Massive Generation of Contextual Quantum Sets

Mladen Pavičić[1,2]

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"What gives quantum computers that extra oomph over their classical digital counterparts? An intrinsic, measurable aspect of quantum mechanics called contextuality, it now emerges." [1] This also gives an additional impetus to a massive generation of Kochen-Specker (KS) contextual sets we started in [2-5].

In this talk we shall present new classes of such sets in 4-, 6-, and 8-dimensional Hilbert spaces with billions of KS sets each as well as methods we make use of to generate them. The main breakthrough in massive generation of KS sets has been achieved by representing them by means of the so-called MMP diagrams which are kinds of hypergraphs in which vertices represent vectors and edges orthogonalities between them. By employing the hypergraphs we reduce the exponential complexity of solving nonlinear equations describing the orthogonalities of vectors to nearly polynomial one of handling the hypergraphs. Figure 1 shows some examples of hypergraph representation of critical KS sets we recently generated.

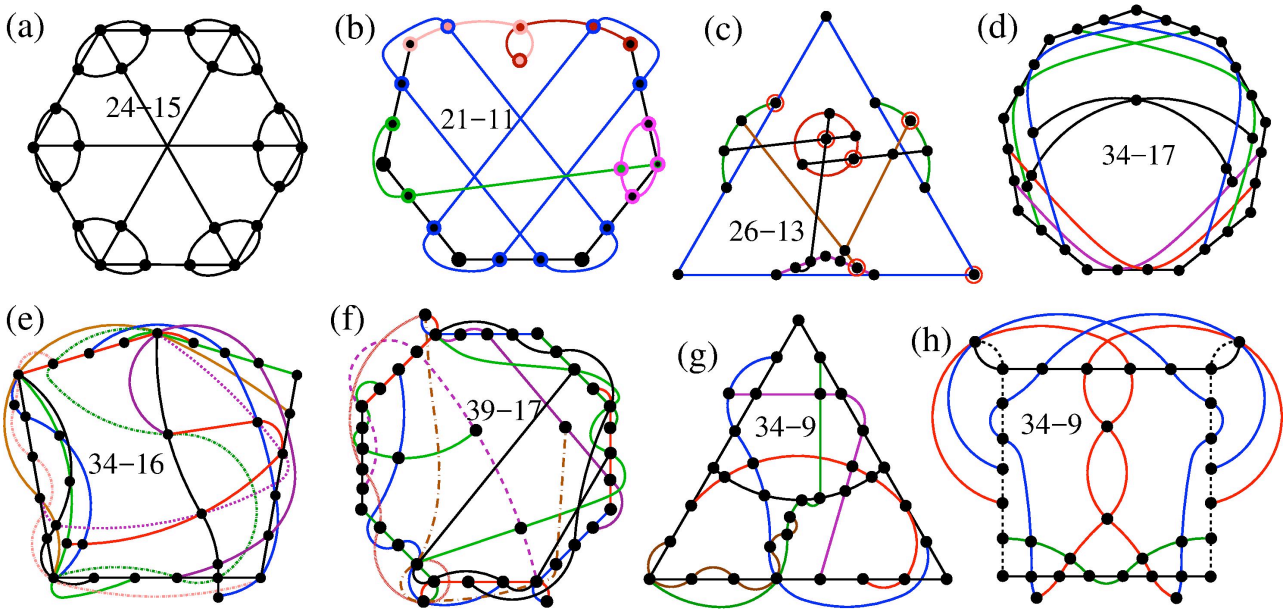

Fig1. MMP diagrams of critical KS sets (ones that would stop being KS sets, if we removed a single vertex from them) from (a) 24-24 4-dim class; (b,c) 60-105 4-dim; (d) 60-74 4-dim; (e,f) 256-1216 6-dim; (g,h) 120-2024 8-dim

As for 3-dim KS sets, we shall present a bottom-up generation of their MMP diagrams. In addition, we consider properties of 3-dim KS sets and show why recently presented 13 vector set [6] and its experimental implementation [7] do not prove the Kochen-Specker theorem.

Supports by the Alexander von Humboldt Foundation and the Croatian Science Foundation (project IP-2014-09-7515) and CEMS funding are acknowledged.

1. S.D. Bartlett, *Nature*, **510,** 345 (19 June, 2014).
2. M. Pavičić, J.-P. Merlet, B.D. McKay, and N.D. Megill**,** *J. Phys. A*, **38**, 1577 (2005).
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6. S. Yu and C.H. Oh, *Phys. Rev. Let.* **108**, 030402 (2012).
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Presentation Method: Invited Oral 30 minutes

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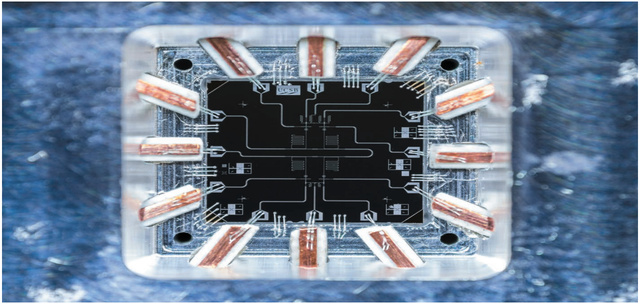
EMN Meeting on QCQI, Aug 23-26, 2016, Berlin, Germany.

EMN QCQI-2016

Quantum Computation Perspectives

NEWS IN FOCUS 426 | NATURE | VOL 532 | 28 APRIL 2016



A €1-billion (US$1.1-billion) European flagship project could advance the state of quantum computing.

FUNDING

Billion-euro boost for quantum tech

Third European Union flagship project will be similar in size and ambition to graphene and human-brain initiatives.

Quantum Computation Importance

QUANTUM COMPUTING

Powered by magic

What gives quantum computers that extra oomph over their classical digital counterparts? An intrinsic, measurable aspect of quantum mechanics called contextuality, it now emerges. SEE ARTICLE P.351

STEPHEN D. BARTLETT

For decades, researchers have struggled with the question of what makes quantum computers so powerful, and the answer has been as elusive as an understanding of quantum physics itself. Is there some unique feature of quantum physics that is responsible for enabling quantum computers to perform certain computations faster than their conventional digital counterparts? Many of the more exotic properties of quantum mechanics have been put forward as possible candidates, but so far none has held up to scrutiny. On page 351 of this issue, Howard *et al.*[1] uncover a remarkable connection between the power of quantum computers and one of the stranger properties of quantum theory known as contextuality.

Designs for quantum computers often mirror those of conventional computers, in that they are built out of basic components such as logic gates that perform elementary operations on quantum bits of information. A commonly used set of operations for a quantum processor

19 JUNE 2014 | VOL 510 | NATURE | 345

Quantum Computation Magic

ARTICLE 19 JUNE 2014 | VOL 510 | NATURE | 351
doi:10.1038/nature13460

Contextuality supplies the 'magic' for quantum computation

Mark Howard[1,2], Joel Wallman[2], Victor Veitch[2,3] & Joseph Emerson[2]

Quantum computers promise dramatic advantages over their classical counterparts, but the source of the power in quantum computing has remained elusive. Here we prove a remarkable equivalence between the onset of contextuality and the possibility of universal quantum computation via 'magic state' distillation, which is the leading model for experimentally realizing a fault-tolerant quantum computer. This is a conceptually satisfying link, because contextuality, which precludes a simple 'hidden variable' model of quantum mechanics, provides one of the fundamental characterizations of uniquely quantum phenomena. Furthermore, this connection suggests a unifying paradigm for the resources of quantum information: the non-locality of quantum theory is a particular kind of contextuality, and non-locality is already known to be a critical resource for achieving advantages with quantum communication. In addition to clarifying these fundamental issues, this work advances the resource framework for quantum computation, which has a number of practical applications, such as characterizing the efficiency and trade-offs between distinct theoretical and experimental schemes for achieving robust quantum computation, and putting bounds on the overhead cost for the classical simulation of quantum algorithms.

Quantum Computation Magic Hypergraph

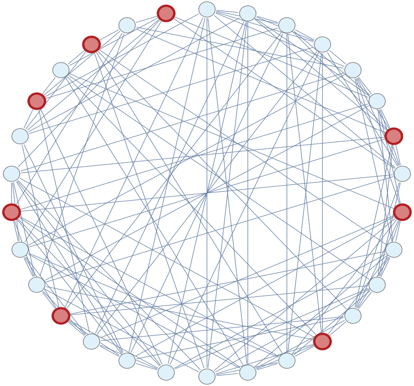
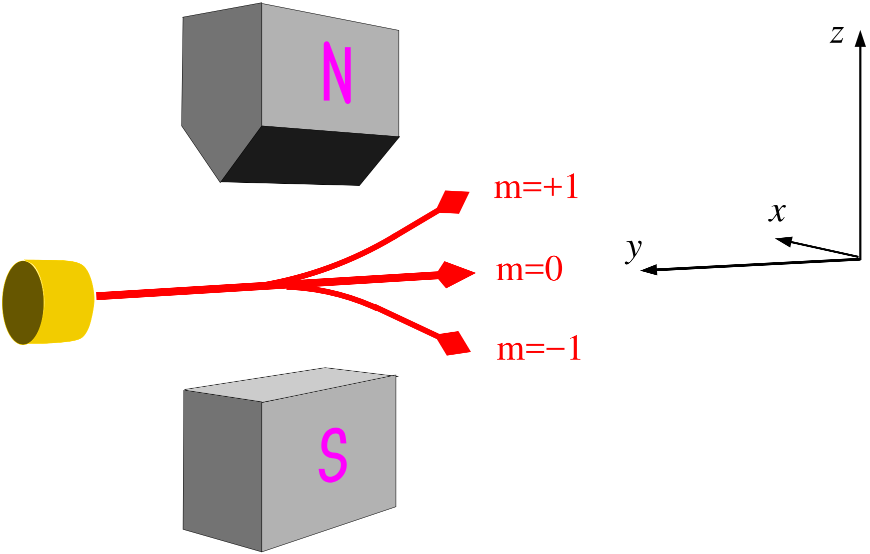


Figure 2 | Our construction applied to two qubits. Each of the 30 vertices in this graph Γ corresponds to a two-qubit stabilizer state; connected vertices correspond to orthogonal states. A maximum independent set (representing mutually non-orthogonal states) of size $\alpha(\Gamma) = 8$ is highlighted in red. As described in Theorem 1 (main text), this value of α identifies all states $\rho \notin \mathcal{P}_{\mathrm{SIM}}$ as exhibiting contextuality with respect to the stabilizer measurements in our construction.

Stern-Gerlach (SG) Experiment



Noncontextuality vs. Contextuality

Classical theories do not depend on arrangements in which measurements are carried out, i.e., on their "context," and we say that classical theories are *non-contextual* and that all their observables can be ascribed predetermined values.

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Quantum theories do depend on arrangements in which measurements are carried out and we say that quantum theories are *contextual* and that their observables cannot be ascribed predetermined values.

What Are Contextual Sets Useful for?

Contextuality can be used to reveal quantum nonlocality

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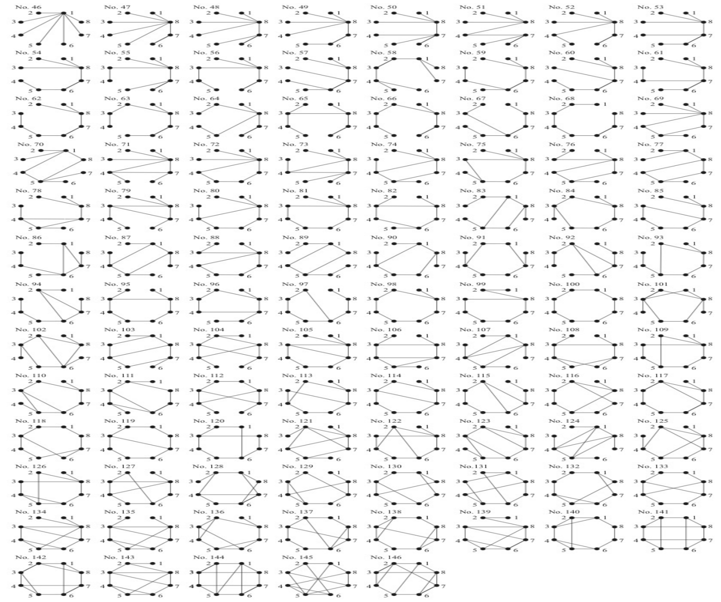
Contextual sets are likely to find applications in the field of quantum information, similar to ones recently found for the graph states implementing entanglements for one-way quantum computing and quantum error correction.

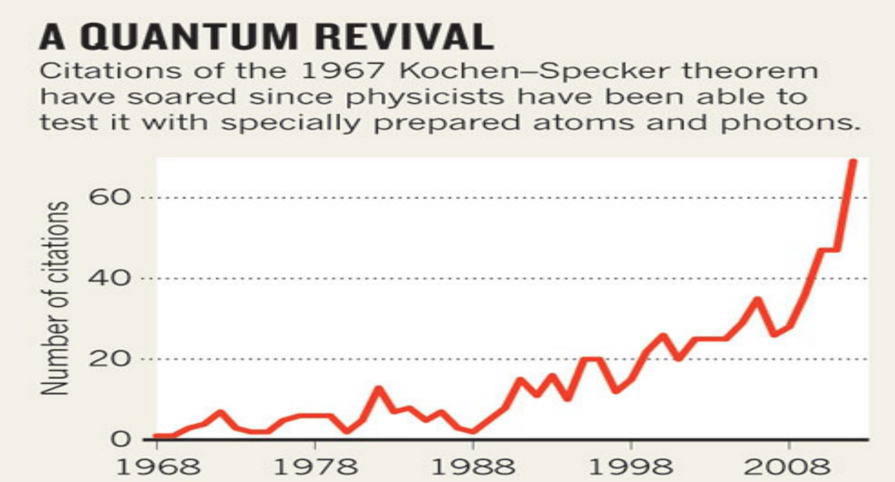
Graph States—Cabello et. al., *Phys. Rev. A* (2009)



Algorithmic resources for the one-way quantum computation.

KS Theorem Cited in Recent Papers on Contextuality

Kochen-Specker sets are the most important examples of contextual sets. Each one of them proves the Kochen-Specker theorem.



KS Contextual Application

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Archive > Volume 496 > Issue 7445 > News > Article

NATURE | NEWS

Photons test quantum paradox

Contextuality theorem could improve secure communication.

Eugenie Samuel Reich

15 April 2013

Photon Experiment

PHYSICAL REVIEW X **3**, 011012 (2013)

Experimental Implementation of a Kochen-Specker Set of Quantum Tests

Vincenzo D'Ambrosio,[1] Isabelle Herbauts,[2] Elias Amselem,[2] Eleonora Nagali,[1]
Mohamed Bourennane,[2] Fabio Sciarrino,[1,3] and Adán Cabello[4,2]
[1]*Dipartimento di Fisica, "Sapienza" Università di Roma, I-00185 Roma, Italy*
[2]*Department of Physics, Stockholm University, S-10691 Stockholm, Sweden*
[3]*Istituto Nazionale di Ottica (INO-CNR), Largo E. Fermi 6, I-50125 Firenze, Italy*
[4]*Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*
(Received 20 September 2012; published 14 February 2013)

The Definition of the Kochen-Specker Sets

Definition 1. Every KS set is a set of vectors in a Hilbert space $\mathcal{H}^n$, $n \geq 3$ to which it is impossible to assign 1's and 0's in such a way that:

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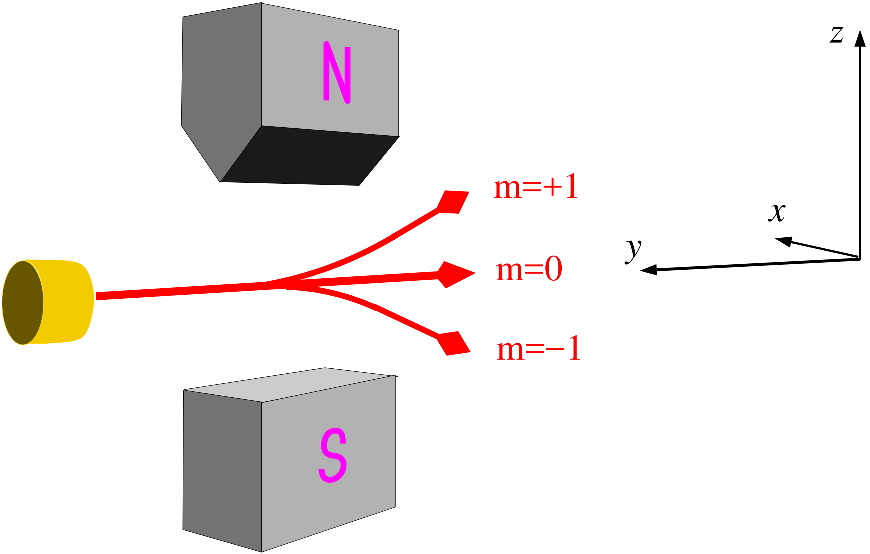
1. No two orthogonal vectors are both assigned the value 1;
2. Not all of any mutually orthogonal vectors are assigned the value 0.

Stern-Gerlach (SG) Experiment



Orthogonality and Nonlinearity; 4-Dim Example

$$\mathbf{a}_A \cdot \mathbf{a}_B = a_{A1}a_{B1} + a_{A2}a_{B2} + a_{A3}a_{B3} + a_{A4}a_{B4} = 0,$$
$$\mathbf{a}_A \cdot \mathbf{a}_C = a_{A1}a_{C1} + a_{A2}a_{C2} + a_{A3}a_{C3} + a_{A4}a_{C4} = 0,$$
$$\mathbf{a}_A \cdot \mathbf{a}_D = a_{A1}a_{D1} + a_{A2}a_{D2} + a_{A3}a_{D3} + a_{A4}a_{D4} = 0,$$
$$\mathbf{a}_B \cdot \mathbf{a}_C = a_{B1}a_{C1} + a_{B2}a_{C2} + a_{B3}a_{C3} + a_{B4}a_{C4} = 0,$$
$$\mathbf{a}_B \cdot \mathbf{a}_D = a_{B1}a_{D1} + a_{B2}a_{D2} + a_{B3}a_{D3} + a_{B4}a_{D4} = 0,$$
$$\mathbf{a}_C \cdot \mathbf{a}_D = a_{C1}a_{D1} + a_{C2}a_{D2} + a_{C3}a_{D3} + a_{C4}a_{D4} = 0.$$

Brute Force — Mission Impossible

McKay-Megill-Pavičić (MMP) Hypergraphs

Fortunately, I realised that these equations can be reduced to a generation and then filtering of hypergraphs, in particular McKay-Megill-Pavičić (MMP) hypergraphs, which Brendan D. McKay, Norman D. Megill, and I defined previously for another purpose.

Definition 2. We define MMP hypergraphs as follows
(i) Every vertex belongs to at least one edge;
(ii) Every edge contains at least 3 vertices;
(iii) Edges that intersect each other in $n-2$ vertices contain at least n vertices.

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This and many other subsequently developed algorithms developed by Brendan D. McKay, Norman D. Megill, Jean-Pierre Merlet, P.K. Aravind, Mordecai Waegell, and Mladen Pavičić (2005-2016) enabled us to generate KS sets exhaustively (in principle).

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Pavičić, M., N. Megill, P. K. Aravind, and M. Waegell, New class of 4-dim Kochen-Specker sets, *J. Math. Phys.*, **52**, 022104-1-9 (2011);

Waegell, M., P. K. Aravind, N. Megill, and M. Pavičić, Parity Proofs of the BKS Theorem Based on the 600-cell, *Found. Physics*, **41**, 883-904 (2011);

Megill, N., K. Fresl, M. Waegell, P. K. Aravind, and M. Pavičić, Probabilistic Generation of Quantum Contextual Sets. *Phys. Lett.* A, **375**, 3419-3424 (2011); Supplementary Material;

Megill, N. and M. Pavičić, Kochen-Specker Sets and Generalized Orthoarguesian Equations, *Ann. Henri Poinc.* , **12**, 1417-1429 (2011);

Pavičić, M., *Companion to Quantum Computation and Communication*, Wiley-VCH (2013); Sec. 1.17

MMP Formalism

We encode MMP hypergraphs by means of alphanumeric and other printable ASCII characters. Each vertex is represented by one of the following characters: 1 2 3 4 5 6 7 8 9 A B C D E F G H I J K L M N O P Q R S T U V W X Y Z a b c d e f g h i j k l m n o p q r s t u v w x y z ! " # $ % & ' ( ) * - / : ; < = > ? @ [\ ] ^ _ ` { | } ~ , and then again all these characters prefixed by '+', then prefixed by '++', etc.

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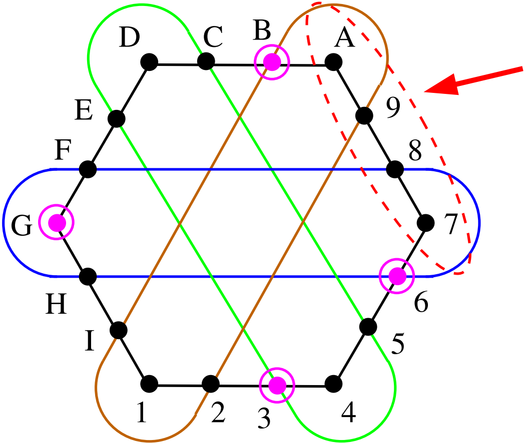
Each edge is represented by a string of characters that represent vertices within a single line. Edges are separated by commas. The line must end with a full stop. Skipping of characters is allowed. A line forms a representation of a hypergraph. The order of the edges is irrelevant. The numbers of vertices and edges are unlimited. We often present MMP hypergraphs starting with edges forming the biggest loop to facilitate their possible drawing.

Contextuality Visualisation



1234,4567,789A,ABCD,DEFG,GHI1,29BI,35CE,68FH.

1={0,0,0,1}, . . . A={0,1,1,0}, . . . C={1,1,-1,-1}, . . . I={0,1,0,0}.

Algorithms Are Statistically Polynomially Complex but Nevertheless Demanding

$$\boxed{\text{brute force}} \qquad \begin{array}{c} 10^{25} \\ \text{nonlinear equations} \\ \Downarrow \\ \boxed{\text{bulk generation}} \\ \Downarrow \end{array} \qquad \boxed{\text{Age of the Universe}}$$

$$\boxed{\text{MMP generation}} \quad 10^{10}\ \text{MMP hypergraphs} \quad \boxed{\begin{array}{c}\text{1 month on} \\ \text{100 CPU cluster}\end{array}}$$

$$\begin{array}{c} \Downarrow \\ \text{1233 KS sets} \end{array} \tag{1}$$

M. Pavičić, J-P. Merlet, B.D. McKay & N.D. Megill (2005)

\	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*total*
18	1																1
19		1															1
20		1	5	1													7
21			2	11	4	1											18
22			1	9	36	23	12	3	1								85
23				2	19	76	79	58	27	11	3	1					276
24				1	6	39	137	187	188	136	83	41	18	6	2	1	845
total	1	2	8	24	65	139	228	248	216	147	86	42	18	6	2	1	1233

Table: KS sets for systems with 4 degrees of freedom with up to 24 vectors with component values from {-1,0,1}.

M. Pavičić, J-P. Merlet, B.D. McKay & N.D. Megill (2005)

\	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*total*
18	1																1
19		1															1
20		1	5	1													7
21			2	11	4	1											18
22			1	9	36	23	12	3	1								85
23				2	19	76	79	58	27	11	3	1					276
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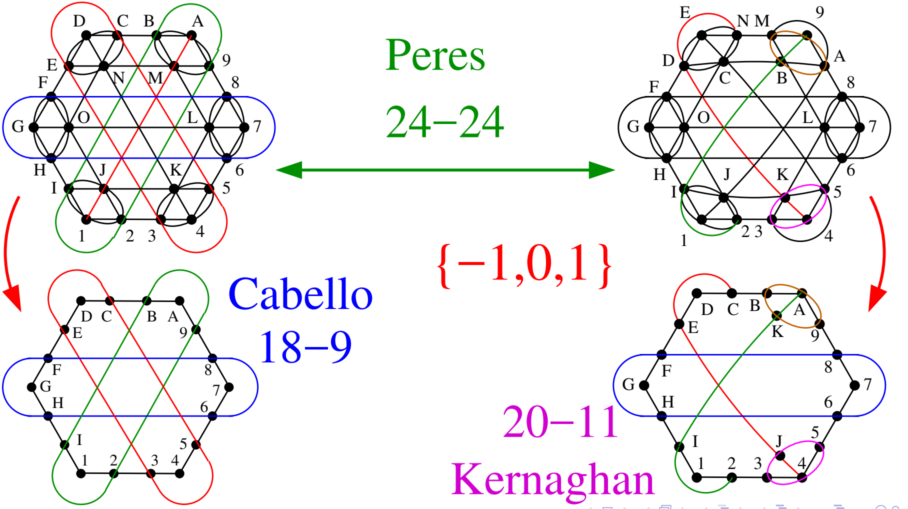
Table: KS sets for systems with 4 degrees of freedom with up to 24 vectors with component values from {-1,0,1}.

We also found 37 new KS sets with 22 through 24 vectors with component values from other sets (not from {-1,0,1})

Asher Peres 24-24 (1991), M. Kernaghan 20-11 (1994), Adán Cabello, J. Estebaranz, & G. García-Alcaine 18-9 (1996)



M. Pavičić, N.D. Megill, & J-P. Merlet (2010)

Stripping Peres' 24-24 KS set gives us the same 1233 KS sets.

$$\begin{array}{ccc} & \boxed{\text{stripping}} & \\ & \Downarrow & \\ \boxed{\text{isomorphic filtering}} & \text{all 1233 KS sets} & \boxed{<\ 1 \text{ min on } 1 \text{ CPU)}} \end{array} \tag{2}$$

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Now, let us consider experimental implementation.

M. Pavičić, N.D. Megill, & J-P. Merlet (2010)

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Experimentally distinguishable are only *critical KS sets*, i.e., those KS sets that do not properly contain any KS subset.

M. Pavičić, N.D. Megill, & J-P. Merlet (2010)

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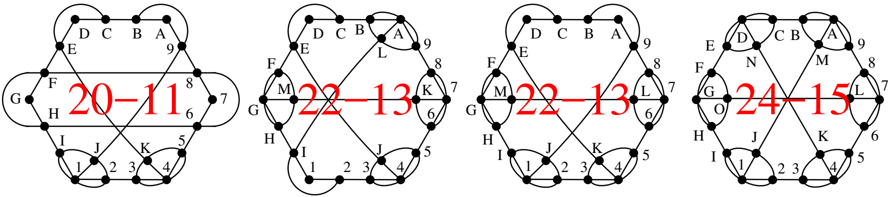
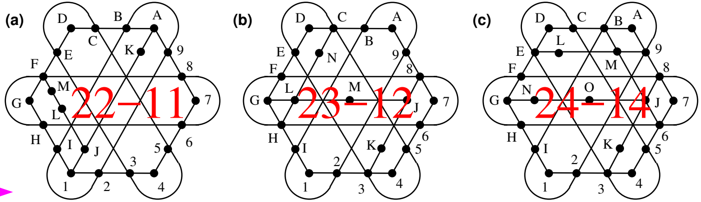
There are altogether six critical subsets of Peres' 24-24 set. These are Cabello's 18-9, Kernagahn's 20-11, and the following 4 of ours $\Longrightarrow$

Critical KS Sets with Components from $\{-1, 0, 1\}$ and Other KS Sets



What about ~~{−1,0,1}~~?



60-74 Vector Class: Billions of Critical KS Sets

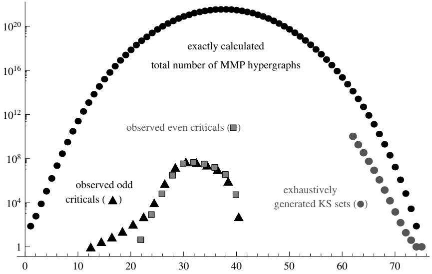


Figure: Statistics calculated for subsets of 60-74 given on a logarithmic scale. There are more than 10^9 critical KS sets. Given numbers of critical KS sets with 13 to 27 edges (on the x-axis) are exhaustive. The number of criticals with 32 edges is the biggest; we estimate that they do not exceed 10^{10}. Given numbers of noncritical KS sets with more than 61 edges are also exhaustive.

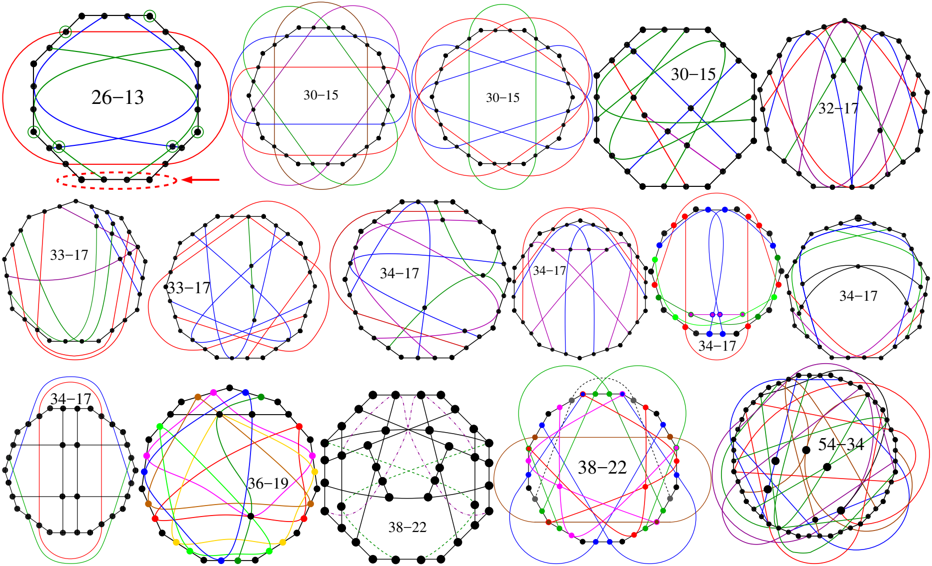
60-74 Vector Class: Billions of Critical KS Sets

Critical KS sets from the 60-74 class (13 to 41 edges & 26 to 60 vertices)

vertices \ edges	13	15	17	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
26	1																								
30		3																							
32			1																						
33			2																						
34			5																						
36				11																					
37				9																					
38				6	11	5																			
39					31		6																		
40					42		74																		
41					22		52	131																	
42					10		126	727	238	1															
43							129	84	4636	162															
44							116	4	3270	21223	152														
45							24		871	37223	61533	66													
46							12		576	13424	284700	160088	17												
47									295	1382	169411	1488043	280073	2											
48									134	95	39418	1451894	6410844	379228	4										
49									21	2	4598	479279	9066784	19893539	185053										
50									14		940	76651	4395338	40017273	34308240	37407									
51											250	6234	975815	24774919	90756609	31426252	451								
52											67	303	120537	7296689	70507109	111015345	20902020								
53											10		7710	1076857	24336519	103127940	95148579	12010450	54						
54											12		731	92442	4476012	42608201	105755960	70800303	6603734						
55													108	3590	423982	8721957	48470624	89663708	48334576	3653990	4				
56													43	128	24845	1016418	11456719	45666317	69872507	31313317	1966077				
57															827	58353	1358794	10884172	36303582	48780264	18078665	878617			
58															103	2326	99201	1410945	8787595	25502173	29571055	8332138	270723		
59																17	2717	79484	939611	5204785	13687824	12863229	2449334	45380	
60																3	162	3905	58412	480428	2204828	4723895	3077397	347579	3519

List of 1540184852 non-isomorphic KS critical sets from the 60-74 class we obtained on our cluster. We conjecture that all possible types of vertex-edge sets are given here, i.e., that an exhaustive generation would not provide us with any new type. An exhaustive generation might give up to about an order of magnitude more samples of these sets.

Chosen Critical Sets from the 60-74 Class

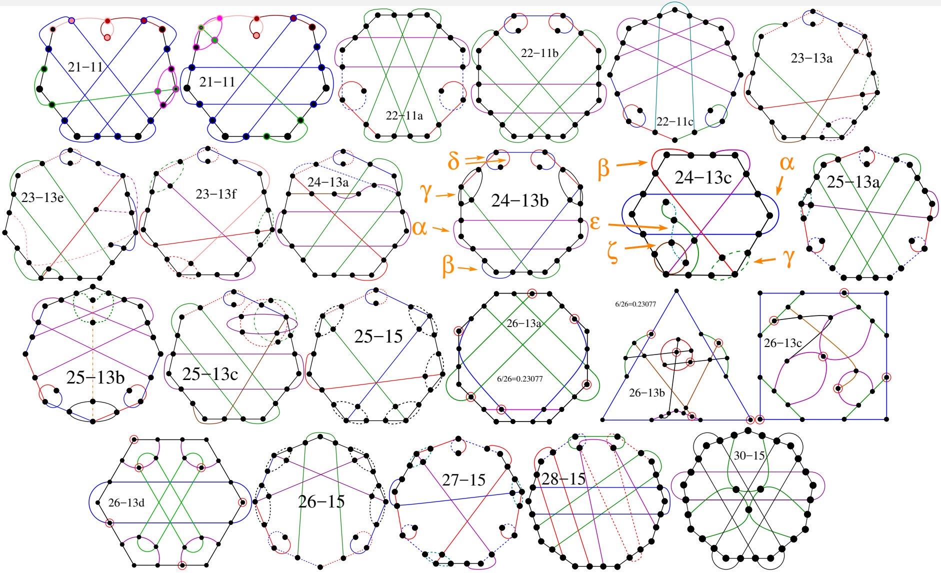


60-105 Vector Class: Millions of Critical KS Sets

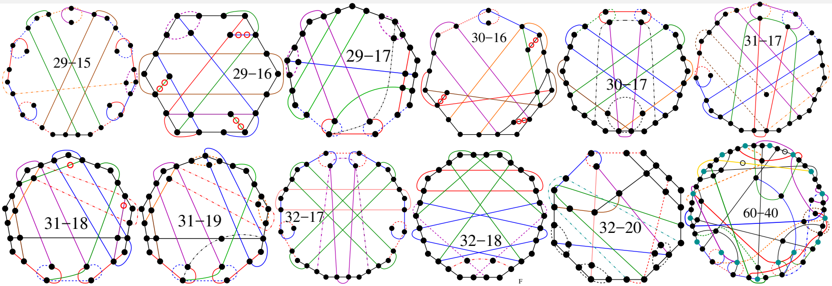
7720539 critical KS sets from the 60-105 class (9 to 40 edges & 18 to 60 vertices)

⊙	9	11	13	15	16	17	18	19	20	21	22	23	24	25	26	27	⊗
⊙	1																18
⊙		2															20
⊙		2															21
⊙		3	2														22
⊙			6														23
⊙			25	1													24
⊙			23	3													25
⊙			15	46													26
⊙				138													27
⊙				252		9											28
	⊙			159	2	123											29
				65	11	890	7										30
		⊙				1812	215	42									31
						1944	444	1173	1								32
			⊙			890	381	5884	42								33
						238	239	13776	3549	665							34
				⊙		1	13	16501	11005	8289	1						35
	28	29	30				10	11606	13459	40535	411	27					36
44	84	1			⊙		3	4059	8183	89747	9477	2162					37
45	3404	41						835	3161	120118	34244	24573	1				38
46	15374	1957		31	32	⊙		12	532	100316	58355	110686	1024	47			39
47	28757	13368	342				⊙	1	170	54486	55221	280813	11634	1648			40
48	30880	37731	4877	77					35	15806	30340	445033	36354	19146			41
49	22962	63876	14899	2516		33	34		3	2623	11214	475675	59429	92755	2085	46	42
50	12687	79656	20713	11260	741			⊙		66	2644	340138	60432	265314	13592	1512	43
51	5410	73059	17174	20368	5458	206				6	641	162232	42123	521889	32795	15500	44
52	1775	47907	9890	22505	10815	2619	18		⊙		97	39997	19078	693125	45112	60379	45
53	394	22611	4317	18025	10589	7314	1226				11	5127	5875	617069	40720	146333	46
54	41	7582	1577	11225	6239	8179	3970	443		⊙		41	1190	353232	26801	248373	47
55	5	1868	457	4947	2611	5388	4534	1920	91				182	137593	12783	297725	48
56	1	706	88	1648	873	2509	2810	2676	872	11	⊙		17	26039	3977	244506	49
57		1	18	497	254	864	1099	1578	1299	399			1	2927	853	137570	50
57		1	18	497	254	864	1099	1578	1299	399		⊙		3	117	48945	51
58				1	45	241	281	612	743	495	128		⊙		17	15678	52
59					8	66	63	120	226	239	131	26		⊙	1	1773	53
60						4	11	15	29	49	40	21	2	⊙		180	54
⊗	28	29	30	31	32	33	34	35	36	37	38	39	40	⊙	⊙	⊙	⊙

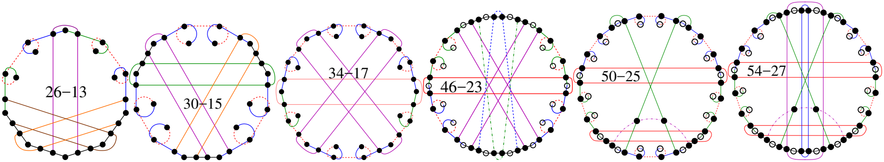
Examples from the 60-105 Operator Class



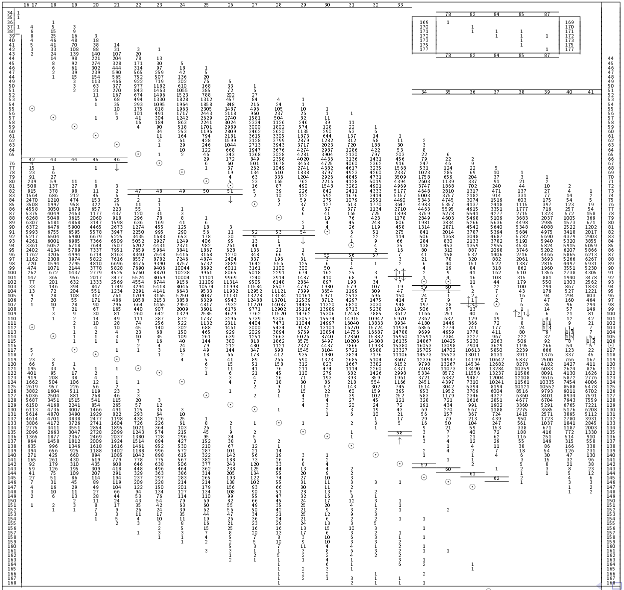
Further Examples from the 60-105 Operator Class

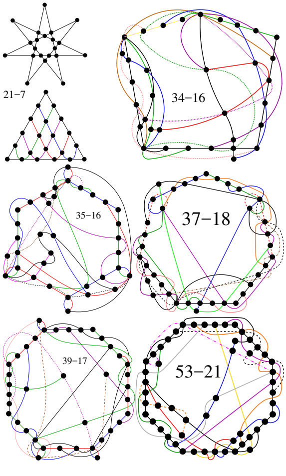


Some 60-105-class KS critical sets with no parity proofs (even number of edges) compared with some sets with parity proofs (odd number of edges).



3714503 6-dim Critical KS Sets from the 236-1216 Class

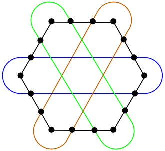


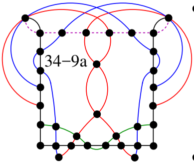


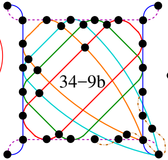
1802014 8-dim Critical KS Sets from the 120-2024 Class

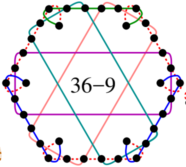
	13	15	17	19	21	22	23	24	25	26
26	1									
30		3								
32			1							
33			2							
34			5							
36				11						
37				9						
38				6	11	5				
39					31		6			
40					42		74			
41					22		52	131		
42					10		126	727	238	1
43							129	84	4636	162
44							116	4	3270	21223
45							24		871	37223
46							12		576	13424
47									295	1382
48									134	95
49									21	2
50									14	

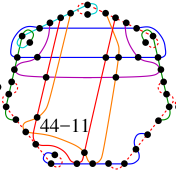
	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
44	152														
45	61533	66													
46	284700	160088	17												
47	169411	1488043	280073	2											
48	39418	1451894	6410844	379228	4										
49	4598	479279	9066784	19893539	185053										
50	940	76651	4395338	40017273	34308240	37407									
51	250	6234	975815	24774919	90756609	31426252	451								
52	67	303	120537	7296689	70507109	111015345	20902020								
53	10		7710	1076857	24336519	103127940	95148579	12010450	54						
54	12		731	92442	4476012	42608201	105755960	70800303	6603734						
55			108	3590	423982	8721957	48470624	89663708	48334576	3653990	4				
56			43	128	24845	1016418	11456719	45666317	69872507	31313317	1966077				
57					827	58353	1358794	10884172	36303582	48780264	18078665	878617			
58					103	2326	99201	1410945	8787595	25502173	29571055	8332138	270723		
59						17	2717	79484	939611	5204785	13687824	12863229	2449334	45380	
60						3	162	3905	58412	480428	2204828	4723895	3077397	347579	3519

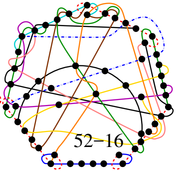










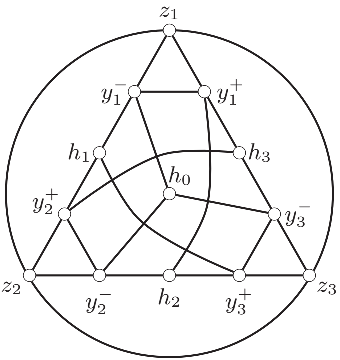


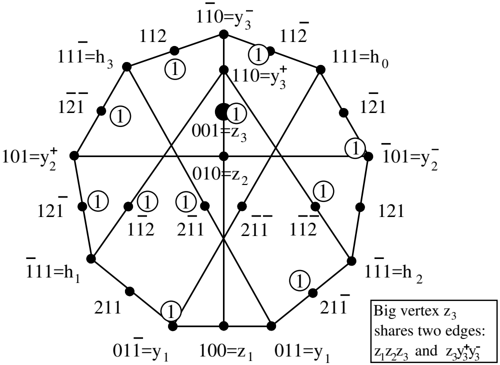
Yu-Oh 13 Vector Set

PRL **108,** 030402 (2012) PHYSICAL REVIEW LETTERS

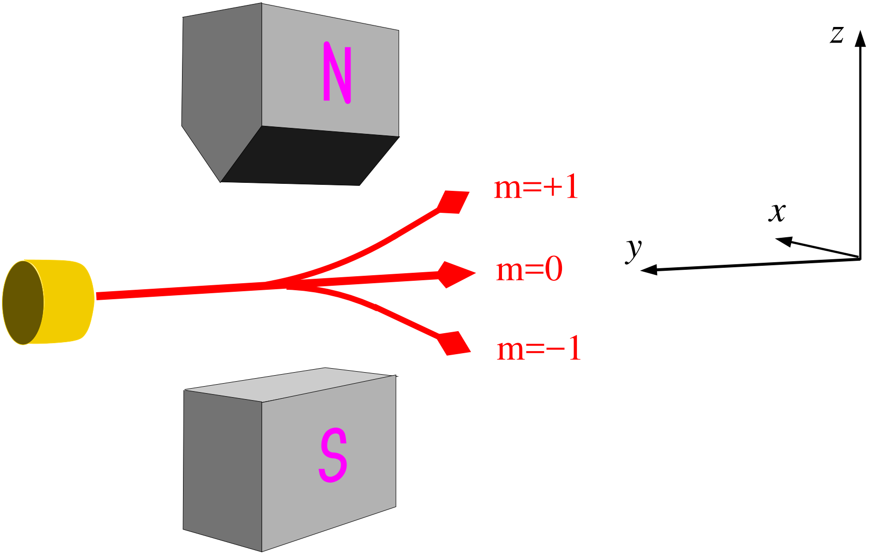
State-Independent Proof of Kochen-Specker Theorem with 13 Rays

Sixia Yu[1,2] and C. H. Oh[1]





Stern-Gerlach (SG) Experiment



Yu-Oh 13 Set Details . . .

Yu and Oh claim that their 13 vector set proves the KS theorem since vertices $h_0 - h_3$ cannot be assigned more than one 1 simultaneously.

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The probability of assigning '1', when any of 25 tests is chosen at random, is between 11/25=0.32 and 9/25=0.48 giving '1' on average with the probability of 2/5=0.4 and since it is greater than quantum 1/3=0.33, there is no contextuality.

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The noncontextuality for $h_0 - h_3$ means that they must be assigned '1', together with all other vertices among 25, with the probability of 2/5 which sets $4{\times}2/5{=}1.6$ and this is $>$ 4/3=1.33. Hence, the set cannot be a proof of the KS theorem by definition since the upper bound for $h_0 - h_3$ values is 1.6 and not 1 as Yu and Oh claim.

...and Consequences—The experiment is Not about the Contextuality

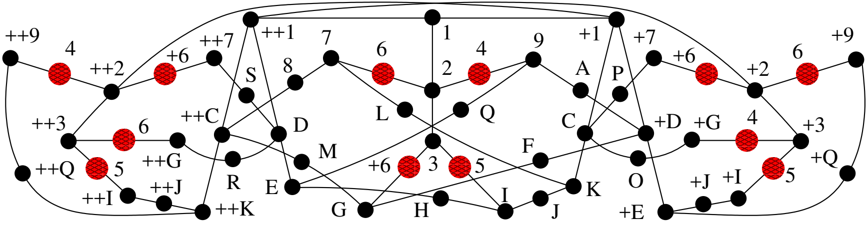
Consequently, the following recent experiment on this Yu-Oh result does not prove the KS theorem either.

Selected for a Viewpoint in *Physics*
PRL **109,** 150401 (2012) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2012

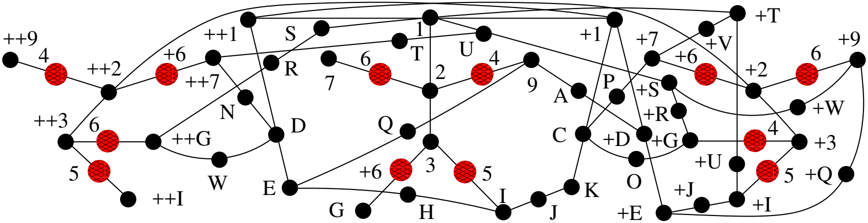
State-Independent Experimental Test of Quantum Contextuality in an Indivisible System

C. Zu,[1] Y.-X. Wang,[1] D.-L. Deng,[1,2] X.-Y. Chang,[1] K. Liu,[1] P.-Y. Hou,[1] H.-X. Yang,[1] and L.-M. Duan[1,2]
[1]*Center for Quantum Information, IIIS, Tsinghua University, Beijing, China*
[2]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*

The smallest 3-Dim KS Sets



Bub's 3-dim KS critical set with 49 vertices and 36 edges.



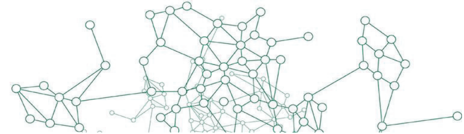
Conway-Kochen 3-dim KS critical set with 51 (not 31 as usually claimed) vertices and 37 edges.

Acknowledgements 😎

Alexander von Humboldt
Stiftung/Foundation

The speaker‘s attendance at this conference was sponsored by the Alexander von Humboldt Foundation.

http://www.humboldt-foundation.de

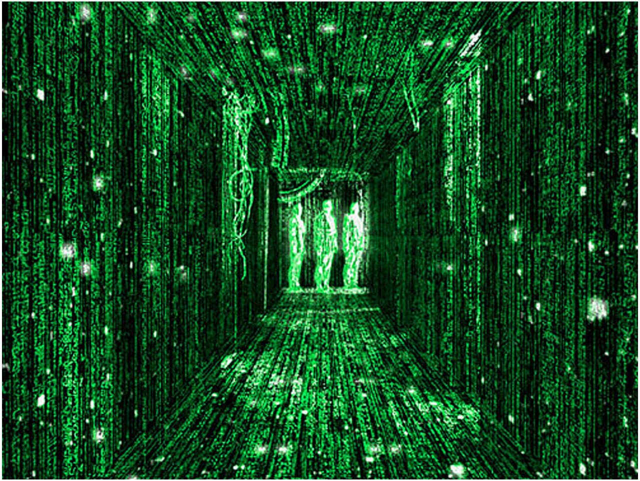


The work presented here is supported by the *Croatian Science Foundation* through project IP-2014-09-7515 and CEMS funding by the *Ministry of Science, Education and Sports of Croatia*.

Computational support was provided by the cluster *Isabella* of the *University Computing Centre* of the *University of Zagreb* and by the *Croatian National Grid Infrastructure*.

Travelling expenses were covered through the project IP-2014-09-7515.

Thanks for your attention 😎



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http://www.irb.hr/users/mpavicic/