# Optimized Coupled Band-Pass Filters 

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#### Abstract

In this paper we compare two versions of fourthorder band-pass (BP) filter section realized as coupled structure, having negative feedback around two Biquadratic sections in cascade, usually designated as Biquartic section (i.e. it is the fourth-order band-pass section). The frequency response of a filter is subject to vary from the nominal values due to the effects of aging, changing working conditions, the fabrication tolerances of passive and active elements, etc. In order to maintain the filter's characteristics inside given specifications at the time of manufacture, and as long as possible, the main requirement is to design filters with reduced sensitivity to component tolerances. The Biquartic section has significantly reduced sensitivities to the changes of passive element values, particularly within the pass band, in comparison to common cascade design. For that purpose this structure is very suitable for use as a building block of narrow band, high order BP filters. The sensitivities are further reduced by using three-amplifier biquadratic block instead of more sensitive single-amplifier block. We investigated the sensitivities of two versions of biquartic section with two extreme combinations of secondorder sections q-factor values, using Schoeffler's measure, the dynamic range, and output thermal noise using analysis in PSpice. Biquartic sections used throughout the examples are realized as reduced amplifier version retaining the low sensitivities of biquartic design. They also possess simple tuning features and simple design; design equations are given.


Keywords: Band-pass filters, Low-sensitivity filters, Low-noise filters, Biquartic section, Cascade realization, Schoeffler's sensitivity.

## I. Introduction

Often in practice band-pass (BP) filters of high-order and narrow pass-band are difficult to be realized. They possess very high pole-Q factors, and therefore have very high sensitivity to component tolerances. This problem is becoming more pronounced if such band-pass filters are realized in integrated-circuit form, because on-chip component tolerances are much larger than those in discrete form. One of the efficient solutions of this problem is reducing the sensitivity using negative feedback: a coupled structure [1], [2].

In this paper we present the reduced-amplifier design of the coupled fourth-order BP filter with low sensitivity which is referred to as Biquart. We analyse the sensitivity of the obtained filter using Schoeffler's measure, on various examples and different designs. In the design we also wish to have the possibility of easy tuning the center frequency and the pass-band width without deteriorating the amplitude-frequency characteristics. It is also desirable that the new structure provides lower signal distortion and output thermal noise, as well.

## II. High-Order BP Filters Design

To design high-order ( $2 N^{\text {th }}$-order) geometrically symmetrical BP filters we start from $N^{\text {th }}$-order low-pass (LP) prototype filter with the transfer function:

$$
T_{L P}(S)=\left[\frac{k_{L P 0} \cdot \gamma}{S+\gamma}\right]^{\eta} \prod_{i=1}^{\eta(N-\eta) / 2} \frac{k_{L P i} \cdot \omega_{L P i}^{2}}{S^{2}+\left(\omega_{L P i} / Q_{L P i}\right) \cdot S+\omega_{L P i}^{2}}, \text { (1) }
$$

where $\eta=1$ if the filter order $N$ is odd, $\eta=0$ for $N$ even, $\gamma$ is the frequency of the negative real pole for $N$ odd, $\omega_{L P_{i}}$ and $q_{L P i}$ are parameters of the $(N-\eta) / 2$ complex-conjugate pole pairs, and the $k_{L P i}$ are pass-band gains. If (1) is realized in the cascade form we take care of the sequence of realizing the individual $Q$-factors such that $Q_{L P_{i}}<Q_{L P i+1}<Q_{L P i+2}<\ldots$. Using common LP-BP transformation:

$$
\begin{equation*}
S=\frac{s^{2}+\omega_{0}^{2}}{B s}, \tag{2}
\end{equation*}
$$

where $\omega_{0}$ is the desired center frequency, and $B$ is the desired pass-band width of the band-pass filter, we realize the BP transfer function of the general form given by:

$$
T_{C B Q}(s)=\left[T_{b 0}(s)\right]^{]^{(N-\eta) / 2}} \prod_{i=1} T_{B Q i}(s), \eta=\left[\begin{array}{ll}
1 & \text { for } N \text { odd }  \tag{3}\\
0 & \text { for } N \text { even }
\end{array},\right.
$$

where we realize $(N-\eta) / 2$ BP fourth-order sections in cascade, and one second-order section if the LP prototype filter was odd. The transfer function of second-order section has the form given by (4) (without index $i$ ).

Each $T_{B Q i}(s)$ in (3) is fourth order transfer function and we will continue to call it Biquartic transfer function, while the network which implements such a function is called Biquartic section or "Biquart" (hereafter BQ). $T_{b 0}(s)$ is second order transfer function and is referred to as Biquadratic section or "Biquad". The BP filter of higher order having the transfer function defined by (3) is referred to as "Cascade of Biquarts" (hereafter CBQ).

## III. Biquartic Section

Biquartic section, which realizes each fourth-order BP transfer function $T_{B Q i}(s)$ in (3) is the main subject of this paper. It will be realized by cascading two BP Biquadratic sections (of second-order) applying a negative feedback, as shown in Fig. 1 (see [1], [2]).

Gain $\beta$ represents the feedback coefficient $(\beta>0), A$ is an input gain, and $T_{b 1}(s), T_{b 2}(s)$, are the second-order BP


Figure 1. Biquartic BP section.


Biquartic (fourth-order) BP filter section

Figure 2. Complete procedure of the calculation of the fourth-order BP Biquartic section starting from second-order LP prototype via cascade.

Biquadratic sections with voltage transfer functions given by:

$$
\begin{equation*}
T_{b i}(s)=\frac{k_{b i} s}{s^{2}+\left(\omega_{b i} / Q_{b i}\right) \cdot s+\omega_{b i}^{2}} ; i=1,2 . \tag{4}
\end{equation*}
$$

Voltage transfer function of the Biquartic section in Fig. 1 is given by:

$$
\begin{equation*}
T_{B Q}(s)=\frac{V_{\text {out }}(s)}{V_{\text {in }}(s)}=\frac{A T_{b 1}(s) T_{b 2}(s)}{1+\beta T_{b 1}(s) T_{b 2}(s)}=\frac{N_{B Q}(s)}{D_{B Q}(s)} \tag{5}
\end{equation*}
$$

The complete procedure in designing of one BP fourth-order BQ section in CBQ , from the second-order LP prototype filter over the cascaded fourth-order BP filter is shown in Fig. 2. The realization starting from the $1^{\text {st }}$-order LP producing the $2^{\text {nd }}$-order BP filters if the filter prototype order $N$ was odd is well-known.

## A. Determination of the coefficients of the cascade

In the first step in Fig. 2 it is shown the common LP-BP transformation applied on the second-order LP prototype filter and the obtained fourth-order BP filter in the cascade structure. The transfer function of a secondorder LP prototype is given by (1) for $N=2$. Combining (2) and (1) we obtain $T_{B P}(s)=N(s) / D(s)$ in the form:

$$
\begin{equation*}
T_{B P}(s)=\frac{k_{L P} \cdot B^{2} \omega_{L P}^{2} \cdot s^{2}}{s^{4}+B \omega_{0}^{2} \frac{\omega_{L P}}{Q_{L P}} s^{3}+\left(2 \omega_{0}^{2}+B^{2} \omega_{L P}^{2}\right) s^{2}+B \frac{\omega_{L P}}{Q_{L P}} s+\omega_{0}^{4}} . \tag{6}
\end{equation*}
$$

Transfer function of the fourth-order BP cascade in Fig. 2 has the form:

$$
\begin{equation*}
T_{C A S}(s)=\prod_{i=1}^{2} T_{c i}(s) \tag{7}
\end{equation*}
$$

where $T_{c 1}(s)$ and $T_{c 2}(s)$ are the second-order Biquadratic sections with BP voltage transfer functions given by:

$$
\begin{equation*}
T_{c i}(s)=\frac{k_{c i} s}{s^{2}+\left(\omega_{c i} Q_{c i}\right) \cdot s+\omega_{c i}^{2}} ; i=1,2 . \tag{8}
\end{equation*}
$$

We substitute (8) into (7) and rewrite it in the form:

$$
\begin{gather*}
T_{C A S}(s)=\frac{k_{c 1} k_{c 2} \cdot}{s^{4}+\left(\frac{\omega_{c 1}}{Q_{c 1}}+\frac{\omega_{c 2}}{Q_{c 2}}\right) s^{3}+} \ldots  \tag{9}\\
+\left(\omega_{c 1}^{2}+\omega_{c 2}^{2}+\frac{\omega_{c 1} \omega_{c 2}}{Q_{c 1} Q_{c 2}}\right) s^{2}+\left(\frac{\omega_{c 1}^{2} \omega_{c 2}}{Q_{c 1}}+\frac{\omega_{c 1} \omega_{c 2}^{2}}{Q_{c 2}}\right) s+\omega_{c 1}^{2} \omega_{c 2}^{2}
\end{gather*} .
$$

If we equate the coefficients multiplying the potentions of complex variable " $s$ " in the denominator of (9) and (6) we obtain the system of equations:

$$
\begin{align*}
\frac{\omega_{c 1}}{Q_{c 1}}+\frac{\omega_{c 2}}{Q_{c 2}} & =B \frac{\omega_{L P}}{Q_{L P}}  \tag{10}\\
\omega_{c 1}^{2}+\omega_{c 2}^{2}+\frac{\omega_{c 1} \omega_{c 2}}{Q_{c 1} Q_{c 2}} & =2 \omega_{0}^{2}+B^{2} \omega_{L P}^{2}  \tag{11}\\
\frac{\omega_{c 1}^{2} \omega_{c 2}}{Q_{c 2}}+\frac{\omega_{c 1} \omega_{c 2}^{2}}{Q_{c 1}} & =B \frac{\omega_{L P}}{Q_{L P}} \omega_{0}^{2}  \tag{12}\\
\omega_{c 1}^{2} \omega_{c 2}^{2} & =\omega_{0}^{4} \tag{13}
\end{align*}
$$

Combining (10) and (12) we obtain the condition for determining solutions:

$$
\begin{equation*}
\left(\omega_{c 2}-\omega_{c 1}\right)\left(Q_{c 2}-Q_{c 1}\right)=0 . \tag{14}
\end{equation*}
$$

Two solutions are possible depending on the poles of the LP prototype:

1. $\omega_{c 1}=\omega_{c 2}=\omega_{0}$ for real poles of the LP prototype.
2. $Q_{c 1}=Q_{c 2}=Q_{c}$ for conjugate-complex poles of the LP prototype.
In what follows we choose conjugate-complex poles of the LP prototype (case 2) and the system of equations $(10)-(13)$ is simplifying into the system:

$$
\begin{gather*}
\frac{\omega_{c 1}+\omega_{c 2}}{Q_{c}}=B \frac{\omega_{L P}}{Q_{L P}}  \tag{15}\\
\omega_{c 1}^{2}+\omega_{c 2}^{2}+\frac{\omega_{0}^{2}}{Q_{c}^{2}}=2 \omega_{0}^{2}+B^{2} \omega_{L P}^{2}  \tag{16}\\
\omega_{c 1} \omega_{c 2}=\omega_{0}^{2} \tag{17}
\end{gather*}
$$

Rearranging and grouping equations of the latest system, we arrive at expressions for $Q_{c}, \omega_{c 1}$ and $\omega_{c 2}$ of the cascade realization (Geffe equations) [3]:

$$
\begin{align*}
Q_{c} & =\sqrt{\frac{Q_{L P} \omega_{0}}{B \omega_{L P}}} \times \sqrt{Q_{L P} X+\sqrt{Q_{L P}^{2} X^{2}-1}}  \tag{18}\\
X & =2 \omega_{0} /\left(B \omega_{L P}\right)+B \omega_{L P} /\left(2 \omega_{0}\right) \\
\frac{\omega_{c 2}}{\omega_{0}} & =\frac{\omega_{0}}{\omega_{c 1}}=\frac{B \omega_{L P}}{2 Q_{L P}} Q_{c}+\sqrt{\left(\frac{B \omega_{L P}}{2 Q_{L P}} Q_{c}\right)^{2}-1} \tag{19}
\end{align*}
$$

If we equate numerator in (9) and (6) we obtain:

$$
\begin{equation*}
k_{c 1} \cdot k_{c 2}=k_{L P} \cdot B^{2} \omega_{L P}^{2} \tag{20}
\end{equation*}
$$

## B. Voltage transfer function of the Biquartic section

Consider the transfer function of the Biquartic section in Fig. 1 given by (5). Since the positions of the poles of $T_{B Q}(\mathrm{~s})$ is determined by the product $\beta k_{b 1} k_{b 2}$, i.e. not only by $\beta$, we will use the designation $F\left(F=\beta k_{b 1} k_{b 2}\right)$ in order to simplify the expression. We rewrite (5) in the form:

$$
\begin{gather*}
T_{B Q}(s)=\frac{A k_{b 1} k_{b 2} \cdot}{s^{4}+\left(\frac{\omega_{b 1}}{Q_{b 1}}+\frac{\omega_{b 2}}{Q_{b 2}}\right) s^{3}+} \cdots  \tag{21}\\
+\left(\omega_{b 1}^{2}+\omega_{b 2}^{2}+\frac{\omega_{b 1} \omega_{b 2}}{Q_{b 1} Q_{b 2}}+F\right) s^{2}+\left(\frac{\omega_{b 1}^{2}}{Q_{b 1}}+\frac{\omega_{b 2}^{2}}{Q_{b 2}}\right) s+\omega_{b 1}^{2} \omega_{b 2}^{2}
\end{gather*}
$$

Poles of the transfer function are determined by the denominator of the Biquartic transfer function:

$$
\begin{equation*}
D_{B Q}(s)=D(s)+F s^{2}=D(s)+\beta k_{b 1} k_{b 2} s^{2} \tag{22}
\end{equation*}
$$

It is evident that for $F=0\left(\beta=0, k_{b 1}, k_{b 2} \neq 0\right)$ the BQ consists of the two Biquads connected in the cascade, and $D(s)$ represents the denominator of (6), i.e. of the cascade.

Consider the commonly used cascade realization and compare it with the Biquartic section (both of fourth order) shown in Fig. 2. If we equate the coefficients multiplying the potentions of complex variable " $s$ " in (21) to (9), we obtain the system of equations:

$$
\begin{align*}
& \frac{\omega_{c 1}}{Q_{c 1}}+\frac{\omega_{c 2}}{Q_{c 2}}=\frac{\omega_{b 1}}{Q_{b 1}}+\frac{\omega_{b 2}}{Q_{b 2}}  \tag{23}\\
& \omega_{c 1}^{2}+\omega_{c 2}^{2}+\frac{\omega_{c 1} \omega_{c 2}}{Q_{c 1} Q_{c 2}}=\omega_{b 1}^{2}+\omega_{b 2}^{2}+\frac{\omega_{b 1} \omega_{b 2}}{Q_{b 1} Q_{b 2}}+F  \tag{24}\\
& \frac{\omega_{c 1}^{2} \omega_{c 2}}{Q_{c 2}}+\frac{\omega_{c 1} \omega_{c 2}^{2}}{Q_{c 1}}=\frac{\omega_{b 1}^{2} \omega_{b 2}}{Q_{b 2}}+\frac{\omega_{b 1} \omega_{b 2}^{2}}{Q_{b 1}}  \tag{25}\\
& \omega_{c 1}^{2} \omega_{c 2}^{2}=\omega_{b 1}^{2} \omega_{b 2}^{2} \tag{26}
\end{align*}
$$

Because it is given above [see (18) and (19)]:

$$
\begin{equation*}
\omega_{c 1} \omega_{c 2}=\omega_{0}^{2}, \quad Q_{c 1}=Q_{c 2}=Q_{c} \tag{27}
\end{equation*}
$$

we have:

$$
\begin{equation*}
\omega_{b 1} \omega_{b 2}=\omega_{0}^{2}, \tag{28}
\end{equation*}
$$

and we choose

$$
\begin{equation*}
\omega_{b 1}=\omega_{b 2}=\omega_{0} . \tag{29}
\end{equation*}
$$

Now the system of equations (23)-(26) is simplifying into the system (note that (23) and (25) have become identical):

$$
\begin{gather*}
\frac{\omega_{c 1}+\omega_{c 2}}{Q_{c 1}}=\omega_{0}\left(\frac{1}{Q_{b 1}}+\frac{1}{Q_{b 2}}\right),  \tag{30}\\
\omega_{c 1}^{2}+\omega_{c 2}^{2}+\frac{\omega_{0}^{2}}{Q_{c}^{2}}=2 \omega_{0}^{2}+\frac{\omega_{0}^{2}}{Q_{b 1} Q_{b 2}}+F . \tag{31}
\end{gather*}
$$

Rearranging it we obtain the condition for determining solutions:

$$
\begin{equation*}
\left(\omega_{b 2}-\omega_{b 1}\right)\left(Q_{b 2}-Q_{b 1}\right)=0 \tag{32}
\end{equation*}
$$

We have two equations (30) and (31) and three unknowns to be determined $Q_{b 1}, Q_{b 2}$ and $F$. One of the unknowns can be freely chosen (therefore we have one degree of freedom). We choose pole Q factors of the sections $T_{b 1}$, $T_{b 2}$ to have two limiting values of Q -factors:

1. Sections $T_{b 1}$, and $T_{b 2}$ have identical Q-factors.
2. One of the sections $T_{b 1}$ or $T_{b 2}$ has an infinite Qfactor.

Between these two limiting cases for Q -factors there are infinite possibilities, which turn to be more sensitive, and we do not treat them separately.

## C. Equal $Q$-factors $Q_{b 1}=Q_{b 2}$ [4]

The sections $T_{b 1}$ and $T_{b 2}$ are becoming identical, and we have:

$$
\begin{align*}
& Q_{b 1}=Q_{b 2}=\frac{2 \sqrt{\omega_{c 1} \omega_{c 2}}}{\omega_{c 1}+\omega_{c 2}} Q_{c} .  \tag{33}\\
& \omega_{b 1}=\omega_{b 2}=\omega_{0}=\sqrt{\omega_{c 1} \omega_{c 2}} . \tag{34}
\end{align*}
$$

Feedback factor $F$ takes the value defined by an expression:

$$
\begin{equation*}
F=\left(\omega_{c 2}-\omega_{c 1}\right)^{2}\left(1-\frac{1}{4 Q_{c}}\right) . \tag{35}
\end{equation*}
$$

## D. One $Q$-factor is in infinity: $Q_{b 2}=\infty$

One Q-factor is becoming infinite, while other has the value:

$$
\begin{equation*}
Q_{b 1}=\frac{\sqrt{\omega_{c 1} \omega_{c 2}}}{\omega_{c 1}+\omega_{c 2}} Q_{c} . \tag{36}
\end{equation*}
$$

Pole frequencies are the same as in the previous case:

$$
\begin{equation*}
\omega_{b 1}=\omega_{b 2}=\omega_{0}=\sqrt{\omega_{c 1} \omega_{c 2}} . \tag{37}
\end{equation*}
$$

Feedback factor $F$ is then given by:

$$
\begin{equation*}
F=\left(\omega_{c 2}-\omega_{c 1}\right)^{2}+\frac{\omega_{c 1} \omega_{c 2}}{Q_{c}^{2}} . \tag{38}
\end{equation*}
$$

## IV. Examples

In this section we realize three examples of the activeRC BP filters: one for the cascade (CAS) case in the Section III.A, and two for the Biquartic (BQ) cases in the Sections III.C and III.D. Consider two approximations: $i$ ) Butterworth (with normalized LP prototype parameters: $\omega_{L P}=1, Q_{L P}=0.707107$ ), and ii) 0.5 dB -ripple Chebyshev ( $\omega_{L P}=1.231342, Q_{L P}=0.863721$ [a higher pole-Q case]) that are transformed into BP characteristics with normalized center frequency $\omega_{0}=1$ and bandwidth $B=0.1$. Filters are then denormalized to the frequency 10 kHz . The active realization is shown in Fig. 3 and the element values are in Table I. Corresponding amplitude-frequency characteristics are shown in Fig. 4.

Generally, the transfer function of each Biquad shown in Fig. 3 has the form given by (4), with:

$$
\begin{gather*}
\omega_{b i}=\sqrt{\frac{R_{F i}}{C_{i 1} C_{i 2} R_{i 1} R_{i 3} R_{b i}}} ; Q_{b i}=C_{i 1} R_{i 2} \sqrt{\frac{R_{F i}}{C_{i 1} C_{i 2} R_{i 1} R_{i 3} R_{b i}}} ;  \tag{39}\\
k_{b i}=\frac{R_{F i}}{C_{i 1} R_{0 i} R_{i 1}} ; i=1,2 .
\end{gather*}
$$

Note that in this realization we do not use a separate operational amplifier for negative feedback. Instead, we lead the output signal to the positive input of the first operational amplifier in the first biquad. As a consequence the feedback factor $F$ is determined by:

$$
\begin{equation*}
F=\beta \cdot k_{b 1} \cdot k_{b 2}=\beta \cdot\left(1+\frac{R_{F 1}}{R_{01}}+\frac{R_{F 1}}{R_{b 1}}\right) \cdot \frac{1}{R_{11} C_{11}} \cdot k_{b 2} \tag{40}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta=R_{F 3} /\left(R_{F 3}+R_{03}\right) \tag{41}
\end{equation*}
$$

It is evident from (39) that for $R_{i 2}=\infty$ the pole-Q factor is infinite (e.g., for $R_{22}=\infty$ in Fig. 3 there is $Q_{b 2}=\infty$ ).
We choose value of $R_{F 3}$ and calculate $R_{03}$ depending on $F$ :

$$
\begin{equation*}
R_{03}=\frac{R_{F 3}}{F}\left[\left(1+\frac{R_{F 1}}{R_{01}}+\frac{R_{F 1}}{R_{b 1}}\right) \cdot \frac{1}{R_{11} C_{11}} \cdot k_{b 2}-F\right] . \tag{42}
\end{equation*}
$$

It is evident from (40) and (41) that for $R_{03}=\infty$ and $R_{F 3}=0$ the feedback factor $F=0$ and the circuit in Fig. 3 becomes cascade of Biquads.

Using (39)-(42) we calculate normalized elements of filter examples and denormalize the filter to the center frequency $f_{0}=10 \mathrm{kHz}$ and bandwidth $B=1 \mathrm{kHz}$. We choose denormalization resistance $R_{0}=1.591 \mathrm{k} \Omega$ in order to obtain denormalization capacitance $C_{0}=10 \mathrm{nF}$. The obtained values are given in Table I. In order to provide maximum dynamic range it will be needed to perform gain optimization in the next step, which is in this case not possible in the simple non-iterative way. It is needed to perform numerical optimization or the optimization in an iterative way.


Figure 3. Realization of Biquartic section with two general purpose active-RC Biquadratic sections and feedback.
TABLE I. Element Values of the Circuit in Fig. 3 for the Butterworth and Chebyshev Examples Denormalized to 10kHz.

| Elements | Butterworth |  |  | Chebyshev 0.5dB |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CAS | BQ $\mathbf{Q}_{1}=\mathbf{Q}_{2}$ | BQ $\mathrm{Q}_{2}=\infty$ | CAS | BQ $\mathrm{Q}_{1}=\mathrm{Q}_{2}$ | BQ $\mathrm{Q}_{2}=\infty$ |
| $\mathrm{R}_{01}$ | $175.923 \mathrm{k} \Omega$ | $116 \mathrm{k} \Omega$ | $95 \mathrm{k} \Omega$ | $502.16 \mathrm{k} \Omega$ | $110 \mathrm{k} \Omega$ | $100 \mathrm{k} \Omega$ |
| $\mathbf{R}_{11}$ | $1.557 \Omega$ | $1.536 \mathrm{k} \Omega$ | $1.515 \mathrm{k} \Omega$ | $0.818 \Omega$ | $1.475 \mathrm{k} \Omega$ | $1.412 \mathrm{k} \Omega$ |
| $\mathrm{R}_{12}$ | $22.521 \mathrm{k} \Omega$ | $22.507 \mathrm{k} \Omega$ | $11.253 \mathrm{k} \Omega$ | $31.05 \mathrm{k} \Omega$ | $22.327 \mathrm{k} \Omega$ | $11.163 \mathrm{k} \Omega$ |
| $\mathrm{R}_{13}$ | $1.895 \mathrm{k} \Omega$ | $1.633 \mathrm{k} \Omega$ | $1.617 \mathrm{k} \Omega$ | $1.681 \mathrm{k} \Omega$ | $1.746 \mathrm{k} \Omega$ | $1.612 \mathrm{k} \Omega$ |
| $\mathbf{R}_{\text {F1 }}$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ |
| $\mathbf{R}_{\text {b1 }}$ | $9.199 \mathrm{k} \Omega$ | $10.086 \mathrm{k} \Omega$ | $10.326 \mathrm{k} \Omega$ | $19.72 \mathrm{k} \Omega$ | $9.821 \mathrm{k} \Omega$ | $11.111 \mathrm{k} \Omega$ |
| $\mathrm{C}_{11}$ | 10nF | 10nF | 10 nF | 10 nF | 10nF | 10 nF |
| $\mathrm{C}_{12}$ | 10 nF | 10 nF | 10 nF | 10 nF | 10 nF | 10 nF |
| $\mathrm{R}_{02}$ | $95.784 \mathrm{k} \Omega$ | $135.714 \mathrm{k} \Omega$ | $214.364 \mathrm{k} \Omega$ | $85.749 \mathrm{k} \Omega$ | $107.843 \mathrm{k} \Omega$ | $144.144 \mathrm{k} \Omega$ |
| $\mathbf{R}_{21}$ | $1.205 \mathrm{k} \Omega$ | $1.533 \mathrm{k} \Omega$ | $1.574 \mathrm{k} \Omega$ | $1.446 \mathrm{k} \Omega$ | $1.504 \mathrm{k} \Omega$ | $1.541 \mathrm{k} \Omega$ |
| $\mathbf{R}_{22}$ | $22.521 \mathrm{k} \Omega$ | $22.507 \mathrm{k} \Omega$ | $\infty$ | $31.05 \mathrm{k} \Omega$ | $22.327 \mathrm{k} \Omega$ | $\infty$ |
| $\mathrm{R}_{23}$ | $1.972 \mathrm{k} \Omega$ | $1.608 \mathrm{k} \Omega$ | $1.591 \mathrm{k} \Omega$ | $1.75 \mathrm{k} \Omega$ | $1.629 \mathrm{k} \Omega$ | $1.591 \mathrm{k} \Omega$ |
| $\mathbf{R}_{\mathbf{F} 2}$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ |
| $\mathbf{R}_{\text {b2 }}$ | $9.918 \mathrm{k} \Omega$ | $10.133 \mathrm{k} \Omega$ | $10.104 \mathrm{k} \Omega$ | $9.317 \mathrm{k} \Omega$ | $10.328 \mathrm{k} \Omega$ | $10.322 \mathrm{k} \Omega$ |
| $\mathrm{C}_{21}$ | 10nF | 10 nF | 10 nF | 10 nF | 10 nF | 10 nF |
| $\mathrm{C}_{22}$ | 10 nF | 10 nF | 10 nF | 10 nF | 10 nF | 10 nF |
| $\mathrm{R}_{03}$ | $\infty$ | $459 \mathrm{k} \Omega$ | $138.438 \mathrm{k} \Omega$ | $\infty$ | $304.758 \mathrm{k} \Omega$ | $149.566 \mathrm{k} \Omega$ |
| $\mathrm{R}_{\text {F3 }}$ | 0 | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ | 0 | $10 \mathrm{k} \Omega$ | $10 \mathrm{k} \Omega$ |



Figure 4. Simulated amplitude-frequency characteristics of filter examples in Table I using Altium Designer. (a) Butterworth. (b) Chebyshev.


Figure 5. Simulated Schoeffler sensitivities of filter examples in Table I using Matlab. (a), (c), (e) Butterworth. (b), (d), (f) Chebyshev.
V. Results of Simulation and Measurement

The sensitivity to component tolerances and output thermal noise, are compared for the cases of cascade (in Section III.A) and for the two BQ realizations: $i$ ) equal Qfactors: $Q_{b 1}=Q_{b 2}$ (in Section III.C); and $i i$ ) one Q -factor is in infinity: $Q_{b 2}=\infty$ (in Section III.D). Sensitivity analysis is performed using MATLAB procedures given in [5], assuming relative changes of resistors and capacitors to be uncorrelated random variables, with a zero-mean Gaussian distribution and $1 \%$ standard deviation. The standard deviation (which is related to Schoeffler's sensitivities) of the variation of the logarithmic gain $\Delta \alpha=8.68588$ $\Delta\left|T_{B P}(\omega)\right| /\left|T_{B P}(\omega)\right|[\mathrm{dB}]$, with respect to passive elements, is calculated for filter examples from Table I and shown in Fig. 5. We conclude that low-sensitivity BP filters can be
designed using CBQ filters with equal Q-factors: $Q_{b 1}=Q_{b 2}$. Using the Altium Designer program output thermal noise spectral density of filter examples in Table I was generated and shown in Fig. 6. In the simulation a model of TL081/TI (Texas Instruments) FET input opamp, having typical values, as found in the data-sheets $e_{\text {na }}(f)=17 \mathrm{nV} / \sqrt{ } \mathrm{Hz}$ and $i_{n a 1}(f) \approx i_{\text {na2 }}(f)=0.01 \mathrm{pA} / \sqrt{H z}$ is used. From Fig. 6 one can conclude that the minimum noise possesses CBQ filters with equal Q-factors: $Q_{b 1}=Q_{b 2}$.

Measurements were performed on the same examples realized on separate printed circuit boards using $1 \%$ passive elements and TL081/TI opamps. For each filter, the output noise spectral density was measured using a typical university lab environment. The measurement equipment consisted of a high-quality HP 4195A Network


Figure 6. Simulated output thermal noise spectral densities of BQ filter examples in Table I using Altium Designer. (a) Butterworth. (b) Chebyshev.


Figure 7. Measured output thermal noise spectral density of BQ filter examples in Table I. (a), (c) Butterworth. (b), (d) Chebyshev.

Analyser, which measures the spectrum of noise (Spectrum mode). Detailed description of measurement procedure and equipment is given in [6]. Measured results are shown in Fig. 7, and reconfirm the results obtained by simulation.

## VI. CONLUSION

In accordance with a simulated sensitivity by Schoeffler in realizations of amplitude-frequency characteristics large discrepancies in implementation with one infinite Q factor are expected. Thus, better results are gained using Biquartic sections with identical Q factors. Deviations in amplitude-frequency characteristics are predictable in shifting of the center frequency and amplitude distortion. Observing Schoeffler sensitivities it is visible that the sections with one infinite Q factor have increased sensitivities at the band-edges.

In order to reduce the overall sensitivities we use the three-amplifier biquads as building blocks for biquartic filter because of their considerably lower sensitivities comparing to single-amplifier structures, at the expense of the increased power consumption.

The sensitivity differences between the two CBQ cases are smaller on Chebyshev filter frequency response. Since Chebyshev transfer function has higher Q-factors than Butterworth it leads us to conclusion that the differences are smaller in the highly selective BP filters. The cascade structure has considerably bigger sensitivities in all cases.

Measured spectral density of circuit output thermal noise is very close to the simulation results. We note better results in Biquartic sections with identical Q factors. Both Biquartic realizations are preferable over the common cascade structure. In all measurements and simulations wins Biquartic section with identical Q factors and imposes as an optimal realization of low- sensitive filters.

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