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Computationally Efficient Finite-Element-Based Methods for the Calculation of Symmetrical Steady-State Load Conditions for Synchronous Generators

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This paper presents an accurate and computationally efficient finite-element calculation and modeling methods for the simulation of symmetrical steady-state load conditions for synchronous generators. For that purpose two iterative methods are presented, implemented and compared. Those are NewtonâĂŹs iterative method and the relaxation parameter iterative method, which were both applied to magneto-static and transient finite-elements simulations. Various methods for obtaining armature voltage and power angle are proposed and analyzed from the computational efficiency and the accuracy point of view. All methods showed good convergence results for solving this particular problem. Calculation results were compared with measurements on two synchronous generators installed in power plants.

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Nomenclature:

A_z	z-component of the vector magnetic potential
f	frequency
f_t	pitch factor (chording factor)
I,	armature current
I_a, I_b, I_c	instantaneus phase armature currents
I_f	field winding current
I_{f0}	open circuit field current
J_z	current density in z-axis direction
k_f, k_ψ	relaxation parameters for the field current and for the angle ψ
k_{Fe}	iron fill factor
l_{ew}	end winding length
l_{stack}, l_i	axial length of the iron stack and ideal stack length
l_{vd},	axial length of ventilation ducts
n	number of iterations
n_{vd}	number and of ventilation ducts
P and Q	active and reactive power
p	number of pole pairs
R_d, L_d	damper winding resistance and inductance
$R_{d,ib}$, $L_{d,ib}$	damper winding inter-bar resistance and leakage inductance
$R_{d,ip}$, $L_{d,ip}$	damper winding inter-polar resistance and leakage inductance
R_{ew}, X_{ew}	armature end winding resistance and end winding leakage reactance
Tol_V, Tol_φ	tolerances of voltage and power factor angle used for iterative methods
V, V_F	terminal phase armature voltage and terminal phase armature voltage obtained from FE calculation
V_a, V_b, V_c	instantaneus phase armature voltages
V_r	rated armature voltage
V_{des}, φ_{des}	desired armature voltage and power factor angle
w	number of armature turns connected in series
α	the angle between magnetic axes of the field winding and the referent phase of
δ	load angle
θ	rotor position angle
ΔI_f, $\Delta\psi$	finite differences of the field current and the angle ψ
Λ_{ew}	armature end winding leakage permeance
μ, μ_{Fe}	permeability, permeability of iron
σ	conductivity
φ, φ_F	the power factor angle on terminals and the power factor angle on terminals obtained from the finite element calculation
φ_V	the phase angle of voltage waveform fundamental harmonic component for an arbitrary operating point
ψ	the angle between the resultant vector of the armature current and the q-axis
ω	angular frequency

Introduction: It is very important for a synchronous generator designer to be able to accurately predict various machine parameters and quantities such as the rated field current or synchronous reactances for a particular operating point. It is also important to predict various characteristics, such as the load angle characteristic, V or regulation curves within a reasonable time period. For the simulation of transients it is often required to start the transient from a particular steady-state operating point which has to be calculated first. For all these purposes it is required to calculate steady-state symmetrical load conditions. In the classical synchronous machine theory, steady-state load operation is calculated using a vector-phasor diagram. However, synchronous reactances are changing due to the change of the level of saturation in the machine which varies with the operating point. Therefore, the calculation of the steady-state load operation for a synchronous machine is often achieved by using the finite element method (FEM). There is a trend in the FEM modeling to develop computationally efficient calculation methods in order to provide accurate solutions as fast as possible. Hence this paper addresses the symmetrical steady-state load conditions calculation for synchronous generators from that point of view.

An accurate solution for the magnetic field in the machine can be obtained by performing the transient finite element (FE) calculation which solves the diffusion equation for the vector magnetic potential with boundary and initial conditions. For a two-dimensional (2D) problem in the Cartesian coordinate system the diffusion equation takes the form

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)=-J_z+\sigma\frac{\partial A_z}{\partial t} \tag{1}$$

To save the computational time a magneto-static computation is often preformed, which is given with the Poisson equation

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)=-J_z \tag{2}$$

However, induced currents are neglected in (2) which decreases the accuracy of the solution compared with the transient computation. In this particular case, it means that damper winding currents are not taken into account.

The sinusoidal symmetrical steady-state load conditions calculations for synchronous generators are analyzed in [1–9]. However, in [1–8] authors analyze steady-state load conditions by magneto-static finite-element calculations only.It is known that the calculation of the load steady-state conditions for synchronous generators requires iterative calculations [1–7]. Papers [1–6] deal with iterative methods in general, reporting that some quantities should be iterated within the calculation process, but do not provide mathematically explicit description of iterative methods by giving iterative equations. For example, papers [1, 4] propose the iteration of the armature and field current and papers [3, 6] propose the iteration of the calculation of reactances. Cross-magnetizing reactances, which were used for the iterative calculation are introduced in [9].

Kunckel and Liese in [7] proposed a method for the iterative calculation of the load conditions using the iterations of the field current and the position of the armature current vector. The authors provided iterative equations and compared the results with measurements. The calculation was carried out with the magneto-static finite integral method for the 520 MVA synchronous hydro-generator. Ashtiani and Lowther in [8] described a method which uses the magneto-static FE calculation to directly obtain the value of the load angle and the field current from the information on terminal operating conditions. However, during the formulation of the FE problem, a non-symmetric system of linear equations was obtained. It eliminated the possibility of using computationally efficient numerical methods for solving the linear system, such as the conjugate gradient method. In addition, authors reported that the proposed method could also have convergence issues. A similar approach was used for the permanent magnet synchronous generator in [10].

Steady-state load conditions and characteristics were calculated using the FE method for permanent magnet synchronous generators in [10–12]. The calculation of the steady-state load conditions for permanent magnet synchronous generators is rather simpler compared to conventional synchronous generators due to the absence of field and damper winding.

A straightforward approach for the steady-state load calculation for synchronous generators uses the time-stepping transient FE simulation coupled with the electrical-network. Armature terminals are connected to the three-phase impedance that consumes the desired active and reactive power. In the beginning of the simulation, the generator is under the condition similar to the three-phase sudden short-circuit which requires simulation of a few tens up to a few hundreds of periods to reach the steady-state, which is computationally inefficient. Armature voltages and currents can be extracted when the machine reaches the steady-state. If the armature voltage does not correspond to the desired armature voltage, the whole process should be repeated with a new value of the field current, which is time consuming.

Transient time-stepping FE calculations are also used for the simulation of non-sinusoidal steady-state conditions. An example of a condition of this kind is when synchronous generators with rectifiers supply DC load. Such analyses were conducted in [13–15].

The electrical network can be coupled with the synchronous machine transient FE model in order to perform detailed simulations. This type of coupling is also known as the coupled finite-element state-space (CFE-SS) [16–19]. This approach provides much better insight into machine non-linearities such as magnetic saturation, losses and performance compared to the standard dynamic model. A similar approach was used in [20] for the detailed analysis of the synchronous turbo-generator using the transient FE calculation, connected to the infinite bus-bar via transmissions lines and the power transformer.

This paper studies accurate and computationally efficient FE calculation methods for steady-state sinusoidal symmetrical load conditions with the use of magneto-static and transient time-stepping FE calculations. The modeling methodology is presented in detail since it is crucial for obtaining correct and accurate results. Two iterative methods for the calculation of load conditions for synchronous machines are presented and tested. Various approaches for obtaining the armature voltage have been were tested using FE methods in order to investigate the accuracy and the computational efficiency of proposed methods. All calculation results have beencompared with measurements conducted on the two real machines: 35 MVA hydro-generator and 247 MVA turbo-generator which were installed in power plants.

Finite Element Models of Analyzed Synchronous Generators: The machines analyzed in this paper are 35 MVA, 10.5 kV, 50 Hz, 12-pole synchronous salient pole generator - Machine A and 247 MVA, 13.8 kV, 50 Hz, 2-pole, turbo-generator - Machine B. Machine B has conductive solid rotor, made of the forged steel with the conductivity $7.7 \times 10^6\ Sm^{-1}$. The main parameters of analyzed machines are given in Table 1 and cross sections of machines for the 2D FEM modeling are shown in Fig. 1(a) and Fig. 1(b).

The axial length of all geometric sub-domains in the 2D FEM model is equal to the ideal machine axial length l_i, obtained by correcting the stack length with the Carter's factor due to ventilation ducts. The permeability of used materials should be corrected due to the iron fill factor k_{Fe} and ventilation ducts:

$$\mu = \frac{l_{stack} - n_{vd} l_{vd}}{l_i} \left[\mu_{Fe} k_{Fe} + \mu_0 (1 - k_{Fe}) \right] \tag{3}$$

Non-linearities due to the magnetic hysteresis were neglected in all analyses conducted in this paper

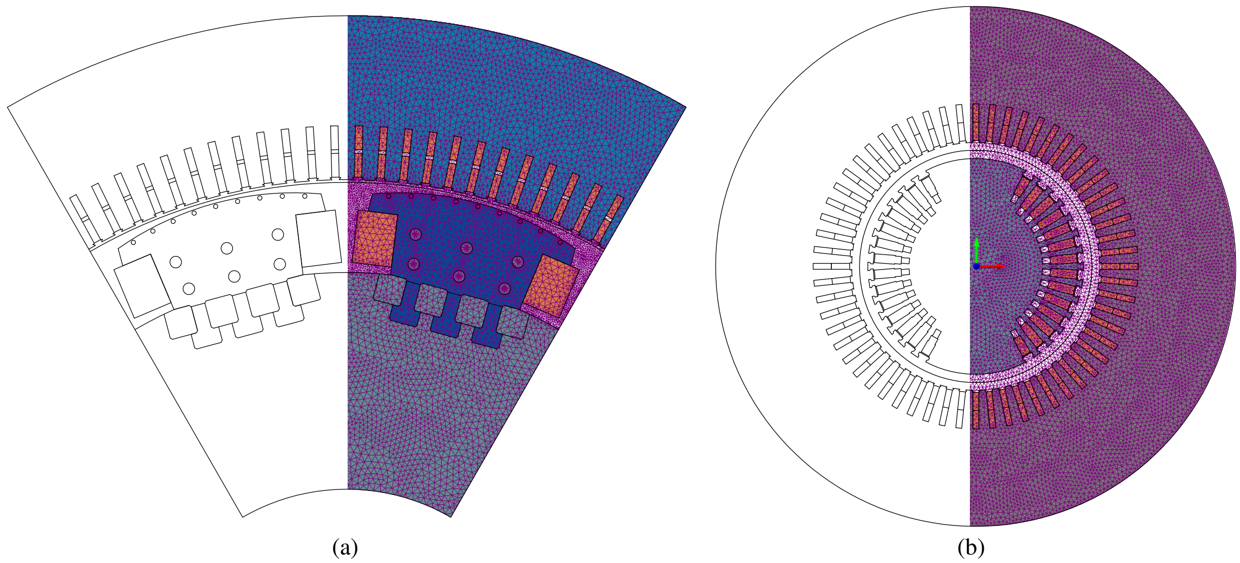


Fig. 1. Cross sections and finite element mashes for analyzed synchronous generators

Two-dimensional FEM models do not take into account the armature end winding resistance and leakage inductance. Therefore, corrections should be made in order to achieve better accuracy. There are many formulas and methods available in literature [21–23] for the calculation of the end winding leakage inductance depending on the armature winding type. For a two-layer winding the following expression [21] can be used

$$X_{ew} = 4\pi f \frac{w^2}{p} \Lambda_{ew} = 4\pi f \frac{w^2}{p} \left(0.43\mu_0 l_{ew} f_t^2\right) \tag{4}$$

The end winding resistance R_{ew} can also be calculated analytically, corrected due to the temperature and taken into the account.

With the known value of the end winding resistance and leakage reactance, it is possible to perform the correction of the armature voltage and power factor angle obtained by the 2D FEM model. According to the Fig. 2., the corrected voltage and the power factor angle should be calculated as:

$$V = \sqrt{\left(V_F \cos\varphi_F - IR_{ew}\right)^2 + \left(V_F \sin\varphi_F - IX_{ew}\right)^2} \tag{5}$$

$$\varphi = \varphi_F - \arctan\frac{V_F \sin\varphi_F - IX_{ew}}{V_F \cos\varphi_F - IR_{ew}} \tag{6}$$

For the fractional slot winding, the number of slots per pole and phase can be written as the fraction $q = a/b$ where a and b are relatively prime integers. $2b$ represents the spatial magnetic period of the machine expressed in pole pitches. Due to damper winding inter-polar currents, it is also required to set the FEM model with at least $2b$ poles. The periodicity of the connection of the damper winding is shown in Fig. 3. The damper winding inter-bar and inter-polar end-region leakage inductance can be calculated as in [24].

Table 1: Main parameters of analyzed synchronous machines

Analyzed machine	A	B	
Rated power	35	247	MVA
Rated voltage	10500	13800	V
Rated current	1925	10334	A
Rated power factor	0.9	0.85	
Number of phases	3	3	
Frequency	50	50	Hz
Rated synchronous speed	500	3000	min^{-1}
Minimal air gap length	17	80	mm
Stator stack length	1350	3700	mm
Number of stator slots	144	60	
Coil pitch to pole pitch ratio, y/τ_p	10/12	25/30	
Turns per pole for field winding	43	12/8	
Number of damper bars per pole	9	14	

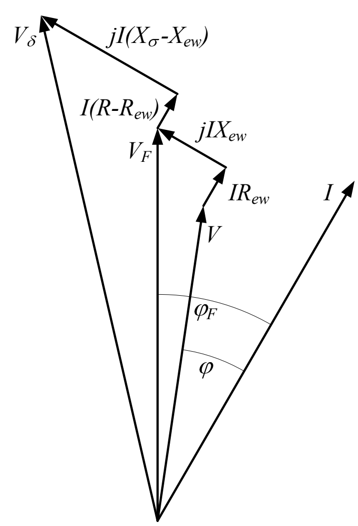


Fig. 2. Phasor relations between the air gap voltage V_δ, the induced armature voltage at terminals obtained by 2D FEM model V_F and the induced armature voltage at the terminals V corrected with respect to armature end-winding resistance and leakage inductance

Calculation of Symmetrical Load Conditions: If the symmetrical sinusoidal load condition is assumed, the operating point of a loaded synchronous generator can be electromagnetically fully defined by the total active power P, the total reactive power Q and the RMS value of the line-to-line armature voltage V. For those three quantities all other quantities such as the field current, armature current or the load angle can be uniquely determined. However, the triple (P, Q, V) is not the only combination of quantities that fully defines the electromagnetic state of the machine. The RMS value of the armature

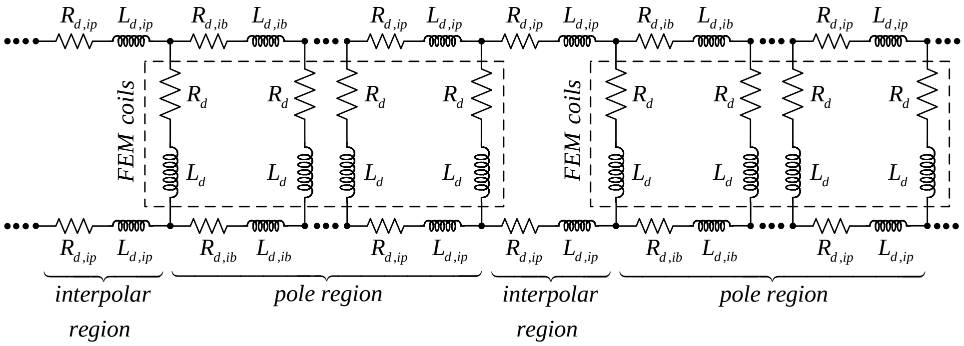


Fig. 3. Connection of the damper winding for FE transient calculations

current I, and the power factor angle φ of the load can be determined from operating point quantities P, Q and V:

$$I = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3}V} \tag{7}$$

and

$$\varphi = \arctan\left(\frac{Q}{P}\right) \tag{8}$$

In electromagnetism, electrical currents are magnetic excitations, thus all quantities in the electromagnetic system are fully defined with known values and geometric positions of electrical currents. For synchronous machines, this means that all quantities are fully defined by the armature and the field current values, rotor position and the angular speed. Furthermore, this means that the armature voltage and the power factor angle can be determined if the mentioned quantities are known. Formally, this can be written as:

$$\begin{aligned} V &= V(I_f, I, \psi, \omega, \theta) \\ \varphi &= \varphi(I_f, I, \psi, \omega, \theta) \end{aligned} \tag{9}$$

By neglecting the change of the permeance due to stator slots, which is a reasonable assumption due to the dominant first harmonic in the machine and by assuming the constant angular speed of the rotor, which is true in the case of the steady-state operation and by supplying the armature winding with the constant sinusoidal current, the armature voltage and the power factor angle φ become dependent on the field current and the angle ψ, only.

$$\begin{aligned} V &= V(I_f, \psi),\ with\ I, \omega = const. \\ \varphi &= \varphi(I_f, \psi),\ with\ I, \omega = const. \end{aligned} \tag{10}$$

Note that the armature voltage, the armature current and the power factor angle are quantities defined by the operating point, while the field current and the angle ψ are not known a priori. Therefore, to perform the load condition calculation, it is required to find out the value of the field current I_f and

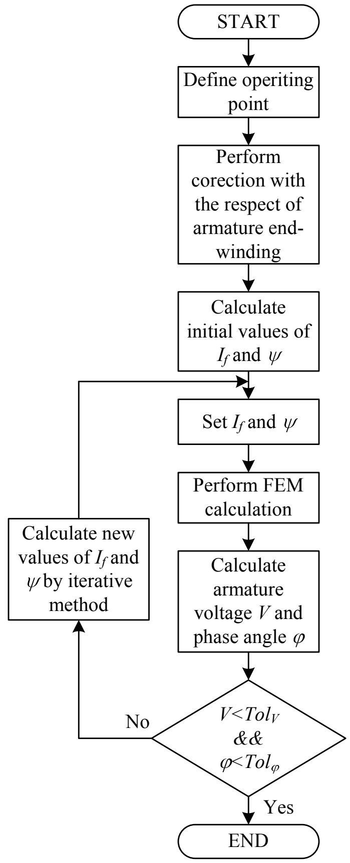


Fig. 4. Flow chart of the algorithm for performing the FE calculation for load conditions for synchronous generators

the angle ψ that will produce the desired armature voltage V_{des} and power factor angle φ_{des} while the armature current I has a value defined by the operating point and the constant angular speed ω. In this case the problem can be formulated as:

$$V_{des} = V(I_f, \psi),\ with\ I, \omega = const.$$

$$\varphi_{des} = \varphi(I_f, \psi),\ with\ I, \omega = const. \quad (11)$$

Functions $V(I_f, \psi)$ and $\varphi(I_f, \psi)$ are non-linear and are not analytically known, but their values are obtained from the magneto-static or the transient FE calculation. Therefore, to solve (11) it is appropriate to apply numerical iterative methods. In general, it is favorable for the iterative method that the initial point is positioned as close as possible to the solution point. Within a few FE calculations it is possible to determine unsaturated synchronous reactances as it is shown in [25], which could be used together with the phasor diagram for the estimation of the initial operating point.

Flow chart (Fig. 4) illustrates the algorithm for the calculation of load conditions for the synchronous generators.

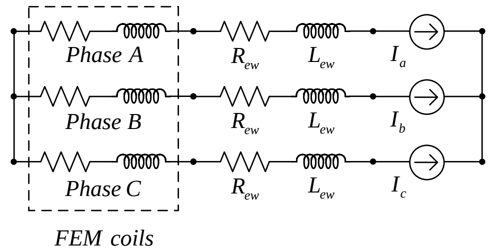


Fig. 5. Connection of armature windings with current sources and the end-winding resistance and the inductances for transient FE load conditions calculation

Obtaining the Armature Voltage and the Power Factor Angle

The sinusoidal steady-state load can be modeled with current sources as it is shown at Fig. 5. During the load operation the armature current is positioned at the angle ψ with the respect to the q-axis, thus the following waveforms should be set to current sources for supplying every phase of the machine

$$
\begin{aligned}
I_a =& \sqrt{2} I \sin(\omega t - \alpha - \psi) \\
I_b =& \sqrt{2} I \sin(\omega t - \alpha - \psi - 120^o) \\
I_c =& \sqrt{2} I \sin(\omega t - \alpha - \psi - 240^o)
\end{aligned} \tag{12}
$$

To perform the FE calculation of the loaded synchronous machine, the field winding should be supplied with the constant value of the field current I_f, the armature winding should be supplied with symmetrical three-phase sinusoidal currents described by the RMS value and the angle ψ. After performing the FE calculation the armature voltage waveform is analyzed to obtain the fundamental harmonic component or the RMS value of the voltage. However, there are differences in obtaining the armature voltage for the transient and the magneto-static FE calculation.

For the magneto-static FE calculation, the armature voltage waveform can be obtained from armature winding flux linkages which are obtained from magnetic vector potentials of stator conductors, as it is described in [26] and [27].

In the transient FE calculation it is required to form the appropriate electrical network circuit. The damper winding should be connected as it is shown in the Fig. 3. To introduce end winding effects in the transient FE calculation, end winding resistances and leakage inductances must be connected in series with phase windings (Fig. 5). The current source voltage corresponds to the armature voltage corrected with the respect to the end winding resistance and the leakage inductance. The voltage waveform is obtained by performing the transient finite-element calculation coupled with the electrical network. For achieving accurate results time steps should be small enough in order to take damper winding currents into account correctly, while the duration of the FE calculation should be long enough to simulate the armature voltage waveform in order to extract the fundamental harmonic component, which are two opposite requirements.

Instead of calculating the first harmonic component, a more computationally efficient way is to calculate the RMS value of the armature voltage, . It is sufficient to perform the calculation for the rotor movement within one stator slot pitch at the synchronous speed because that corresponds to one period of damper winding currents. This approach significantly reduces the computation time. The instantaneous line-to-line value of the armature voltage RMS can be calculated as

$$V_{RMS}(t) = \sqrt{V_a^2(t) + V_b^2(t) + V_c^2(t)} \tag{13}$$

and the RMS value used for further calculations can be obtained by averaging (13).

The power factor angle can be calculated directly from the active and the reactive power which are obtained from the FE calculation using armature voltage and current waveforms or using the load angle. The load angle δ can be calculated using

$$\delta = \arctan\left(\frac{V_d}{V_q}\right) \tag{14}$$

With the familiar load angle δ it is possible to determine the power factor angle φ:

$$\varphi = \psi - \delta \tag{15}$$

Iterative Methods for the Steady-State Load Calculation:

Relaxation Parameter Iterative Method

This method is suitable for the use in both magneto-static and time-stepping transient FE calculations. The system (11) can be solved by the following iterative scheme:

$$I_{f(n+1)} = I_{f(n)} + k_f\big(V_{des} - V(I_{f(n)}, \psi_n)\big) \tag{16}$$

$$\psi_{n+1} = \psi_n + k_\psi\big(\varphi_{des} - \varphi(I_{f(n)}, \psi_n)\big) \tag{17}$$

Motivation for using this iterative scheme is that the field current has major influence to the armature voltage, while the shifting of the current vector has the main influence in the change of the power factor angle φ. Basically, the higher values of relaxation parameters k_f and k_ψ will lead to a faster, but less stable convergence or even divergence. On the contrary, by choosing the lower values of k_f and k_ψ the iterative method will have better stability properties and slower speed of convergence which could be computationally expensive. After testing numerous combinations for relaxation parameters values, choosing

$$\begin{aligned} k_f &= \frac{I_{f0}}{V_r} \\ k_\psi &= 1 \end{aligned} \tag{18}$$

has been proven as the reasonable choice.

Newton's Iterative Method

Newton's iterative method can also be applied to the magneto-static and the time-stepping transient FE calculation. The system (11) is linearized in the vicinity of the operating point given by I_f and ψ. The obtained linear system is solved and the solutions are new values of I_f and ψ that are closer to the solution of (11). This procedure of the linearization and solving the linear system in the vicinity of the new operating point is repeated until the desired accuracy of the solution of (11) is met. Linearization requires the calculation of the approximation of the Jacobian matrix for the system (11). The Jacobian matrix can be approximated from three different FE calculations. Described procedure can be formulated by iterative set of equations.

$$\begin{bmatrix} I_{f(n+1)} \\ \psi_{n+1} \end{bmatrix} = \begin{bmatrix} I_{f(n)} \\ \psi_n \end{bmatrix} - \begin{bmatrix} \dfrac{V(I_{f(n)}+\Delta I_f,\psi_n)-V(I_{f(n)},\psi_n)}{\Delta I_f} & \dfrac{V(I_{f(n)},\psi_n+\Delta\psi)-V(I_{f(n)},\psi_n)}{\Delta\psi} \\ \dfrac{\varphi(I_{f(n)}+\Delta I_f,\psi_n)-\varphi(I_{f(n)},\psi_n)}{\Delta I_f} & \dfrac{\varphi(I_{f(n)},\psi_n+\Delta\psi)-\varphi(I_{f(n)},\psi_n)}{\Delta\psi} \end{bmatrix}^{-1} \times \begin{bmatrix} V(I_{f(n)},\psi_n)-V_{des} \\ \varphi(I_{f(n)},\psi_n)-\varphi_{des} \end{bmatrix} \tag{19}$$

ΔI_f can be set to 1% of the rated field current or lower and $\Delta\psi$ can be set to less than 1^o. For a wide range of load operating points and initial condition points, as far as it was tested, the method has always been convergent.

The advantage of this method, compared to the relaxation parameter method, is in the estimation of the contributions of the change of the armature voltage due to the change of the angle ψ, and the change of the power factor angle due to the change of the field current which has positive effects to the stability of the method and the speed of convergence.

Results and Comparison with Measurements: The measurements of the load operation were conducted on synchronous generators installed in power plants. Armature voltages were measured with the overall accuracy of 0.7 % for both machines. Armature currents were measured with the overall accuracy of 0.7 % and 1.7 % for Machine A and B respectively and field currents were measured with the overall accuracy of 0.3 % and 0.4 % for Machine A and B respectively. The load angle was measured by the custom-made microcontroller-based device with the accuracy of 0.5^o electrical for both machines. This device uses a proximity signal from the rotor and line-to-line armature voltage measurement. It measures time from the moment when proximity signal crosses 0 V threshold to the moment when the armature voltage signal crosses 0 V threshold and converts that time into an electrical angle. This angle is calibrated in no load condition and archived accuracy equals to 0.5 degrees when measuring load angle. The method used for the measurement of the load angle is described in [28]. Armature voltages and currents were measured using power quality analyzer NORMA 4000, which measures active, reactive power and power angle φ. Angle ψ was indirectly measured using $\psi = \varphi - \delta$. All calculations were carried out for real operational temperature which means that armature resistance was corrected due to the temperature.

Five different symmetrical load operating points were analyzed for each machine listed in tables 2 and 5. Operating point numbers in tables 3 and 4 correspond to the operating point numbers defined in table 2 and operating point numbers in tables 6 and 7 correspond to operating point numbers in table 5.

For FE calculations, the voltage tolerance was set to 0.1 % of the desired voltage and the power factor angle tolerance was set to 0.01^o for all iterative methods. Initial field currents and angles ψ for all analyzed operating points were calculated using phasor diagram with unsaturated values of direct and quadrature reactances. Infolytica MagNet® software was used together with MATLAB® for scripting and implementation of proposed iterative methods.

The Newton's iterative method and the relaxation parameter iterative method were applied on the magneto-static and transient time-stepping FE calculations. Tables 3 and 6 show the number of iterations and the number of the conducted FE simulations required to find the solution for a given operating point. Figure 6 shows a typical convergence plot for Newton's and relaxation parameter method.

Since the iteration methods iterate the field current and the angle ψ until the desired accuracy of the armature voltage and the power factor angle is met, it is obvious that final values of field currents and angles ψ will not depend on the iteration method (i.e. Newton's method or relaxation parameter). It will onyl depend on the type of FE calculation method used (i.e. magneto-static or transient). Iterative methods have influence on the speed of convergence. Since Newton's method showed better convergence results, it was chosen for further investigations.

Three different methods were tested for the FE load condition calculation in order to investigate accuracy of the proposed methods. They are listed in tables 4 and 7. Methods were different in the way of obtaining the armature voltage waveform. In tables4 and 7, Method 1 - magneto-static refers to the magneto-static FE calculation method which calculates the armature voltage waveform and extracts the first harmonic for further iterations. For Machine A the armature voltage waveform was reconstructed using two different magneto-static FE calculations for two different rotor positions and for Machine B the armature voltage was reconstructed from only one magneto-static FE calculation. Method 2 - transient RMS refers to the method that simulates the armature voltage waveform using the transient time-stepping FE calculation and uses the RMS value of the waveform for further iterations. RMS value of the armature voltage was extracted from ten waveform points that correspond to the ten equally distributed rotor positions within one stator slot pitch. Finally, Method 3 - transient 1h denotes to the method that simulates one period of armature voltage using the transient time-stepping FE calculation and uses the first harmonic for further iterations. The time step was set to 0.25 ms for both machines.

Tables 4 and 7 show the comparison of the measured and calculated values of field currents and ψ angles. Figure 7 shows flux lines and the magnetic flux density for the first operating point from tables 2 and 5 for Machines A and B.

Table 2: Analyzed load operating points for the Machine A

Point No.	1	2	3	4	5
V, V	10500	10680	11254	10679	10724
P, MW	31.50	33.53	22.07	6.62	12.16
Q, MVAr	15.25	-0.78	18.92	-9.33	1.22
I, A	1925	1813	1491	619	658
$\cos(\varphi)$	0.90	0.99	0.76	0.58	0.99
$\varphi,^o$	25.8	-1.4	40.6	-54.6	5.8
$\delta,^o$	23.4	30.7	13.8	10.6	13.8

Table 3: Number of iterations and conducted FE simulations required to calculate steady-state operating points for the Machine A

	Point No.	1	2	3	4	5
MS Newton's m.	Iter.	2	2	2	1	1
	FE calc.	14	14	14	8	8
MS relaxation param.	Iter.	6	7	11	14	5
	FE calc.	14	16	24	30	12
TR Newton's m. (1h)	Iter.	2	2	2	1	1
	FE calc.	7	7	7	4	4
TR Newton's m. RMS	Iter.	2	2	2	2	1
	FE calc.	7	7	7	7	4
TR relaxation param.	Iter.	3	9	9	7	3
	FE calc.	4	10	10	8	4

Table 4: Final values of calculated field currents I_f and angles ψ using three different methods for the Machine A. All methods use Newton's iterations

	Method 1 - Magneto-static		Method 2 - Transient RMS		Method 3 - Transient 1h		Test	
Point No.	$I_f,\ A$, (err., %)	$\psi,\ ^o$, (err., ^{o})	$I_f,\ A$, (err., %)	$\psi,\ ^o$, (err., ^{o})	$I_f,\ A$, (err., %)	$\psi,\ ^o$, (err., ^{o})	$I_f,\ A$	$\psi,\ ^o$
1	755.2, (3.4 %)	52.0 , (2.7)	718.5, (-3,0 %)	47.6, (-1.8)	728.0, (-0.3 %)	48.4, (-0.9)	730.5	49.3
2	590.9, (-5.4 %)	32.8, (3.5)	550.9, (-0.1 %)	28.5 , (-1.0)	560.8, (2.3 %)	29.1, (-0.2)	548.0	29.3
3	782.9, (2.2 %)	58.9, (4.5)	751.6, (-1.9 %)	53.8, (-0.7)	760.0, (-0.8 %)	54.8, (0.4)	766.3	54.4
4	280.9, (5.6 %)	-36.6, (7.4)	256.6, (-0.1 %)	-44.4, (-0.4)	268.7, (1.1 %)	-42.6, (1.4)	265.9	-44.0
5	442.6, (3.7 %)	24.5, (4.9)	423.5, (2.1 %)	18.5, (-1.2)	427.6, (0.2 %)	19.8, (0.2)	426.9	-19.6

Table 5: Analyzed load operating points for the Machine B

Point No.	1	2	3	4	5
V, V	13617	13407	13606	13469	13891
P, MW	200.45	202.08	88.01	123.76	158.89
Q, MVAr	47.81	1.61	-3.42	-21.32	-16.43
I, A	8751	8715	3750	5384	6685
$\cos(\varphi)$	0.971	1.000	1.000	0.983	0.994
$\varphi,^o$	13.8	0.7	-1.7	-10.5	-6.4
$\delta,^o$	45.1	53.6	29.2	45.9	49.2

Table 6: Number of iterations and conducted FE simulations required to calculate steady-state operating points for the Machine B

	Point No.	1	2	3	4	5
MS Newton's m.	Iter.	2	2	2	2	2
	FE calc.	7	7	7	7	7
MS relaxation param.	Iter.	7	37	10	30	28
	FE calc.	8	38	11	31	29
TR Newton's m. 1h	Iter.	2	2	2	2	2
	FE calc.	7	7	7	7	7
TR Newton's m. RMS	Iter.	2	2	2	2	2
	FE calc.	7	7	7	7	7
TR relaxation param.	Iter.	8	29	8	24	31
	FE calc.	9	30	9	25	32

Table 7: Final values of calculated field currents I_f and angles ψ using three different methods for the Machine B. All methods use Newton's iterations

	Method 1 - Magneto-static		Method 2 - Transient RMS		Method 3 - Transient 1h		Test	
Point No.	$I_f,\ A$, (err., %)	$\psi,\ ^o$, (err., ^{o})	$I_f,\ A$, (err., %)	$\psi,\ ^o$, (err., ^{o})	$I_f,\ A$, (err., %)	$\psi,\ ^o$, (err., ^{o})	$I_f,\ A$	$\psi,\ ^o$
1	2037, (1.1 %)	57.5, (-1.1)	2062, (2.4 %)	57.6, (1.3)	2056, (2.1 %)	57.7, (-1.1)	2014	58.8
2	1788, (-0.2 %)	54.6, (-0.3)	1791, (0.0 %)	53.7, (-0.6)	1789, (-0.1 %)	53.9, (-0.4)	1791	54.3
3	1088, (-2.9 %)	31.3, (3.7)	1106, (-1,4 %)	30.4, (2.9)	1101, (-1.7 %)	30.7, (3.1)	1120	27.6
4	1171, (-2.2 %)	31.3, (1.4)	1169, (-2.3 %)	35.5, (0.1)	1172, (-2.0 %)	35.8, (0.4)	1197	35.4
5	1419, (-1.4 %)	42.3, (0.5)	1429, (0.7 %)	42.5, (-0.3)	1426, (-0.9 %)	42.7, (-0.1)	1439	42.8

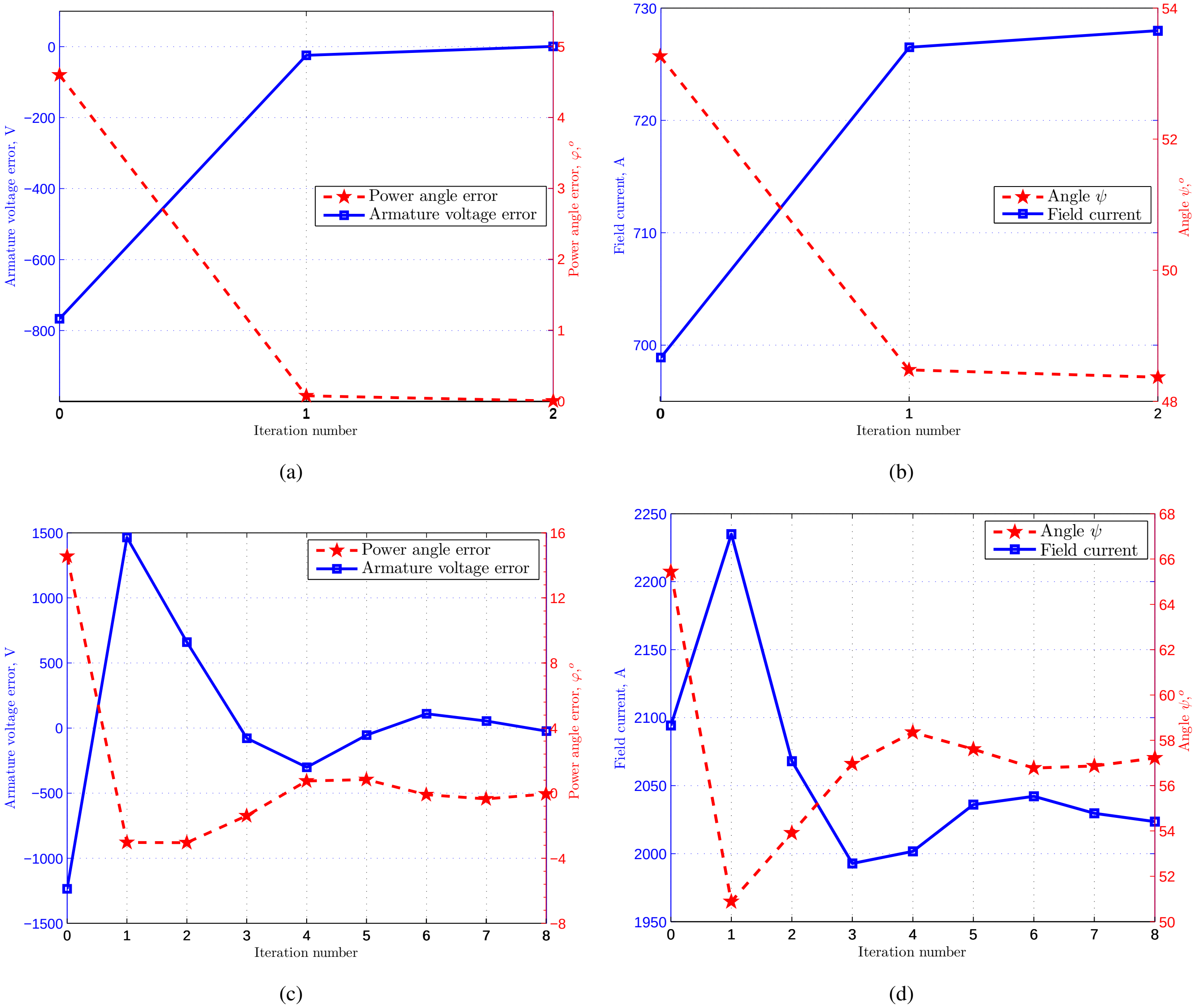
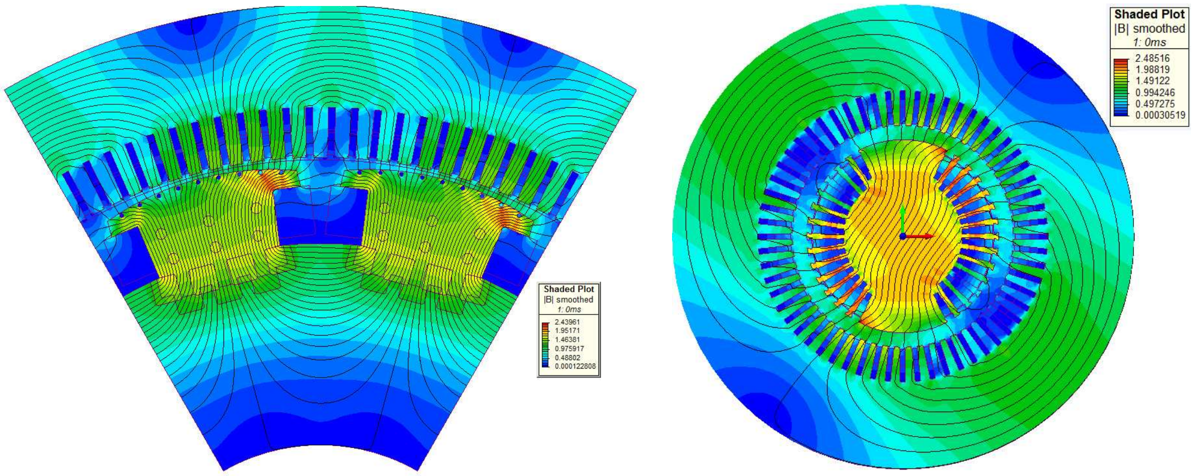


Fig. 6. Typical convergence plots of the error and solution vector for Newton's and the relaxation parameter method using time-stepping transient FE calculation a) convergence of the error vector for Machine A for load point No. 1 using Newton's method, b) convergence of the solution vector for machine Machine A for load point No. 1 using Newton's method, c) convergence of the error vector for Machine B for load point No. 1 using relaxation parameter method d) convergence of the solution vector for Machine B for load point No. 1 using relaxation parameter method



(a) (b)

Fig. 7. Flux lines and the magnetic flux density for the load point No. 1 for Machine A (a) and Machine B (b). (as given in tables 2 and 5)

Conclusion: The paper presents computationally efficient finite element calculation methods for simulating symmetrical steady-state load conditions for synchronous generators. Newton's and relaxation parameter iterative methods were applied for the solution of this problem. Both, Newton's and the relaxation parameter method showed good convergence results and were convergent for a wide range of initial values of the field current and the angle ψ. For the majority of analyzed operating points Newton's method showed faster convergence compared to the relaxation parameter method. However, the relaxation parameter method is easier to implement.

Three different FE calculation methods were tested for the load-condition calculation: magneto-static which uses the first harmonic, transient which uses the first harmonic and transient which uses the RMS value of the armature voltage. The transient time-stepping FE calculation which uses the first harmonic of the armature voltage for iterations is the most computationally expensive, however, it turns out to be the most accurate. The transient time-stepping FE calculation that uses the RMS value of the armature voltage for iterations represents a good trade-off between the computation efficiency and the accuracy. In this particular case, the RMS value of the armature voltage was calculated from ten different time-steps in the transient FE simulation. Magneto-static calculations are typically performed several tens of times faster compared to transient FE calculations, depending on defined time step in the transient calculation. However, the transient FE calculation shows better agreement with measurements when compared with the magneto-static FE calculation. This means that damper winding currents can locally influence the level of saturation and can influence the steady-state performance of synchronous generators. Therefore, for obtaining high accuracy results for the load conditions calculation, damper winding currents should be taken into account. The differences between calculated and measured values of the field current and the angle ψ can be caused by the nonlinearities due to the magnetic hysteresis which was not taken into the account in this analysis.

Developed methods enable calculation of steady-state load conditions within time of a few minutes up to a few hours per an operating point depending on the method used.

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