Bounded memory Dolev-Yao adversaries in collaborative systems

Max Kanovich[1], Tajana Ban Kirigin[2], Vivek Nigam[3], and Andre Scedrov[3]

[1] Queen Mary, University of London, UK
mik@dcs.qmul.ac.uk
[2] University of Rijeka, HR
bank@math.uniri.hr
[3] University of Pennsylvania, Philadelphia, USA
{vnigam,scedrov}@math.upenn.edu

Abstract. This paper extends existing models for collaborative systems. We investigate how much damage can be done by insiders alone, without collusion with an outside adversary. In contrast to traditional intruder models, such as in protocol security, all the players inside our system, including potential adversaries, have similar capabilities. They have bounded storage capacity, that is, they can only remember at any moment a bounded number of facts. This is technically imposed by only allowing balanced actions, that is, actions that have the same number of facts in their pre and post conditions. On the other hand, the adversaries inside our system have many capabilities of the standard Dolev-Yao intruder, namely, they are able, within their bounded storage capacity, to compose, decompose, overhear, and intercept messages as well as update values with fresh ones. We investigate the complexity of the decision problem of whether or not an adversary is able to discover secret data. We show that this problem is PSPACE-complete when all actions are balanced and can update values with fresh ones. As an application we turn to security protocol analysis and demonstrate that many protocol anomalies, such as the Lowe anomaly in the Needham-Schroeder public key exchange protocol, can also occur when the intruder is one of the insiders with bounded memory.

1 Introduction

A major concern in any system where agents do not trust each other completely is whether or not the system is secure, that is, whether or not any confidential information or secret of any agent can be leaked to a malicious agent. This paper investigates the complexity of this problem in the context of collaborative system with confidentiality policies [17, 18].

Following [18], we assume here that all actions in our system are *balanced*, that is, they have the same number of facts in their pre and post conditions. This implies that all players inside our system, including adversaries, have a bounded storage capacity, that is, they can only remember at any moment a bounded number of facts. This contrasts with traditional intruder models, which normally include a powerful Dolev-Yao intruder [11] that has an unbounded memory. On the other hand, our adversaries and the standard Dolev-Yao intruder [11] share many capabilities, namely, they are able,

within their bounded storage capacity, to compose, decompose, overhear, and intercept messages as well as update values with fresh ones.

This paper shows that the secrecy problem of whether or not an adversary can discover a secret is PSPACE-complete when actions are balanced and can update values with fresh ones. This contrasts with previous results in protocol security literature [12], where it is shown that the same problem is undecidable. However, there the actions of the intruder were possibly unbalanced, or in other words, they assumed that the intruder's memory is not necessarily bounded.

In order to obtain a secret, an adversary might need to perform exponentially many actions. Since actions might update values with fresh ones, there might be an exponential number of fresh constants involved in an anomaly, which in principle precludes PSPACE membership. To cope with this problem, we show in Section 3 how to reuse obsolete constants instead of updating with fresh constants.

Although our initial efforts were in collaborative systems, we realized that our results have important consequences for the domain of protocol security. In particular, we demonstrate that when our adversary has *enough* storage capacity, then many protocol anomalies, such as the Lowe anomaly [19] in the Needham-Schroeder public key exchange protocol, can also occur in the presence of a bounded memory intruder. We believe that this is one reason for the successful use in the past years of model checkers in protocol verification. Moreover, we also provide some *quantitative measures* for the security of protocols, namely, the smallest amount of memory needed by the intruder to carry out anomalies for a number of protocols.

This paper is structured as follows: in Section 2 we review the main definitions of local state transition systems used to model collaborative systems. We formalize the notion of fresh values in Section 3, and in Section 4 we summarize the main theoretical results involving the complexity of the different problems considered. We show in Sections 5 that many protocol anomalies can also be carried by our bounded memory intruder. Finally, in Sections 6 and 7, we discuss related work and conclude by pointing out some future work.

Full details of our results are in a technical report [15].

2 Preliminaries

In this section we review the main vocabulary and concepts introduced in [17, 18], extend their definitions to accommodate actions that can update values with fresh ones, and we introduce an adversary.

Local State Transition Systems At the lowest level, we have a first-order signature Σ that consists of a set of sorts together with the predicate symbols $P_1, P_2, \ldots$, function symbols $f_1, f_2, \ldots$, and constant symbols $c_1, c_2, \ldots$ all with specific sorts. The multi-sorted terms over the signature are expressions formed by applying functions to arguments of the correct sort. Since terms may contain variables, all variables must have associated sorts. A *fact* is a ground, atomic predicate over multi-sorted terms. Facts have the form $P(\boldsymbol{t})$ where P is an n-ary predicate symbol and $\boldsymbol{t}$ is an n-tuple of terms, each with its own sort. A *state*, or *configuration* of the system is a finite multiset W of facts. We use both WX and W, X to denote the multiset resulting from the multiset union of W and X.

Definition 1. *The* size of a fact *is the number of term and predicate symbols it contains. We count one for each predicate and function name, and one for each variable or constant symbol. We use $|P|$ to denote the size of a fact P.*

For example, $|P(x, c)| = 3$, and $|P(f(x, n), z)| = 5$. In this paper, we will assume an upper bound on the size of facts, as in [12, 17, 18].

Following [17, 18], we assume that the global configuration is partitioned into different local configurations each of which is accessible only to one agent. There is also a public configuration, which is accessible to all agents. This separation of the global configuration is done by partitioning the set of predicate symbols in the signature and it will be usually clear from the context. However, differently from [17, 18], we assume that among the agents in the system, there is an adversary M. We also assume the existence of a special constant s in Σ denoting the secret that should not be discovered by the adversary.

As in [17, 18], each agent has a finite set of *actions* or *rules* which transform the global configuration. Here, as in [12, 16], we allow agents to have more general actions which can update values with fresh ones. These values are often called *nonces* in protocol security literature. Such fresh values are often used in administrative processes. For example, when one opens a new bank account, the number assigned to the account has to be fresh, that is, it has to be different from all other existing bank account numbers. Similarly, whenever a bank transaction is initiated, a fresh number is assigned to the transaction, so that it can be uniquely identified. Fresh values are also used in the execution of protocols. At some moment in a protocol run an agent might need to update a value with a fresh one, or *nonce*, that is not known to any other agent in the network. This nonce, when encrypted in a message, is then usually used to establish a secure communication among agents.

Actions that belong to an agent A have the form: $X_A X_{pub} \rightarrow_A \exists \boldsymbol{t}.Y_A Y_{pub}$. The multisets X_A and Y_A contain facts belonging to the agent A and the multisets X_{pub} and Y_{pub} contain only public facts. Actions work as multiset rewrite rules. All free variables in a rule are treated as universally quantified. $X_A X_{pub}$ are the pre-conditions of the action and $Y_A Y_{pub}$ are the postconditions of the action. By applying the action for a ground substitution (σ), the pre-condition applied to this substitution ($X_A \sigma X_{pub} \sigma$) is replaced with the post-conditions applied to the same substitution ($Y_A \sigma Y_{pub} \sigma$). In this process, the existentially quantified variables ($\boldsymbol{t}$) appearing in the post-condition are replaced by fresh variables. The rest of the configuration remains untouched. Thus, we can apply the action $P_A(x)\ Q_{pub}(y) \rightarrow_A \exists z.R_A(x, z)\ Q_{pub}(y)$ to the global configuration $V\ P_A(t)\ Q_{pub}(s)$ to get the global configuration $V\ R_A(t, c)\ Q_{pub}(s)$, where the constant c is fresh. For simplicity, we often omit the name of the agent from the action and predicates when the agent is clear from the context.

Definition 2. *A* local state transition system *(LSTS) T is a tuple $\langle \Sigma, I, M, R_T, s \rangle$, where Σ is the signature of the language, I is a set of agents, $M \in I$ is the adversary, R_T is the set of actions owned the agents in I, and s is the secret.*

We classify a rule as *balanced* if the number of facts in its precondition is the same as the number of facts in its postcondition. As discussed in [18], if we restrict actions to be balanced, then the size of the configurations in a run remains the same as in the initial configuration. Since we assume facts to have a bounded size, the use of balanced actions imposes a bound on the storage capacity of the agents in the system.

We use the notation $W \rhd_T U$ or $W \rhd_r U$ to mean that there is an action r in T which can be applied to the configuration W to transform it into the configuration U. We let $\rhd_T^+$ and $\rhd_T^*$ denote the transitive closure and the reflexive, transitive closure of $\rhd_T$ respectively. Usually, however, agents do not care about the entire configuration of the system, but only whether a configuration contains some particular facts. Therefore we use the notion of partial goals. We write $W \rightsquigarrow_T Z$ or $W \rightsquigarrow_r Z$ to mean that $W \rhd_r ZU$ for some multiset of facts U. For example with the action $r : X \rightarrow_A Y$, we find that $WX \rightsquigarrow_r Y$, since $WX \rhd_r WY$. We define $\rightsquigarrow_T^+$ and $\rightsquigarrow_T^*$ to be the transitive closure and the reflexive, transitive closure of $\rightsquigarrow_T$ respectively. We say that the partial configuration Z is reachable from configuration W using T if $W \rightsquigarrow_T^* Z$. Finally, given an initial configuration W and a partial configuration Z, we call a *plan* any sequence of actions that leads from configuration W to a configuration containing Z.

In order to achieve a final goal, it is often necessary for an agent to share some private knowledge with another agent. However, although agents might be willing to share some private information with some agents, they might not be willing to do the same with other agents. For example, a patient might be willing to share his medical history with his doctor, but not with all agents, such as the doctor's secretary. One is, therefore, interested in determining if a system complies with some *confidentiality policies*, such as a patient's medical history should not be publicly available. We call *critical configuration* any configuration that conflicts with some given confidentiality policies, and we classify any plan that does not reach any critical configuration as *compliant*.

In this paper, we make an additional assumption that critical configurations are closed under renaming of nonce names, that is, if W is a critical configuration and $W\sigma = W'$ where σ is substitution renaming the nonces in W, then W' is also critical. This is a reasonable assumption since critical configurations are normally defined without taking into account the names of nonces used in a particular plan, but only how they relate in a configuration to the initial set of symbols in Σ and amongst themselves. For instance, in the medical example above consider the following configuration $\{Paul(n_1, hist), Sec(n_1, hist), Sec(n_1, paul)\}$. This configuration is critical because the secretary knows Paul's medical history, $hist$, since she knows his idendity number, denoted by the nonce n_1, and the medical history associated to this identifier. Using the same reasoning, one can easily check that the configuration resulting from renaming n_1 is also critical. This paper additionally assumes that the initial and the goal configurations are also closed under renaming of nonces.

In [17, 18] several notions of plan compliances were proposed. Here, we consider only the weakest one, called *weak plan compliance*: Given a local state transition system T, an initial configuration W, a (partial) goal configuration Z, and a set of critical configurations, is there a compliant plan which leads from W to Z?

Regarding protocol security, we will be interested in the following *secrecy problem*, which is basically an instantiation of the weak plan compliance problem with no critical configurations: Is there a plan from the initial configuration to a configuration in which the adversary M owns the fact $M(s)$ where s is a secret originally owned by another participant? It is interesting to note that this problem can also be seen as a kind of dual to the weak plan compliance problem; is there a plan from the initial configuration to a *critical configuration* in which the adversary M owns the fact $M(s)$ where s is a secret originally owned by another participant?

3 Formalizing Freshness for LSTSes with Balanced Actions

In principle a plan can be exponentially long. Consider the following example encoding the Towers of Hanoi puzzle.[1]

Example 1. Towers of Hanoi is a well-known mathematical puzzle. It consists of three pegs b_1, b_2, b_3 and a number of disks a_1, a_2, a_3, ... of different sizes which can slide onto any peg. The puzzle starts with the disks neatly stacked in ascending order of size on one peg, the smallest disk at the top. The objective is to move the entire stack stacked on one peg to another peg, obeying the following rules:

(a) Only one disk may be moved at a time.
(b) Each move consists of taking the upper disk from one of the pegs and sliding it onto another peg, on top of the other disks that may already be present on that peg.
(c) No disk may be placed on top of a smaller disk.

The puzzle can be played with any number of disks and it is known that the minimal number of moves required to solve a Tower of Hanoi puzzle is $2^n - 1$, where n is the number of disks.

The problem can be represented by an LSTS: We introduce the type $disk$ for the disks, type $diskp$ for either disks or pegs, with $disk$ being a subtype of $diskp$. The constants a_1, a_2, a_3, ..., a_n are of type $disk$ and b_1, b_2, b_3 of type $diskp$. We use facts of the form $On(x, y)$, where x is of type $disk$ and y is of type $diskp$, to denote that the disk x is either on top of the disk or on the peg y, and facts of the form $Clear(x)$, where x is of type $diskp$, to denote that the top of the disk x is clear, *i.e.*, no disk is on the top of or on x, or that no disk is on the peg x. Since disks need to be placed according to their size, we also use facts of the form $S(x, y)$, where x is of type $disk$ and y is of type $diskp$, to denote that the disk x can be put on top of y. In our encoding, we make sure that one is only allowed to put a disk on top of a larger disk or on an empty peg, *i.e.*, that x is smaller than y in the case of y being a disk. This is encoded by the following facts in the initial configuration:

$$\begin{array}{rcl} S(a_1,a_2)\ S(a_1,a_3)\ S(a_1,a_4) & \dots & S(a_1,a_n)\ S(a_1,b_1)\ S(a_1,b_2)\ S(a_1,b_3) \\ S(a_2,a_3)\ S(a_2,a_4) & \dots & S(a_2,a_n)\ S(a_2,b_1)\ S(a_2,b_2)\ S(a_2,b_3) \\ & \vdots & \\ S(a_{n-1},a_n)\ S(a_{n-1},a_n)\ S(a_{n-1},b_1)\ S(a_{n-1},b_2)\ S(a_{n-1},b_3) & & \end{array}$$

The initial configuration also contains the facts that describe the initial placing of the disks:

$$On(a_1,a_2)\ On(a_2,a_3) \dots On(a_{n-1},a_n)\ On(a_n,b_1)$$
$$Clear(a_1)\ \ Clear(b_2)\ \ Clear(b_3)\,,$$

The goal configuration consists of the following facts and encodes the state where all the disks are stacked on the peg b_3:

$$On(a_1,a_2)\ On(a_2,a_3) \dots On(a_{n-1},a_n)\ On(a_n,b_3)$$
$$Clear(a_1)\ \ Clear(b_1)\ \ Clear(b_2)$$

[1] In the technical report [15], we also provide an example of an exponentially long anomaly in the context of protocol security.

Finally, the only action in our system is:

$$Clear(x)\ On(x,y)\ Clear(z)\ S(x,z) \rightarrow Clear(x)\ Clear(y)\ On(x,z)\ S(x,z)$$

where x has type $disk$, while y and z have type $diskp$. Notice that the action above is balanced. This action specifies that if there is a disk, x, that has no disk on top, it can be either moved to the top of another disk, z, that also has no disk on top, provided that x is smaller than y, specified by predicate $S(x,z)$, or onto a clear peg.

The encoding above of the Towers of Hanoi illustrates that plans can be exponentially long. Moreover, we can easily adapt that example to illustrate that in such plans exponentially many fresh values are used.

Example 2. We modify the LSTS used before to model the Towers of Hanoi puzzle so that each move is identified/accompanied by replacing an old identifier, t, with a fresh identifier, t':

$$P(t)\ Clear(x)\ On(x,y)\ Clear(z)\ S(x,z) \rightarrow \exists t'.P(t')\ Clear(x)\ Clear(y)\ On(x,z)\ S(x,z)$$

As already stated, given n disks, all plans must be of the exponential length $2^n - 1$, at least. Consequently, within the modified version, a plan includes an exponential number of fresh values.

The use of an exponential number of fresh values seems to prelude PSPACE membership of the secrecy and weak compliance problems. We circumvent this problem by showing how to reuse obsolete constants instead of updating with fresh values.

Consider as an intuitive example the scenario where customers are waiting at a counter. Whenever a new customer arrives, he picks a number and waits until his number is called. Since only one person is called at a time, usually in a first come first serve fashion, a number that is picked has to be a fresh value, that is, it should not belong to any other customer in the waiting room. However, since only a bounded number of customers wait at the counter in a period of time, one only needs a bounded number of tickets: once a customer is finished, his number can be in fact reused and assigned to another customer.

We can generalize the idea illustrated by the above example to systems with balanced actions. Since in such systems all configurations have the same number of facts and the size of facts is bounded, in practice we do not need an unbounded number of new constants in order to reach a goal, but just a small number of them. This is formalized by the following theorem:

Theorem 1. *Given an LSTS with balanced actions that can update nonces, any plan leading from an initial configuration W to a partial goal Z can be transformed into another plan also leading from W to Z that uses only a polynomial number of nonces with respect to the number of facts in W and an upper bound on the size of facts.*

The proof of Theorem 1 relies on the observation that from the perspective of an insider of the system two configurations can be considered the same whenever they only differ in the names of the nonces used. Consider for example the following two configurations, where the n_is are nonces and t_is are constants in the initial signature:

$\{A(t_1,n_1), B(n_2,n_1), C(n_3,t_2)\}$ and $\{A(t_1,n_4), B(n_5,n_4), C(n_6,t_2)\}$

Since these configurations only differ in the nonce's names used, they can be regarded as equivalent: the same fresh value, n_1 in the former configuration and n_4 in the latter, is shared by the agents A and B, and similarly, for the new values n_2 and n_5, and n_3 and n_6. Inspired by a similar notion in λ-calculus [8], we say that these configurations above are α-equivalent.

Definition 3. *Two configurations $\mathcal{S}_1$ and $\mathcal{S}_2$ are* α-equivalent*, denoted by $\mathcal{S}_1 =_\alpha \mathcal{S}_2$, if there is a bijection σ that maps the set of all nonces appearing in one configuration to the set of all nonces appearing in the other configuration, such that the set $\mathcal{S}_1\sigma = \mathcal{S}_2$.*

The two configurations given above are α-equivalent because of the following the bijection $\{(n_1, n_4), (n_2, n_5), (n_3, n_6)\}$. It is easy to show that the relation $=_\alpha$ is indeed an equivalence, that is, it is symmetric, transitive, and reflexive.

The following lemma formalizes the intuition described above, namely, from the point of view of an insider two α-equivalent configurations are the same. That is, one can apply the same action to one or the other and the resulting configurations are also equivalent. This is similar to the notion of bisimulation in process calculi [20].

Lemma 1. *Let m be the number of facts in a configuration $\mathcal{S}_1$ and a be an upper bound on the size of facts. Let $\mathcal{N}_{m,a}$ be a fixed set of $2ma$ nonce names. Suppose that the configuration S_1 is α-equivalent to a configuration S_1' and, in addition, each of the nonce names occurring in S_1' belongs to $\mathcal{N}_{m,a}$. Let an instance of the action r transform the configuration S_1 into the configuration S_2. Then there is a configuration S_2' such that: (1) an instance of action r transforms S_1' into S_2'; (2) S_2' is α-equivalent to S_2; and (3) each of the nonce names occurring in S_2' belongs to $\mathcal{N}_{m,a}$.*

Proof The most interesting case is when a rule updates nonces. Let r be a balanced action that updates nonces. Suppose that some occurrences of nonces $\boldsymbol{n_1}$ within S_1 are updated with fresh nonces $\boldsymbol{n_2}$ resulting in S_2. Note that other places may still keep some of these *old* nonces $\boldsymbol{n_1}$. Take the corresponding occurrence of say $\boldsymbol{n_1}\sigma$ in S_1' (in accordance with our α-equivalence). Since the number of all places is bounded by ma, we can find enough elements (at most ma in the extreme case where all nonces are supposed to be updated simultaneously) $\boldsymbol{n_2'}$ in $\mathcal{N}_{m,a}$ that do not occur in $\mathcal{S}_1'$. We update the particular occurrences in question with $\boldsymbol{n_2'}$, resulting in the desired S_2'. Moreover, from the assumption that critical configurations are closed under renaming of nonces and that S_2 is not critical, the configuration S_2' is also not critical. □

We are now ready to prove Theorem 1:

Proof (of Theorem 1). The proof is by induction on the length of a plan and it is based on Lemma 1. Let T be a LSTS with balanced actions that can update nonces, m the number of facts in a configuration, and a the bound on size of each fact. Let $\mathcal{N}_{m,a}$ be a fixed set of $2ma$ nonce names. Given a plan P leading from W to a partial goal Z we adjust it so that all nonces updated along the plan P are taken from $\mathcal{N}_{m,a}$.

For the base case, assume that the plan is of the length 0, that is, the configuration W already contains Z. Since we assume that goal and initial configurations are closed under renaming of nonces, we can rename the nonces in W by nonces from $\mathcal{N}_{m,a}$. The resulting plan is compliant.

Assume that any plan of length n can be transformed into a plan that uses the fixed number of nonces. Let a plan P of the length $n+1$ be such that $W \rhd_T^* ZU$. Let r be the last action in P and $Z_1 \rhd_r ZU$. By induction hypothesis along $W \rhd_T^* Z_1$, we only

have nonces from the set $\mathcal{N}_{m,a}$. We can then apply Lemma 1 to the configuration Z_1 and conclude that all nonces in ZU belong to $\mathcal{N}_{m,a}$. Therefore all nonces updated along the plan P are taken from $\mathcal{N}_{m,a}$. Notice that no critical configuration is reached in this process, since we assume that critical configurations are closed under nonce names. □

Corollary 1. *For LSTSes with balanced actions that can update nonces, we only need to consider the planning problem with a polynomial number of* fresh *nonces, which can be fixed in advance, with respect to the number of facts in the initial configuration and the upper bound on the size of facts.*

4 Complexity Results

We start this section by improving the PSPACE lower-bound in [18, Theorem 6.1] for the weak plan compliance and secrecy problems. While their result allowed LSTSes with any type of balanced actions, we show next that the weak plan compliance and secrecy problems are also PSPACE-hard for LSTSes with balanced actions that can modify a single fact and in the process check whether another fact is present in the configuration. The main challenge here is to simulate operations over a non-commutative structure by using a commutative one. *i.e.*. to simulate the behavior of a Turing machine that uses a sequential, non-commutative tape in our formalism that uses commutative multisets. Please note that in this theorem no nonce updates are allowed.

Theorem 2. *Given an LSTS with only actions of the form $ab \rightarrow a'b$, the weak plan compliance problem and the secrecy problem are PSPACE-hard.*

The PSPACE upper bound for this problem can be inferred directly from [18].

Proof In order to prove the lower bound, we encode a non-deterministic Turing machine $\mathcal{M}$ that accepts in space n within actions of the form $ab \rightarrow a'b$.

In our proof, we do not use critical configurations and we need just one agent A.

For each n, we design a local state transition system T_n as follows:

First, we introduce the following propositions: $R_{i,\xi}$ which denotes that *"the i-th cell contains symbol ξ"*, where $i=0,1,..,n+1$, ξ is a symbol of the tape alphabet of $\mathcal{M}$, and $S_{j,q}$ denotes that *"the j-th cell is scanned by $\mathcal{M}$ in state q"*, where $j=0,1,..,n+1$, q is a state of $\mathcal{M}$. Assume without loss of generality that $\mathcal{M}$ has only one *accepting* state, q_f, and that all *accepting* configurations in space n are of one and the same form.

Given a *machine configuration* of $\mathcal{M}$ in space n such that $\mathcal{M}$ scans j-th cell in state q and that the string $\xi_0\xi_1\xi_2..\xi_i..\xi_n\xi_{n+1}$ is written left-justified on the tape, we will represent such a state of the machine by a configuration of T_n of the form (here ξ_0 and ξ_{n+1} are the end markers):

$$S_{j,q}R_{0,\xi_0}R_{1,\xi_1}R_{2,\xi_2}\cdots R_{n,\xi_n}R_{n+1,\xi_{n+1}}. \tag{1}$$

Second, each instruction γ in $\mathcal{M}$ of the form $q\xi \rightarrow q'\eta D$, denoting *"if in state q looking at symbol ξ, replace it by η, move the tape head one cell in direction D along the tape, and go into state q'"*, is specified by the set of $5(n+2)$ actions of the form:

$$\begin{array}{lll} S_{i,q}R_{i,\xi} \rightarrow_A F_{i,\gamma}R_{i,\xi}, & F_{i,\gamma}R_{i,\xi} \rightarrow_A F_{i,\gamma}H_{i,\gamma}, & F_{i,\gamma}H_{i,\gamma} \rightarrow_A G_{i,\gamma}H_{i,\gamma}, \\ G_{i,\gamma}H_{i,\gamma} \rightarrow_A G_{i,\gamma}R_{i,\eta}, & G_{i,\gamma}R_{i,\eta} \rightarrow_A S_{i_D,q'}R_{i,\eta}, & \end{array} \tag{2}$$

where $i=0,1,..,n+1$, $F_{i,\gamma}$, $G_{i,\gamma}$, $H_{i,\gamma}$ are auxiliary atomic propositions, $i_D := i+1$ if D is *right*, $i_D := i-1$ if D is *left*, and $i_D := i$, otherwise.

The idea behind this encoding is that by means of such five actions, applied in succession, we can simulate any successful non-deterministic computation in space n that leads from the initial configuration, W_n, with a given input string $x_1x_2..x_n$, to the accepting configuration, Z_n.

The *faithfulness* of our encoding heavily relies on the fact that any machine configuration includes exactly one machine state q. Namely, because of the specific form of our actions in (2), any configuration reached by using a plan $\mathcal{P}$, leading from W_n to Z_n, has exactly one occurrence of either $S_{i,q}$ or $F_{i,\gamma}$ or $G_{i,\gamma}$. Therefore the actions in (2) are necessarily used one after another as below:

$S_{i,q}R_{i,\xi} \rightarrow_A F_{i,\gamma}R_{i,\xi} \rightarrow_A F_{i,\gamma}H_{i,\gamma} \rightarrow_A G_{i,\gamma}H_{i,\gamma} \rightarrow_A G_{i,\gamma}R_{i,\eta} \rightarrow_A S_{i_D,q'}R_{i,\eta}$. Moreover, any configuration reached by using the plan $\mathcal{P}$ is of the form similar to (1), and, hence, represents a configuration of $\mathcal{M}$ in space n.

Passing through this plan $\mathcal{P}$ from its last action to its first v_0, we prove that whatever intermediate action v we take, there is a successful non-deterministic computation performed by $\mathcal{M}$ leading from the configuration reached to the *accepting* configuration represented by Z_n. In particular, since the first configuration reached by $\mathcal{P}$ is W_n, we can conclude that the given input string $x_1x_2..x_n$ is accepted by $\mathcal{M}$. □

We turn our attention to the case when actions can update nonces. We show that the weak plan compliance and the secrecy problems for LSTSes with balanced actions that can update nonces are in PSPACE. From Theorem 2, we can infer that these problems are indeed PSPACE-complete.

To determine the existence of a plan we only need to consider plans that never reach α-equivalent configurations more than once. If a plan loops back to a previously reached configuration, there is a cycle of actions which could have been avoided. The following lemma imposes an upper bound on the number of different configurations given an initial finite signature. Such a upper bound provides us with the maximal length of a plan one needs to consider.

Lemma 2. *Given an LSTS T under a finite signature Σ, then the number of configurations with m facts (counting repeatitions), $L_T(m, a)$, that are pairwise not α-equivalent is such that $L_T(m, a) \leq J^m(D + 2ma)^{ma}$, where J and D are, respectively, the number of predicate and the number of constant and function symbols in the initial signature Σ; and a is an upper bound on the size of facts.*

Proof There are m slots for predicate names and at most ma slots for constants and function symbols. Constants can be either constants in the initial signature Σ or nonce names. Following Theorem 1, we need to consider only $2ma$ nonces. □

Clearly, the upper bound above on the number of configurations is an overestimate. It does not take into account, for example, the equivalence of configurations that only differ in the order of facts. For our purposes, however, it will be enough to assume such a bound. In particular, we show next that the secrecy problem for LSTSes with balanced actions that can update nonces is in PSPACE.

Although the secrecy problem is stated as a decision problem, we prove more than just PSPACE decidability. Ideally we would also be able to generate a plan in PSPACE when there is a solution. Unfortunately, the number of actions in the plan may already be exponential in the size of the inputs, see Example 1, precluding PSPACE membership of plan generation. For this reason we follow [18] and use the notion of "scheduling" a plan in which an algorithm will also take an input i and output the i-th step of the plan.

Definition 4. *An algorithm is said to* schedule *a plan if it (1) finds a plan if one exists, and (2) on input i, if the plan contains at least i actions, then it outputs the i^{th} action of the plan, otherwise it outputs* no.

Following [18], we assume that when given an LSTS, there are three programs, $\mathcal{C}, \mathcal{G}$, and $\mathcal{T}$, such that they return the value 1 in polynomial space when given as input, respectively, a configuration that is critical, a configuration that contains the goal configuration, and a transition that is valid, that is, an instance of an action in the LSTS, and return 0 otherwise.

Theorem 3. *The weak compliance problem and the secrecy problem for LSTSes with balanced actions that can update nonces are in PSPACE.*

Proof Assume as inputs an initial configuration W containing m facts, an upper bound, a, on the size of facts, programs $\mathcal{G}, \mathcal{C}$, and $\mathcal{T}$, as described above, and a natural number $0 \leq i \leq L_T(m, a)$.

We modify the algorithm proposed in [18] in order to accommodate the updating of nonces. The algorithm must return "yes" whenever there is compliant plan from the initial configuration W to a goal configuration, that is, a configuration S such that $\mathcal{G}(S) = 1$. In order to do so, we construct an algorithm that searches non-deterministically whether such configuration *is reachable*. Then we apply Savitch's Theorem to determinize this algorithm.

The algorithm begins with $W_0 := W$. For any $t \geq 0$, we first check if $\mathcal{C}(W_t) = 1$. If this is the case, then the algorithm outputs "no". We also check whether the configuration W_t is a goal configuration, that is, if $\mathcal{G}(W_t) = 1$. If so, we end the algorithm by returning"yes". Otherwise, we guess a transition r such that $\mathcal{T}(r) = 1$ and that is applicable using the configuration W_t. If no such action exists, then the algorithm outputs "no". Otherwise, we replace W_t by the configuration W_{t+1} resulting from applying the action r to W_t. Following Lemma 2 the goal configuration is reached in at most $L_T(m, a)$ steps. We use a global counter, called step-counter, to keep track of the number of actions used in a partial plan constructed by this algorithm.

In order to accommodate nonce update, we need a way to enforce that whenever an action updates nonces, these are considered fresh. This is done, as in the proof of Theorem 1, by replacing the relevant nonce occurrence(s) with nonces from a fixed set of nonce names so that they are different from any of the nonces in the enabling configuration.

We now show that this algorithm runs in polynomial space. We start with the step-counter: The greatest number reached by this counter is $L_T(m, a)$. When stored in binary encoding, this number takes only space polynomial to the given inputs:

$$\begin{aligned}\log_2(L_T(m,a)) \;&\leq\; \log_2(J^m(D+2ma)^{ma}) = \log_2(J^m) + \log_2((D+2ma)^{ma}) \\ &= m\log_2(J) + ma\log_2(D+2ma).\end{aligned}$$

Therefore, one only needs polynomial space to store the values in the step-counter.

Following Theorem 1 there are at most polynomialy many nonces updated in any run, namely at most $2ma$. Hence nonces can also be stored in polynomial space.

We must also be careful to check that any configuration, W_t, can also be stored in polynomial space with respect to the given inputs. Since our system is balanced and we assume that the size of facts is bounded, the size of a configuration remains the same throughout the run. Finally, the algorithm needs to keep track of the action r guessed when moving from one configuration to another and for the scheduling of a plan. It has

Table 1. Summary of the complexity results for the weak compliance and the secrecy problems. We mark the new results appearing here with a $\star$.

		Weak Plan Compliance Problem	**Secrecy Problem**
Balanced Actions	Bounded N^{o} of Nonces	PSPACE-complete [18]	PSPACE-complete [18]
	Any N^{o} of Nonces	PSPACE-complete*	PSPACE-complete*
Possibly Unbalanced Actions		Undecidable [17]	Undecidable [12]

to store the action that has been used at the i^{th} step. Since any action can be stored by remembering two configurations, one can also store these actions in space polynomial to the inputs.

A similar algorithm can be used for the secrecy problem. The only modification to the previous algorithm is that one does not need to check for critical configurations as in the secrecy problem there are no such configurations. □

Our PSPACE-complete result contrast with results in [12], where the secrecy problem is shown to be undecidable. Although they also impose an upper bound on the size of facts, they did not restrict the actions of their systems to be balanced. Therefore, it is possible for their intruder to remember an unbounded number of facts, while the memory of all our agents is bounded. Moreover, in the DEXPTIME result in [12] a fixed bound is imposed on the number of nonces that can be created, whereas we do not impose such a bound here.

Table 1 summarizes the complexity results for the weak plan compliance and secrecy problems.

5 Application: Protocol theories with a bounded memory intruder

Although the results above were motivated by collaborative systems, we then noticed that our results have important consequences for protocol security analysis. In particular, we show that many protocol anomalies, such as Lowe's anomaly [19], can also occur when using our bounded memory adversary. We assume that the reader is familiar with such anomalies, see [9, 12, 19]. The complete details appear in [15].

As in [12], we assume that all messages are transmitted by passing first through the intruder, that is, the intruder acts as the network of the system. We use the public predicate names N_S and N_R to denote messages that are, respectively, sent from an agent to the intruder and from the intruder to another agent. On the other hand, the predicates C, D, and M are private to intruder. The first two are used when he is composing and decomposing messages, respectively, while the third predicate is used to denote some data learned by the intruder. Since the memory of agents is bounded, it is important to keep track of how many facts they can store. In particular, the public fact $P(*)$ denotes a free memory-slot available to any agent and the private fact $R(*)$ denotes a free memory-slot available only to the intruder. The use of the two distinct facts for free memory-slots helps us to formalize precise upper-bounds on the space needed by the intruder to realize an anomaly, see [15]. There, we also prove that the secrecy problem is PSPACE-hard when using intruder models, similar to those in [12], but that contain only balanced actions.

Table 2. Table containing the total number of facts, the number of $R(*)$ facts, and the largest size of facts needed to encode protocol runs and known anomalies when using LSTSes with balanced actions. The largest size of facts needed to encode an anomaly is the same as in the corresponding normal run of the protocol. In the cases for the Otway-Rees and the Kerberos 5 protocols, we encode different anomalies, which are identified by the numbering, as follows: [(1)] The type flaw anomaly in [9]; [(2)] The replay anomaly from [24]; [(3)] The ticket anomaly and [(4)] the replay anomaly in [5]; [(5)] The PKINIT anomaly also for Kerberos 5 described in [6].

Protocol		Needham Schroeder	Yahalom	Otway Rees	Woo Lam	Kerberos 5	PKINIT[(5)]
Normal	N° of facts	9	8	8	7	15	18
	Size of facts	6	16	26	6	16	28
Anomaly	N° of facts	19	15	$11^{(1)}$, $17^{(2)}$	8	$22^{(3)}$, $20^{(4)}$	31
	N° of $R(*)$	7	9	$5^{(1)}$, $9^{(2)}$	2	$9^{(3)}$, $4^{(4)}$	10

We use balanced actions to model the intruder's actions. In particular, our bounded memory Dolev-Yao intruder is also two-phased [12], that is, he first decomposes messages that are intercepted in the network and only then he starts composing new messages. For example, the following rules belong to the intruder:

REC : $N_S(x)R(*) \rightarrow D(x)P(*)$ SND : $C(x)P(*) \rightarrow N_R(x)R(*)$
DCMP : $D(\langle x, y\rangle)R(*) \rightarrow D(x)D(y)$ COMP : $C(x)C(y) \rightarrow C(\langle x, y\rangle)R(*)$
USE : $M(x)R(*) \rightarrow C(x)M(x)$ LRN : $D(x) \rightarrow M(x)$
GEN : $R(*) \rightarrow \exists n.M(n)$

The rules REC and SND specify, respectively, the intruder's actions of intercepting a message from and sending a message to the network. The rules DCMP and COMP specify the intruder's actions of decomposing and composing messages. The rules USE and LRN specify the intruder's actions of using known data to compose a message and learn some data from an intercepted message. Finally, the rule GEN specifies that the intruder can update with fresh values. Notice the role of the facts $P(*)$ and $R(*)$ in the rules. For instance, in the REC rule, when the intruder intercepts a message from the network, one of the the intruder's free memory slots, $R(*)$, is replaced by a decomposable fact, $D(x)$, while the fact representing data on the network, $N_S(x)$, is replaced by a free memory slot, $P(*)$, belonging to the other agents. The intruder is not allowed to intercept a new network fact if he does not have any free memory slot left.

Therefore, differently from [12] where the intruder had only *persistent* facts, the bounded memory intruder might have to forget data. That is, he has actions that replace some facts stored in his memory with the empty fact $R(*)$, allowing hence the adversary to store eventually new information. For instance, the following rule specifies the intruder's action of forgeting data known to the intruder: $M(x) \rightarrow R(*)$. The complete set of rules for the adversary, including rules involving encryption and decryption, is given in [15].

Regarding protocol anomalies, the main observation is that when the adversary has enough $R(*)$ facts, then anomalies can also occur using adversaries with bounded memory. We believe that this is one reason for the successful use in the past years of model checkers for protocol verifications. In the technical report [15], we show that many anomalies can be realized using our bounded memory intruder. Table 2 summarizes the number of $P(*)$ and $R(*)$ facts and the upper bound on the size of facts needed to

encode normal runs, where no intruder is present, and to encode the anomalies where the bounded memory intruder is present. We specify protocols using rules that handle encryption and decryption, as in [12]. For instance, to realize the Lowe anomaly to the Needham-Schroeder protocol, the intruder requires only seven $R(*)$ facts.[2]

Since all players in our system have bounded memory, the role generation phase in well-founded theories [12] necessarily yields a bounded number of protocols roles in our system, using here the terminology from [12]. This is because in such theories all protocol roles that are used in a run are created at the beginning. Since the size of configurations when using balanced actions is bounded, the number of roles that can be created is also bounded. Thus, under well founded theories, our PSPACE upper bound result (Theorem 3) reduces to the NP upper bound from [12, Theorem 3]. We therefore do not use well-founded theories, but rather allow protocol roles to be created not necessarily at the beginning of the run, but also after a protocol session is finished. Once a protocol session is finished it can be deleted, creating a free memory slot to be (possibly) used to create new protocol roles. Existing protocol analysis tools seem to proceed in a similar fashion.

6 Related Work

As previously discussed, we build on the framework described in [18, 17]. In particular, here we investigate the use of actions that can update values with nonces, providing new complexity results for the partial reachability problem. In [3, 4], a temporal logic formalism for modeling organizational processes is introduced. In their framework, one relates the scope of privacy to the specific roles of agents in the system. We believe that our system can be adapted or extended to accommodate such roles depending on the scenario considered.

In [22], Roscoe formalized the intuition of reusing nonces to model-check protocols where an unbounded number of nonces could be used, by using methods from data independence. We confirm his initial intuition by providing tight complexity results and demonstrating that many protocol anomalies can be specified when using our model that reuses nonces.

Harrison *et al.* present a formal approach to access control [14]. In their proofs, they faithfully encode a Turing machine in their system. However, in contrast to our encoding, they use a non-commutative matrix to encode the sequential, non-commutative tape of a Turing machine. We, on the other hand, encode Turing machine tapes by using commutative multisets. Specifically, they show that if no restrictions are imposed to the systems, the reachability problem is undecidable. However, if actions are not allowed to update values with fresh ones, then they show that the same problem is PSPACE-complete. Furthermore, if actions can delete or insert exactly one fact and in the process one can also check for the presence of other facts and even update values with nonces, then they show the problem is NP-complete, but in their proof they implicitly impose a bound on the number of nonces that can be used. In their proofs, the non-commutative nature of their encoding plays an important role.

Our paper is closely related to frameworks based on multiset rewriting systems used to specify and verify security properties of protocols [1, 2, 7, 10, 12, 23]. While

[2] Notice that here we only encode standard anomalies described in the literature [5, 9, 24]. This does not mean, however, that there are not any other anomalies that can be carried out by an intruder with less memory, that is, with less $R(*)$ facts.

here we are concerned with systems where agents are in a *closed room* and collaborate, in those papers, the concern was with systems in an *open room* where an intruder tries to attack the participants of the system by manipulating the transmitted messages. This difference is reflected in the assumptions used by the frameworks. In particular, the security research considers a powerful intruder that has an unbounded memory and that can, for example, accumulate messages at will. On the other hand, we assume here that each agent has a bounded memory, technically imposed by the use of balanced actions.

Much work on reachability related problems has been done within the Petri nets (PNs) community, see *e.g.*, [13]. Specifically, we are interested in the *coverability problem* which is closely related to the partial goal reachability problem in LSTSes [17]. To our knowledge, no work that captures exactly the conditions in this paper has yet been proposed. For instance, [13, 21] show that the coverability problem is PSPACE-complete for 1-conservative PNs. While this type of PNs is related to LSTSes with balanced actions, it does not seem possible to provide direct, *faithful* reductions between LSTSes and PNs in this case.

7 Conclusions and Future Work

This paper extended existing models for collaborative systems with confidentiality policies to include actions that can update values with fresh ones. Then, given a system with balanced actions, we showed that one only needs a polynomial number of constants with respect to the number of facts in the initial configuration and an upper bound on the size of facts to formalize the notion of fresh values. Furthermore, we proved that the weak plan compliance problem and the secrecy problem for systems with balanced actions that can update values with fresh ones are PSPACE-complete. As an application of our results, we showed that a number of anomalies for traditional protocols can be carried by a bounded memory intruder, whose actions are all balanced.

There are many directions to follow from here, which we are currently working on. Here, we only prove the complexity results for the secrecy problem. We are searching for complexity bounds for the weak plan compliance and other policy compliances problems proposed in [17]. We would also like to understand better the impact of our work to existing protocol analysis tools, in particular, our PSPACE upper-bound result. Moreover, we are currently working on determining more precise bounds on the memory needed by an intruder to find an attack on a given protocol. Finally, despite of our idealized model, we believe that the numbers appearing in Table 2 provide some measure on the security of protocols. In general, we seek to provide further quantitative information on the security of protocols. Some of these parameters appear in existing model checkers, such as Murϕ. We are investigating precise connections to such tools.

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