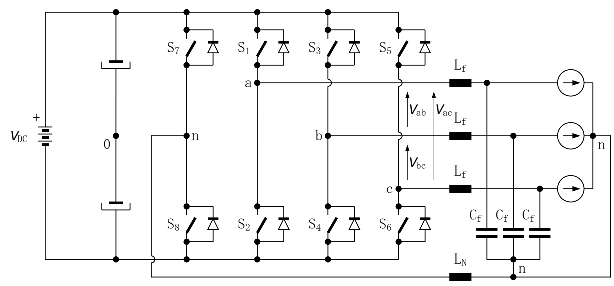

Fig 4. Three-phase four-leg inverter

output voltage harmonics, higher utilization of DC voltage and easy digital implementation [9]. Other modulation techniques like hysteresis modulation are also being used. Sinusoidal and hysteresis PWM can be analogously implemented, while others are microprocessor based [10]. Comparison criteria among different modulation techniques is based on the spectrum of the output voltages and currents, switching losses, output current oscillations and maximum available output voltages for a given input DC voltage [11].

Since three-phase four-leg inverter varies from the standard three-phase three-leg inverter only in one additional leg, some authors propose to use two types of modulation in a following way. Standard space vector modulation is used for three-phase legs, while zero leg is separately modulated [12] and [13]. The benefit is a simple modulation algorithm, but it does not explore all the possibilities of a three-phase four-leg inverter.

New modulation strategies were also developed i.e. expanded to four legs:

- Three Dimensional Space Vector PWM and
- Carrier based PWM

A. 3D Space Vector PWM

The main idea behind the space vector modulation is to find vectors adjacent to reference output vector, calculate ON-time and select appropriate switching sequence for each switching period.

Each switch in a three-phase four-leg inverter has two states. Let us assume that two switches in one leg cannot be in active or deactivate state simultaneously. Sixteen (2^4) switching states are shown in Table I. Leg voltage is 0 or v_{DC} ($v_{DC} \le V_{DC}$). Since the control of each leg is independent, this means that the each leg voltages can be placed on separate perpendicular line. The so-defined space is called leg voltage space. Points of this four dimensional object are actually sixteen switching states as seen Table I. Output voltage values can be calculated from (4). Three dimensional space with coordinates a, b and c can be obtained by projecting orthogonally to vector $(1\ 1\ 1\ 1)^T$. Coordinate transformation from abc to Cartesian αβ0 coordinate system is done in (5).

$$\begin{pmatrix} V_{an} \\ V_{bn} \\ V_{cn} \\ V_{n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \\ V_{n0} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} V_{\alpha} \\ V_{\beta} \\ V_{0} \\ V_{z} \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{-3}{\sqrt{2}} \\ k_1 & k_1 & k_1 & k_1 \end{pmatrix} \begin{pmatrix} V_{an} \\ V_{bn} \\ V_{cn} \\ V_{n} \end{pmatrix} \tag{5}$$

Voltage values of V_{α}, V_{β} and V_0 are also given in Table I1. Voltage V_z is called the placeholder value and it is not used from this point on. Consequently, the last row in transformation matrix can be omitted. Fourteen active vectors comprise polyhedron (dodecahedron) while the two zero vectors are located at the origin of the coordinate axes, Fig. 5. There are 24 different sectors. Separating planes and boundary planes can also be determined as shown in [14] − [16].

Let us now assume that the reference vector V_{ref} is located in sector I (gray area on Fig. 5). Three adjacent active

TABLE I. SWITCHING STATES AND LEG VOLTAGES OF A FOUR-LEG INVERTER

S_{a1}	S_{b1}	S_{c1}	S_{n1}	V_{a0}	V_{b0}	V_{c0}	V_{n0}	V_{an}	V_{bn}	V_{cn}	V_{α}	V_{β}	V_0	Vector
0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\mathbf{V_0}$
0	0	0	1	0	0	0	V_{DC}	$-V_{DC}$	$-V_{DC}$	$-V_{DC}$	0	0	$-\sqrt{3}V_{DC}$	$\mathbf{V_1}$
0	0	1	0	0	0	V_{DC}	0	0	0	V_{DC}	$-V_{DC}/\sqrt{6}$	$-V_{DC}/\sqrt{2}$	$V_{DC}/\sqrt{3}$	$\mathbf{V_2}$
0	0	1	1	0	0	V_{DC}	V_{DC}	$-V_{DC}$	$-V_{DC}$	0	$-V_{DC}/\sqrt{6}$	$-V_{DC}/\sqrt{2}$	$-2V_{DC}/\sqrt{3}$	$\mathbf{V_3}$
0	1	0	0	0	V_{DC}	0	0	0	V_{DC}	0	$-V_{DC}/\sqrt{6}$	$V_{DC}/\sqrt{2}$	$V_{DC}/\sqrt{3}$	$\mathbf{V_4}$
0	1	0	1	0	V_{DC}	0	V_{DC}	$-V_{DC}$	0	$-V_{DC}$	$-V_{DC}/\sqrt{6}$	$V_{DC}/\sqrt{2}$	$-2V_{DC}/\sqrt{3}$	$\mathbf{V_5}$
0	1	1	0	0	V_{DC}	V_{DC}	0	0	V_{DC}	V_{DC}	$-V_{DC}\sqrt{(2/3)}$	0	$2V_{DC}/\sqrt{3}$	$\mathbf{V_6}$
0	1	1	1	0	V_{DC}	V_{DC}	V_{DC}	$-V_{DC}$	0	0	$-V_{DC}\sqrt{(2/3)}$	0	$-V_{DC}/\sqrt{3}$	$\mathbf{V_7}$
1	0	0	0	V_{DC}	0	0	0	V_{DC}	0	0	$V_{DC}\sqrt{(2/3)}$	0	$V_{DC}/\sqrt{3}$	$\mathbf{V_8}$
1	0	0	1	V_{DC}	0	0	V_{DC}	0	$-V_{DC}$	$-V_{DC}$	$V_{DC}\sqrt{(2/3)}$	0	$-2V_{DC}/\sqrt{3}$	$\mathbf{V_9}$
1	0	1	0	V_{DC}	0	V_{DC}	0	V_{DC}	0	V_{DC}	$V_{DC}/\sqrt{6}$	$-V_{DC}/\sqrt{2}$	$2V_{DC}/\sqrt{3}$	$\mathbf{V_{10}}$
1	0	1	1	V_{DC}	0	V_{DC}	V_{DC}	0	$-V_{DC}$	0	$V_{DC}/\sqrt{6}$	$-V_{DC}/\sqrt{2}$	$-V_{DC}/\sqrt{3}$	$\mathbf{V_{11}}$
1	1	0	0	V_{DC}	V_{DC}	0	0	V_{DC}	V_{DC}	0	$V_{DC}/\sqrt{6}$	$V_{DC}/\sqrt{2}$	$2V_{DC}/\sqrt{3}$	$\mathbf{V_{12}}$
1	1	0	1	V_{DC}	V_{DC}	0	V_{DC}	0	0	$-V_{DC}$	$V_{DC}/\sqrt{6}$	$V_{DC}/\sqrt{2}$	$-V_{DC}/\sqrt{3}$	$\mathbf{V_{13}}$
1	1	1	0	V_{DC}	V_{DC}	V_{DC}	0	V_{DC}	V_{DC}	V_{DC}	0	0	$\sqrt{3}V_{DC}$	$\mathbf{V_{14}}$
1	1	1	1	V_{DC}	V_{DC}	V_{DC}	V_{DC}	0	0	0	0	0	0	$\mathbf{V_{15}}$

switching vectors (8, 12 and 14) and two zero vectors (0 and 15) form V_{ref}. During each switching period average value of an output voltage must be equal to the average value of reference vectors as shown in (6). The duration (ON-time) of active and zero vectors can be derived from (7) and (8) respectively. Every space vector modulation up to this point has mainly identical procedure: determine the switching vectors, identify sectors and boundary planes and calculate vector duration time. The main difference between numerous space vector modulation schemes is in the last step – switching sequence selection. Output voltage THD, switching losses, speed, algorithm complexity, etc., they all depend on switching sequence. A symmetrical switching sequence is chosen (Fig. 6).

Now it is possible to implement vector PWM in Matlab/ Simulink environment. The block diagram is shown in Fig 7. Input variables are reference voltage values or values deriving from inverter voltage control in abc system and switching frequency. By using (5) voltage values are transformed to V_α, V_β and V_0 and feed to the sector identification block where one of 24 sectors is identified for each sample period. V_α, V_β, V_0, switching frequency, vectors from Table I and sector number

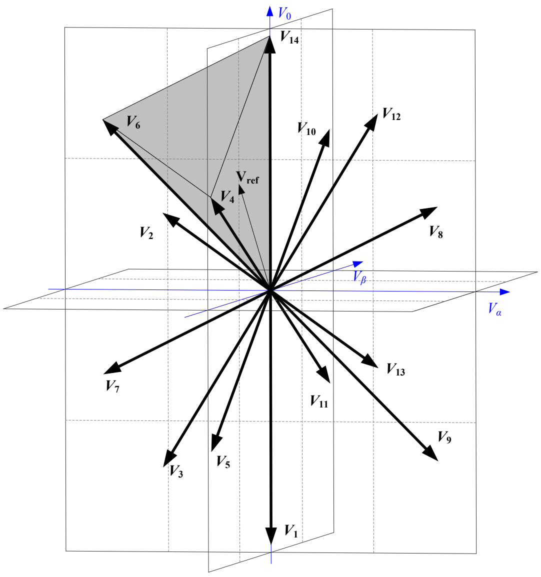

Fig 5. Switching state vectors of a four-leg inverter

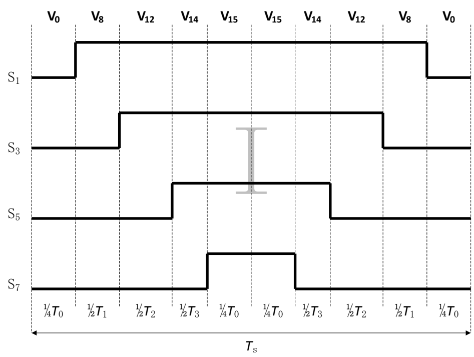

Fig 6. Symmetrical switching sequence for sector I

are used to calculate the duration time of active and zero vectors according to (7) and (8). ON-time of upper (or lower) switch in each leg is calculated based on chosen switching sequence. For sector I in Fig 6.: $t_{S1} = T_1 + T_2 + T_3 + ½\, T_0$, $t_{S3} = T_2 + T_3 + ½\, T_0$, $t_{S5} = T_3 + ½\, T_0$ and $t_{S7} = ½\, T_0$. A different value of ON-time for each leg is calculated depending on a sector number. The output of the ON-time calculation block is shown in Fig. 8. A triangular carrier waveform with frequency f_{sw} and amplitude of $1 / f_{sw}$ is compared with the block output in order to get pulses corresponding to the symmetrically align 3D SV PWM can be obtained.

$$\mathbf{V_{ref}} = \frac{1}{T_s}\mathbf{V_8}\Delta t_8 + \frac{1}{T_s}\mathbf{V_{12}}\Delta t_{12} + \frac{1}{T_s}\mathbf{V_{14}}\Delta t_{14} \tag{6}$$

$$\begin{pmatrix} \Delta t_8 \\ \Delta t_{12} \\ \Delta t_{14} \end{pmatrix} = \begin{pmatrix} \mathbf{V_8} & \mathbf{V_{12}} & \mathbf{V_{14}} \end{pmatrix}^{-1} \begin{pmatrix} V_\alpha \\ V_\beta \\ V_0 \end{pmatrix} T_s \tag{7}$$

where $\Delta t_8 + \Delta t_{12} + \Delta t_{14} \le T_s$.

$$\Delta t_0 = T_s - \left(\Delta t_8 + \Delta t_{12} + \Delta t_{14}\right) \tag{8}$$

Since the value of the input voltage vector is constantly changing (e.g. various load changes, start-up, etc.) and in some cases it is possible for the vector to have values that are outside the inverter operating range. To avoid this, it is important to limit the maximum value of the vector. Two methods are presented in [14] and [15]. Inscribed ellipsoid method limits the vector to the largest ellipsoid inscribed in the polyhedron. Boundary plane limiting method limits the magnitude of the vector that is outside the polyhedron by using the polyhedron boundary plane equations. The first method is easier to implement due to only one equation, but it is considered somehow conservative. The second method uses a higher

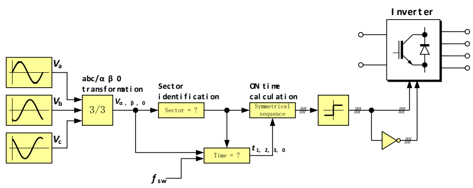

Fig 7. Block diagram scheme for three-phase four-leg space vector PWM

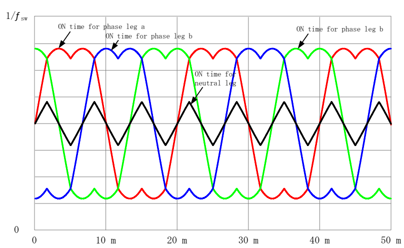

Fig 8. ON-time for each leg

number of equations and lower harmonics can be found in voltage THD, but fully exploits the space vector space and gives 5 % higher output rms voltage.

B. Carrier based PWM

This modulation is similar to sinusoidal PWM where switching states are determined by comparing triangular carrier and three sinus reference waveforms. The only difference is in the reference waveform generation where additional, so-called offset voltage is added to sinusoidal waveform. Using this modification it is possible to have any type of switching sequence [17]. In [18] it is also shown that this type of modulation is identical to symmetrically align 3D SV PWM.

Output line to neutral voltages V_{an}, V_{bn} and V_{cn} can be rewritten as shown in (10) by using respective leg voltages and common offset voltage V_{n0}. Maximum values of V_{an}, V_{bn} and V_{cn} are $\pm V_{DC}$. In three-phase three-leg inverter (Fig. 1), offset voltage is selected as (9). With the help of the fourth leg, V_{n0} can be actively manipulated by the control of the gating signal of the additional leg. If the offset voltage is fixed to a certain value, three respective pole voltages can be calculated as (11). Offset voltage constrains are in (12) and (13) [19].

$$V_{n0} = -\frac{\max(V_{an}, V_{bn}, V_{cn}) + \min(V_{an}, V_{bn}, V_{cn})}{2} \tag{9}$$

$$V_{an} = V_{a0} - V_{n0} \tag{10}$$

$$V_{bn} = V_{b0} - V_{n0} \qquad (-)$$

$$V_{cn} = V_{c0} - V_{n0} \qquad (-)$$

$$V_{a0} = V_{an} + V_{n0}^{*} \tag{11}$$

$$V_{b0} = V_{bn} + V_{n0}^{*} \qquad (-)$$

$$V_{c0} = V_{cn} + V_{n0}^{*} \qquad (-)$$

$$-V_{DC}/2 \le V_{n0} \le V_{DC}/2 - V_{max}, \text{ for } V_{max} > 0 \tag{12}$$

$$-V_{DC}/2 - V_{min} \le V_{n0} \le V_{DC}/2, \text{ for } V_{min} < 0 \qquad (-)$$

$$-V_{DC}/2 - V_{min} \le V_{n0} \le V_{DC}/2 - V_{max}, \text{ otherwise} \qquad (-)$$

$$V_{max} - V_{min} \le V_{DC} \tag{13}$$

where $V_{min} = \min(V_{an}, V_{bn}, V_{cn})$ and $V_{max} = \max(V_{an}, V_{bn}, V_{cn})$.

The optimum switching sequence, which is equivalent to symmetrically align 3D SV PWM, can be achieved by selecting the offset voltages as given in (14), Fig. 9. The ON-times of the upper switch of respective legs can be obtained as in (15) Implementation with a triangular carrier and the offset voltage calculation is shown as a block diagram in Fig. 10.

$$V_{n0}^{*} = \begin{cases} -\dfrac{V_{max}}{2}, & V_{min} > 0 \\ -\dfrac{V_{min}}{2}, & V_{max} < 0 \\ -\dfrac{V_{max} + V_{min}}{2}, & \text{otherwise} \end{cases} \tag{14}$$

$$T_a = \frac{T_s}{2} + \frac{V_{a0}}{V_{DC}} T_s \tag{15}$$

$$T_b = \frac{T_s}{2} + \frac{V_{b0}}{V_{DC}} T_s \qquad (-)$$

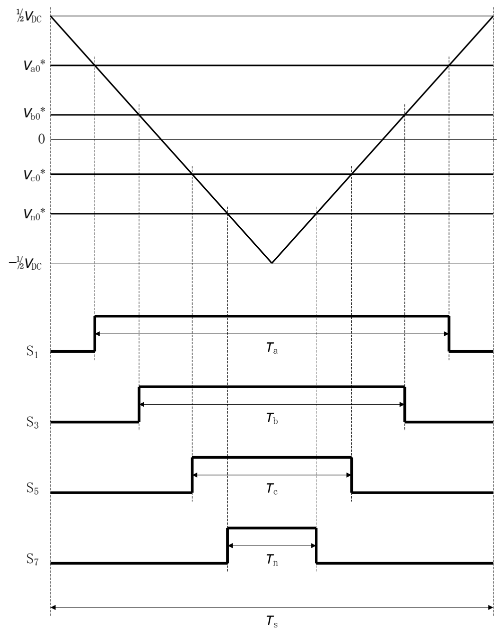

Fig 9. Implementation of the SVPWM in a three-phase four-leg inverter

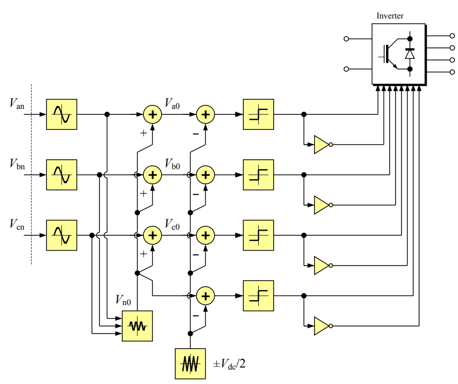

Fig 10. Block diagram scheme for three-phase four-leg carrier based PWM

$$T_{\mathrm{c}} = \frac{T_{\mathrm{s}}}{2} + \frac{V_{\mathrm{c0}}}{V_{\mathrm{DC}}} T_{\mathrm{s}} \qquad (-)$$

$$T_{\mathrm{n}} = \frac{T_{\mathrm{s}}}{2} + \frac{V_{\mathrm{n0}}}{V_{\mathrm{DC}}} T_{\mathrm{s}} \qquad (-)$$

If the vector is located outside the operating range of inverters, new reference must be selected. The algorithm in [19] and [20] limits the voltage vector to the boundary planes of polyhedron, while simultaneously achieving the maximization of a dq component voltage rather than zero sequence voltage of an original voltage reference vector.

IV. Mathematical Model of Three-Phase Four-Leg Inverter

Three models: time discrete model, average mode and linear model are extensively used in literature for inverter design [21] and [22]. In time discrete model current and voltage waveforms are mathematically represented for each interval. This model is mainly used in simulations and for detailed inverter analysis. Mathematical expressions, depending on a model, can sometimes be complicated and since the model is nonlinear, it is not suitable to control. For that reason linear frequency domain model is used. Since switching frequency is much higher than the fundamental output frequency, voltage and current ripples can be neglected and averaging process can be done (average value of current and voltage are obtained) [23]. Three-phase four-leg averaged model is in Fig. 11. Expression for output voltages van, vbn and vcn is in (16). Current − voltage equation of inverter model are in (17) to (19), while DC current is in (20).

$$\begin{pmatrix} v_{\mathrm{an}} \\ v_{\mathrm{bn}} \\ v_{\mathrm{cn}} \end{pmatrix} = V_{\mathrm{DC}} \begin{pmatrix} d_{\mathrm{an}} \\ d_{\mathrm{bn}} \\ d_{\mathrm{cn}} \end{pmatrix} \qquad (16)$$

where d_{an}, d_{bn} and d_{cn} are duty ratios.

$$\mathrm{L}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} i_{\mathrm{a}} \\ i_{\mathrm{b}} \\ i_{\mathrm{c}} \end{pmatrix} = \mathrm{L_n}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} i_{\mathrm{n}} \\ i_{\mathrm{n}} \\ i_{\mathrm{n}} \end{pmatrix} + V_{\mathrm{DC}}\begin{pmatrix} d_{\mathrm{an}} \\ d_{\mathrm{bn}} \\ d_{\mathrm{cn}} \end{pmatrix} - \begin{pmatrix} v_{\mathrm{a\,load}} \\ v_{\mathrm{b\,load}} \\ v_{\mathrm{c\,load}} \end{pmatrix} \qquad (17)$$

$$\mathrm{C}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} v_{\mathrm{a\,load}} \\ v_{\mathrm{b\,load}} \\ v_{\mathrm{c\,load}} \end{pmatrix} = \begin{pmatrix} i_{\mathrm{a}} \\ i_{\mathrm{b}} \\ i_{\mathrm{c}} \end{pmatrix} - \begin{pmatrix} i_{\mathrm{a\,load}} \\ i_{\mathrm{b\,load}} \\ i_{\mathrm{c\,load}} \end{pmatrix} \qquad (18)$$

$$i_{\mathrm{a}} + i_{\mathrm{b}} + i_{\mathrm{c}} + i_{\mathrm{n}} = 0 \qquad (19)$$

where L and C are the inductance and capacitance of the output filter, v_{an}, v_{bn} and v_{cn} phase to neutral values of output voltage and i_{a}, i_{b} and i_{c} line to line output current and i_{n} is neutral current.

$$i_{\mathrm{DC}} = \begin{pmatrix} d_{\mathrm{an}} & d_{\mathrm{bn}} & d_{\mathrm{cn}} \end{pmatrix}\begin{pmatrix} i_{\mathrm{a}} \\ i_{\mathrm{b}} \\ i_{\mathrm{c}} \end{pmatrix} \qquad (20)$$

Transformation from abc to dq0 system is used when developing the control algorithm. This is done because classical control techniques work better with DC values, rather than sinusoidal. By transforming (17) and (18) to dq0 system we get (21) and (22). dq0 averaged model is shown in Fig 12.

$$\mathrm{L}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \\ i_{0} \end{pmatrix} = \mathrm{L_n}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} 0 \\ 0 \\ -3i_{0} \end{pmatrix} + V_{\mathrm{DC}}\begin{pmatrix} d_{\mathrm{dn}} \\ d_{\mathrm{qn}} \\ d_{\mathrm{0n}} \end{pmatrix} - \begin{pmatrix} v_{\mathrm{d}} \\ v_{\mathrm{q}} \\ v_{\mathrm{o}} \end{pmatrix} + \mathrm{L}\omega\begin{pmatrix} i_{\mathrm{q}} \\ -i_{\mathrm{d}} \\ 0 \end{pmatrix} \qquad (21)$$

$$C\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} v_{\mathrm{d}} \\ v_{\mathrm{q}} \\ v_{0} \end{pmatrix} = \begin{pmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \\ i_{0} \end{pmatrix} - \begin{pmatrix} i_{\mathrm{d\,load}} \\ i_{\mathrm{q\,load}} \\ i_{\mathrm{0\,load}} \end{pmatrix} + \mathrm{C}\omega\begin{pmatrix} v_{\mathrm{q}} \\ -v_{\mathrm{d}} \\ 0 \end{pmatrix} \qquad (22)$$

Third model (small signal model) is obtained from the average model using a process of linearization and perturbation at around an operating point. In literature this model is sometimes also called linear model [24].

V. Control

The simplified control scheme is shown in Fig. 13. Measured currents and voltages are transformed to dq0

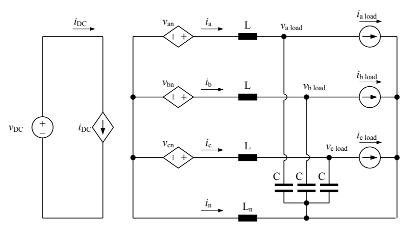

Fig 11. Averaged model of three-phase four-leg inverter in abc system

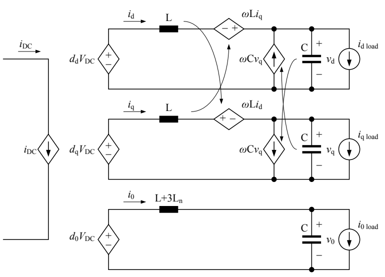

Fig 12. Averaged model of three-phase four-leg inverter in dq0 system

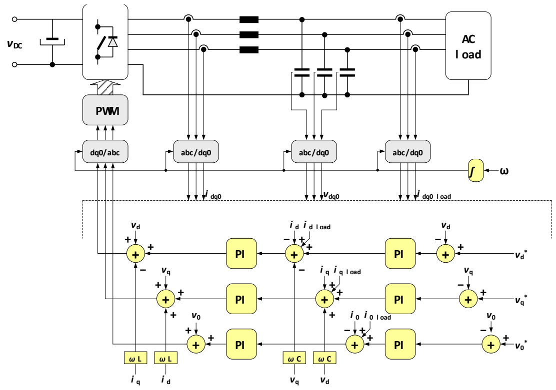

Fig 13. Simplified control block diagram

reference frame in order to get DC values. Each loop is controlled independently. Outer, also called the voltage control loop is used for voltage reference tracking (e.g. determining the amplitude and the phase of the output voltage) and low order harmonic elimination. Voltage error is used as an input to a controller whose output is current reference. Inner or current control loop is used for protection of the inverter against overload and for improvement of the control system response. PI or PID controllers, sliding mode [25], fuzzy logic or neural network [26] based controllers are often used. The output values of current controller are fed to inverse dq0 transformation and PWM block.

There are two basic types of controllers:

- Linear (controllers using constant switching frequency: PI controller, predictive controller)
- Nonlinear (controllers using adjustable switching frequency: hysteresis controller, fuzzy based or neural network based controller)

With the development of faster microprocessors and estimation algorithms, current loop is often omitted and the current is estimated, not measured; so controllers can be than grouped based on control type [27]:

- Indirect Current Control (ICC)
- Direct Current Control (DCC)

Some of the most widely used controllers for three-phase four-leg inverter control are described below.

A. Proportional − integral controller

In [28] three independent PI controller in the abc coordinate system for equal current, equal power and equal impedance per phase are used. However, it is noted that there is a problem with eliminating steady state current when loads are non-symmetrical. A zero error sinusoidal controller is used in [15] and voltage error remains zero even in the case of non-symmetrical loads. It is known that PI controller has an infinitive gain only for DC values. For that reason transformation from abc to dq0, where AC quantities become dc, is used in [28] and [29]. Many papers dealing with control propose feedback PI controller in dq0 system. When the load is not symmetrical, d and q components are oscillating with frequency $2f$, and 0 component is not equal to zero and oscillates with frequency f. In that case, PI controller cannot eliminate steady state error. As a solution in [23] and [30] − [32] a transformation to symmetrical components (direct, inverse and zero) is proposed, which then gives DC quantities in dq0 system. An inverse transformation is used before entering the PWM block.

Some of the disadvantages of the classic control strategies, such as PI controller are [33]:

- Digital implementation of the controller leads to a delay in control loop.
- Most of the classic control techniques use frequency domain factors for designing. However, this does not lead to a precise control in time domain and control cannot adjust transient performance of the system. Modern control techniques can be used to achieve better performance.
- The system parameters vary along the time. Moreover, classic control techniques design a controller only for one operating point. Robust and adaptive control techniques are then required.

In [33] and [34] a model considering digital delay is proposed. PI controller and pole placement controller are compared in [33]. It was shown that the pole placement controller has faster response. When controller parameters are changing and during transients PI controller can become unstable. In [34] adaptive self-tuning controller is used. Compared with non adaptive, adaptive controller has faster response and lower overshoot. In [21] 4 zero / 5 pole controller is used. It is possible to obtain lower overshoot than with PI controller by selecting appropriate parameters.

B. Proportional – resonant controller

Proportional – resonant (P − resonant) controller used in [35] eliminates oscillations of measured quantities during non-symmetrical loads in dq0 system. In every leg (d, q and 0) there is one P − resonant controller on fundamental frequency and additionally one for every higher frequency (22).

$$G(s) = K_P + \frac{K_I s}{s^2 + \omega^2} + \sum_{h=3,5,7} \frac{K_{Ih} s}{s^2 + (h \cdot \omega)^2} \tag{23}$$

where K_P is the proportional gain, K_I is the integrational gain and h are selected harmonics to compensate: 3rd, 5th, 7th.

In comparison to PI controller, P − resonant controller has better steady state response. In dynamical state, proportional part alone is not able to properly respond to a change. For that reason an additional feedback loop consisting of the output filter capacitor current is added in [36]. In [36] and [37] control using P − resonant controller is in abc system. In [38] P − resonant controller is used to control direct, inverse and zero component of current in αβ0 system.

C. Hysteresis controller

This robust and relatively simple method does not need complex circuits, has fast response and generally good properties [25], [39]. The switching frequency is not fixed and depends on load parameters and operating point. In [40] two, three and multi level hysteresis controllers are compared. In [41] it is shown that if a switching frequency of a fourth leg is fixed to a certain value, switching frequency of a phase legs automatically adapts the same frequency. Compared with PI controller, hysteresis controller exhibits lower switching losses, but higher harmonic distortion [35].

Development of microprocessors has enabled the usage of advanced control methods with predictive control, deadbeat controller [36] and servo controller, for example discrete linear quadratic controller in average state [14]. Predicative control broad concept and includes many methods further listed in [8]. In [8] reduced order observer is used for estimating the load current and in [10] predictive control for the same purpose. Generally speaking predictive methods have fast response.

VI. Conclusion

There are at the moment three ways a three-phase inverter can supply single-phase and three-phase loads:

- Three-phase inverter with isolated transformer
- Three-phase inverter with split DC bus capacitors
- Three-phase four-leg inverter

In three-phase four-leg inverter neutral current flows through fourth leg. This topology offers higher DC voltage utilization, does not need high values of DC capacitors and also there is no need for isolating transformer. The disadvantage is complex modulation scheme. Two modulation schemes are extensively used in the literature, namely 3D space vector and carrier based PWM. Control is done in abc, αβ0 or dq0 reference frame. Using transformation matrix from αβ0 to dq0 rotating coordinate system, the controller operates with DC quantities. In case of imbalance, DC quantities oscillate with frequency $2f$ (d, q) and f (0). For that reason, additional transformations to symmetrical coordinate system or other types of controllers like P − resonant controller are used.

References

[1] N. Ciurro, “Harmonics in Industrial Power Systems“, [Online] Available http://ecmweb.com/design_engineering/harmonics-industrial-power-systems-20090601/

[2] M. Elbar et all, “Power Quality Improvement Using Four-Leg Three-level Inverter and Cross Vector Theory“, EFEEA’10 International Symposium on Environment Friendly Energies in Electrical Applications, 2-4 November 2010, Ghardaïa, Algeria

[3] The Electricity Grid Rules, NN br. 36/2006.

[4] EN 50160, Voltage Characteristics in Public Distribution Systems

[5] IEEE 519-1992, IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems

[6] UIC 550 Power supply installations for passenger stock

[7] IEC 60038, IEC standard voltages

[8] S. El-Barbari and W. Hofmann, “Digital control of a four leg inverter for standalone photovoltaic systems with unbalanced load”, IECON‘ 2000 Nagoya Proceedings

[9] N.-Y. Dai, M.-C. Wong and Y.-D. Han, “Application of a Three-level NPC Inverter as a Three-Phase Four-Wire Power Quality Compensator by Generalized 3DSVM”, IEEE Transactions on Power Electronics, March 2006. Volume: 21, Issue: 2, pp. 440. – 449.

[10] J. Rodriguez, “Predictive current control of three-phase two-level four-leg inverter“, 14th International Power Electronics and Motion Control Conference (EPE/PEMC), 2010, Ohrid, 6-8 Sept. 2010.

[11] V. H. Prasad, “Analysis and Comparison of space vector modulation schemes for three-leg and four-leg voltage source inverters“, Thesis (M.S.) – Virginia Polytechnic Institute and State University, 1998.

[12] A. Bellini and S. Bifaretti, “Modulation techniques for three-phase four-leg inverters”, Proceedings of the 6th WSEAS International Conference on Power Systems, Lisbon, Portugal, September 22-24, 2006, pp. 398. – 403.

[13] N.-V. Nho, M.-B. Kim, G.-W. Moon and M.-J. Youn, “A novel carrier based PWM method in three phase four wire inverters”, The 30th Annual Conference of the IEEE Industrial Electronics Society, November 2-6, 2004, Busan, Korea, vol. 2., pp. 1458. – 1462.

[14] R. F. de Camargo et. all, “New Limiting Algorithms for space vector modulated three-phase four-leg voltage source inverter”, Power Electronics Specialists Conference, 2002. PESC 02. 2002 IEEE 33rd Annual Issue Date: 2002 Volume: 1 pp. 232. – 237.

[15] F. Botteron et all, “On the space vector modulation and limiting algorithms for three-phase four-leg voltage source inverter in abc coordinates”, IEEE 2002 28th Annual Conference of the IECON 02 [Industrial Electronics Society, vol. 1, Issue Date: 5-8 Nov. 2002., pp. 12. – 17.

[16] H. Pinheiro et all., “Space Vector Modulation for Voltage-Source Inverters- A Unified Approach”, IECON 02, 28th Annual Conference of the Industrial Electronics Society, 5-8 Nov. 2002., Volume: 1, pp. 23. – 29.

[17] J.-H. Kim and S.-K. Sul, “A Carrier-based PWM Method for Three-Phase Four-Leg Voltage Source Converters”, , IEEE Transactions on Power Electronics, Issue Date: Jan. 2004 Volume: 19 Issue: 1, pp. 66. – 75.

[18] Zhou K. and Wang D., “Relationship between Space-vector Modulation and Three-phase Carrier-based PWM: A Comprehensive Analysis”, IEEE Transactions on Industrial Electronics, 49(1), pp. 186. – 196.

[19] J.-H. Kim and S.-K. Sul, “A Carrier-Based SVPWM Strategy to Exploit the full capability along with 3DOF in voltage synthesis using three-phase four-leg VSC”, 2nd Nagoya University of Technology & SNU Joint Workshop, 2004. Nagoya in Japan 2004.

[20] J.-H. Kim and S.-K. Sul, “Overmodulation strategy for a three-phase four-leg voltage source converter”, 38th IAS Annual Meeting. Conference Record of the Industry Applications Conference, 2003., 12. – 16. Oct. 2003., pp. 656. – 663.

[21] J. I. Corcau, T. L. Grigorie, N Jula, C Cepisca. and N Popoviciu, “Dynamics and control of three-phase four-leg inverter”, EHAC'10 Proceedings of the 9th WSEAS international conference on Electronics, hardware, wireless and optical communications, pp. 26. – 27.

[22] R. A. Gannett, “Control strategies for high power four-leg voltage source inverters“, Master's Thesis, Virginia Polytechnic Institute and State University, 2001.

[23] I. Vechiu, H. Camblong, G. Tapia, B. Dakyo and O. Curea, “Control of four leg inverter for hybrid power system applications with unbalanced load”, Energy Conversion and Management, Volume 48, Issue 7, July 2007, pp. 2119. –2128.

[24] Q.-C. Zhong, J. Liang, G. Weiss, C. Feng and T. C. Green, “H∞ Control of the Neutral Point in Four-Wire Three-Phase DC–AC Converters”, IEEE Transactions on Industrial Electronics, 2006, Vol 53; Numb 5, pp. 1594. – 1602.

[25] V. Soares and P. Verdelho, “Linear and Sliding-Mode Controllers for Three-Phase Four-Wire Power Converters”, Proceedings of the IEEE International Symposium on Industrial Electronics, 2005. ISIE 2005. Issue Date: 20-23 June 2005 Volume: 2, pp. 799. – 804.

[26] D. K. Setiawan, M. H. Purnomo and M. Ashari, “Diagonal Recurrent Neural Network Control of Four-Leg Inverter for Hybrid Power System Under luctuating Unbalanced Loads“

[27] A. H. Bhat and P. Agarwal, “Three-phase, power quality improvement ac/dc converters”, Electric Power Systems Research, Volume 78, Issue 2, February 2008, pp 276. – 289.

[28] B. Korich, K. Aliouane, B. Abdelkader and M. Aissani, “Control of a Four-Legged Three-phase ac/dc converter in unbalanced conditions”,