Multiscale computational approach using strain gradient formulation at microlevel

Tomislav Lesičar, Jurica Sorić, Zdenko Tonković

Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb

Ivana Lučića 5, 10000 Zagreb, Croatia

[tomislav.lesicar@fsb.hr](mailto:tomislav.lesicar@fsb.hr), [jurica.soric@fsb.hr](mailto:jurica.soric@fsb.hr), [zdenko.tonkovic@fsb.hr](mailto:zdenko.tonkovic@fsb.hr),

A more realistic description of the deformation responses of heterogeneous materials demands more accurate modeling at both macroscopic and microscopic scales. The size, shape, spatial distribution, volume fraction and the properties of the constituents making up the microstructure have a significant impact on the material behavior observed at the macroscale. Strain localization phenomena and material softening as results of extreme loading conditions, may significantly decrease structural load-carrying capacity. Therefore, in order to assess structural integrity and reliability as well as to predict structural lifetime, an analysis on the microlevel is unavoidable.

Multiscale techniques employing several homogenization schemes have been proposed. The two-scale second-order homogenization approach has mostly been used, which requires *C*1 continuity in the discretization at macrolevel. The standard *C*0 continuity has been hold at microlevel, where the solution of the boundary value problem of the representative volume element (RVE) has been performed. However, this *C*1 - *C*0 transition has some shortcomings. The microlevel second-order gradient cannot be related to the macrolevel as volume average, and a modified second-order stress is extracted from the Hill-Mandel energy condition, which bring some inconsistences in the formulations and disturb accuracy. Furthermore, the localization and the material softening cannot be modeled at microlevel without loss of ellipticity of governing field equations.

The present contribution is concerned with a multiscale second-order computational homogenization algorithm employing *C*1 continuity at both macro- and microlevels under assumptions of small strains and linear elastic material behavior. Discretization is performed by means of the *C*1 continuity finite element developed using strain gradient theory. A new scale transition methodology is derived in which the volume average of the macrolevel variables prescribed at the microlevel is explicitly satisfied. The Hill Mandel condition yields the true state variables. The macroscopic consistent constitutive matrices are computed from the RVE global stiffness matrix using the standard procedures. The implemented strain gradient theory enables the modeling of damage response at the microstructural level, which is connected with strain localization and softening. The algorithms derived are implemented into FE software ABAQUS via user subroutines. The robustness and accuracy of the proposed homogenization approach is demonstrated by numerical examples.

# Acknowledgements

This work has been fully supported by Croatian Science Foundation under project Multiscale Numerical Modeling of Material Deformation Responses from Macro- to Nanolevel (2516).