STRESS AND DEFORMATION ANALYSIS OF COMPOUND CYLINDERS IN ELASTIC AND PLASTIC RANGE

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1. Introduction

Thick-walled vessels and pipes have found their use in systems with extreme loads and aggressiveness of the surrounding media. Thick-walled cylinders can be loaded with internal overpressure, external overpressure, heat, bending, twisting, and combinations of these load features [1].

In this paper, the analytical and numerical analysis of a compound thick walled cylinder in elastic and plastic range is given. In addition, the condition after completely unloading the cylinder is analysed and the analytical and numerical results are compared.

2. Analytical analysis of the compound cylinder

The analytical analysis is performed on a compound cylinder with rising inner pressure until the complete plastification of the inner tube, see Fig. 1. The presumption for this analysis is that there is no overlap in the initial position of the inner and outer cylinder. For the analytical and numerical analysis of this problem a linear elastic-ideal plastic material model is used. The following were the values of material properties (TStE500): $E = 206855$ MPa and $\sigma_T = 575$ MPa [2].

According to [3] and [4] equations for circular and radial stress for a thick-walled cylinder with internal load in elastic range are given:

$$\sigma_\varphi = \frac{\sigma_T}{\sqrt{3}}\left(\frac{r_T}{r_2}\right)^2\left[1+\left(\frac{r_2}{r}\right)^2\right], \qquad (1)$$

$$\sigma_r = \frac{\sigma_T}{\sqrt{3}}\left(\frac{r_T}{r_2}\right)^2\left[1-\left(\frac{r_2}{r}\right)^2\right]. \qquad (2)$$

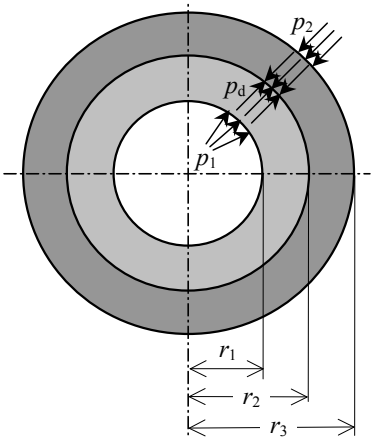

Fig. 1. Symbolic representation of compound tube dimensions and loads

With rising inner pressure, plastification slowly progresses through the inner cylinder, so for further analysis the equations describing the radial and circular stress in the plastic range with considering the von Mieses yield criterion are needed [3, 4]:

$$\sigma_r = \frac{\sigma_T}{\sqrt{3}}\left[2\ln\frac{r}{r_T}+\left(\frac{r_T}{r_2}\right)^2-1\right], \qquad (3)$$

$$\sigma_\varphi = \frac{\sigma_T}{\sqrt{3}}\left[2\ln\frac{r}{r_T}+\left(\frac{r_T}{r_2}\right)^2+1\right]. \qquad (4)$$

Additionally, the condition after unloading the compound cylinder after the complete plastification of the inner cylinder is also described and analysed analytically.

3. Finite Element Method analysis

For the given compound cylinder, see Fig. 1., the numerical analysis using FEM is also performed with additional attention directed to modelling the contact problem [5]. The numerical analysis is performed as an axis-symmetric problem [6] where the schematic of the finite element mesh used is shown, see Fig. 2.

In addition, the state after unloading the cylinder is analysed numerically and compared to the analytical results.

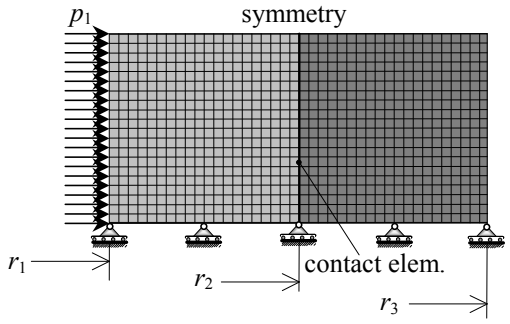

Fig. 2. Axis-symmetric computational domain and the schematic of the finite element mesh for the FEM analysis

4. Elaboration of obtained results

The obtained numerical and analytical results, for the compound cylinder loaded with gradually rising inner pressure, are given in this chapter of the paper.

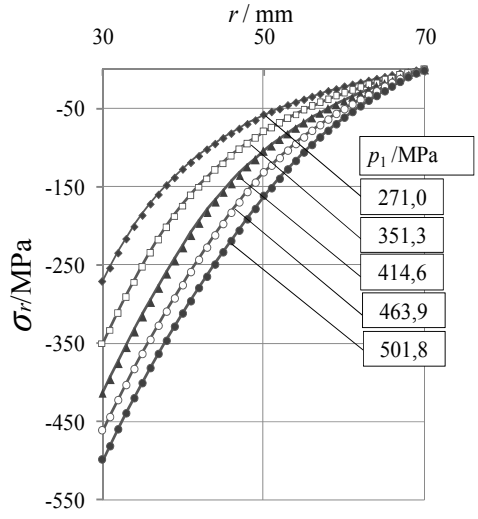

Fig. 3. Comparison of analytical and FEM results of radial stress distribution

Radial and circular stress distributions are compared, see Fig. 3. and Fig. 4., additionally to the comparison of the analytical and numerical results for the condition after completely unloading the compound cylinder after the complete plastification of the inner tube.

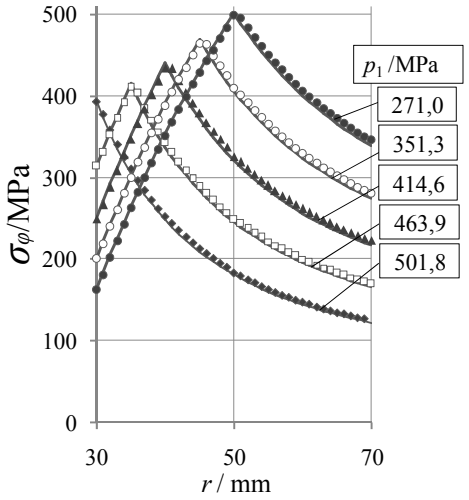

Fig. 4. Comparison of analytical and FEM results of circular stress distribution

5. Conclusion

The aim of this paper was to compare the analytical and numerical results in elastic and plastic range for a compound thick-walled cylinder exposed to rising inner pressure until the complete plastification of the inner tube. Besides that, the results for the state after completely unloading the cylinder are also compared.

References

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