MODELLING IN TEACHING PROBABILITY AND STATISTICS AT THE FACULTIES OF TECHNOLOGY IN THE REPUBLIC OF CROATIA

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At all levels of teaching mathematics it is necessary to implement as many as possible examples of idealised real problems which require the application of mathematical knowledge and development of mathematical models. The course probability and statistics, taught at the faculties of technology in Croatia, should therefore include examples which are connected with technologies studied at respective faculties. This paper describes the method of developing new models that can be implemented in teaching probability and statistics at the faculties of technology. As the result of this method, the examples of new tasks are obtained, from the area of printing and graphic design, which are suitable for use in teaching at the Faculty of Graphic Arts of the University of Zagreb. The described approach to the teaching process is in line with the latest guidelines of the International Mathematical Union as well as the European Executive Education Agency.

Key words: education, modelling, probability and statistics, technology.

INTRODUCTION

The International Mathematical Union founded the ICMI (International Commission on Mathematical Instruction [1]) at the international congress of mathematicians in Rome in 1908. The aim of this organisation is to improve the standard of teaching mathematics in all countries of the world. The basic idea about teaching mathematical content represented by the ICMI can be expressed in two mottos: “via applications” and “by real world” [2-3]. The ICMI believes that it is necessary to bridge the gap between mathematical knowledge and the world around us. Mathematical modelling of “real problems” is the process of linking mathematical knowledge with situations from the “real life”. It is therefore necessary to harmonize the teaching of mathematical courses with the needs of individual or group learners of mathematical courses [4-6].

According to the document published by the European Executive Education Agency [7], all mathematical curricula of the
EU countries should include, among key mathematical competences, the application of mathematics in real life [8]. It is also recommended to use the real life context in teaching in order to make mathematics closer to students’ experience. The use of real life context should be one of especially prominent methods used in teaching mathematics.

The teaching process in courses probability and mathematical statistics at all level of education should include probability modelling of familiar random phenomena from all areas of real life [9] related to the examples from economy, social and cultural life [10]. It is appropriate to implement random phenomena models from famous sports or even games of chance [11] such as bingo, lottery, roulette or betting [12] into teaching lessons. This approach is especially important for the students of non-mathematical schools and faculties where probability and statistics is taught [13-15].

According to the above, there is the need to integrate real problems related to the respective areas of study into courses of probability and statistics at faculties [16-18]. There are numerous and detailed discussions about the specificities of teaching probability at different non-mathematical study programmes whereby attention should be paid to special needs of respective areas of study. For example, expectations from teaching probability to students of physics [19-20] differ from those for students of biology [21]. Many papers are published related to teaching probability and statistics to engineers [22-24].

Currently, the teaching of mathematics at the faculties of technology in Croatia rarely uses the examples and tasks directly related to other non-mathematical courses. Instead, the emphasis is put on theoretical mathematical aspects of probability and mathematical statistics. In this way students gain basic knowledge and develop abstract logical thinking. They are however not able to sufficiently use the knowledge in practice what is expected from them at the labour market. It is therefore necessary that teaching probability and statistics includes the examples related to the content of other non-mathematical courses. In this way the teaching process is created which links mathematical knowledge with real problems appearing in different technologies. Students acquire the skill of recognizing, describing and solving real technological problems by probability methods. Students educated in such way will be the leaders in developing science and new technologies in the region.

The described strategy is in line with the national framework curricula for pre-school education and general education in elementary and secondary schools in the Republic of Croatia [25]. The basic ideas from this document can also be applied to teaching mathematics at the faculties of technology.

STRATEGY AND AIMS OF MODELLING IN TEACHING PROBABILITY AND STATISTICS AT THE FACULTY OF GRAPHIC ARTS

For creating models in real context at the faculties of technology it is necessary to determine in what areas probability and statistical tools are applied within corresponding technological problems. It is thereby necessary to study expert and scientific literature from the respective area of technology.

The following text describes the method of creating examples from the real
context of graphic technology. Probability and statistics have an incredibly wide spectrum of applications within printing industry and graphic design. Only some of them are mentioned in this chapter. In printing, for example, Bayesian networks are applied when estimating the quality of graphical reproductions in dependence with the chosen print parameters [26-27]. Bayesian methodology is also used when determining limits of control cards for determining the quality of graphical reproductions [28]. Modern quality control of graphic products is based on six sigma methodology that is based on Gaussian probability distribution [29]. The control of production process within graphic industry uses control cards which use different probability distributions [30-31]. Chauchy distribution is very often used in graphic technology when describing noise and other similar phenomena used when analysing raster prints. One example is the approximation of subsurface light scattering in subsurface prints that is best modelled by Chauchy density function. The introduction of new graphic protection implies testing of statistical hypotheses among participants in a survey in order to raise the level of their efficiency [32-33]. In large printing companies it is impossible to imagine the organisation of production without statistical data analysis. Management of graphic products generally uses the results of statistical data processing as a form of help in reaching business decisions. The interpretation of results of laboratory measurements of graphic parameters with the aim of promoting printing process is not efficient without detailed statistical analyses. The regression analysis is used to determine curves describing the strength of book block [34-35].

As can be seen from the above text the labour market in graphic industry requires from graphic technology engineers the knowledge of probability and statistical tools. Therefore the teaching process should include more examples of probability and statistical modelling of real problems from graphic practice. Only in this way can students acquire the necessary experience in solving specific problems they will face in graphic industry where they will work. It is also necessary to educate students for the use of efficient statistical programme tools such as SPSS or STATISTICA 10.

The realisation of the described programme is only possible through active interdisciplinary cooperation of mathematics teachers with scientists from the area of graphic technology. When finding real examples from printing and graphic technology it is necessary to use the relevant scientific and expert literature from graphic technology and to find those examples where probability or statistical models are used. Models found in scientific or expert articles are usually too complex to be used in teaching. It is therefore necessary to idealize them and make them simpler and to present them in an appropriate way to students [36-38]. Students should also be included in finding examples and their probability and statistical modelling. Their task would be to actively participate in teaching through holding a number of interdisciplinary seminars. They would present there areas from graphic technology and appropriate probability models. It is also necessary to introduce a larger number of students’ projects and stimulate students to do their own research. Students should be motivated for writing papers. In this way models can be created in all other faculties of technology.

It can be expected that the implementation of real examples in teaching probability and statistics would have numerous positive effects such as the following:

a) Increased students’ interest for the course probability and statistics;

b) More interesting teaching process for students;
c) Increased students’ capability for probability and statistical modelling of technological problems;

d) Increased students’ understanding for the application of probability and statistics within respective technologies;

e) Increased level of understanding for theoretical mathematical content of probability and statistics;

f) Students would learn how to think in complex systems and how to connect contents of different courses;

g) Increased recognition and understanding of the role of mathematics for the development of science and new technological achievements;

h) Students would learn how to cope with technological problems in a different, better way;

i) Students would become more competitive at the labour market.

Based on the above it can be concluded that the most prominent method of teaching probability and statistics at the faculties of technology in the Republic of Croatia should be the use of real problems. This would increase the current level of mathematical education at the faculties in question and would also bring the teaching process more in line with the teaching process in the EU countries.

**PROPOSITION TASKS OF PROBABILITY AND STATISTICAL MODELS**

Following all the above the following tasks were created at the Faculty of Graphic Arts in Zagreb for the academic years 2011/12, 2012/13 and 2013/14 which are in line with the teaching curriculum (see Table 1);

**Table 1.** Proposed tasks which include probability modelling of real problems from graphic technology

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<th>Combinatorics</th>
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<td><strong>Task (The problem of distributing workers in a printing facility, [39]):</strong> In how many ways can 9 workers be distributed in a printing facility so that 4 workers do graphic preparation, 3 workers in printery and 2 in processing?</td>
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<th>Combinatory probability</th>
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<td><strong>Task (EAN barcodes for product protection, [40]) EAN 13 barcode is the global system of protecting and encrypting products which consists of 12 digits and the 13th control number obtained by the algorithm of module 10 from the first twelve digits. The first three digits relate to the manufacturing country of the product to be protected. In the case of Croatia the first three digits are 385. What is the number of all possible EAN 13 barcodes that can protect Croatian products?</strong></td>
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<th>Combinatory probability</th>
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<td><strong>Task (Quality control in printing facilities, [41], [42]) Quality control of graphic products</strong></td>
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implements testing of products of a printing facility. In this facility out of 100 products there are 7 products with fault. Supervisors test three products out of the possible 100. They reach a positive evaluation if at least two out of three tested products are correct. What is the probability that this printing facility gets a positive evaluation by quality supervisors?

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<th>Geometric probability</th>
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<td>Task (The problem of transporting graphic products) Two trucks for transporting graphic products arrive to the warehouse of a printing facility to take over the goods. Times of truck arrival are independent and equally probable at any point of time in the time period between 8:00 and 14:00. It takes two hours to load the first truck and 3 hours to load the second truck. Both trucks cannot be loaded at the same time so that the truck that comes later must wait till the truck that arrives earlier is loaded. What is the probability that one of these two trucks will have to wait to be loaded?</td>
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<th>Conditional probability and independence</th>
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<td>Task (Detecting mistakes in printing process [39], [42]) Five different graphic laboratories look for mistakes created in a printing process. The detection of one mistake is independent from the detection of another mistake. Also, the detection of a certain mistake in all laboratories is independent. The number of mistakes in this process was 7. The probability that one laboratory detects a mistake is ( p = 0.8 ). Determine the probability that at least one mistake created in the printing process will not be found.</td>
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<th>Complete probability formula and Bayes formula</th>
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<td>Task (Tone value increase, [42]) In visual estimate of the quality control of graphic reproduction the examinees were given the task to evaluate the given prints as being of high quality or of lower quality. If the target print does not have tone value increase the probability that the print is evaluated as being of high quality is 0.9. If the print has an average tone value increase, the probability that such a print will be evaluated as being of high quality is 0.5. If tone value increase is high the probability that such a print will be evaluated as of high quality is 0.1. The printing machine prints 70 percent of prints with no tone value increase, 20 percent of prints have average tone value increase and 10 percent of prints have high tone value increase. What is the probability that examinees will evaluate a randomly chosen print as being of high quality? If the chosen print is evaluated as being of high quality, what is the probability that the tone value increase was average?</td>
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<th>Binomial distribution</th>
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<td>Task (Relative print contrast, [43]) The value of the relative print contrast for black colour in one edition is beyond the limits of allowed deviation in 15 percent of the cases. Three different prints from this edition were chosen. What is the probability that for at least one of those prints the value of the relative print contrast for black colour will be beyond the limits of allowed deviation?</td>
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| Poisson distribution |
Task (The number of printing errors per book page) A book contains in average 2.3 printing errors per page. What is the probability that 3 randomly chosen pages will have more than 3 printing errors?

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<th>Constant random variable</th>
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Task (The time of work of a printing machine spare part, [42], [43]) The time of work of a printing machine spare part is described by random variable $T$ which expresses time in hours. Density function of $T$ variable is

$$ f(x) = \begin{cases} 
  cx(10 - x), & 0 \leq x \leq 10 \\
  0, & \text{inac}
\end{cases} $$

a) Find $c$ constant and draw the diagram of density function.

b) Find distribution function and draw its diagram.

c) What is the probability that the spare part will work less than 3 hours?

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<th>Uniform distribution</th>
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Task (Print speed, [44], [45]) Print speed at the machine with a movable printing roll is uniformly distributed random variable $X$ with the values ranging between 1000 and 5000 prints per hour.

a) What is the probability that print speed of a randomly chosen edition will range between 2000 and 3000 prints per hour?

b) Find the expectancy and variance of this random variable.

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<th>Exponential distribution</th>
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Task (Lifetime of printing machine spare parts [44], [45]) Printing machine contains 5 spare parts the lifetime of which is exponentially distributed. The expected lifetime of each part is 1000 hours. Malfunction of any spare part is independent from the malfunction of any other spare part and causes the interruption of work of the whole machine.

a) What is the probability that the interruption of work of the printing machine will occur in 300 hours?

b) What is the expected time till the interruption of work of the printing machine?

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<th>Normal distribution</th>
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Task (Edge crush test in corrugated cardboard, [46], [47], [48]) The edge crush test (ECT) in five-layered corrugated cardboard from one production series is a normally distributed random variable $X$ with the expectancy $\mu = 7 \, kN/m$ and standard deviation $\sigma = \sqrt{\sigma^2} = 2 \, kN/m$.

a) Find the probability that a randomly chosen corrugated cardboard sample has the ECT higher than $9 \, kN/m$.

b) What percentage of corrugated cardboard samples has the ECT between $4 \, kN/m$ and $10 \, kN/m$?

c) Find the limit underneath which is 5 percent of corrugated cardboard samples with the lowest ECT.

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Task (Resistance to rupture according to Mullen, [49]) Measurements of resistance to rupture according to Mullen of one type of five-layered corrugated cardboard are random variables with the expectancy $\mu = 750$ and variance $\sigma^2 = 55$. In the experiment 250 measurements were made. Estimate the probability that the difference between arithmetic mean of all measurements and expectancy $\mu$ is lower than 10.

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Task (Intergrafika fair, [50]) The number of visitors entering Intergrafika fair within one minute is of Poisson distribution with the parameter $\lambda = 12$.

a) What is the probability that within one hour at least 700 visitors will enter Intergrafika fair premises?

b) How much time should pass so that the probability that at least 700 visitors visit Intergrafika fair amounts to 99 percent?

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Task (Evaluation of the Munker-White effect, [51], [52], [53]) When evaluating the Munker-White effect in certain conditions colorimetric differences $\Delta E^*$ on the obtained graphic reproductions are measured. This effect influences the visual quality of reproduction if the value of colorimetric differences $\Delta E^*$ is higher than the limit value of 3.5. Graphic laboratory is trying to determine whether the value of colorimetric differences $\Delta E^*$ of one series of graphic reproduction is higher than the limit 3.5 at which level the Munker-White effect influences the quality of reproduction. The mean value of the obtained results on the sample of 60 measurements of colorimetric differences $\Delta E^*$ amounts to 3.7 with the standard deviation of 2.1. Test the hypothesis on the influence of Munker-White effect on the quality of reproduction with the level of significance of $\alpha = 0.1$.

With the help of the described methodology similar tasks can be created for other faculties of technology as well.
CONCLUSION

What can be perceived is the necessity of introducing new mathematical models, which are connected with non-mathematical contents appearing in technology, into teaching probability and statistics. The models are created via interdisciplinary cooperation with technological scientists and experts for new technologies. In this process expert and scientific literature from a certain area is used. It is also necessary to establish cooperation with students in order to take into consideration their ideas and expectations. The result is new tasks and examples which can be used in teaching probability and statistics. This approach would raise the level of students’ education and of their capacity of critical engineering thinking and would introduce them into the methodology of scientific work. It would also raise students’ competences and their competitiveness at the labour market. This is the direction the organisation of teaching probability and statistics at the faculties of technology in the Republic of Croatia should be headed in order to be in line with the countries of the European Union.

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