Target Temperature Effect on Eddy-Current Displacement Sensing

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Abstract—Target temperature effect on eddy current displacement sensing is analyzed and evaluated by simulation. The equivalent target quality factor is detected as the main factor, along with the probe equivalent quality factor, determines this effect. It manifests in ambiguity of displacement measurement, as well as, masking the displacement variation by target temperature variation, and vice versa. The analysis and the simulation show that there is an optimal operating frequency for minimum sensitivity over an acceptable displacement range.

Keywords—eddy-currents; displacement; sensing; target; temperature

I. INTRODUCTION

Among various approaches to improvement of performance and efficiency of power turbomachinery (e.g. turbines, compressors, pumps), the reduction of rotor-to-stator clearances, deflections, vibration, displacements of shafts, bearings and seals [1] is one of the goals in contemporary turbomachinery design trends. Measuring and monitoring the reduced clearances and displacements are vital for optimal exploitation of these machines, as well as, for preventing catastrophic failures like rotor-to-stator rub, vibration caused bearing damage etc. The eddy current displacement transducers have been used successfully for this task for many decades and this method and the corresponding commercial equipment are considered to be a reliable and matured technology. However, its application has been so far limited to rather narrow temperature range, typically -50°C to +200°C [2]. New demands, like measurement of gas turbine disc axial movement and rotor trajectory in the engine core, close to the combustion chamber; blade tip clearance over shrouded turbine rotors [3]; bearing clearances in rocket engine cryogenic fuel pump [4] etc., push the limits of required temperature range to extremes (-252°C for a liquid hydrogen pump, over 500°C for a gas or steam turbine). Development of the transducer system for such a wide temperature range is very challenging and the effect of temperature in various aspects must be analyzed very carefully. So far, only the temperature effect on transducer electronics [4], [10] and the eddy current displacement probe [4], [5] have been addressed, while the target temperature effect on measurement has not been widely recognized, although noticed [5], [6] and so far has been mostly ignored. The purpose of this paper is to provide an insight into the target temperature effect and to propose methods for minimization of this effect on eddy current displacement sensing.

II. PROBE-TARGET MODEL

A. Probe-target set-up

The only active part of the eddy current probe is a simple coil placed in the tip of the probe. The coil may have a ferromagnetic core, but usually it is coreless. In the following analysis and simulation the coil parameters are as for the probe referred to as “Probe No.2” in [8-10] (coreless, thin ring-shaped coil, coil mean radius \( R_{CM} = 3.46 \) mm, winding height \( h = 0.92 \) mm, winding width \( w = 0.8 \) mm, number of turns \( N_c = 70 \), Cu-wire diameter \( 0.1 \) mm, inductance \( L_0 = 51 \mu \text{H} \), Q-factor \( Q_0 = 14 \)@1MHz). The probe is placed perpendicularly to the target surface at varying displacement (Fig. 1). The inset in Fig. 1 is the close-up view of the probe tip, clearly showing the shape of the coil.

Fig. 1 Probe-target set-up, probe tip close-up view is in the inset.
B. Probe-target equivalent circuit

The coil turns are distributed over the coil width, \( w \) and the coil height, \( h \) (Fig. 2). Each turn carries the excitation current, \( I e^{j\omega t} \) which contributes to the vector magnetic potential, \( \mathbf{A}(r, \psi, t) \) in the target, where \( I \) is the amplitude and \( \omega \) is the frequency of the excitation current. The eddy current density \( \mathbf{J}(r, \psi, t) \) at a point in the target is with the target conductivity \( \sigma \) proportional to the time derivative of the vector magnetic potential at this point [9]

\[
\mathbf{J}(r, \psi, t) = -\sigma \frac{\partial \mathbf{A}(r, \psi, t)}{\partial t}. \tag{1}
\]

The solution for the total vector magnetic potential [9] shows that it has no radial component and that the azimuth component is independent of the azimuth at any point in the target. Therefore, the eddy currents induced in the target follow circular paths. Due to skin-effect, eddy currents penetrate the target to the equivalent depth \( \Delta \) (skin depth). For the purpose of discrete simulation, the whole target area carrying significant eddy currents is segmented into \( K \) rings of width \( \Gamma \) and thickness \( \Delta \). Each ring is characterized with its resistance and inductance which vary with the ring radius. Because there is no radial component of the induced eddy currents, each ring can be modeled as the single turn of the equivalent transformer secondary, loaded with the ring’s resistance and inductance, insulated from other rings. Consequently, mutual inductances exist between each of the probe coil \( N_c \) turns and each ring, as well as, between every turn of the coil and the rest of the turns, and also, between every ring and the rest of the rings. Such a magnetic coupling results in the probe-target distributed equivalent circuit in Fig. 3. \( C_T \) is the external capacitor added for tuning the equivalent probe impedance into resonance [9], [10]. All other parasitic capacitances are neglected in this model. Neglecting \( C_T \) for the moment (to be included later), the total voltage across the probe terminals is derived from the distributed equivalent circuit [9]

\[
V = (R_1 + j\omega L_1)I + U. \tag{2}
\]

where \( U \) is the total voltage induced in coil turns coupled with the eddy currents in the target, \( R_1 \) and \( L_1 \) are resistance and inductance of the coil far from target (no eddy currents influence)

\[
U = \sum_{\ell=0}^{N_c-1} u_\ell. \tag{3}
\]

\[
R_1 = \sum_{\ell=0}^{N_c-1} r_\ell. \tag{4}
\]

\[
L_1 = \sum_{\ell=0}^{N_c-1} l_\ell + \sum_{\ell=0}^{N_c-1} \sum_{\xi=0}^{N_c-1} e_{\ell\xi} \delta_{\ell\xi} \quad \ell \neq \xi. \tag{5}
\]

\[
u_\ell = j\omega \sum_{k=0}^{K-1} \sum_{\ell=0}^{N_c-1} M_{k\ell} i_k = -\omega e^{2} \sum_{k=0}^{K-1} \sum_{\ell=0}^{N_c-1} \sum_{\rho=0}^{L-1} M_{k\ell}\delta_{\rho} + j\omega \delta_{\rho}. \tag{6}
\]

Insertion of these summation results into (2), and application of the Ohm’s law yield the equivalent impedance of the probe

\[
\text{Fig. 3 Probe-target distributed equivalent circuit.}
\]

\[
\text{Fig. 4 Probe-target lumped equivalent circuit.}
\]
where from the probe-target lumped equivalent circuit can be derived as in Fig. 4. The equivalent impedance real and imaginary parts both consist of two components: resistance ($R_i$) and inductance ($L_i$) not dependent on mutual inductance between the probe and the target, i.e. intrinsic probe resistance and inductance far from target, and the resistance and capacitive reactance which reflect target equivalent impedance ($R_2 + j\omega L_2$) to the probe terminals by equivalent probe-target mutual inductance ($M$). Adding a proper valued $C_T$ parallel to the probe, tunes the probe into resonance. The resonant probe impedance is then purely resistive and equals
\[ \text{Re}(Z_{res}) = Z_{res} = \frac{R_2^2 + X_2^2}{R_{eq}}, \quad \text{Im}(Z_{res}) = 0. \]

The required $C_T$ reactance for resonant tuning equals
\[ X_T = -\frac{1}{\omega C_T} = -\frac{R_2^2 + X_2^2}{X_{eq}}. \]  

In practice, variation of $R_{eq}$ and $X_{eq}$ with displacement would require automatic resonance tuning [10].

### III. ANALYSIS

 Resistances $R_1$ and $R_2$ are directly temperature dependent through the corresponding resistance temperature coefficients of the materials they are made of. The probe coil is usually made of copper and its resistance temperature compensation has recently been approached by several research teams. As $R_1$ is in (8) the additive factor, the analysis of $R_{eq}$ temperature dependence can be separated into two independent analyses yielding two temperature sensitivities that can be summed into total sensitivity. In this paper only the sensitivity to target temperature is analyzed. The analysis and the accompanied simulation is performed for the target made of AISI4140 steel which is the common material used for calibration of the commercial eddy current displacement transducers [2]. In Fig.5 there are the resistivity and relative resistivity temperature coefficient for this material [7]. Data for low temperatures are not available, thus the analysis and simulation are restricted only to high temperatures (the most common case in practice).

Differentiation of (10) in respect to temperature ($T$) yields
\[ \frac{dZ_{res}}{dT} = \frac{(R_{eq}^2 - X_{eq}^2)\frac{dR_{eq}}{dT} + 2R_{eq}X_{eq}\frac{dX_{eq}}{dT}}{R_{eq}^2}. \]

Combining (8) and (9) follow the quotients (13) and (14)
\[ Q_{eq} = \frac{X_{eq}}{R_{eq}}, \]
\[ Q_2 = \frac{X_2}{R_2} = \frac{\alpha L_2}{R_2} = \frac{X_1 - X_{eq}}{R_{eq} - R_1}. \]

which are identified as the probe equivalent quality factor ($Q_{eq}$) and the target equivalent quality factor ($Q_2$). Inserting (13) and (14) into (12), yields relative probe impedance deviation in respect to target resistance temperature deviation
\[ \frac{dZ_{res}}{Z_{res}} = \frac{\left(\frac{X_{eq}}{R_{eq}}\right)^2}{\left(1 - Q_{eq}^2\right)\left(Q_2^2 - 1\right) + 4Q_{eq}Q_2} \cdot \frac{dR_2}{dT}. \]

Inserting (8) into (15) and dividing it by $R_2$ eliminates $X_M$ and yields the relative sensitivity of the probe impedance to the target resistance temperature sensitivity
\[ \frac{(dZ_{res}/Z_{res})/dT}{(dR_2/R_2)/dT} = S^P_{R_{eq}} = \frac{1 - \frac{X_1}{R_2}}{\frac{1 - Q_{eq}^2\left(Q_2^2 - 1\right) + 4Q_{eq}Q_2}{\left(1 - Q_{eq}^2\right)\left(1 + Q_2^2\right)}}. \]

Parameters $R_1, X_1, R_{eq}$ and $Q_{eq}$ can be easily determined by measurement: $R_1, X_1$ far from target and $R_{eq}, X_{eq}$ at selected probe-to-target displacements. Solving (16) for $S^P_{R_{eq}} = 0$ yields the optimum target quality factor for zero impedance temperature sensitivity
\[ Q_{2, opt} = \frac{Q_{eq} + 1}{Q_{eq} - 1}. \]

$Q_2$ can be adjusted by the selection of operating frequency, but unfortunately $Q_{eq}$ is also dependent on operating frequency and, as well as, on displacement. Thus, the selection of optimal frequency is also displacement sensitive what makes $S^P_{R_{eq}} = 0$ impossible simultaneously for all displacements. The goal is to achieve reasonably small sensitivity over the adequate displacement range.

### IV. SIMULATION

Experimental verification of the analysis results would require quite complicated test rig consisting of precision slide
table and controlled high temperature target heater (both commercially available), arranged in a manner where only the target would be heated while the probe would stay cool (how to guaranty that with the probe only few millimeters away from the hot target?). Therefore, instead of experimental verification, simulation was decided upon for the first confirmation of the analysis, leaving the experiments for the future work. All the simulation results to follow are for the “Probe No.2” [8-10]. The simulation model is the same as in [9] which validity is confirmed with probe voltage vs. displacement measurement [9]. In Fig. 6 there is the set of probe-displacement transfer curves evaluated for target temperature range 20°C - 600°C. It is apparent that there is a range of displacement values corresponding to each particular probe voltage. Therefore, there is ambiguity in measured displacement (over ±0.3mm or equivalently over ±5% at the displacement range limits). Another problem is that the change of actual displacement can be completely masked by the target temperature change, making the measurement useless [6]. In Fig. 7 there are the optimal target quality factors ($Q_2^{opt}$) for various displacements and temperatures calculated from (17), as well as, the equivalent target quality factors ($Q_2$) derived by simulation [9]. Although $Q_2^{opt}$ doesn’t change much with temperature, $Q_2$ changes considerably with both temperature and displacement. The ideal target temperature compensation would be achieved at points of intersection of optimal and equivalent target quality factor curves. Unfortunately, it is displacement dependent. The next step in search for optimum compensation is investigation of frequency influence. In Figs. 8 and 9 there are the simulation results for $S^\text{res}_{R_2}$ at 20°C, but for varying frequency and displacement. In Fig. 8 there is the sensitivity versus displacement with frequency as a parameter, while in Fig. 9 there is the sensitivity versus frequency with displacement as a parameter. It is apparent that for this particular probe the operating frequency of 500 kHz would be optimal. The resulting sensitivity is quite low over the whole displacement

![Fig. 6 Displacement-probe voltage transfer curves.](image1.png)

![Fig. 7 Optimal and equivalent target quality factors at 1 MHz.](image2.png)

![Fig. 8 Displacement dependence of normalized sensitivity to target temperature ($S^\text{res}_{R_2}$) at 20°C. Frequency is the parameter.](image3.png)

![Fig. 9 Frequency dependence of normalized sensitivity to target temperature ($S^\text{res}_{R_2}$) at 20°C. Displacement is the parameter.](image4.png)
range, restricted to 0.1 over the 1mm - 5 mm displacement range. Such a low sensitivity could be considered acceptable for industrial use.

V. FUTURE WORK

The frequency and displacement dependent differential sensitivities displayed in Figs. 8 and 9 are for temperature of 20°C. At other temperatures sensitivities will be different because all the parameters in (16) (except $R_t$) vary with the target temperature. The future work will concentrate on the simulation of the probe parameters at different temperatures and then repeating simulation with different frequencies for each of the selected temperatures. The result will hopefully indicate the optimal operating frequency for a particular probe over the selected range of temperatures.

The other aspect of the future work will be the search for a method for minimizing the ambiguity of the measured displacement and prevention of masking the displacement change with the target temperature variation. It will probably be based on multiple frequency operation and an adequate algorithm for measured data fusion.

VI. CONCLUSION

The effect of target temperature variation on eddy current displacement sensing is analyzed and evaluated by simulation. The equivalent target quality factor (the ratio of the equivalent inductive reactance to the equivalent resistance of the target area carrying significant eddy currents) is detected as the main factor that along with the probe equivalent quality factor determine the effect of the target temperature on eddy current displacement sensing. This effect manifests in ambiguity of displacement measurement, as well as, masking the displacement variation by target temperature variation, and vice versa. This is all due to the sensitivity of the probe impedance to the target temperature. The analysis and the simulation show that there is an optimal operating frequency for minimum sensitivity over an acceptable displacement range. Minimization of the sensitivity will automatically reduce the displacement ambiguity and possible masking, as well.

REFERENCES