Non-stationary Friedrichs systems

Krešimir Burazin
(joint work with Marko Erceg)

Thursday, 10:00

Abstract

A symmetric positive systems (also known as Friedrichs systems) consist of a first order system of partial differential equations (of a specific type) and an *admissible* boundary condition. They were introduced by Kurt Otto Friedrichs (1958) in order to treat the equations that change their type, like the equations modelling transonic fluid flow. This class of problems encompasses a wide variety of classical and neoclassical initial and boundary value problems for various linear partial differential equations.

More recently, Ern, Guermond and Caplain (CPDE, 2007) suggested another approach to the Friedrichs theory, which was inspired by their interest in the numerical treatment of Friedrichs systems. They expressed the theory in terms of operators acting in abstract Hilbert spaces and proved well-posedness result in this abstract setting. Although some evolution (non-stationary) problems can be treated within this framework, their theory is not suitable for problems like the initial-boundary value problem for the non-stationary Maxwell system, or the Cauchy problem for the symmetric hyperbolic system.

We develop an abstract theory for non-stationary Friedrichs systems that can address these problems as well. More precisely, we consider an abstract Cauchy problem in a Hilbert space, that involves a time independent abstract Friedrichs operator. We use the semigroup theory approach, and prove that the operator involved satisfies the conditions of the Hille-Yosida generation theorem. We also address the semilinear problem and apply the new results to symmetric hyperbolic systems, the unsteady Maxwell system, the unsteady div-grad problem, and the wave equation. The theory can be extended to the complex space setting, as well, with application to the Dirac system.