Life-Cycle of Engineering Systems: Emphasis on Sustainable Civil Infrastructure – Bakker, Frangopol & van Breugel (Eds)
 ISBN 978-1-138-02847-0

A new method for structural damage detection and localization based on modal shapes

I. Duvnjak, M. Rak & D. Damjanović

University of Zagreb Faculty of Civil Engineering
Zagreb, Croatia

ABSTRACT: Damage assessment of structures includes estimation of location and severity of damage using changes of dynamic properties (natural frequencies, damping ratios and modal shapes) determined on undamaged and damaged structure. The basic principle is to use dynamic properties of a structure as indicators of any change of its stiffness and/or mass. In this paper, a new method for damage detection based only on mode shape data is presented to identify damage for plate like structures. A new method is based on comparison of normalised modal shape vectors determined in undamaged and damaged state of the structure. The validity of the proposed method is demonstrated on numerical model. Accuracy of the method is analysed on nine different damage scenarios simulated by local change of elasticity modulus and change in boundary conditions.

1 INTRODUCTION

Imperfections in design and construction of structures, lack of maintenance, natural disasters, environmental effects, human negligence or some other influences may result in structural damage. Structural damage can cause loss of load bearing capacity with far-reaching consequences. In order to prevent that risk, early identification of damage is of great importance. There are many non-destructive techniques (ultrasonic, radiography, magnetic particles, eddy currents and acoustics emissions) which are capable to detect and locate damages even if they are not visible on the surface of the structure. These experimental methods have a number of disadvantages such as the small inspection area in time, prediction of the damage location in advance and also, at the same time that part of the structure must be accessible (Farrar et al. 1997).

The difficulties in damage identification can be overcome by measuring dynamic response of structure. Global behaviour of the structure is described by dynamic properties (natural frequencies, damping ratios and modal shapes) which can be used for identification of damage. Basic assumption is that the structural damage changes physical properties such as stiffness and/or mass properties which cause changes in its vibration characteristics. It is obvious that the introduction of damage in structure will cause a local change in stiffness. Damage assessment in structure consists of four levels (Rytter 1993): damage existence at very early stage (level 1), damage localization (level 2), damage severity (level 3) and remaining life prediction (level 4). This paper is focused on vibration based structural damage detection on the first three levels.

There is a variety of vibration based methods that are being widely investigated by many researchers because of their global sensitivity. For example most commonly used methods are: changes in natural frequencies (Salawu 1997; Salehi 2010; Yang et al. 2013), modal flexibility change (Pandey & Biswas 1994), modal curvature change (Wahab & De Roeck 1999) and change in modal strain energy (Shi et al. 1998).

This paper presents a new method for structural damage detection and localization using changes in mode shapes. This method, named Mode Shape Damage Index *(MSDI)*, is based on comparison of normalised modal shape vectors determined in undamaged and damaged state of the structure (Duvnjak 2015). The validity of the proposed method was demonstrated using numerical models (shell elements). Different damage scenarios were simulated by reducing the modulus of elasticity of selected finite elements in numerical models to 80 % and 50 %. In this stage, nine damage cases were investigated whilst changing three different boundary conditions.

2 METODOLODY OF MODAL SHAPE DAMAGE DETECTION

Procedure of damage detection (level 1 to level 3), which consists of four stages, will be presented on numerical models of plate like structures.

2.1 *Stage 1: mass normalized mode shape*

If damage occurs, structural and modal parameters will be changed. Comparison of the modal shapes is based on two states undamaged and damaged state. In order to compare modal shapes firstly they need to be mass normalized in the form:

$$\phi_i^T [M] \phi_i = 1 \tag{1}$$

2.2 *Stage 2: trace of MAC matrix*

The Modal Assurance Criterion (MAC) is used to determine the level of correlation between mode shapes determined for undamaged and damaged state of the structure. MAC value varies between 0 and 1. MAC criteria that composes mode *i* and *j* has the following form:

$$MAC_{(i,j)} = \frac{\left| \{\phi_u\}_i^T \{\phi_d\}_j \right|^2}{\left(\{\phi_u\}_i^T \{\phi_u\}_i \right)\left(\{\phi_d\}_j^T \{\phi_d\}_j \right)} \tag{2}$$

where ϕ_u and ϕ_d represent mass normalized sets of vectors for undamaged and damaged conditions. Values near unity means that two mode shapes of two states are identical, which means modal vectors are consistent. While values close to zero show that compared mode shapes are dissimilar.

The diagonal elements of the MAC matrix contains only pair mode shapes and indicate which modes are most affected by the damage. Basic idea of the MSDI method is to use only diagonal elements of MAC matrix in form:

$$\gamma_{trMAC} = \left(\sum_{i=1}^{n} trMAC_{(ii)} \right)^2 \tag{3}$$

where γ_{trMAC} represents squared value of the summed MAC matrix trace. Value γ_{trMAC} varies between 0 and n^2, where value n denotes number of compared mode shapes vectors. Values equal n^2 means that compared mode shapes for two states are identical. If values are different from n^2 and streaming towards zero it means that compared modal shapes are dissimilar and structure is affected by damage. Previous statement can be valid only if there are no measurement errors or uncertainties.

2.3 *Stage 3: modification of MAC matrix*

Original MAC criterion is a statistical indicator that is sensitive to large differences and relatively insensitive to small differences in mode shapes. The aim of modified MAC matrix (ΔMAC) is to exclude or decrease influence of those modal shapes which are dissimilar when comparing undamaged and damaged state of structure. Therefore, diagonal elements of ΔMAC matrix are defined as follow:

$$\Delta\alpha_{ii} = \alpha_{ii}^{\gamma_{trMAC}} \tag{4}$$

where a_{ii} and Δa_{ii} represent diagonal elements of original MAC and modified ΔMAC matrix.
If there is no damage in between two stages of measurement, stiffness of structure will not be changed. Therefore, if there is no change in stiffness modified MAC matrix is equal to original MAC. In this way, modified MAC matrix ensures that only those modal shapes which are almost identical can be useful for damage detection. Modified MAC matrix can be useful to overcome difficulties caused by measurement uncertainties and errors.

2.4 *Stage 4: Mode shape damage index*

A new method for structural damaged detection and localization based only on modal shapes is given in this form:

$$MSDI = \left| \frac{\bar{\phi}_u - \bar{\phi}_d}{\bar{\phi}_u} \right| \tag{5}$$

The coefficients $\bar{\phi}_u$ and $\bar{\phi}_d$ represent modification of squared normal mode shape vectors and can be computed using the following equitation's:

$$\bar{\phi}_u = \sum_{i=1}^{n} \left(\{\phi_u\}_i^2 \cdot \Delta\alpha_i \right) \tag{6}$$

$$\bar{\phi}_d = \sum_{i=1}^{n} \left(\{\phi_d\}_i^2 \cdot \Delta\alpha_i \right) \tag{7}$$

Proposed MSDI method utilized coefficients related to global behaviour of the structure. When a structural stiffness matrix is changed (damage induced) original modal shape vectors are changed as well. In order to emphasize difference between two sets of vectors their value is squared and multiplied with corresponding element from modified MAC matrix. Finally, all sets of modified mode shape vectors are summarized thereby representing coefficients for damage detection.
The presented method only considers modal shapes that are almost identical and excluding dissimilar modal shapes. Theoretically, if there are no damages in between two stages stiffness of structure will not be changed, provided there are no measurement uncertainties and errors. Therefore, if there is no change in stiffness modified MSDI index is equal to zero.

3 APLICATION OF MSDI METHOD TO DAMGE IDENTIFICATION

Three numerical examples of plate like structures are presented to demonstrate effectiveness of the proposed method. Selected numerical model of steel plate is analysed. Natural frequencies and mode shapes of the plate determined by numerical modal analysis are compared to experimental results provided by authors Ulz and Semercigil. Numerical values are in good agreement with experimental results presented by mentioned authors. Analysed rectangular steel plate is 2.5 m in long, 1.0 m wide and 2 mm in thick, elasticity modulus is 210 GPa and mass density 7800 kg/m^3. The finite element software SAP2000 was used to model plate, which is discretised by 200 shell elements. Damage to the structure is induced by reducing the elasticity modulus of selected elements to 80% and 50%. In this paper nine damage scenarios are presented. Among these cases, three different boundary conditions with different damage severity are simulated. A Finite Element (FE) analysis is performed for both the undamaged and damaged case. It is assumed that structural mass remains unchanged and no structural damping is used in FE analysis.

3.1 *Clamped plate*

FE model of steel plate clamped along all edges is analysed. In total three damage scenarios are simulated in order to investigate capability of method for damage detection (Fig.1.). The first ten vibration modes of two states are taken into analysis. Figure 2 presents first ten modal shapes of undamaged clamped plate.

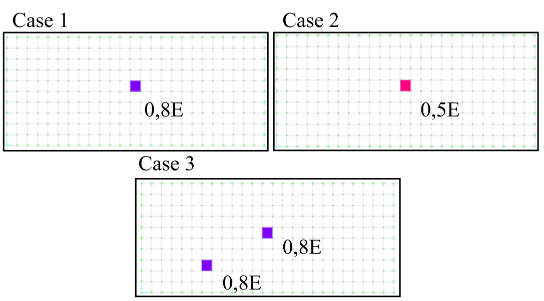

Figure 1. Damage cases for plate with all edges clamped

The peak values of MSDI indicate the location of the damage in the plate. Comparison of case 1 (Fig.3.) and case 2 (Fig.4.) shows that if the severity of the single damage at mid-span increases, corresponding MSDI also increases.

The results presented in case 3 (Fig.5.) show that proposed procedure is capable to locate multiple damages in plate like structures. If we look closer to inspection case 3, it can be seen that MSDI index for two damaged locations are different although the imposed damage is equal at booth locations. The reason for these discrepancies is that severity of damage is dependent to boundary conditions and damage location. It can be seen from the results that proposed method is capable to determine location and to some extent the severity of damage.

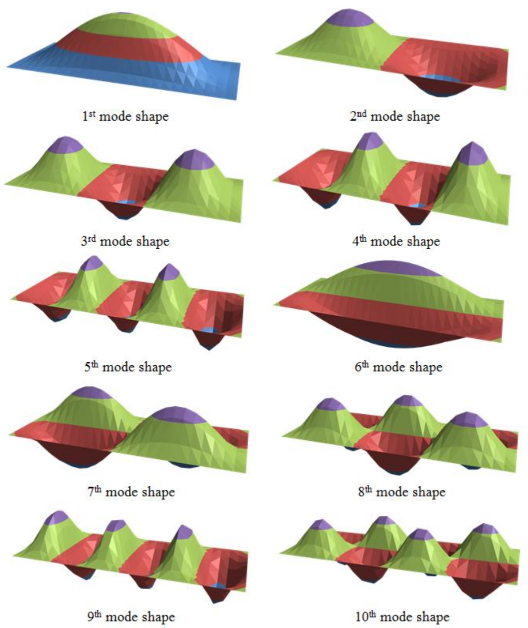

Figure 2. First ten mode shapes of clamped plate

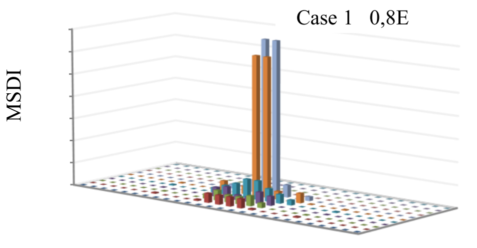

Figure 3. Plot of damage for clamped plate case 1

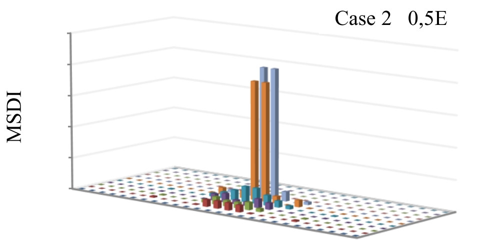

Figure 4. Plot of damage for clamped plate case 2

Trace of original and modified MAC matrices are shown in Fig. 6 to Fig. 8. The results showed that modified MAC matrix is equal to original MAC if there is small change in stiffness (case 1). Further reduction of stiffness causes decrease of modified MAC matrix (case 2 and case 3).

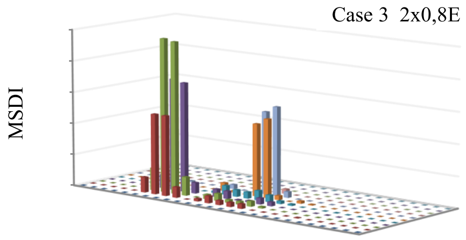

Figure 5. Plot of damage for clamped plate case 3

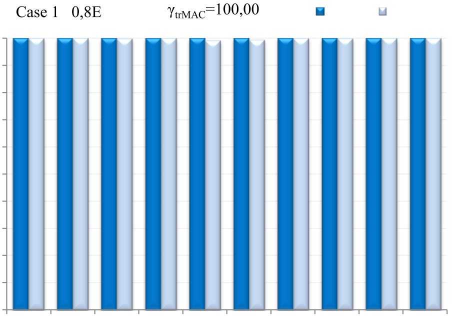

Figure 6. Plot of MAC for clamped plate case 1

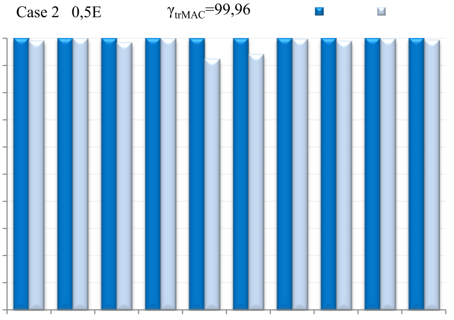

Figure 7. Plot of MAC for clamped plate case 2

Table 1. Changes of natural frequencies (%) from undamaged state

Frequency	Damage cases		
	Case 1	Case 2	Case 3
f_1	0.08	0.34	0.08
f_2	0.00	0.00	0.00
f_3	0.11	0.40	0.17
f_4	0.00	0.00	0.04
f_5	0.13	0.43	0.17
f_6	0.03	0.10	0.10
f_7	0.03	0.06	0.15
f_8	0.00	0.05	0.08
f_9	0.00	0.03	0.05
f_{10}	0.05	0.14	0.05

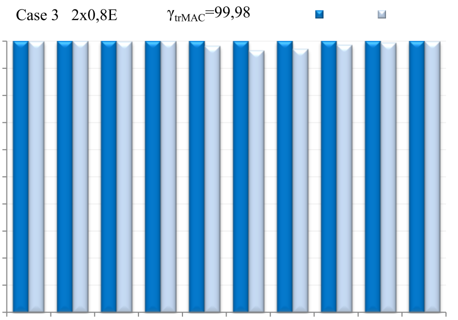

Figure 8. Plot of MAC for clamped plate case 3

From the results shown in case 2 (Fig.7), it can be seen that fifth and sixth mode shapes are less sensitive, and their contribution in damage detection is lower for damage detection. This is because the location of damage corresponds to point in the mode shapes where the mode shape is zero.

Natural frequencies of first ten mode shapes of the plate before and after damage are presented in table 1. As expected, presence of damage causes decrease of natural frequencies.

3.2 *Simply supported plate*

For the simply supported plate five damage scenarios were simulated by reducing the elastic modulus of selected elements to 80% and 50%. The first ten mode shapes of two states are taken into analysis (Fig. 10).

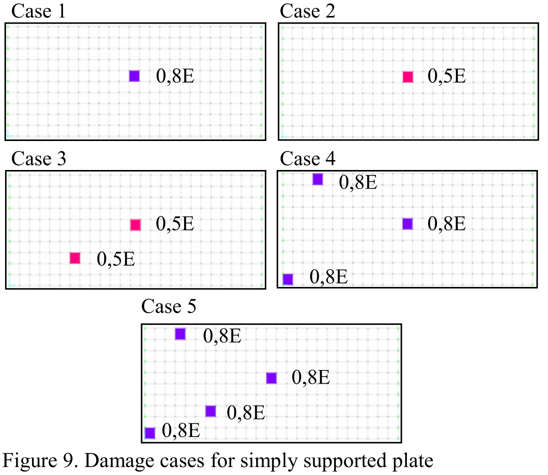

Figure 9. Damage cases for simply supported plate

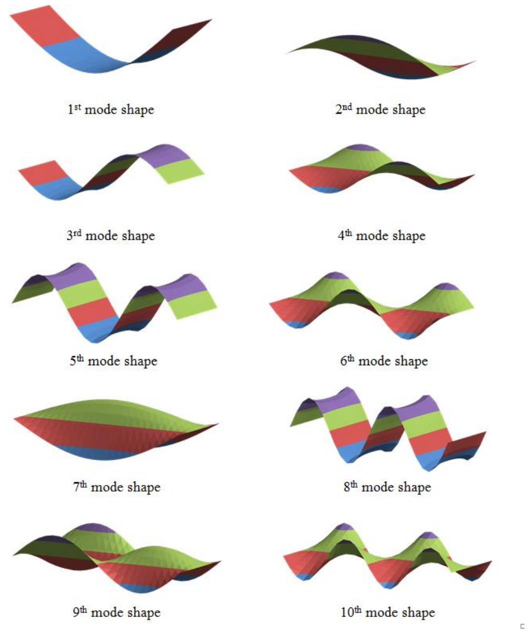

Figure 10. First ten mode shapes of simply supported plate

Comparison of case 1 (Fig.11.) and case 2 (Fig.12.) shows that if the severity of the single damage at min-span increases, corresponding MSDI also increases.

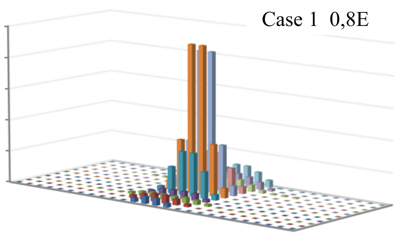

Figure 11. Plot of damage for simply supported plate case 1

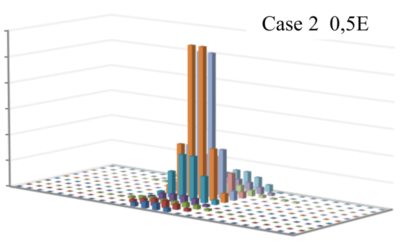

Figure 12. Plot of damage for simply supported plate case 2

Results showed good sensitivity and ability of MSDI method to correctly locate damage in all single and multiple damage cases for simply supported plate regardless of its severity and location.

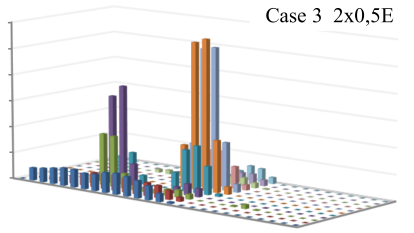

Figure 13. Plot of damage for simply supported plate case 3

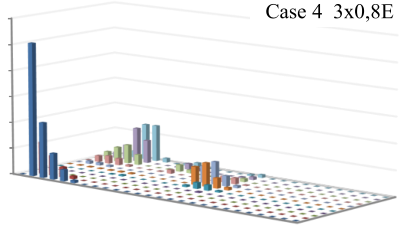

Figure 14. Plot of damage for simply supported plate case 4

It can be seen, for multiple damages in case 4 (Fig.14.) and case 5 (Fig.15.) that severity of damage is not equal for all damage locations. As discussed previously, the reason for these discrepancies is that severity of damage is dependent to boundary conditions and damage location.

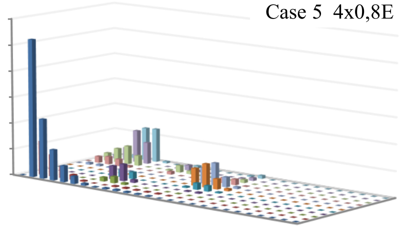

Figure 15. Plot of damage for simply supported plate case 5

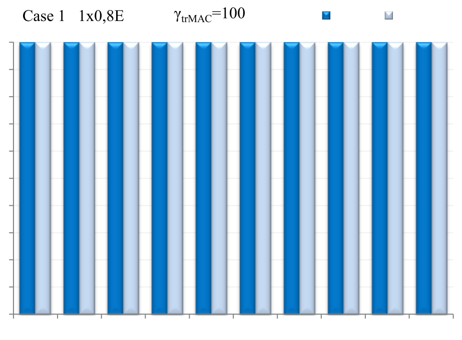

Figure 16. Plot of MAC for simply supported plate case 1

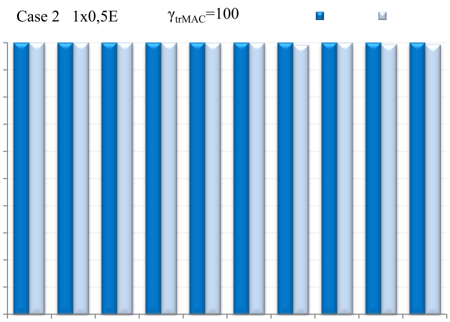

Figure 17. Plot of MAC for simply supported plate case 2

Trace of original and modified MAC matrices are shown in Fig. 16 to Fig. 20. It can be seen from the results that modified MAC matrix is almost equal to original MAC if there is small change in stiffness (case 1 and case 2). Further reduction of stiffness causes very slightly decrease of modified MAC matrix (case 3, case 4 and case 5). Plots of MAC matrix traces show damage induced changes in higher order modal shapes.

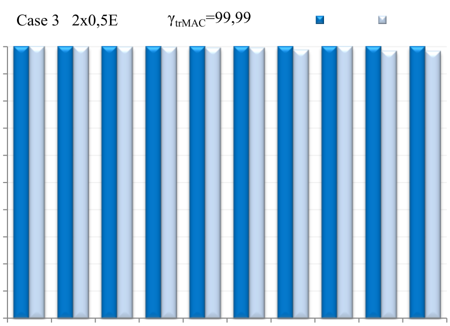

Figure 18. Plot of MAC for simply supported plate case 3

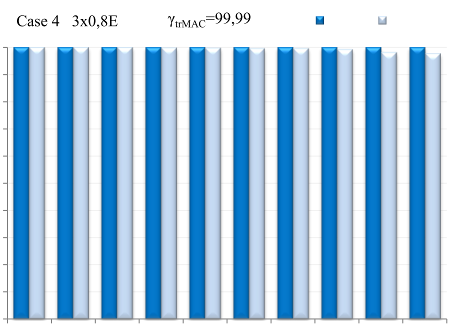

Figure 19. Plot of MAC for simply supported plate case 4

Differences in the natural frequencies between the undamaged and damaged state, expressed in percentage, are presented in table 2. Presence of damage in all cases causes small decreases of natural frequencies. It can be assumed that changes of natural frequencies can be used for detection of existence of the damage. Therefore, it is possible to use change of the natural frequencies in damage assessment of structures at level 1.

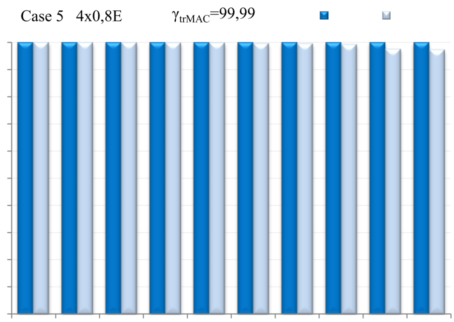

Figure 20. Plot of MAC for simply supported plate case 5

Table 2. Changes of natural frequencies (%) from undamaged state

Frequency	Damage cases				
	Case 1	Case 2	Case 3	Case 4	Case 5
f_1	0.08	0.24	0.37	0.11	0.15
f_2	0.00	0.00	0.11	0.15	0.19
f_3	0.00	0.00	0.24	0.07	0.15
f_4	0.06	0.17	0.23	0.19	0.21
f_5	0.08	0.23	0.33	0.18	0.22
f_6	0.00	0.00	0.12	0.15	0.18
f_7	0.17	0.51	0.61	0.22	0.26
f_8	0.00	0.01	0.02	0.11	0.12
f_9	0.01	0.01	0.10	0.10	0.13
f_{10}	0.04	0.10	0.19	0.20	0.23

3.3 *Simply supported plate over two spans*

FE model of simply supported two span plate is analysed. A single damage scenario is simulated in order to investigate capability and robustness of method. As in previous cases first ten vibration modes of two states are taken into analysis (Fig. 22). Damage is induced at each span with stiffness reduction of selected elements to 80% (Fig.21).

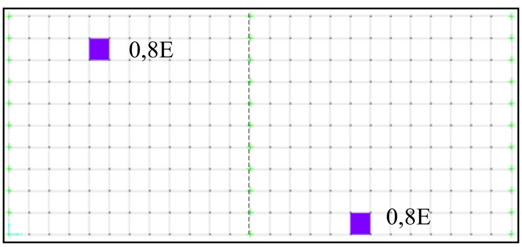

Figure 21. Damage case for simply supported plate over two spans

Presented results of damage detection, for the case of multiple damage scenario imposed on simply supported plate over two spans, suggest that the MSDI method is able to correctly locate the damage. Two peak values correspond to location of the damage. The severity of detected damage is not equal for both damage locations, as previously concluded these discrepancies are caused by boundary conditions and damage location.

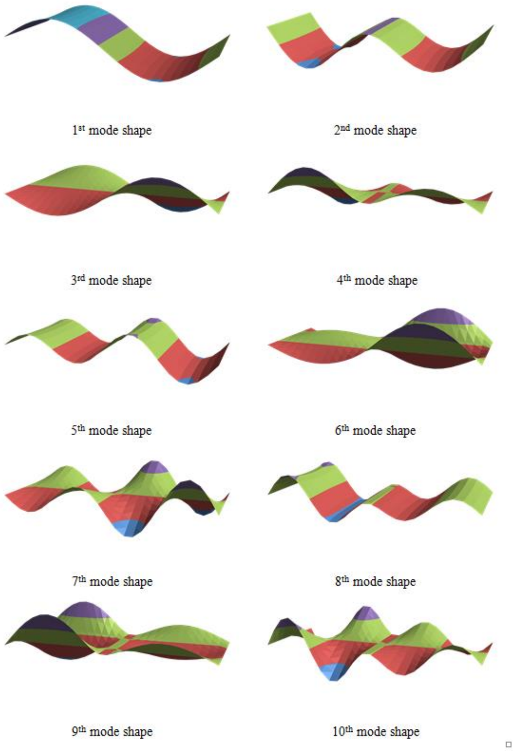

Figure 22. First ten mode shapes of simply supported plate over two spans

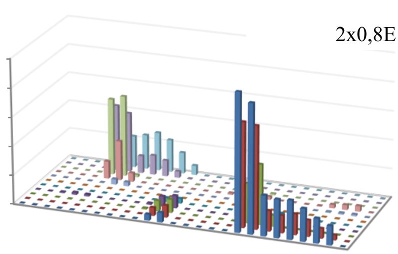

Figure 23. Plot of damage for simply supported plate over two spans

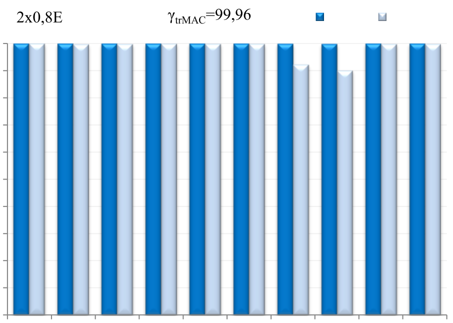

Figure 24. Plot of MAC for simply supported plate over two spans

Plots of MAC matrix trace show damage induced changes mostly in seventh and eighth mode shape.

Table 3. Changes of natural frequencies (%) from undamaged state

Frequency	Damage case
f_1	0.33
f_2	0.00
f_3	0.17
f_4	0.00
f_5	0.08
f_6	0.00
f_7	0.13
f_8	0.06
f_9	0.00
f_{10}	0.10

Changes of natural frequencies between undamaged and damaged state for simply supported plate over two spans is presented in table 3. Presence of damage in all cases causes small decreases of natural frequencies. Most affected natural frequencies are first and third because the damaged elements are located at the maximum of these mode shape.

4 CONCLUSION

In this paper, damage detection of plate like structures using, mode shape data based indicator, has been investigated. The new method is based on comparison of normalised mode shape vectors determined in undamaged and damaged state of the structure.

In order to validate proposed method different damage scenarios were simulated by reducing the modulus of elasticity of selected finite elements in numerical models to 80 % and 50 %. A total of nine different damage scenarios were investigated,

It was found that the Modal Shape Damage Index method is robust and effective to accurately locate single and multiple damages on plate-like structures. The results indicate that proposed method is successful in not only predicting the location of damage but also in determining the severity of damage to some extent.

It was shown that proposed method is able to locally detect the severity of the damage. One of the important characteristics of proposed method that it is capable to detect damages adjacent to plate supports. Disadvantage of the method is that damage location affects the severity of damage that is being detected.

It is shown that structural damage on plate like structures is related to the change of modal shape and also natural frequencies. The reason for using natural frequencies in structural damage assessment is that natural frequencies are easy to determine with relatively high accuracy.

Further research will be focused on effect of measurement noise and validation of the proposed method on experimental results.

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