Spectral representation of nonperturbative quark propagators – for microscopic calculations in strong matter

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Dubravko Klabučar (1) in collaboration with **Dalibor Kekez**(2)

(1)Physics Department, Faculty of Science – PMF, University of Zagreb, Croatia
(2)Rudjer Bošković Institute, Zagreb, Croatia

Overview

Introduction

- Many QFT studies (on lattice, large majority of Schwinger–Dyson studies, etc.) are **not done in the physical, Minkowski spacetime, but in 4-dim Euclidean space.**
- $\Rightarrow$ **Situation with Wick rotation (relating Minkowski with Euclid) must be under control, but this is highly nontrivial in the nonperturbative case – most importantly, the nonperturbative QCD.**
- **Do nonperturbative Green's functions permit Wick rotation?**
- **For solving Bethe-Salpeter equation and calculation of processes, extrapolation to complex momenta is necessary. $\Rightarrow$ Knowledge of the analytic behavior in the whole complex plane is needed.**
- **Very complicated matters $\Rightarrow$ studies of Ansatz forms are instructive and can be helpful to *ab initio* studies of nonperturbative QCD Green's functions. ... (and vice versa of course) ...**
- **Among them, quark propagator is the "most unavoidable" one, especially for quark matter applications.**

How Schwinger–Dyson approach generates quark propagators

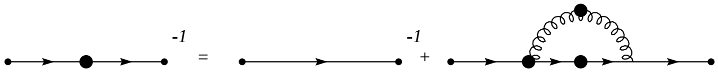
- Schwinger-Dyson (SD) approach: ranges from solving SD equations for Green's functions of non-perturbative QCD *ab initio*, to higher degrees of phenomenological modeling, *esp.* in applications including $T, \mu > 0$.

  e.g., [Alkofer, v.Smekal Phys. Rept. 353 (2001) 281], and [Roberts, Schmidt Prog.Part.Nucl.Phys. 45 (2000)S1]

- SD approach to quark-hadron physics = nonpertubative, covariant bound state approach with strong connections with QCD.

The "gap" Schwinger–Dyson equation "dressing" the quark propagator:

$$A(p^2)\not{p} - B(p^2) \equiv S^{-1}(p) = \not{p} - m - iC_F g^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k) \Gamma^\nu(k,p) G_{\mu\nu}(p-k) \, . \tag{1}$$

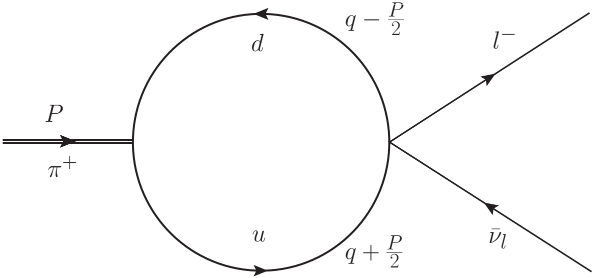


$$S(q) = \frac{A(q^2)\not{q} + B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} = Z(-q^2)\frac{\not{q} + M(-q^2)}{q^2 - M^2(-q^2)} = -\sigma_V(-q^2)\not{q} - \sigma_S(-q^2)$$

[$M(x)$ = dressed quark mass function, $Z(x)$ = wave-function renormalization]

Already just $S(p)$ enables calculations of some observables; e.g., pion decay constant

... since **close to the chiral limit**, $\Gamma_\pi^{BS} \approx -\frac{2B(q^2)}{f_\pi}\gamma_5$ is a good **approximation**

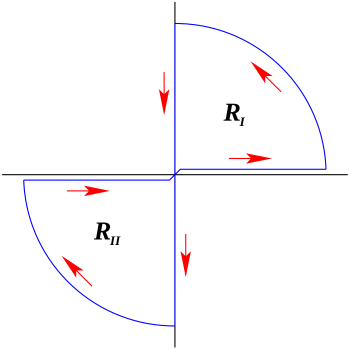


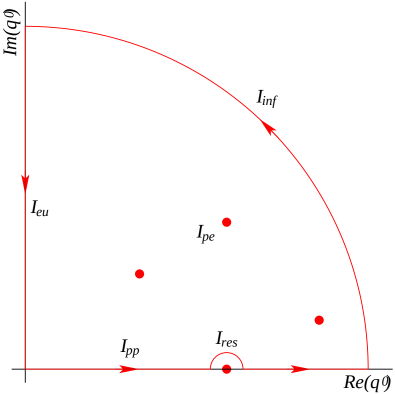
$$f_\pi = i\frac{N_c}{2}\frac{1}{M_\pi^2}\int \frac{d^4q}{(2\pi)^4}\mathrm{tr}\left[\not{P}\gamma_5 S(q+\frac{P}{2})\left(-\frac{2B(q^2)}{f_\pi}\gamma_5\right) S(q-\frac{P}{2})\right]$$

$$\Gamma(\pi^+ \to e^+\nu_e) = \frac{1}{4\pi}G^2 f_\pi^2 \cos^2\theta_c(1-\frac{m_e^2}{M_\pi^2})^2 M_\pi m_e^2 \ .$$

Wick rotation

- Even with much approximating & modeling, solving SD equations like Eq. (1), and related calculations (e.g., of f_π) with Green's functions like $S(q)$, are technically very hard to do in the physical, Minkowski space-time.
- $\Rightarrow$ additional simplification sought by transforming to 4-dim. Euclidean space by the Wick rotation to the imaginary time-component: $q^0 \to i\, q^0$:





Unlike the perturbative case, propagator singularities can cause problems!

General properties a quark propagator should have:

- $S(q) \to S_{\rm free}(q)$ because of asymptotic freedom
  $\Rightarrow \sigma_{V,S}(-q^2) \to 0$ for $q^2 \in \mathbb{C}$ and $q^2 \to \infty$
- $\sigma_{V,S}(-q^2) \to 0$ cannot be analytic over the whole complex plane
- positivity violating spectral density $\leftrightarrow$ confinement

Try Ansätze of the form (meromorphic parametrizations, like Alkofer&al. [1]):

$$S(p) = \frac{1}{Z_2} \sum_{j=1}^{n_p} r_j \left(\frac{\not p + a_j + ib_j}{p^2 - (a_j + ib_j)^2} + \frac{\not p + a_j - ib_j}{p^2 - (a_j - ib_j)^2} \right) \tag{2}$$

Dressing f′nctns are thus

$$\sigma_V(x) = \frac{1}{Z_2} \sum_{j=1}^{n_p} \frac{2r_j(x + a_j^2 - b_j^2)}{(x + a_j^2 - b_j^2)^2 + 4a_j^2 b_j^2}$$

(e.g., see Ref. [1]) and

$$\sigma_S(x) = \frac{1}{Z_2} \sum_{j=1}^{n_p} \frac{2r_j a_j(x + a_j^2 + b_j^2)}{(x + a_j^2 - b_j^2)^2 + 4a_j^2 b_j^2}$$

Constraints:

$$\sum_{j=1}^{n_p} r_j = \frac{1}{2} \qquad \sum_{j=1}^{n_p} r_j a_j = 0$$

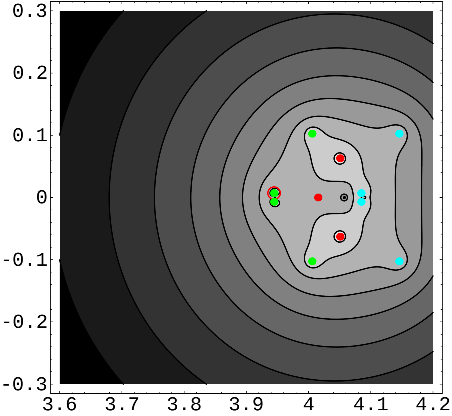
2CC quark propagator - with just 2 pairs of complex conj. poles

Parameters (yielding 2CC propagator of Alkofer & al.): $n_p = 2$, $a_1 = 0.351$, $a_2 = -0.903$, $b_1 = 0.08$, $b_2 = 0.463$, $r_1 = 0.360$, $r_2 = 0.140$, $Z_2 = 1$.

- **Naive Wick rotation impossible**
- **In calculated quantities, proliferation of poles from propagators:**
- **contour plot of the f_π integrand - the complex function**

$q^0 \mapsto |\mathrm{trace}(q^0, \xi)|$
for $\xi \equiv |\mathbf{q}| = 4.0$.

Dots are the poles of this function.



Are quark propagators with CC poles really OK for nonperturbative QCD?

2CC propagator Ansatz by Alkofer&al. gives at $\xi = 0.5$, these integrals over q^0:

$$I_{pp} = 0.184102 \qquad I_{pe} = 0.184102 + 0.0397371i$$

$$I_{inf} \approx 0 \qquad I_{res} = 0.0166534i \qquad I_{eu} = 0.0230837i$$

$\Rightarrow$ Cauchy's residue theorem is satisfied: $|I_{pp} + I_{res} + I_{eu} - I_{pe}| = 1.4 \cdot 10^{-9}$

$f_\pi = 0.071 \ \mathrm{GeV}$ = Euclidean result (from I_{eu}), cannot be reproduced in Minkowski space (i.e., from $I_{pp} + I_{res}$).

* Quark propagators with CC poles have been "popular" since they have **no Källen-Lehmann representation** $\Rightarrow$ corresponding states cannot appear in the physical particle spectrum $\Rightarrow$ **seem appropriate for confined QCD** ... **but,** $\exists$ **objections besides not allowing naive Wick rotation: possible problems for causality and unitarity of the theory.**

* Also, Benić *et al.*, PRD 86 (2012) 074002, have shown that CC poles of the quark propagator cause thermodynamical instabilities at nonvanishing temperature and density.

Quark propagators with singularities only on the real axis?

Investigations (e.g. [1]Alkofer et al., Phys.Rev. D70 (2004) 014014) based on the available *ab initio* SD and lattice results indicate: **gluon, but probably also quark, propagators have singularities (isolated poles or branch cuts) only for real momenta.** $\Rightarrow$ **Use such Ansätze !**

This brings also a practical calculational advantage: **No obstacles to Wick rotation** $\Rightarrow$ π **decay constant** f_π **calculated equivalently**

* **in the Minkowski space:**

$$f_\pi^2 = -i\frac{N_c}{4\pi^3 M_\pi^2}\int_0^\infty \xi^2\, d\xi \int_{-\infty}^{+\infty} dq^0\, B(q^2)\,\mathrm{tr}\left(\not{P}\gamma_5 S(q+\frac{P}{2})\gamma_5 S(q-\frac{P}{2})\right)$$

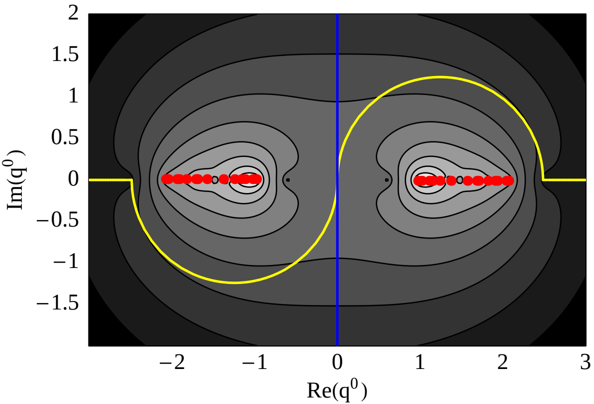
where $q^2 = (q^0)^2 - \xi^2$**,** $\xi \equiv |\mathbf{q}|$**, and** $q\cdot P = M_\pi q^0$, **OR**

* **in the Euclidean space:**

$$f_\pi^2 = \frac{3}{8\pi^3 M_\pi^2}\int_0^\infty dx\, x \int_0^\pi d\beta\, \sin^2\beta\, B(q^2)\mathrm{tr}\left(\not{P}\gamma_5 S(q+\frac{P}{2})\gamma_5 S(q-\frac{P}{2})\right)$$

where $q^2 = -x$ **and** $q\cdot P = -iM_\pi\sqrt{x}\cos\beta$**.**

Example: f_π calculation with a propagator singular only on the real axis



Contour plot of the integrand and path of integration in the complex q^0–plane.

$\Rightarrow$ $2\frac{f_\pi - f_\pi}{f_\pi + f_\pi} \sim 10^{-5}$ ... great ... but $f_\pi = 67\ \mathrm{MeV}$ too small $\Rightarrow$ try out various **such** models (Ansätze)! **But how to make such Ansätze more generally? ...**

… revisit Källén–Lehmann spectral representation

Spectral density $\rho(q^2)$: the sum-integral over a complete set of states $|n\rangle$:

$$\theta(q^0)\rho(q^2) = (2\pi)^3 \sum_n \delta^{(4)}(q - P_n)|\langle 0|\phi(0)|n\rangle|^2$$

Spectral representation of the scalar field (ϕ) propagator:

$$\Delta(q) = \int_0^\infty d\nu^2 \; \frac{\rho(\nu^2)}{q^2 - \nu^2 + i\epsilon} \quad ,$$

and of Dirac **fermion propagator** : $\quad S(q) = \int_0^\infty d\nu^2 \; \frac{\rho_1(\nu^2)\, \not{q} + \rho_2(\nu^2)}{q^2 - \nu^2 + i\epsilon}$

Quark propagator decomposition: $\quad S(q) = -\sigma_V(-q^2)\not{q} - \sigma_S(-q^2)$

Quark dressing functions (of $x \equiv -q^2$):

$$\sigma_V(x) = \int_0^\infty d\nu^2 \; \frac{\rho_1(\nu^2)}{x + \nu^2} \qquad \sigma_S(x) = \int_0^\infty d\nu^2 \; \frac{\rho_2(\nu^2)}{x + \nu^2} \tag{3}$$

Properties of spectral densities in various spectral representations

All $\rho_{,1,2} \in \mathbb{R}$ but only positive spectral densities correspond to states in the physical particle spectrum! For a Dirac field, Källén–Lehmann spectral representation exists only if

- $\nu\rho_1(\nu^2) - \rho_2(\nu^2) \geq 0$. (Permits simplifying Ansatz $\rho_2(\nu^2) = \nu\rho_1(\nu^2)$.)
- $\rho_1(\nu^2) \geq 0$

But a spectral representation need not be a Källén–Lehmann one!

If $\rho_{1,2}$ are not positive, Eqs. (3) are still spectral representations, but appropriate to confined quarks, absent from physical particle spectrum.

Then, use σ_V and σ_S from Eqs. (3) to express functions A_E, B_E, and M_E:

$$
\begin{aligned}
A_E(x) \equiv A(-x) &= \frac{\sigma_V(x)}{\sigma_S^2(x) + x\,\sigma_V^2(x)} ,\\
B_E(x) \equiv B(-x) &= \frac{\sigma_S(x)}{\sigma_S^2(x) + x\,\sigma_V^2(x)} ,\\
M_E(x) \equiv M(-x) &= \frac{\sigma_S(x)}{\sigma_V(x)} .
\end{aligned}
$$

Stieltjes transform

The Stieltjes transformation $F(z)$, for $\mathrm{Im}(z) \neq 0$ or $\mathrm{Re}(z) > 0$ is

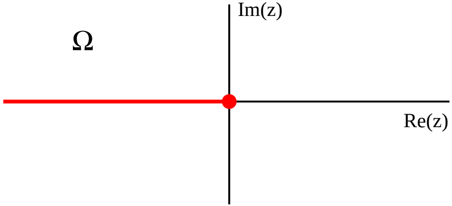
$$F(z) = \int_0^\infty \frac{d\alpha(t)}{t+z} = \lim_{R\to\infty} \int_0^R \frac{d\alpha(t)}{t+z} \tag{4}$$

Remarkable: **If (4) converges, function $z \mapsto F(z)$ is analytic $\forall z \in \Omega$.** $\Rightarrow$ it cannot exist for, e.g., propagators with CC poles, not analytic in the whole Ω.
Spectral representations like Eqs. (3) – just a special case of Eq. (4):

$$F(z) = \int_0^\infty dt\, \frac{\rho(t)}{t+z} \ .$$

Cut plane Ω:

$\Omega \equiv \mathbb{C} -$ *negative axis*



Conversely, the density function $\rho(t) = \dfrac{1}{2\pi i} \lim_{\epsilon\to 0^+} [F(-t-i\epsilon) - F(-t+i\epsilon)]$

3R quark propagator - 3 poles on the real axis

Parameters (**yielding through Eq. (2) 3R propagator of Alkofer & al., and giving larger f_π**): $n_p = 3$, $a_1 = 0.341$, $a_2 = -1.31$, $a_3 = -1.35919$, $b_1 = 0$, $b_2 = 0$, $b_3 = 0$, $r_1 = 0.365$, $r_2 = 1.2$, $r_3 = -1.065$, $Z_2 = 0.982731$.

To have such poles on the real axis, the spectral density $\rho(\nu^2)$ must be given by distributions - delta functions. Concretely for the 3R quark propagator:

$$\rho(\nu^2) = \sum_{j=1}^{3} A_j^{-1} \delta(\nu^2 - M_j^2)$$

where $A_1 = 1.35$, $A_2 = 0.41$, $A_3 = -0.46$

Although the propagator functions A and B from $S^{-1}(q) = A(-q^2)\not{q} - B(-q^2)$ exhibit structure of poles different from σ_V and σ_S,

$$A(x) = \frac{\sigma_V(x)}{\sigma_S^2(x) + x\,\sigma_V^2(x)} \quad , \quad B(x) = \frac{\sigma_S(x)}{\sigma_S^2(x) + x\,\sigma_V^2(x)}$$

their poles are still on the real axis.

Some results with 3R quark propagator Ansatz

Cauchy theorem checks out well; for $\xi = 0.5$

$$|I_{\rm pp} + I_{\rm res} + I_{\rm inf} + I_{\rm eu} - I_{\rm pe}| \sim 10^{-8}$$

Poles at real axis $\Rightarrow I_{\rm pe} = 0$, and particular numerical integrals for $\xi = 0.5$ are

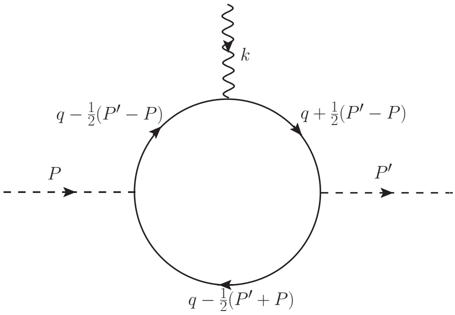
$$\begin{aligned} I_{\rm pp} &\approx 4 \cdot 10^{-12}\,, \qquad I_{\rm res} = -0.0221314i \\ I_{\rm inf} &\approx 4 \cdot 10^{-12}\,, \qquad I_{\rm eu} = 0.0221314i \end{aligned}$$

$\Rightarrow$ **In both Euclidean and Minkowski space, the same** $f_\pi = 0.072\ \mathrm{GeV}$**.**

Also, this value of f_π **is not too far from experimental value of** $0.092\ \mathrm{GeV}$**.**

However, the values for the pion form factors (charge and transition) are less successful phenomenologically with 3R quark propagator Ansatz.

Quark loops: e.g., electromagnetic form factor:



Simplifications:

a) chiral-limit pion Bethe–Salpeter vertex: $\Gamma_\pi(q,P) \propto \gamma_5 B(q^2)_{\mathrm{c.l.}}/f_\pi$

b) *Ansatz* $\Gamma^\mu(p',p) \;\rightarrow\; \Gamma^\mu_{\mathrm{BC}}(p',p) = \frac{1}{2}[A(p'^2)+A(p^2)]\,\gamma^\mu \;+$

$$+ \;\; \frac{(p'+p)^\mu}{(p'^2-p^2)}\left\{[A(p'^2)-A(p^2)]\frac{(\not p' + \not p)}{2} - [B(p'^2)-B(p^2)]\right\}$$

Charged pion electromagnetic form factor

$$\langle\pi^+(P')|J^\mu(0)|\pi^+(P)\rangle = (P^\mu + P'^\mu)F_\pi(Q^2) = i(\mathcal{Q}_u - \mathcal{Q}_d)\frac{N_c}{2}\int\frac{d^4q}{(2\pi)^4}\times$$

$$\times\mathrm{tr}\Big\{\bar\Gamma_\pi(q-\tfrac{P}{2},P')S(q+\tfrac{1}{2}(P'-P))\Gamma^\mu(q+\tfrac{1}{2}(P'-P),q-\tfrac{1}{2}(P'-P))$$

$$\times S(q-\tfrac{1}{2}(P'-P))\Gamma_\pi(q-\tfrac{1}{2}P',P)S(q-\tfrac{1}{2}(P+P'))\Big\}$$

- $\Gamma^\mu(p',p)$ = dressed quark-γ vertex, modeled by Ball-Chiu vertex Ansatz
- The proper perturbative QCD asymptotics cannot be expected with this Ansatz ... but in the future this will be the goal:

$$F_\pi(Q^2) = 16\pi\frac{\alpha_s(Q^2)}{Q^2}f_\pi^2 \propto \frac{1}{Q^2\ln(Q^2)} \quad \text{for} \quad Q^2\to\infty$$

Perturbative QCD result

Asymptotic form of the quark mass function

$$M_E(Q^2) \sim \begin{cases} -\frac{2\pi^2 d}{3}\,\frac{\langle\bar{q}q\rangle_{\mathrm{R.G.inv.}}}{Q^2}\left[\frac{1}{2}\ln(Q^2/\Lambda^2_{\mathrm{QCD}})\right]^{d-1} & m = 0 \qquad \text{(chiral limit)} \\ m\left[\frac{\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}{\ln(Q^2/\Lambda^2_{\mathrm{QCD}})}\right]^{d} & m \neq 0 \end{cases},$$

where

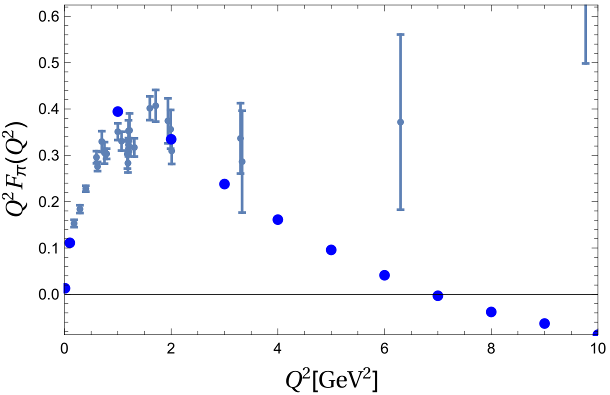
$$d = \frac{2\gamma_0}{\beta_0} = \frac{12}{33 - 2N_f}\ .$$

The exact perturbative asymptotics for $N_f = 3$ $x \to \infty$:

$$\sigma_V(x) \sim \frac{1}{x}\,,$$

$$\sigma_S(x) \sim \frac{M_E(x)}{x} \propto \begin{cases} \frac{1}{x^2 \ln^{5/9} x} & \text{in the chiral limit} \\ \frac{1}{x \ln^{4/9} x} & \text{otherwise } (m \neq 0) \end{cases}.$$

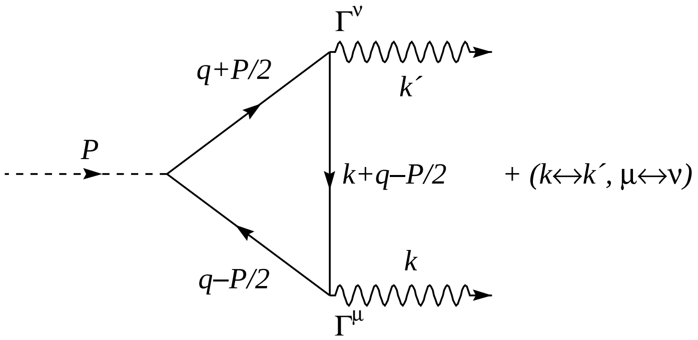
Charged pion electromagnetic form factor



Charged pion electromagnetic form factor $\times \mathbf{Q^2}$.

- Experimental points: a compilation from Zweber [4]
- Round blue dots: calculated with "3R Quark Propagator" *Ansatz*.

Transition form factor for flavorless pseudoscalar mesons



- Diagram for $\pi^0 \to \gamma\gamma$ decay, and for the $\gamma^\star\pi^0 \to \gamma$ process if $k'^2 \neq 0$
- ... also for η and η', but even just π^0 is challenging enough for now ...

Transition form factor in the chiral limit

$$S_{fi} = (2\pi)^4 \delta^{(4)}(P - k - k')e^2\, \varepsilon^{\alpha\beta\mu\nu} \varepsilon_\mu^\star(k,\lambda)\varepsilon_\nu^\star(k',\lambda')\, T_{\alpha\beta}(k^2, k'^2)$$

$$\begin{aligned}
T^{\mu\nu}(k,k') \;&=\; \varepsilon^{\alpha\beta\mu\nu}\, k_\alpha k'_\beta\; T(k^2, k'^2) \\
&= -N_c\, \frac{\mathcal{Q}_u^2 - \mathcal{Q}_d^2}{2} \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\{\Gamma^\mu(q - \frac{P}{2}, k + q - \frac{P}{2}) S(k + q - \frac{P}{2}) \\
&\times \Gamma^\nu(k + q - \frac{P}{2}, q + \frac{P}{2}) S(q + \frac{P}{2}) \left(-\frac{2B(q^2)}{f_\pi}\gamma_5\right) S(q - \frac{P}{2})\} \\
&+ (k \leftrightarrow k', \mu \leftrightarrow \nu)\ .
\end{aligned}$$

The π^0 transition form factor: $F_{\pi\gamma}(Q^2) = |T(-Q^2, 0)|$

UV limit: asymptotically, $F_{\pi\gamma}(Q^2) \to \frac{2f_\pi}{Q^2}$ for $Q^2 \to \infty$

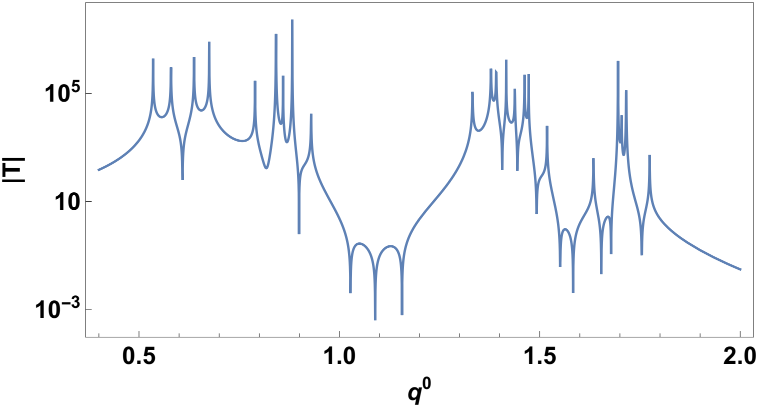
In the chiral limit, the π^0 decay amplitude to two real photons: $T(0,0) = \frac{1}{4\pi^2 f_\pi}$

Transition form factor

Loop integration:

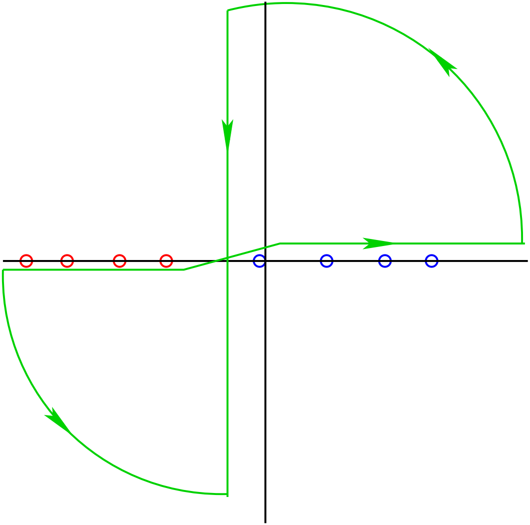
$$q = (q^0,\ \xi\ \sin\vartheta\cos\varphi,\ \xi\ \sin\vartheta\sin\varphi,\ \xi\ \cos\varphi) \qquad (\xi = |\mathbf{q}|)$$

$$Q^2 = 0,\ \xi = 0.5,\ \vartheta = \pi/3 \qquad \text{(values giving } |T \text{ integrand}| \text{ below)}$$

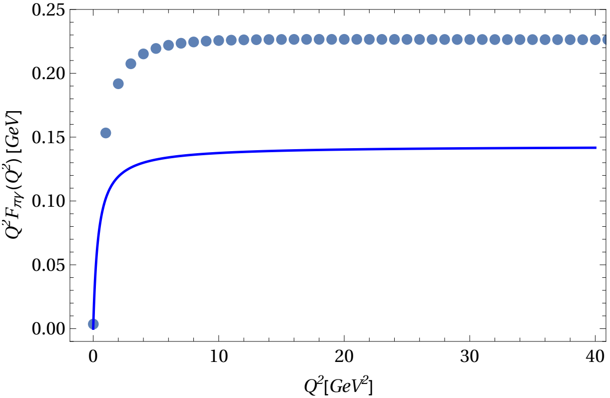


Transition form factor

Loop integration: q^0 complex plane



Transition form factor from 3R QP *Ansatz*



- Blue dots: π^0 transition form factor calculated using 3R QP *Ansatz*
- Blue curve: the Brodsky-Lepage interpolation formula [2, 3] $F_{\pi\gamma}(Q^2) = (1/4\pi^2 f_\pi) \times 1/(1 + Q^2/8\pi^2 f_\pi^2)$ with $f_\pi = 72\ \mathrm{MeV}$.

Quark Propagator with Branch Cut

For simplicity, we choose $\rho_2(\nu^2) = \nu\rho_1(\nu^2) \equiv \nu\rho(\nu^2)$

$$\Rightarrow \qquad S(q) = \int_0^\infty dt\, \rho(t)\, \frac{\not{q} + \sqrt{t}}{q^2 - t + i\epsilon} \ .$$

Then, the quark dressing functions are:

$$\sigma_V(x) = \int_0^\infty dt\, \frac{\rho(t)}{x+t} \ , \qquad \sigma_S(x) = \int_0^\infty dt\, \frac{\sqrt{t}\,\rho(t)}{x+t} \ .$$

To get just a branch cut and no poles, the spectral density Ansatz contains no delta-functions, but is an analytic function:

$$\rho(t) = \frac{t^2(b-t)(c-t)}{(a+t)^7} \ , \quad a, b, c > 0$$

where the parameters c and b are $b = (9 - 2\sqrt{15})\, a$, and $c = (9 - 2\sqrt{15})\, a$, whereas the parameter $a = 1/2\sqrt{3}$, being fixed by

$$\lim_{x\to\infty} A_E(x) = 12\, a^2 = 1 \ .$$

Quark Propagator with Branch Cut

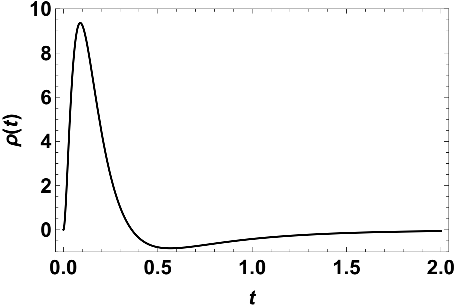
Quark dressing functions:

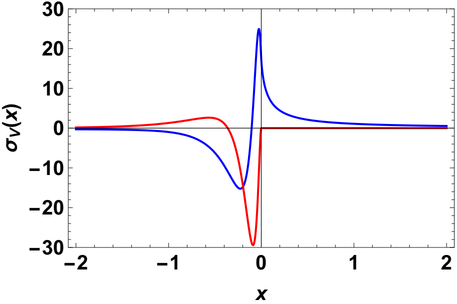
$$
\begin{aligned}
\sigma_S(x) &= \frac{\pi(a+16x+7\sqrt{ax})}{16(\sqrt{a}+\sqrt{x})^7}\ , \\
\sigma_V(x) &= \Big[12a^2\left(21a^2+18xa+x^2\right)(\log(a)-\log(x))x^2 \\
&+ (a-x)\left(5a^5-65xa^4-368x^2a^3-48x^3a^2-5x^4a+x^5\right)\Big] \\
&\times \left[12a^2(a-x)^7\right]^{-1}\ .
\end{aligned}
$$

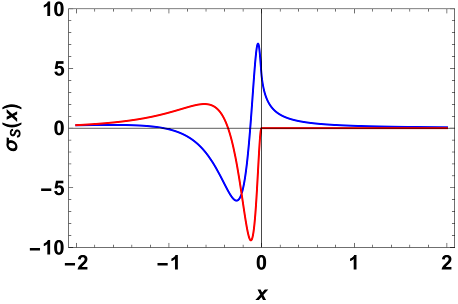
Asymptotic behavior for $x \to \infty$:

$$
\sigma_S(x) \sim \frac{\pi}{x^{5/2}}\ ,\quad \sigma_V(x) \sim \frac{1}{x}\ ,\quad A_E(x) \sim 1\ ,\quad B_E(x) \sim \frac{\pi}{x^{3/2}}\ .
$$

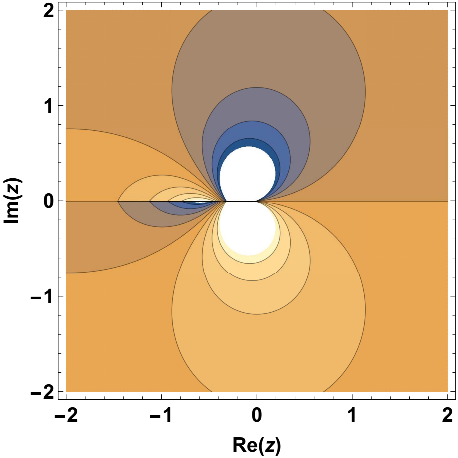
Quark Propagator with Branch Cut– graphs of the functions





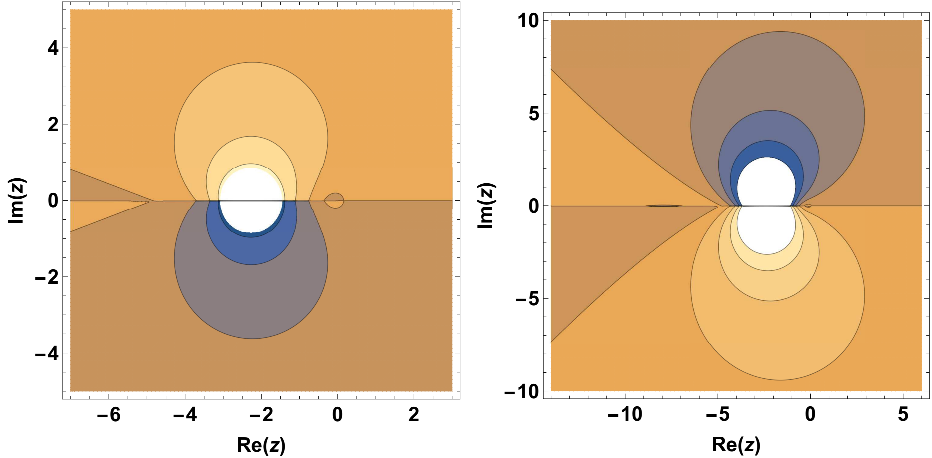


Quark Propagator with Branch Cut– contour plot



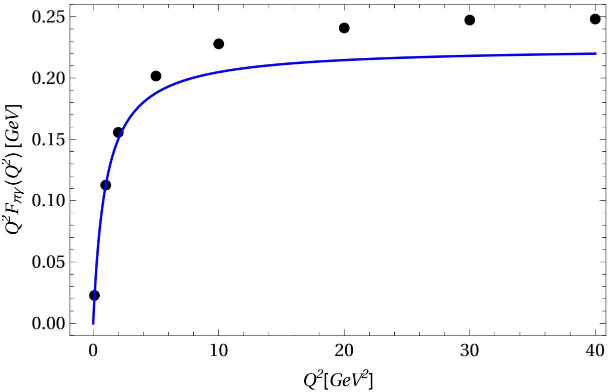
Contour plot of $\mathrm{Im}(\sigma_V(z))$ in the complex z-plane.

Quark Propagator with Branch Cut– contour plots



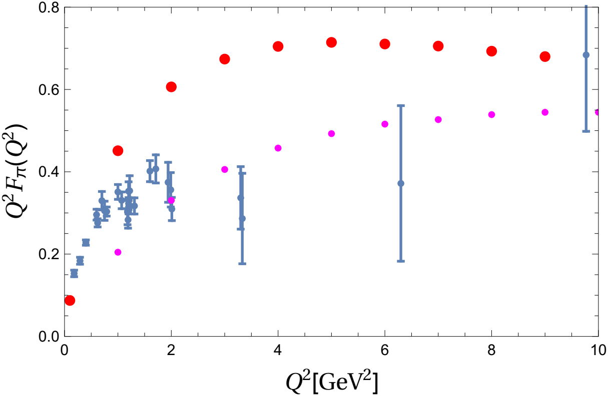
Contour plot of $\mathrm{Im}(A_E(z))$ and $\mathrm{Im}(B_E(z))$ in the complex z-plane.

Transition form factor



- Black dots: π^0 transition form factor calculated using "Quark Propagator with Branch Cut" *Ansatz*
- Blue curve: represents the Brodsky-Lepage interpolation with $f_\pi = 112.7\ \mathrm{MeV}$.

Electromagnetic form factor



Charged pion electromagnetic form factor.

- Experimental points: a compilation of Zweber [4]
- Magenta ("violet") points: “Quark Propagator with Branch Cut” *Ansatz*.

Extension to finite density may be simplified?

For application to quark matter/stars, extension to finite density needed!

Medium breaks the original space symmetry ⇒ propagators have more independent tensor structures than in the vacuum! **However**, Zong *et al.*, PRC 71 (2005) 015205, assuming analyticity of the dressed quark propagator at $\mu = 0$ and neglecting the μ-dependence of the dressed gluon propagator, argued that within the rainbow approximation, the dressed quark propagator at $\mu \neq 0$ is obtained from the $\mu = 0$ one by a simple shift $q_4 \to q_4 + i\mu \equiv \tilde{q}_4$:

$$S[\mu](q) = -\sigma_V(\tilde{q}^2)\,\tilde{\not{q}} - \sigma_S(\tilde{q}^2)$$

- Only two tensor structures again, in spite of medium! Looks like a very severe truncation!
- Nevertheless, Jiang *et al.*, PRD 78 (2008) 116005, used this simplification in their $\mu > 0$ extension of the present Ansatze 3R and 2CC.
- They obtained reasonable μ-dependence of f_π and m_π, in accordance with other, independent predictions on general grounds (Halasz *et al.*, PRD 58 (1998) 096007).
- ⇒ The present Minkowski-vs-Euclidean analysis probably can be extended to $\mu > 0$ in this simplifed way (under study now ... hopefully less truncations in the future ...)

Summary

- We motivated and studied comparatively simple Ansätze for **quark propagators $S(q)$ with poles and cuts on the negative (time-like) half axis in the complex p^2-plane**, enabling equivalent Minkowski and Euclidean calculations.

- We find it is possible to construct spectral densities $\rho(t)$ such that both **$S(q)$ and $S^{-1}(q)$ are analytic on the cut plane Ω**, and that **these quark Ansatz-propagators lead to fairly successful phenomenology.**

Ongoing work:

- The calculations of form factors still need improvements: *i)* **obtain correct UV asymptotics in agreement with the perturbative QCD**, and *ii)* **to include pseudoscalar mesons heavier than pions, the presently used chiral-limit approximations should be surpassed.**

- Extension to finite density – first try Zong-Jiang simplified approach.

[1] R. Alkofer, W. Detmold, C. Fischer, and P. Maris, "Analytic properties of the Landau gauge gluon and quark propagators," *Phys.Rev.* **D70** (2004) 014014, arXiv:hep-ph/0309077 [hep-ph].

[2] G. P. Lepage and S. J. Brodsky, "Exclusive processes in perturbative quantum chromodynamics," *Phys. Rev.* **D22** (1980) 2157.

[3] S. J. Brodsky and G. P. Lepage, "Large Angle Two Photon Exclusive Channels in Quantum Chromodynamics," *Phys.Rev.* **D24** (1981) 1808.

[4] P. K. Zweber, *Precision measurements of the timelike electromagnetic form factors of the pion, kaon, and proton*. PhD thesis, Northwestern U., 2006. arXiv:hep-ex/0605026 [hep-ex]. http://wwwlib.umi.com/dissertations/fullcit?p3212999.

[5] R. J. Holt and C. D. Roberts, "Distribution Functions of the Nucleon and Pion in the Valence Region," *Rev.Mod.Phys.* **82** (2010) 2991–3044, arXiv:1002.4666 [nucl-th].