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MODELING OF DAMAGE PHENOMENON USING STRAIN GRADIENT BASED FINITE ELEMENTS

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It is well-known that damage phenomenon cannot be objectively modeled using the classical continuum theory because the differential equations which describe the deformation process may lose the elliptic characteristic once the damage is initiated. Mathematical description of the model in that case becomes ill-posed and numerical solutions do not converge to a physically meaningful solution [1]. To overcome these problems, most of the regularization techniques developed are based on the improvement of the classical continuum model by its enrichment with the internal length scale parameters in various forms. Among these techniques nonlocal theories are especially known, which have been shown to be the most versatile. One of them is so called full strain gradient theory, which is a special case of the general Mindlin's first-order gradient theory [2], and is usually used for the homogenization purposes in multiscale analyses. The nonlocal behavior is therein introduced in the model with the size of the representative volume element (RVE), which additionally allows the analysis of arbitrary heterogeneous materials.

In this contribution, the two dimensional C^1 continuity triangular finite element used in the small-strain, multi-scale computational approach [3] is extended to the modeling of damage and strain localization phenomena. The element is based on the aforementioned strain gradient continuum theory which incorporates additional higher-order term in the strain energy density function necessary for the description of material nonlocality. It consists of three nodes, each having twelve degrees of freedom, including the two displacements and their first- and second-order derivatives with respect to the Cartesian coordinates. The displacement field is approximated by the full fifth order polynomial, thus easily allowing the formation of high strain gradients usually arising in the localization zones. When it comes to the modeling of damage, the isotropic law is used for the reduction of the elastic stiffness properties. The linear elastic material behavior is considered, where the linear and exponential evolution laws of quasi-brittle damage are employed [4]. The damage variable D is calculated as a function of the highest equivalent strain value reached in the integration point through entire deformation history, where the equivalent elastic strain measure depends only on the tensile principal strains. The strain gradient constitutive relations with the damage variable included may be written in the incremental form as

$$\begin{aligned}\Delta\boldsymbol{\sigma} &= \Delta\left[(1-D)\left(\mathbf{C}_{\sigma\varepsilon}\boldsymbol{\varepsilon}+\mathbf{C}_{\sigma\eta}\boldsymbol{\eta}\right)\right]\\ \Delta\boldsymbol{\mu} &= \Delta\left[(1-D)\left(\mathbf{C}_{\mu\varepsilon}\boldsymbol{\varepsilon}+\mathbf{C}_{\mu\eta}\boldsymbol{\eta}\right)\right],\end{aligned} \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\mu}$ are the stress and second-order stress tensors, while $\boldsymbol{\varepsilon}$ and $\boldsymbol{\eta}$ are the strain and second-order strain tensors, respectively. The values $\mathbf{C}_{\sigma\varepsilon}$, $\mathbf{C}_{\sigma\eta}$, $\mathbf{C}_{\mu\varepsilon}$ and $\mathbf{C}_{\mu\eta}$ are the constitutive matrices which are computed from the appropriate RVE using the second-order homogenization procedure [3] and are kept constant during the analysis. In case of material homogeneity, $\mathbf{C}_{\sigma\eta}$ and $\mathbf{C}_{\mu\varepsilon}$ are assumed to be zero, and the remaining two constitutive matrices can be found analytically. The block diagram demonstrating the analysis procedure is presented in Fig. 1.

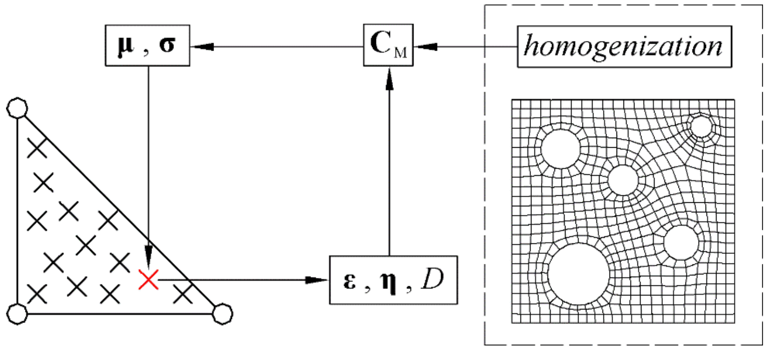

Figure 1. Scheme of the damage algorithm

Before the start of the incremental-iterative loop, the values of the aforementioned stiffness tensors appearing in relation (1) have to be homogenized from the appropriate RVE that represents the heterogeneous material. As the linear elastic material behavior is considered here, the homogenization has to be performed only once in each analysis. Once computed, homogenized tensors enter the constitutive relations where the isotropic damage law governs the stiffness degradation in softening stage. The presented damage formulation has been embedded into the aforementioned two-dimensional C^1 continuity triangular finite element which is implemented into the FE program ABAQUS by means of the user subroutines.

The verification of the presented computational strategy is made on a benchmark example consisting of a rectangular plate with an imperfect zone under tension, already studied in [5] under assumption of a homogeneous material. Here, the analysis is extended to heterogeneous materials. Besides, an analysis of the shear band formation along an imperfect plate subjected to compressive load is performed.

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