Wave Diffraction CFD Nonlinear Time–Spectral Simulations in foam–extend

Inno Gatin, Gregor Cvijetić, Vuko Vukčević and Hrvoje Jasak
University of Zagreb, Zagreb/Croatia, {inno.gatin, gregor.cvijetic, vuko.vukcevic, hrvoje.jasak}@fsb.hr

1 Introduction

Harmonic Balance (HB) method is applied to the problems of surface gravity waves in the `Naval Hydro` pack in foam–extend, which enables time–spectral simulations of wave diffraction. HB transforms a transient problem into a set of coupled steady state equations by assuming a temporally periodic flow. The method is extensively used in the field of turbomachinery [Cvijetić et al., 2016], however no application in field of naval hydrodynamics has been published to our knowledge.

In CFD wave related problems are often simulated in time domain, which requires significant number of periods to be simulated in order to reach periodic steady state. HB assumes the flow variables to be temporally periodic, expanding them into finite Fourier series, resulting in $2N + 1$ coupled steady–state equations, where N is the arbitrary number of resolved harmonics in the Fourier series. Depending on the flow characteristics, the number of harmonics, i.e. spectral resolution, can be adjusted to capture relevant higher order phenomena. For a large number of problems in naval hydrodynamics, solving $2N + 1$ steady–state problems is less computationally demanding than running a full transient simulation. The present HB two–phase method is especially adequate for regular wave problems without violent free surface effects with non–zero mean velocity.

The two–phase HB method is implemented by combining the existing single–phase model for turbomachinery application [Cvijetić et al., 2016] and the existing two–phase flow framework comprising SWENSE (Spectral Wave Explicit Navier–Stokes) solution decomposition [Vukčević et al., 2016a], Ghost Fluid Method (GFM) [Huang et al., 2007] accounting for the pressure and density jump conditions at the interface, and implicitly redistanced Level Set (LS) interface capturing method [Sun and Beckermann, 2007].

In this paper the mathematical model of the time–spectral two–phase HB method is briefly described, followed by the governing equations in the HB form. A detailed mathematical model of the HB method can be found in [Cvijetić et al., 2016]. Next an outline of the numerical framework is given, and finally a working example is shown for regular wave diffraction of a DTMB hull model.

2 Harmonic Balance method

In HB the solution of flow variables is assumed to be periodic in time and expanded in finite Fourier series:

$$\phi(t) = \Phi_0 + \sum_{i=1}^{N} \Phi_{S_i} \sin(i\omega t) + \Phi_{C_i} \cos(i\omega t), \tag{1}$$

where ϕ stands for a general flow variable in time–domain, while Φ denotes its spectral counterpart. ω is the base frequency of the harmonic oscillation, while indices S_i and C_i denote the sine and cosine Fourier coefficients, respectively. A general transport equation for ϕ can be written in abbreviated form as:

$$\frac{\partial \phi}{\partial t} + \mathcal{R} = 0, \tag{2}$$

where $\mathcal{R}$ represents convection, diffusion and source/sink terms, which are also periodic in time, and hence expanded into Fourier series as well. HB is based on equating the corresponding harmonics after inserting the Fourier series of ϕ and $\mathcal{R}$ in Eq. (2), which results in a system of equations that has the following form in the matrix notation:

$$\omega \underline{\underline{A}}\, \underline{\Phi} + \underline{R} = \underline{0}, \tag{3}$$

where $\underline{\underline{A}}$ is the coefficient square matrix with dimensions $2N + 1$, while $\underline{\Phi}$ and $\underline{R}$ represent vectors of Fourier coefficients of ϕ and $\mathcal{R}$, respectively. In the matrix form the Fourier expansion Eq. (1) of ϕ can be written as $\phi = \underline{\underline{E}}^{-1}\underline{\Phi}$, where $\underline{\underline{E}}$ represents a discrete Fourier transformation operator. By applying inverse Fourier transform Eq. (3) the time–spectral form of the equations is obtained, which is used in this work:

$$\omega \underline{\underline{E}}^{-1}\, \underline{\underline{A}}\, \underline{\underline{E}}\, \underline{\phi} + \underline{\mathcal{R}} = \underline{0}. \tag{4}$$

Eq. (4) represents a set of $2N+1$ coupled steady–state equations in discrete tie–domain. The HB method effectively replaces the time derivative term with by the harmonic coupling source term:

$$\underline{S\ (\phi)} = \omega \underline{\underline{E}}^{-1}\, \underline{\underline{A}}\, \underline{\underline{E}}, \tag{5}$$

which written in the expanded form reads:

$$S_j\,(\phi) = -\frac{2\omega}{2N+1}\left(\sum_{i=1}^{2N} P_{i-j}\phi_{t_i}\right),j = 1 \ldots 2N+1\,, \tag{6}$$

where P denotes inter–equation coupling matrix defined as:

$$P_i = \sum_{k=1}^{N} k \sin(ik\omega\Delta t)\,,\ \text{for}\ i = 1 \ldots 2N\,, \tag{7}$$

with $\Delta t = T/(2N+1)$, where T presents the base period of oscillation.

3 Numerical model

HB method is applied to the existing two–phase incompressible numerical model in the `Naval Hydro` pack [Vukčević and Jasak, 2015a, Vukčević and Jasak, 2015b, Gatin et al., 2015, Vukčević et al., 2016b].

SWENSE decomposition is used to decompose the flow field into the incident and perturbed component, where the incident is known from an external potential flow model such as analytical wave solution or nonlinear potential flow solution, while the perturbed component represents the difference between the forced incident component and the full CFD solution. The reader is referred to [Vukčević et al., 2016a] for more details.

In order to prevent wave reflection perturbed component is gradually damped to zero towards the far–field boundaries using implicit relaxation zones.

The kinematic free surface boundary condition and normal stress balance at the free surface are modelled using the GFM [Huang et al., 2007], which implicitly imposes the jump conditions in the cells adjacent to the interface via interface–corrected interpolation schemes.

LS method derived from the Phase Field equation [Sun and Beckermann, 2007] with implicit redistancing is used for interface capturing.

In the present work the coupling between equations is performed implicitly in a block linear system. Implicit coupling enables stable simulations with small or zero mean velocities with respect to the magnitude of oscillation variable ϕ, which cannot be achieved by explicit coupling [Hall et al., 2013]. This is specially important for the application in naval hydrodynamics where the ship velocity can be of the same order of magnitude of the orbital wave velocity in the case of low Froude numbers. The above described HB method is applied on the momentum equation and the LS transport equation, while the pressure equation has no temporal derivative, hence no special treatment is needed.

4 DTMB regular wave diffraction

In this section a HB simulation of regular head wave diffraction for a DTMB hull is presented. Results and computational times are compared to a transient simulation. Hull model with scale 49.59 is used, with length $L_{PP} = 3.05$ m, draught $T = 1.7$ m, and velocity of 1.53 m/s for $F_r = 0.28$. Realistic wave parameters are chosen with wave height $H = 0.036$ m, wave length $\lambda = 4.57$ m and period $T = 1.09$ s. Mesh with 521 000 cells is used in both HB and transient simulations with only half of the domain being simulated. In the HB simulations two harmonics are used, while 20 encounter wave periods are used in the transient simulation with 200 time steps per encounter period, resulting in $dt = 0.005$ s.

Hydrodynamic forces acting on the hull in the longitudinal and vertical direction, denoted with F_x and F_z respectively, are compared for zeroth and first order of harmonic oscillation, which are obtained using a discrete Fourier transform of the HB steady state equation results, while a moving window Fast Fourier Transform (FFT) is performed on the time history from the transient simulations, where successive FFT's are performed for each encounter wave period. Forces convergence in the HB simulations is shown on Fig. 1, where F_x and F_z are shown against the number of iterations of the steady state solver. It takes $\approx$ 3000 iterations for the longitudinal forces to converge and 2000 iterations for the vertical. Periodic convergence from the transient simulation is show on Fig. 2, where zeroth and first order of F_x and F_z are shown against the number of simulated periods, showing that 20 periods of simulation time suffices in order to reach convergence for all items.

Fig. 3 shows the perspective view of the free surface in the transient and HB simulation corresponding to $t = T$. Free surface elevation in the HB simulation agrees well with the transient simulation, although minor differences can be observed. The comparison of zeroth and first order harmonic amplitudes of F_x and F_z is shown in Table 1, where the relative difference between the transient and HB solution is expressed as $\epsilon = S_t - S_{hb}/S_t S_t$, with S_t and S_{hb} denoting the transient and HB solution, respectively. The relative difference ranges from -0.11% and -10.2 %, while for most items the difference is below 10 %.

The required CPU time 20 periods of simulated time in the transient simulation is 18.9 hours, while it took 8.6 hours for 3000 iterations in the HB simulation which represents a significant savings in terms of CPU time. Moreover, larger savings could be achieved with coarser mesh and larger number of resolved harmonics, since the convergence rate of implicitly coupled HB equation system improves with the increase of cell size and number of resolved harmonics.

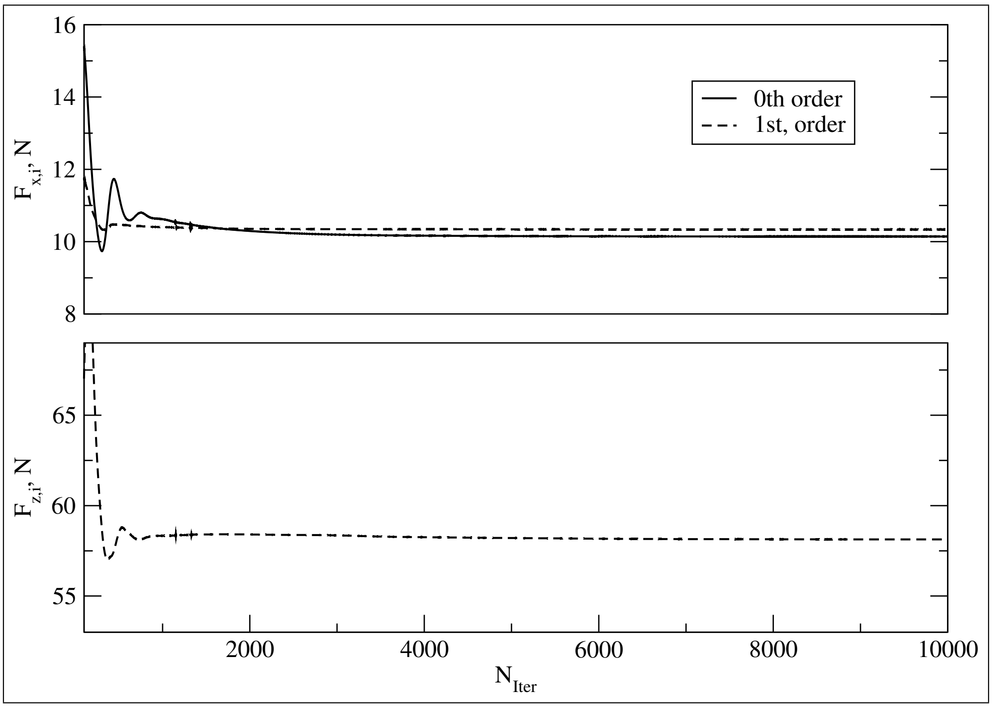


Fig. 1: Convergence of 0th and 1st longitudinal F_x and vertical F_z force harmonic amplitudes in the harmonic balance simulation of DTMB wave diffraction.

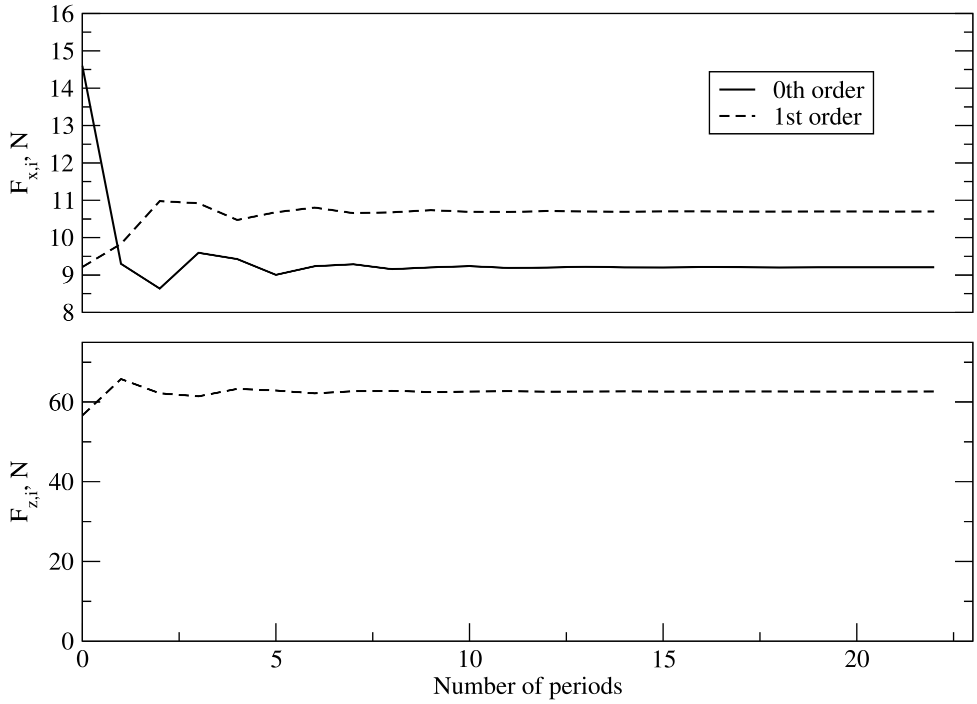
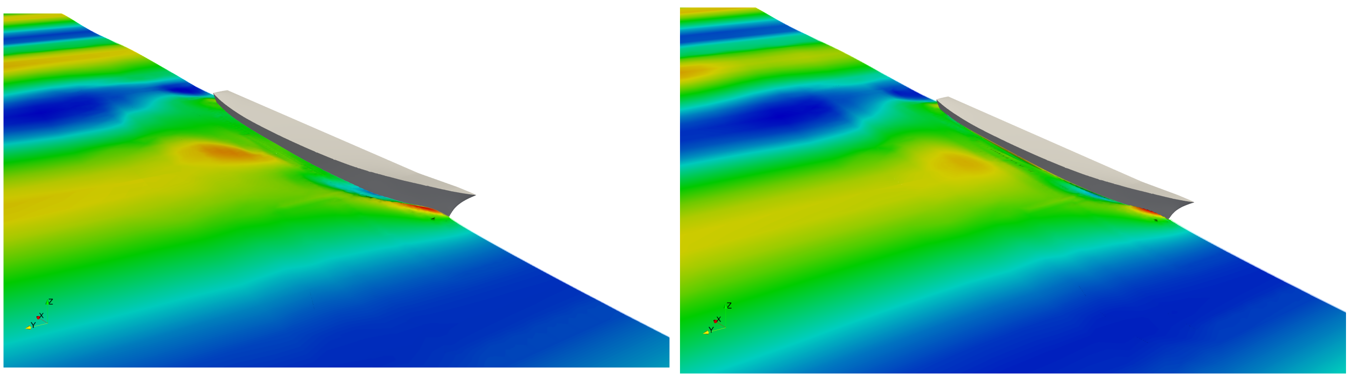


Fig. 2: Convergence of 0th and 1st longitudinal F_x and vertical F_z force harmonic amplitudes in the transient simulation of DTMB wave diffraction.



(a) (b)

Fig. 3: Perspective view of the DTMB diffraction simulation at $t = T$: a) transient, b) harmonic balance simulation.

Table 1: Comparison of harmonic amplitudes of horizontal F_x and vertical F_z wave forces on DTMB hull.

Item	Transient	Harmonic balance	ϵ, %
$F_{x,0}$, N	9.20	10.14	-10.2
$F_{x,1}$, N	10.70	10.34	3.36
$F_{z,0}$, N	784.88	785.72	-0.11
$F_{z,1}$, N	62.63	58.14	7.17

5 Conclusion

The two–phase Harmonic Balance method proved to be applicable on naval hydrodynamics problems regarding wave–structure interaction. The advantage of HB is in the steady state formulation of the periodic problem, reducing the required computational effort to reach a periodic steady state solution.

Comparison of HB and transient DTMB head wave diffraction simulations showed that accurate results can be achieved in less than half of the computational time required for the transient simulation. Larger savings could be achieved if coarser mesh and more harmonics are used.

New step is to formulate a spectral rigid body motion model to enable full seakeeping simulations. The stability and rate of convergence of implicitly coupled HB steady state equations increases with coarser mesh, increasing the savings with respect to the transients simulation. This characteristic renders the present method a perfect choice for early design stages.

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