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Influence of constitutive and process parameters on the cylindrical cup deep drawing predictions for AA2090-T3 sheet

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Abstract: Constitutive modelling of the anisotropic plastic behaviour of the sheet material is of the great importance in the design and analysis of the forming processes intended for these materials. In this article influence of the constitutive parameters, directional dependences of the tensile and compressive yield stresses and Lankford parameters, on the cylindrical cup deep drawing predictions for AA2090-T3 sheet is analyzed. Finite element simulations of the cup drawing process are performed with the finite element program ADINA upgraded with the several orthotropic elasto-plastic constitutive formulations that are adjusted to the different sets of the material data. The analyzed constitutive formulations are based on isotropic hardening, associated or non-associated flow rule and symmetric orthotropic Hill (1948) stress function. In order to evaluate the impact of the strenght diffential effect on the final cup predictions, an investigation is carried out with a recently proposed constitutive formulation based on non-associated flow rule and asymmetric orthotropic yield function derived by extending the original orthotropic Hill (1948) equivalent stress term with a linear combination of the normal stress components. Furthermore, influence of the blankholder force and friction coefficient on the cup earing profile is assessed.

Utjecaj konstitutivnih i procesnih parametara na predviđanja postupka cilindričnog dubokog vučenja legure AA2090-T3

Izvorni znanstveni rad

Sažetak: U postupku planiranja i analize postupaka oblikovanja limova plastičnom deformacijom važno je poznavanje opisa anizotropnog ponašanja materijala. U ovom radu analiziran je utjecaj materijalnih parametara, orijentacijske ovisnosti vlačnih i tlačnih naprezanja tečenja kao i Lankfordova parametra, na predviđanja postupka cilindričnog dubokog vučenja aluminijske legure AA2090-T3. Simulacije postupka dubokog vučenja izvršene su korištenjem programa za analizu metodom konačnih elemenata ADINA nadograđenim s nekoliko ortotropnih konstitutivnih formulacija. Analizirane konstitutivne formulacije temelje se na modelu izotropnog očvršćavanja, pridruženom ili nepridruženom pravilu tečenja i simetričnoj ortotropnoj Hill (1948) funkciji naprezanja. Parametri analiziranih konstitutivnih formulacija određeni su iz različitih skupova materijalnih podataka. S ciljem određivanja utjecaja asimetrije naprezanja tečenja na rezultate predviđanja, analizirana su predviđanja postupka dobivena konstitutivnom formulacijom koja se temelji na nepridruženom pravilu tečenja i asimetričnoj ortotropnoj funkciji tečenja. Analizirana asimetrična funkcija tečenja izvedena je proširivanjem izraza za ekvivalentno naprezanje Hill (1948) funkcije tečenja linearnom kombinacijom normalnih naprezanja. Nadalje, ispitan je utjecaj vrijednosti sile na tlačnom prstenu i faktora trenja na predviđanja ušičavosti u postupku cilindričnog dubokog vučenja.

1. Introduction

In many manufacturing areas the optimization of the sheet metal forming processes is a key factor to reduce product development time and final cost. Considerable research work has been carried out, based on various approaches involving experimental, analytical and computational methods, in an effort to be able to predict material behaviour in these complex processes. Computational methods, especially finite element method, have made significant contribution in this area in last few decades. This contribution is certainly result of the rapid advancement of the computer capabilities but it is also caused by significantly improved knowlege in the constitutive modeling of the anisotropic materials. Since most sheet metals exibit plastic anisotropy, the

Symbols/Oznake

$\mathbf{C}^{\mathbf{e}}$	- tensor of elastic moduli - tenzor elastičnosti
$d\boldsymbol{\varepsilon}$	- strain tensor increment - inkrement tenzora deformacije
$d\boldsymbol{\varepsilon}^{\mathbf{e}}$	- elastic strain tensor increment - inkrement elastičnog tenzora deformacije
$d\boldsymbol{\varepsilon}^{\mathbf{p}}$	- plastic strain tensor increment - inkrement plastičnog tenzora deformacije
$d\bar{\varepsilon}^{p}$	- equivalent plastic strain increment - inkrement ekvivalentne plastične deformacije
$d\lambda$	- plastic multiplier, consistency parameter - plastični množitelj, parametar konzistentnosti
$d\boldsymbol{\sigma}$	- stress tensor increment - inkrement tenzora naprezanja
$F,\ f_y$	- yield function, equivalent stress - funkcija tečenja, ekvivalentno naprezanje
f_p	- plastic potential function - funkcija plastičnog potencijala
r	- Lankford parameter - Lankfordov parametar
$r_0,\ r_{45}, r_{90}$	- Lankford parameter for specimen orientations 0°, 45°, 90° - Lankfordov parametar za uzorke orijentacija 0°, 45°, 90°
	Greek letters/Grčka slova
$\delta_1,\ \delta_2$	- asymmetry parameters - parametri asimetrije
$\bar{\varepsilon}^{p}$	- hardening parameter, equivalent plastic strain - parameter očvršćavanja, ekvivalentna plastična deformacija
κ	- hardening function - funkcija očvršćavanja
$\lambda_1, \lambda_2, \nu, \rho$	- anisotropy parameters of Hill (1948) stress function - anizotropni parametri Hill (1948) funkcije naprezanja
$\boldsymbol{\sigma}$	- stress tensor - tenzor naprezanja
σ_b	- equibiaxial yield stress - ujednačeno dvoosno naprezanje tečenja
$\sigma_t,\ \sigma_c$	- tensile, compressive yield stress - vlačno, tlačno naprezanje tečenja
σ_y	- yield stress for referent direction - naprezanje tečenja za referentni pravac
$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{yx}$	- stress components - komponente naprezanja
$\sigma_0, \sigma_{45}, \sigma_{90}$	- yield stress for specimens orientations 0°, 45°, 90° - naprezanja tečenja za uzorke orijentacija 0°, 45°, 90°

use of an appropriate anisotropic constitutive model in a finite element model of the sheet forming process is important to accurately predict material behaviour.

There are several possibilities to describe plastic anisotropy. The phenomenological plasticity theories based on associated flow rule are most frequently applied for metal materials. A realistic constitutive model for an anisotropic sheet material based on the associated flow rule requires a yield function with complex functional form and with considerable number of parameters in order to simultaneously describe anisotropy of yielding and anisotropy of plastic flow [1], [2], [3], [4]. Recently, several studies considering aspects of the application of the non-associated flow rule in sheet metal forming have been conducted [5], [6], [7], [8], [9], [10], [11], [12], [13]. It has been shown that non-associated formulations intended for orthotropic materials can result in acceptable predictions of the uniaxial material behaviour and forming processes even if stress functions with a simple functional form and low number of material parameters are utilized as yield function and plastic potential function. Furthermore, in order to describe experimentally observed difference of the tensile and compressive yield stresses, so called strength differential effect, the non-associated formulations for sheet materials based on asymmetric orthotropic yield functions have been developed [10], [11]. These asymmetric orthotropic yield functions have been derived by extending the original orthotropic quadratic Hill (1948) [14] and non-quadratic Karafillis-Boyce (1993) [15] equivalent stress term with a linear combination of the normal stress components.

The objective of the present paper is to further investigate finite element formulations based on associated and non-associated constitutive formulations that utilize simple orthotropic four parametric Hill stress function [9] or its asymmetric modification [10]. Analyzed associated and non-associated formulations might be adjusted to the different sets of the material data obtained in uniaxial tensile and compressive tests. The analyzed formulations are evaluated in predicting the cylindrical cup deep drawing process for AA2090-T3 sheet sample. By comparing the results obtained by the analyzed formulations adjusted to the different sets of material data, impact of the representation of the directional dependences of the tensile yield stresses, compressive yield stresses and Lankford parameter on the final cup predictions is assessed. Furthermore, influence of the blankholder force and friction factor value on the final drawn cup predictions is analyzed.

2. Analyzed anisotropic elasto-plastic constitutive formulations

2.1. Basic equations

Assuming isotropic linear elasticity and additive decomposition of the strain tensor increment $d\boldsymbol{\varepsilon}$ into elastic $d\boldsymbol{\varepsilon}^{\mathbf{e}}$ and plastic part $d\boldsymbol{\varepsilon}^{\mathbf{p}}$, the stress tensor increment $d\boldsymbol{\sigma}$ reads

$$d\boldsymbol{\sigma} = \mathbf{C}^{\mathrm{e}} : d\boldsymbol{\varepsilon}^{\mathrm{e}} = \mathbf{C}^{\mathrm{e}} : (d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}^{\mathbf{p}}) \tag{1}$$

where $\mathbf{C}^{\mathrm{e}}$ is the tensor of elastic moduli. If isotropic hardening is assumed, the yield criterion is defined as

$$F = f_y(\boldsymbol{\sigma}) - \kappa(\bar{\varepsilon}^p) = 0 \tag{2}$$

where $f_y(\boldsymbol{\sigma})$ is a continuously differentiable function called the yield function, scalar $\bar{\varepsilon}^p$ is a hardening parameter and $\kappa(\bar{\varepsilon}^p)$ is a scalar function called hardening function representing a stress-strain relation. According to the plastic potential theory, the plastic part of the strain tensor increment is proportional to the gradient of the stress function $f_p(\boldsymbol{\sigma})$ named plastic potential function

$$d\boldsymbol{\varepsilon}^{\mathbf{p}} = d\lambda \frac{\partial f_p(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \tag{3}$$

where $d\lambda$ is a non-negative scalar called plastic multiplier or consistency parameter. If plastic potential and yield function are not identical above evolution equation for the plastic strain tensor increment becomes the so-called non-associated flow rule. If the plastic potential and yield function are identical the so-called associated flow rule is reproduced. In the present formulation, hardening parameter is considered as an equivalent plastic strain that obeys the principle of the plastic work equivalence

$$f_y(\boldsymbol{\sigma})\, d\bar{\varepsilon}^p = \boldsymbol{\sigma} : d\boldsymbol{\varepsilon}^{\mathbf{p}} \tag{4}$$

If the plastic potential is a first order homogeneous function that obeys Euler's identity, by using Eqs. (3) and (4) evolution equation for the hardening parameter reads

$$d\bar{\varepsilon}^p = d\lambda \frac{\boldsymbol{\sigma} : \partial f_p(\boldsymbol{\sigma}) / \partial \boldsymbol{\sigma}}{f_y(\boldsymbol{\sigma})} = d\lambda \frac{f_p(\boldsymbol{\sigma})}{f_y(\boldsymbol{\sigma})} \tag{5}$$

Since, for the elastic processes incremental changes of the internal variables (plastic strain tensor and hardening parameter) vanish and $d\lambda = 0$ applies, for a hardening material plastic multiplier obeys complementary conditions

$$d\lambda \geq 0, \qquad F \leq 0, \qquad d\lambda F = 0 \tag{6}$$

and consistency condition

$$d\lambda dF = 0 \tag{7}$$

Details about the developed computational procedure of the above constitutive description based on the incremental deformation theory and implicit return mapping can be found in [9], [10].

2.2. Hill (1948) stress function

In sheet metal forming it is common practice to assume that the sheet is approximately subjected to plane stress conditions and that material exhibits orthotropic symmetry in plastic properties. Therefore, yield function and plastic potential are considered as functions of the in-plane stress components, $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ and σ_{yx}, where *x*-axis denotes the original sheet rolling direction and *y*-axis denotes the direction in sheet plane transverse to the rolling direction. The *z*-axis denotes the sheet normal direction. Analyzed constitutive formulations utilize yield function and plastic potential with functional form as orthotropic quadratic Hill (1948) yield function [14]. This yield function was originally derived for a three-dimensional anisotropic solid, but it can be written in the following form if expressed in the two-dimensional stress space for a rolled sheet material

$$f_y = \sqrt{\lambda_1 \sigma_{xx}^2 + \lambda_2 \sigma_{yy}^2 - 2\nu \sigma_{xx}\sigma_{yy} + \rho \sigma_{xy}^2 + \rho \sigma_{yx}^2} \qquad (8)$$

In the above expression, λ_1, λ_2, ν and ρ are anisotropic material parameters. Yielding condition can be stated as $f_y = \sigma_y$ where σ_y is the yield stress for the referent direction. Hill (1948) stress function can be adjusted to the yield stresses or the experimental data indicating plastic flow. If the yield stress for rolling direction is adopted as the referent value, the parameters of Hill (1948) yield function adjusted to yield stresses read

$$\lambda_1 = 1, \quad \lambda_2 = (\sigma_0 / \sigma_{90})^2$$
$$\nu = 0.5(1 + (\sigma_0/\sigma_{90})^2 + (\sigma_0/\sigma_b)^2)$$
$$\rho = 2(\sigma_0/\sigma_{45})^2 - 0.5(\sigma_0/\sigma_b)^2 \qquad (9)$$

where σ_b is equibiaxial yield stress and σ_0, σ_{45} and σ_{90} are the yield stresses obtained in uniaxial loading of the sheet specimens oriented at 0°, 45° and 90° to the rolling direction. The parameters of the Hill (1948) stress function can be defined in terms of Lankford parameter that reads

$$r = d\varepsilon_{22}^p / d\varepsilon_{33}^p \qquad (10)$$

where $d\varepsilon_{22}^p$ and $d\varepsilon_{33}^p$ are width and thickness plastic strain increments obtained in uniaxial loading of the sheet specimen. In calculating parameters of Hill (1948) stress function, three strain ratios, r_0, r_{45} and r_{90}, obtained in uniaxial tension along 0°, 45° and 90° to the rolling direction and the yield stress for the rolling direction can be used. The adopted calibration procedure results in the following expressions for function parameters

$$\lambda_1 = 1, \quad \lambda_2 = \frac{1 + 1/r_{90}}{1 + 1/r_0}$$
$$\nu = \frac{1}{1 + 1/r_0}, \quad \rho = \frac{(1 + 2r_{45})(1/r_0 + 1/r_{90})}{2(1 + 1/r_0)} \qquad (11)$$

2.3. Asymmetric orthotropic yield stress function

In the present study, yielding of a sheet material with strength differential effect is described by the yield criterion that was derived by extending the original orthogonal Hill (1948) stress function by the linear combination of the normal stress components [16], [10]. For the plane stress conditions, adopted criterion can be written in the following form

$$f_y = \sqrt{\lambda_1 \sigma_{xx}^2 + \lambda_2 \sigma_{yy}^2 - 2\nu \sigma_{xx}\sigma_{yy} + 2\rho \sigma_{xy}^2} + (\delta_1 \sigma_{xx} + \delta_2 \sigma_{yy}) = \sigma_y \qquad (12)$$

where λ_1, λ_2, ρ, ν, δ_1, δ_2 are material parameters to be determined experimentally. Obviously, if $\delta_1 = \delta_2 = 0$ yield function defined by Eq. (8) is reproduced. Furthermore, if $\lambda_1 = \lambda_2 = 1, \rho = 3/2, \nu = 1/2$ and $\delta_1 = \delta_2 \neq 0$ in addition, well-known isotropic Drucker-Prager criterion [17] is obtained. Therefore, the adopted criterion can also be considered as the Drucker-Prager formulation slightly modified by adding anisotropy parameters.

The parameters in Eq. (12) can be defined in terms of selected initial yield stresses in uniaxial tension and compression and equibiaxial tension. If σ_y corresponds to the tensile yield stress for the rolling direction, the adopted calibration procedure results in following expressions for the asymmetry parameters

$$\delta_1 = 0.5(1 - \sigma_{t0}/\sigma_{c0})$$
$$\delta_2 = 0.5(\sigma_{t0}/\sigma_{t90} - \sigma_{t0}/\sigma_{c90}) \qquad (13)$$

while parameters of the pressure-insensitive term are defined as

$$\lambda_1 = (1 - \delta_1)^2, \ \lambda_2 = (\sigma_{t0}/\sigma_{t90} - \delta_2)^2$$
$$\nu = 0.5((\lambda_1 + \lambda_2) - ((\sigma_{t0}/\sigma_b) - (\delta_1 + \delta_2))^2)$$

for σ_{45} in tension:

$$\rho = 0.5((2\sigma_{t0}/\sigma_{45} - \delta_1 - \delta_2)^2 - \lambda_1 - \lambda_2 + 2\nu)$$

for σ_{45} in compression:

$$\rho = 0.5((2\sigma_{t0}/\sigma_{45} + \delta_1 + \delta_2)^2 - \lambda_1 - \lambda_2 + 2\nu) \qquad (14)$$

In the present paper, influence of the uniaxial material parameters on the cup drawing predictions is assessed by testing associated and non-associated formulations based on Hill (1948) stress function and adjusted to the different sets of material data. Analyzed associated formulation utilize Hill (1948) stress function adjusted to the yield stresses or Lankford parameters as yield function and plastic potential, while non-associated formulation utilizes Hill (1948) stress function adjusted to the yield stresses as yield function and Hill (1948) stress function adjusted to the Lankford parameters as plastic potential. Furthermore, non-associated formulations based on the presented asymmetric orthotropic yield function adjusted to the tensile and compressive yield stresses are considered.

3. Predictions of planar anisotropy and yielding asymmetry

In this section capabilities of the analyzed associated and non-associated formulations in predicting uniaxial behaviour for AA2090-T3 sheet sample are considered. Descriptions of the directional dependences of the tensile and compressive yield stresses and Lankford parameter obtained by the stress functions used in analyzed formulations are investigated.

In the previous study conducted by authors [9] capabilities of the Hill (1948) stress function adjusted solely to the tensile properties were investigated for the considered material. In the present study, the strength differential effect is considered and predictions of the symmetric Hill (1948) stress function adjusted to the compressive yield stresses are investigated. The parameters of the symmetric Hill (1948) stress functions denoted as Hill-1 and Hill-2 are defined in terms of three directional yield stresses obtained in the uniaxial tension and compression, respectively, of the specimens oriented at 0°, 45° and 90° to the rolling direction and the equibiaxial tensile yield stress according to Eq. (9). Parameters of Hill (1948) stress function denoted as Hill-3 are calculated according to Eq. (11) using three Lankford parameters obtained in uniaxial tensions along 0°, 45° and 90° to the rolling direction. The analyzed asymmetric stress function defined by Eq. (12) contains six adjustable parameters $\lambda_1, \lambda_2, \rho, \nu, \delta_1, \delta_2$ that may be evaluated according to the Eqs. (13) and (14) using tensile and compressive yield stresses along the rolling and the transverse directions, tensile or compressive yield stress at 45° to the rolling direction and the equibiaxial yield stress. In the following, asymmetric yield function adjusted to the tensile yield stress at 45° to the rolling direction is denoted as ASYMM-Hill-1 and yield function adjusted to the compressive yield stress at 45° is denoted as ASYMM-Hill-2. Material data for AA2090-T3 sheet sample are given in Table 1. Table 2 shows calculated anisotropy and asymmetry parameters of the analyzed functions for the considered material.

Table 1. Material data for AA2090-T3 sheet sample [18]

Tablica 1. Materijalni podaci za uzorak lima AA2090-T3 [18]

specimen orientation/ orijentacija uzorka	tensile yield stress σ_t (MPa)/ vlačno naprezanje tečenja σ_t (MPa)	compressive yield stress σ_c (MPa)/ tlačno naprezanje tečenja σ_c (MPa)	Lankford parameter/ Lankfordov parametar
0°	279.6	248.0	0.211
15°	268.7	256.0	0.327
30°	254.4	253.0	0.692
45°	226.8	240.0	1.577
60°	226.4	248.5	1.039
75°	246.6	257.0	0.538
90°	254.4	266.5	0.692

thickness/debljina: 1.60 mm, equibiaxial tensile yield stress/ujednačeno dvoosno naprezanje tečenja: σ_b= 289.4 MPa

Young modulus/Youngov modul: 69 000 MPa

Poisson ratio/Poissonov koeficijent: 0.33

true stress-strain curve for rolling direction/zakon očvršćavanja za pravac valjanja: $\kappa(\bar{\varepsilon}^p) = 646(0.025 + \bar{\varepsilon}^p)^{0.227}$ MPa

Table 2. Calculated anisotropy and asymmetry parameters of the analyzed stress functions for AA2090-T3 sheet sample

Tablica 2. Vrijednosti parametara anizotropije i asimetrije analiziranih funkcija naprezanja za uzorak lima AA2090-T3

function label / oznaka funkcije	material data used / korišteni materijalni podaci	λ_1	λ_2	ν	ρ	δ_1	δ_2
Hill-1	$\sigma_{t0}, \sigma_{t45}, \sigma_{t90}, \sigma_b$	1.0000	1.2070	0.6368	2.5729	—	—
Hill-2	$\sigma_{c0}, \sigma_{c45}, \sigma_{c90}, \sigma_b$	1.0000	0.8660	0.5658	1.7684	—	—
Hill-3	$r_0, r_{45}, r_{90}, \sigma_{t0}$	1.0000	0.4251	0.1736	2.2422	—	—
ASYMM -Hill-1	$\sigma_{t0}, \sigma_{t45}, \sigma_{t90}, \sigma_b, \sigma_{c0}, \sigma_{c90}$	1.1315	1.1532	0.6372	2.6313	-0.0637	0.0247
ASYMM- Hill-2	$\sigma_{c0}, \sigma_{c45}, \sigma_{c90}, \sigma_b, \sigma_{t0}, \sigma_{t90}$	1.1315	1.1532	0.6372	2.1193	-0.0637	0.0247

Figs. 1 and 2 show predictions of the uniaxial properties and yield loci obtained by the symmetric Hill (1948) stress functions. It can be observed that functions Hill-1 and Hill-2 adjusted to the yield stresses result in a good description of the directional yield stresses with corresponding sign but poor description of the strain ratios. Function Hill-3 adjusted to the strain ratios results in a reasonable description of directional strain ratios but fails in predicting the yield stresses. These predictions clearly indicate poor capability of the Hill (1948) stress function under associated flow rule in predicting material behaviour even if one type of deformation, tension or compression, prevails. As shown in the previous study [9] some improvements in predicting material behaviour in forming processes can be assessed by using non-associated formulations based on stress function adjusted to tensile yield stresses as yield function and stress function adjusted to Lankford parameter as plastic potential. In this paper non-associated formulations based on symmetric Hill-2 or asymmetric ortotropic yield function adjusted to the compressive yield stresses are considered.

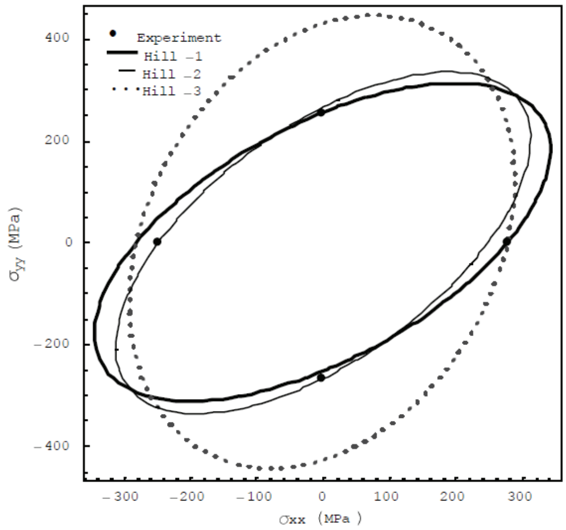


Figure 2. Contours of the symmetric Hill functions for AA2090-T3. Contours correspond to σ_{xy}=0.0

Slika 2. Konture simetrične Hill funkcije za AA2090-T3. Konture odgovaraju vrijednosti σ_{xy}=0.0

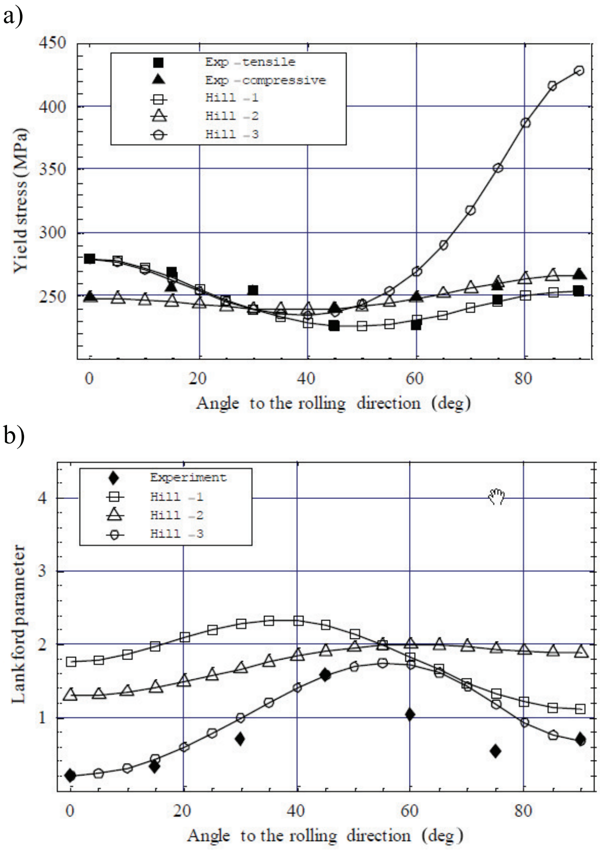


Figure 1. Experimental values and predictions of symmetric Hill stress functions: a) yield stresses; b) Lankford parameters for AA2090-T3

Slika 1. Eksperimentalne vrijednosti i predviđanja simetrične Hill funkcije naprezanja: a) naprezanja tečenja; b) Lankfordovi parametri za AA2090-T3

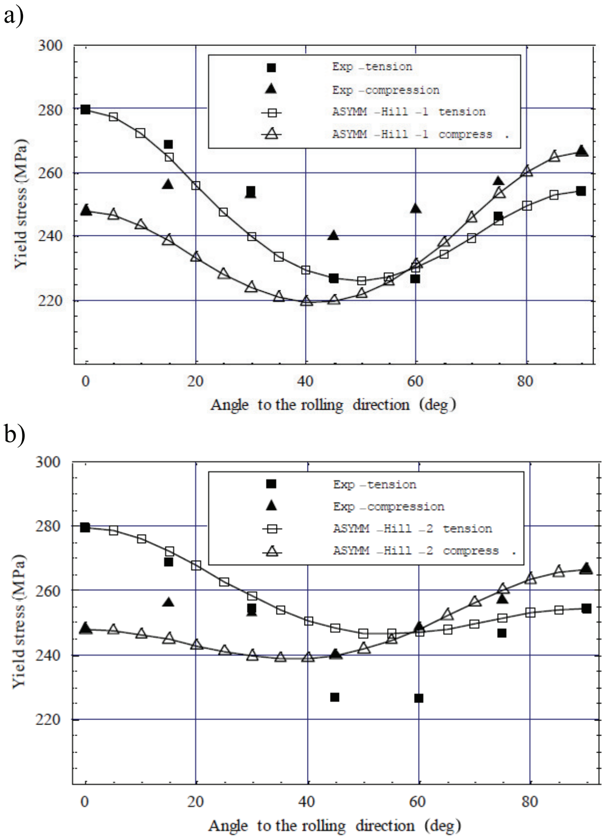
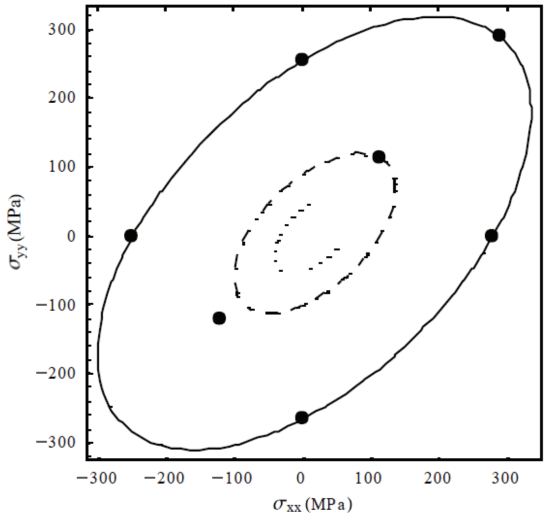


Figure 3. Experimental values and predictions of asymmetric yield functions for AA2090-T3: a) ASYMM-Hill-1; b) ASYMM-Hill-2

Slika 3. Eksperimentalne vrijednosti i predviđanja asimetrične funkcije tečenja za AA2090-T3: a) ASYMM-Hill-1; b) ASYMM-Hill-2

Fig. 3 shows predictions of the uniaxial yield stresses obtained by the asymmetric yield functions. Fig. 4 shows contours of these functions for the zero shear stress and for the shear stress values corresponding to the uniaxial tension/compression of the specimen oriented at 45° to the rolling direction.

a)



b)

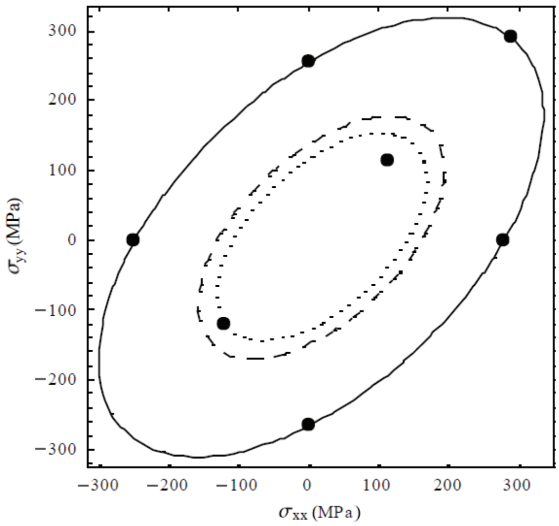


Figure 4. Contours of asymmetric yield functions for AA2090-T3:a)ASYMM-Hill-1; b)ASYMM-Hill-2 $-\sigma_{xy}$=0.0, - - σ_{xy}=113.4 MPa, …. σ_{xy}=-120.0 MPa

Slika 4. Konture asimetričnih funkcija tečenja za AA2090-T3:a)ASYMM-Hill-1;b)ASYMM-Hill-2 $-\sigma_{xy}$=0.0, - - σ_{xy}=113.4 MPa, …. σ_{xy}=-120.0 MPa

As shown in Fig. 3 directional dependences of tensile and compressive yield stresses cannot be completely captured simultaneously with the analyzed asymmetric yield function. The function ASYMM-Hill-1 with shear parameter adjusted to σ_{45} in tension fails in predicting compressive stress amplitude while the function ASYMM-Hill-2 adjusted to σ_{45} in compression fails in predicting tensile stress amplitude. It can be concluded that two asymmetry parameters involved in the analyzed yield function are insufficient for the accurate uniaxial yield stress representation for the considered material. Nevertheless, it can be expected that by comparing the associated formulations and the non-associated formulations based on symmetric or asymmetric yield function impact of the uniaxial material parameters on the final predictions can be assessed.

4. Cylindrical cup deep drawing for AA2090-T3 sheet sample

In this section, predictions of the associated and non-associated formulations adjusted to the different sets of material data are compared in predicting cylindrical cup deep drawing for AA2090-T3 sheet sample. The simulations are performed with FE code ADINA 8.6 [19] upgraded with the updated Lagrangian formulation of the CBR shell element and the stress integration procedure of the analyzed constitutive formulations based on the incremental deformation theory and implicit return mapping [9], [10]. A schematic view of the cup drawing problem is presented in Fig 5.

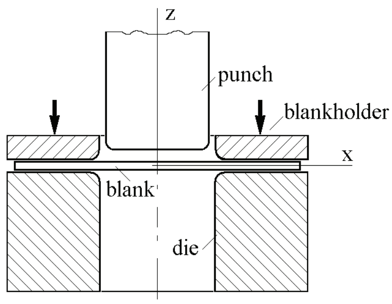


Figure 5. Schematic view of the cylindrical cup deep drawing in *x-z* plane

Slika 5. Shematski prikaz cilindričnog dubokog vučenja u *x-z* ravnini

For the considered cylindrical cup drawing, material and process parameters are those reported in [18]. The initial blank diameter is 158.76 mm with 1.60 mm in thickness. The punch diameter is 97.46 mm with shoulder radius of 12.70 mm. The die opening diameter is 101.48 mm with shoulder radius of 12.70 mm. The blank-holding force is 22.0 kN and the friction factor of 0.1 for blank/punch, blank/die and blank/blank holder contact surfaces is utilized. Due to the shape geometry and material orthotropy, a one-quarter model of the cup is analyzed. One-quarter section of the blank is modeled with 369 4-node CBR shell elements while the blankholder, die and punch are modelled as rigid surfaces. For the frictional contact treatment, the constraint function algorithm available in ADINA 8.6 is employed. The classical Coulomb friction law and constant friction factor are used. The isotropic hardening is assumed and power hardening curve, specified in Tab.1, describing the experimental stress-strain data for the rolling direction is utilized.

The contours of drawn cups obtained by the analyzed associated and non-associated formulations are compared with experimental data reported in [18]. The formulations are label according to the utilized yield function and flow rule (AFR for associated and NAFR non-associated flow rule). Full meaning of the labels is presented in Tab. 3.

Table 3. Formulation labels

Tablica 3. Oznake formulacija

formulation label / oznaka formulacije	yield function / funkcija tečenja	potential/ potencijal
AFR-Hill-1	Hill-1	Hill-1
AFR-Hill-2	Hill-2	Hill-2
AFR-Hill-3	Hill-3	Hill-3
NAFR-Hill-1	Hill-1	Hill-3
NAFR-Hill-2	Hill-2	Hill-3
NAFR-ASYMM-Hill-1	ASYMM-Hill-1	Hill-3
NAFR-ASYMM-Hill-2	ASYMM-Hill-2	Hill-3

In the previous study conducted by the authors [9] simulations of the cylindrical cup drawing for AA2090-T3 were performed by the associated and non-associated formulations based on symmetric Hill stress function and adjusted solely to the uniaxial tensile properties (AFR-Hill-1, AFR-Hill-3 and NAFR-Hill-1). Since the analysis of the thickness strain distribution indicated that compressive deformations prevail, in the present study associated and non-associated formulations adjusted to the compressive yield stresses are tested. The experimental earing profiles and simulated profiles obtained by the associated formulations are compared in Fig. 6. By considering Fig. 1 and Fig. 6 it can be concluded that the predicted earing trend (location of peaks and valleys) is the mirror image of the Lankford parameter representation obtained by the material formulation with respect to the transverse direction. The same correlation applies between the experimental earing profile and the experimental strain ratio distribution. As shown in Fig. 6 analyzed associated formulations utilize as plastic potential stress functions Hill-1, Hill-2 and Hill-3 that cannot capture the trend of the experimental strain ratio distribution. Therefore, these models fail in predicting small ears near 0° (and 180°) as well as the location of more pronounce ears near 50°. Also form Fig. 6 it can be seen that the earing amplitude (the maximum and minimum height difference) is best captured by the associated formulation adjusted to the tensile yield stresses.

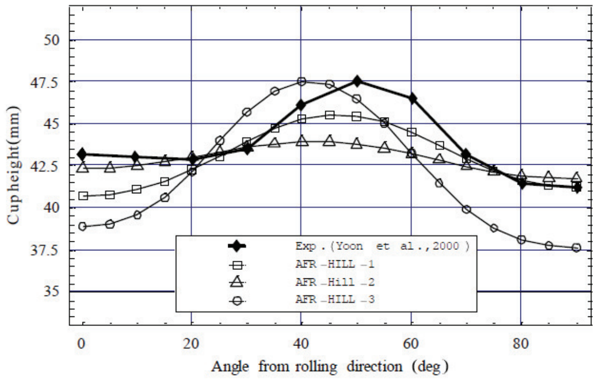


Figure 6. Predictions of the cup heights obtained by the associated formulations based on the symmetric Hill stress function.

Slika 6. Predviđanja visina posudice dobivena formulacijama temeljenim na pridruženom pravilu tečenja i simetričnoj Hill funkciji naprezanja.

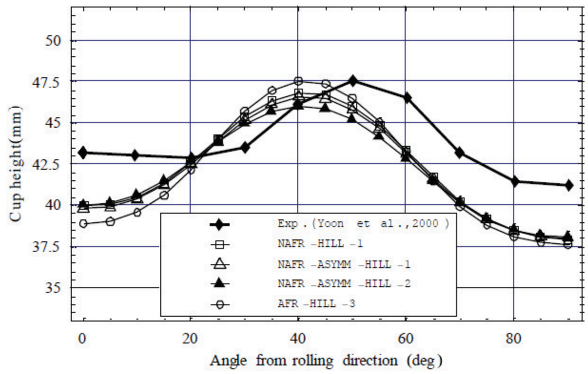


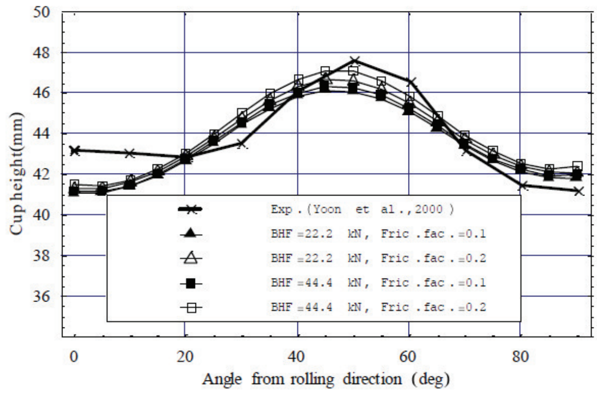
Figure 7. Predictions of the cup heights obtained by the non-associated formulations based on the symmetric or asymmetric yield function.

Slika 7. Predviđanja visina posudice dobivena formulacijama temeljenim na nepridruženom pravilu tečenja i simetričnoj ili nesimetričnoj funkciji tečenja.

Fig. 7 shows predictions of the non-associated formulations based on Hill-3 plastic potential and symmetric or asymmetric yield stress function adjusted to the different sets of yield stresses. Predicted cup heights clearly indicate that formulations based on the same plastic potential result in almost identical earing trend and that the difference in the yielding description influences the earing profile amplitude. It can be observed that formulations NAFR-ASYMM-Hill-1 and NAFR-ASYMM-Hill-2 with the improved representation of the orientation dependence of the yield stresses and yielding asymmetry results in improved predictions of earing amplitude compared to AFR-Hill-3 formulation.

In Fig. 8 influence of the blankholder force and the friction factor on the predicted earing profile for the associated formulations adjusted to the tensile yield stresses or Lankford parameters is presented. The presented results are obtained by the corresponding constitutive formulations available in ADINA 8.6. These ADINA constitutive formulations are marked according to the calibration procedure as the formulations considered in the rest of the paper. Therefore, ADINA associated formulation adjusted to the tensile yield stresses is marked as AFR-Hill-1, while ADINA associated formulation adjusted to the Lankford parameters is marked as AFR-Hill-3. It can be observed that the increase of the blankholder force and friction factor results in increased predicted earing amplitude, i.e. increase of the cup heights near predicted pronounced ears. The increase of the earing amplitude due to increased values of the considered process parameters is more pronounced for the formulation adjusted to the strain ratios, AFR-Hill-3. Also it can be observed that for the associated formulation adjusted to the yield stresses AFR-Hill-1 double increase of the blankholder force and friction factor results in small ears near 0° and 90°.

a)



b)

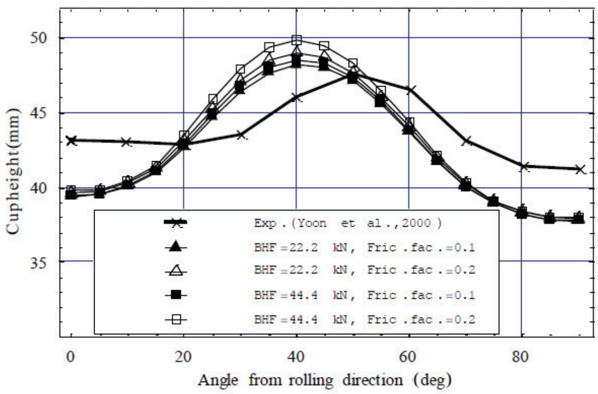


Figure 8. Influence of the blankholder force and friction factor on the predicted cup heights, a) formulation AFR-Hill-1, b) formulation AFR-Hill-3.

Slika 8. Utjecaj sile na tlačnom prstenu i faktora trenja na predviđanja visina posudice, a) formulacija AFR-Hill-1, b) formulacija AFR-Hill-3.

5. Conclusions

In the present paper associated and non-associated constitutive formulations adjusted to the different sets of material data are examined in predicting cylindrical cup drawing process for AA2090-T3 sheet sample. By comparing obtained predictions, influence of the representation of the uniaxial material properties, i.e. directional dependences of the tensile and compressive yield stresses and Lankford parameter on the earing predictions is assessed. It can be concluded that the final predicted earing trend is strongly influenced by the strain ratio representation i.e. by the choice of the plastic potential function. In the non-associated formulations the improved description of the directional dependence of yield stresses and yielding asymmetry results in some improvements of the earing amplitude. Furthermore, by considering predictions obtained with various values of the blankholder force and friction factor it can be concluded that the increase of these processes parameters results in certain increase of the earing amplitude.

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