Special workshop
MULTISCALE MODELING OF HETEROGENEOUS STRUCTURES
September 21 - 23, 2016, Dubrovnik, Croatia

MIXED MESHLESS ANALYSIS OF HETEROGENEOUS STRUCTURES USING STAGGERED GRADIENT ELASTICITY

Boris Jalušić*[1], Jurica Sorić[1], Tomislav Jarak[1], Christian Weißenfels[2] and Peter Wriggers[2]

[1] Faculty of Mechanical Engineering and Naval Architecture,
University of Zagreb, Ivana Lučića 5, 10002 Zagreb, Croatia,
e-mail: boris.jalusic@fsb.hr, jurica.soric@fsb.hr, tomislav.jarak@fsb.hr
[2] Institut für Kontinuumsmechanik, Leibniz Universität Hannover,
Appelstraße 11, 30167 Hannover,
e-mail: weissenfels@ikm.uni-hannover.de, wriggers@ikm.uni-hannover.de

Keywords: *Meshless Methods, Mixed Approach, Gradient Elasticity.*

In recent decades, an increasing interest in using meshless methods has existed due to their beneficial properties in comparison to more commonly used numerical methods such as the Finite Element Method (FEM). In this class of numerical approaches, the discretization of geometry and the approximation of unknown field variables have been done by using only points that are not connected into elements. Hence, there is no need for a time-consuming mesh creation process and the problems associated with the distorsion of elements are avoided. Despite these attractive properties of meshless methods, high numerical costs and low accuracy associated with the calculation of high order derivatives of approximation functions, which are particularly needed for solving problems involving the gradient elasticity, still represent a severe setback. The use of meshless methods based on the mixed approach [1] can alleviate the aforementioned drawbacks since they require a lower continuity degree of the approximation functions.

In this contribution, the mixed Meshless Local Petrov Galerkin (MLPG) collocation method [1] is considered for modeling the deformation of heterogeneous structures. Thereby, the strain gradient elasticity based on the Aifantis theory [2] with only one unknown microstructural parameter is employed. The gradient theory is used here in order to more accurately capture the material behaviour near the interface between regions with different material properties and to remove jumps in the strain fields that can be observed when a classic theory of linear elasticity is used.

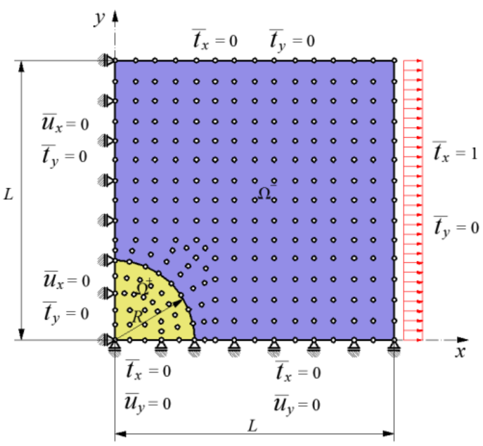

Figure 1. Plate with circular inclusion

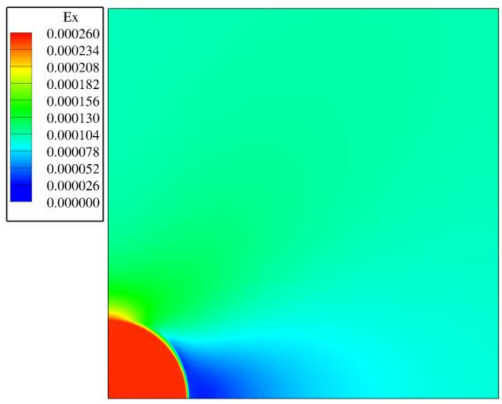

Figure 2. Contours of strain ε_x

As an example to portray the mentioned jumps in the strain field, a rectangular square plate of $2L \times 2L$ with the circular inclusion of radius $R = 1$, subjected to the horizontal traction at the opposite vertical edge is considered, as depicted in Figure 1. Due to the symmetry, only one quarter of the plate consisting of two subdomains Ω^- and Ω^+ is discretized. As obvious, the symmetry boundary conditions are used along the left and bottom edges, while the tractions are prescribed along other edges. The material properties of the plate are $E^- = 10000$, $\nu^- = 0.3$, while the values of $E^+ = 1000$ and $\nu^+ = 0.25$ are chosen for the inclusion. The solution for the heterogeneous plate is obtained by using the second order approximation functions and discretization with 606 nodes. The strain contour plot is presented in Figure 2.

As can be seen, here the heterogeneous structures consist of homogeneous parts with linear elastic material properties which are discretized by grid points, where equilibrium equations may be imposed. The solution for the entire heterogeneous structure is determined by enforcing appropriate essential and natural boundary conditions along the interfaces of the homogeneous subdomains. Herein, all independent variables are approximated in such a manner that each homogeneous subdomain is treated as a separate problem [3].

The numerical solution of fourth order problems arising in non-classic theories requires a high order of approximation functions. Hence, using FEM for solving this type of problems is not a wise choice since standard formulations need to possess C^1 continuity, which leads to complicated shape functions with large number of nodal degrees of freedom, even if mixed elements are utilized [4]. Therefore, these procedures are considered to be highly numerically inefficient [2]. The required C^1 continuity is easily obtainable in the meshless methods since the approximation functions of arbitrary order can be formulated without increasing the nodal number of unknowns [1]. In this contribution, the fourth order equilibrium equations of gradient elasticity are solved as an uncoupled sequence of two sets of the second order differential equations according to [5]. The application of the staggered solution scheme and the mixed meshless approach should result in a more stable numerical formulation. The proposed computational strategy can be extended on the modeling of material localization phenomena, and it can be used for the modeling of deformation responses on the macro as well as microlevel in the frame of multiscale computational procedures.

Acknowledgement
This work has been supported in part by Croatian Science Foundation under the project 2516. The investigations are also part of the project supported by Alexander von Humboldt Foundation.

REFERENCES

[1] S.N. Atluri, H.T. Liu, Z.D. Han. Meshless Local Petrov-Galerkin (MLPG) Mixed Collocation Method for Elasticity Problems. CMES: Computer Modeling in Engineering & Sciences, 14, 141-152, 2006.

[2] H. Askes, E.C. Aifantis. Gradient elasticity in statics and dynamics: An overview of formulations, length scale identification procedures, finite element implementations and new results. International Journal of Solids and Structures, 48, 1962-1990, 2011.

[3] J.-S. Chen, L. Wang, H.-Y. Hu, S.-W. Chi. Subdomain radial basis collocation method for heterogeneous media. International Journal for Numerical Methods in Engineering, 80, 163–190, 2009.

[4] E. Amanatidou, N. Aravas. Mixed finite element formulations of strain-gradient elasticity problems. Computer Methods in Applied Mechanics and Engineering, 191, 1723-1751, 2002.

[5] H. Askes, I. Morata, E.C. Aifantis. Finite element analysis with staggered gradient elasticity. Computers & Structures, 86, 1266-1279, 2008.