NUMERICAL MODELING AND FIELD MEASUREMENT OF STRATIFIED ESTUARY: RJEČINA RIVER CASE STUDY

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Abstract: A detail sampling campaign was conducted in 2015 to determine salinity structure in the lower reaches of Rječina River, a small karstic river in the coastal part of Croatia. The measurements confirmed a presence of a highly stratified salt wedge. Furthermore, a one-dimensional two layer numerical model based on a finite volume scheme was applied to simulate dynamic processes in the estuary. Field measurements were used for comparison and calibration of the interfacial drag coefficient. For one considered case where freshwater discharge was the dominant factor agreement between the numerical model and measurements was excellent.

INTRODUCTION

Most estuaries in Mediterranean Sea, such as Rhone, Po, Ebro and Neretva are stratified as a result of a microtidal environment (Ibanez et al. 1997, Ljubenkov and Vranješ 2012). Highly stratified estuary, or a salt wedge estuary, is characterized by an upper layer of fresh water flowing towards the river mouth over a lower layer of salt water. In ideal conditions strong stratification is present and two layers of different density are divided by a sharp interface.

There are several numerical models describing a one-dimensional two shallow layer water flow developed in the last few years that are mostly based on a finite volume scheme. We adapted the numerical model used in (Castro et al., 2004) which is based on the Q-scheme of Van Leer and Roe to solve a coupled system of two Shallow Water Equations with source terms. This method was developed to study the exchange flow in channels with irregular geometry and tested on Strait of Gibraltar. In this work, the proposed method is used to study a highly stratified salt wedge, namely Rječina Estuary.

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STUDY AREA

Rječina River is a short karst river, located in the northern coastal part of Croatia, and it flows into Adriatic Sea in the center of Rijeka City (Figure 1). Total length of the river is 18,6 km, with a catchment area of 246 km^2. In lower reaches of the river there are two main inflows; overflown waters from Zvir spring which is used by the public water supply system, and from hydroelectric power plant HE Rijeka. Both are located upstream from the hydrological station "Sušak tvornica".

Fig. 1. Location map for Rječina Estuary, located in Rijeka City in Croatia

Based on the analysis of hydrological data from the period 1999-2011, the average annual discharge is 10,4 m^3/s. Rječina River is characterized by strong seasonal oscillations; maximum mean monthly discharges are usually observed in winter months (51,1 m^3/s in December), while minimum mean monthly discharges are observed during summer months (0,14 m^3/s in August). Moreover, due to the periodic operation of the HE Rijeka, prominent daily oscillations are common in the lower reaches of Rječina River. Tidal oscillations here are semi-diurnal with a mean daily amplitude of approximately 30 cm, which categorizes Adriatic Sea as a microtidal sea. (Krvavica et al, 2012)

FIELD MEASUREMENTS

Field measurements were conducted in Rječina Estuary throughout 2014 and first half of 2015. For the purpose of this work, the measurements made on July 3rd 2015 are considered, when the conditions were especially favorable and suitable for verification of the numerical model. Several measurements were made on that day. There was almost no wind or waves present, freshwater flow was changing, but the salt wedge in its whole length remained inside the observed reach for both high and low discharge.

The haline properties, i.e. vertical profiles of temperature and salinity, were measured at 10 different locations from the mouth (Figure 1). Although two-layer models presume a sharp interface between the upper and lower layer, in field conditions a halocline of some thickness is always present. Figure 2 shows two measured vertical profiles of temperature and salinity, one at the beginning of the salt wedge and the other one near the river mouth, both measured on July 3rd 2015 during a lower discharge. Strong stratification is present in all profiles, therefore a numerical model based on a two-layer salt wedge theory can be considered as sufficiently adequate in this case. The point of the strongest salinity gradient was chosen to represent the interface depth in comparison with the numerical model.

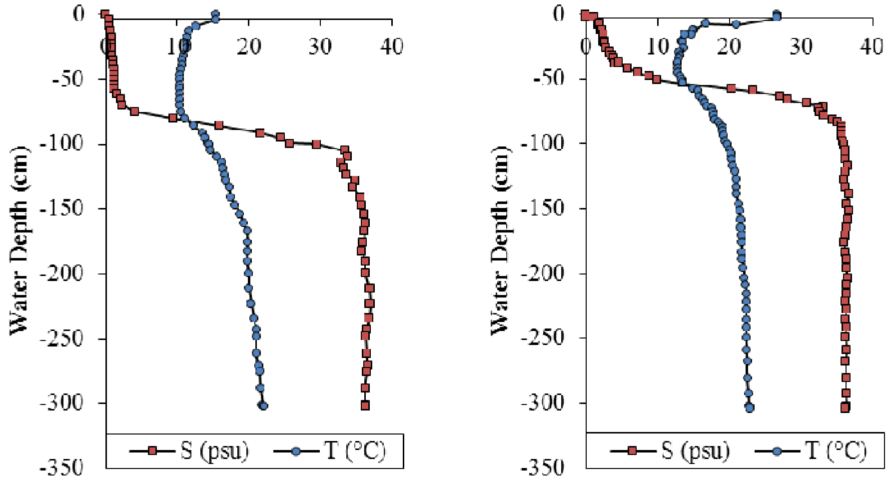


Fig. 2. Salinity and temperature profiles in Rječina Estuary on 3rd July 2015, during a freshwater discharge of 5,7 m^3/s

MATEMATICAL MODEL

A mathematical model based on a one-dimensional Saint-Venant system of equations written in a conservative form for two shallow layers of immiscible fluids is described and used to simulate salt wedge dynamics in Rječina Estuary.

Governing equations

In (Castro et al., 2004) a PDE system for one-dimensional flow along a channel with arbitrary shaped cross sections was presented and successfully used to simulate the exchange flow through Strait of Gibraltar. The fluid was assumed to be composed of two shallow layers of immiscible fluids with constant densities, without mixing between the layers. Resulting equations had the form of a system of two coupled conservation laws with source terms. Every term in the momentum equations is expressed as a function of main variables; A_i (wetted cross section area) and Q_i (discharge), where $i = 1$ corresponds to the upper layer, and $i = 2$ to the lower one. This PDE system is written as follows:

$$\frac{\partial w}{\partial t}+\frac{\partial f(\sigma,w)}{\partial x}=B(\sigma,w)\frac{\partial w}{\partial x}+v(\sigma,w)+s(x,\sigma,w)\,, \qquad (1)$$

and

$$w(x,t)=\{A_1(x,t),Q_1(x,t),A_2(x,t),Q_2(x,t)\}^T\,, \qquad (2)$$

$$
f(\sigma,w)=\begin{Bmatrix} Q_1 \\ \frac{Q_1^2}{A_1}+\frac{g}{2\sigma_1}A_1^2 \\ Q_2 \\ \frac{Q_2^2}{A_2}+\frac{g}{2\sigma_2}A_2^2 \end{Bmatrix}, \tag{3}
$$

$$
B(\sigma,w)=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{g}{\sigma_1}A_1 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{g}{\sigma_1}rA_2 & & 0 & 0 \end{bmatrix}, \tag{4}
$$

$$
v(\sigma,w)=\begin{Bmatrix} 0 \\ \frac{g}{2}\frac{d}{dx}\left(\frac{1}{\sigma_1}\right)A_1^2 \\ 0 \\ \frac{g}{2}\frac{d}{dx}\left(\frac{1}{\sigma_2}\right)A_2^2 \end{Bmatrix}, \tag{5}
$$

$$
s(x,\sigma,w)=\left\{s^g(x,\sigma,w)+s^f(x,\sigma,w)\right\}, \tag{6}
$$

where x = horizontal coordinate along the axis of the channel (m); y = horizontal coordinate normal to the channel axis (m); z = vertical coordinate (m); t = time (s); g = acceleration of gravity (m/s^2); ρ_i = density of the i-th layer (kg/m^3); r = density ratio (ρ_1/ρ_2); σ_1 = breadth at the water surface (m); σ_3 = breadth at the interface (m); $1/\sigma_2 = (1-r)/\sigma_3 + r/\sigma_1$; h_i = water depth of the i-th layer (m). Figure 3 shows the notations on a longitudinal and cross section of the channel.

The source term accounts for bed slope and cross section variations, as well as for drag forces between layers, and bottom and wall friction, denoted by s^g and s^f, respectively:

$$
s^g(x,\sigma,w)=\begin{Bmatrix} 0 \\ gA_1\left[\frac{1}{\sigma_1}\frac{\partial}{\partial x}(A_1+A_2)-\frac{db}{dx}-\frac{\partial}{\partial x}(h_2+h_1)\right] \\ 0 \\ gA_2\left[\frac{1}{\sigma_2}\frac{\partial A_2}{\partial x}+\frac{r}{\sigma_1}\frac{\partial A_1}{\partial x}-\frac{db}{dx}-\frac{\partial}{\partial x}(h_2+rh_1)\right] \end{Bmatrix}, \tag{7}
$$

$$
s^f(x,\sigma,w)=\begin{Bmatrix} 0 \\ \frac{\tau_{int}}{\rho_1}\sigma_3+\frac{\tau_{b,1}}{\rho_1}O_1' \\ 0 \\ \frac{\tau_{int}}{\rho_2}\sigma_3+\frac{\tau_{b,2}}{\rho_2}O_2 \end{Bmatrix}, \tag{8}
$$

Interfacial drag force is expressed in the form of a quadratic law, bottom and wall friction by the Manning's equation:

$$\tau_{int} = -\rho_1 C_d |u_1 - u_2|(u_1 - u_2), \tag{9}$$

$$\tau_{b,1} = -\rho_1 g n^2 |u_1| u_1 R_{W1}^{-1/3}, \tag{10}$$

$$\tau_{b,2} = -\rho_2 g n^2 |u_2| u_2 R_2^{-1/3}, \tag{11}$$

where u_i = depth averaged velocity of the i-th layer (m/s); C_d = interfacial drag coefficient; n = Manning's coefficient ($m^{-1/3}s$); R_{w1} = reduced hydraulic radius for upper layer (m) ($R_w = A_1 / (O_1 - \sigma_3)$); R_2 = hydraulic radius for lower layer (m).

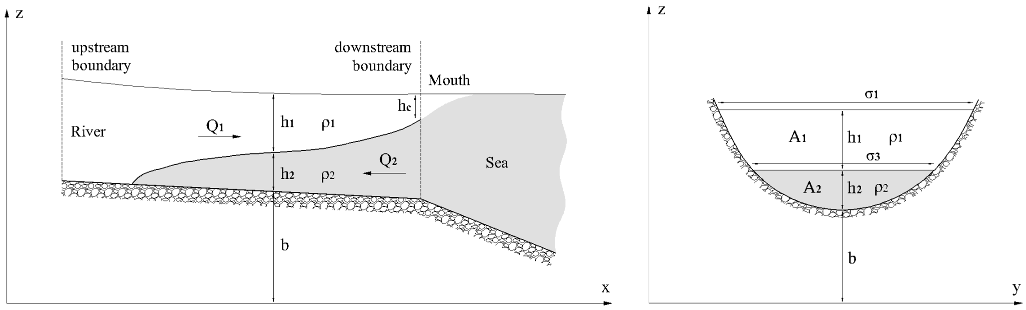


Fig. 3. Notations for a longitudinal and cross section of the channel

Numerical scheme

A first-order upwind finite volume scheme with explicit time discretization is used to solve the problem defined by the system (Eq. 1). Castro et al. (2001) proposed a Q-scheme based on Approximate Reimann Solvers for general coupled systems of conservation laws with a source term. That scheme was further developed by Castro et al. (2004) to deal with the problems where the flux depends also on x as a result of irregular channel geometry. The latter scheme is applied in this work to construct a suitable numerical scheme for the system described by (Eq. 1).

The following matrices are also used in the description of the numerical scheme:

$$\mathrm{A}(\sigma, w) = J(\sigma, w) - B(\sigma, w), \tag{12}$$

where

$$J(\sigma, w) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{Q_1^2}{A_1^2} + \frac{g}{\sigma_1} A_1 & 2\frac{Q_1}{A_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{Q_2^2}{A_2^2} + \frac{g}{\sigma_2} A_2 & 2\frac{Q_2}{A_2} \end{bmatrix}. \tag{13}$$

The eigenvalues of $A(\sigma,w)$ relate to the internal and external propagation speeds, so that the flow is subcritical when the internal eigenvalues have different signs, critical if one of them is equal to zero and supercritical otherwise. The internal eigenvalues can become complex, which corresponds to the increased shear instabilities at the interface, i.e. development of the Kelvin-Helmholtz or Holmboe waves (Zhu and Lawrence, 2000). The numerical scheme presented in this work is stable only when there are four distinct real eigenvalues.

The first step is to divide the spatial domain into N finite volume cells $I_i = [x_{i-1/2}, x_{i+1/2}]$ for $i = 1$ to N. A constant cell size Δx and temporal step Δt are assumed. Once the solution is known at time t, the intermediate states are calculated between cells in order to linearize the system. The intermediate states between cells x_i and x_{i+1} are calculated at the interface $x_{i+1/2}$, as follows:

$$w_{i+1/2} = \left\{A_{i+1/2,1}, Q_{i+1/2,1}, A_{i+1/2,2}, Q_{i+1/2,2}\right\}^T , \tag{14}$$

$$u_{i+1/2,j} = \frac{u_{i,j}\sqrt{A_{i,j}} + u_{i+1,j}\sqrt{A_{i+1,j}}}{\sqrt{A_{i,j}} + \sqrt{A_{i+1,j}}} , \tag{15}$$

$h_{i+1/2,j}$ are water depths corresponding to $A_{i+1/2,j}$, $\sigma_{i+1/2,j}$ are breadths corresponding to $h_{i+1/2,j}$ at $x_{i+1/2}$, $A_{i+1/2,j}$ and $B_{i+1/2,j}$ represent matrices A and B corresponding to $\sigma_{i+1/2,j}$ and $w_{i+1/2}$.

After the intermediate states are calculated, Roe's method is applied by solving the linearized Riemann Problem at each interface. The source term is upwinded, as suggested by Castro et al. (2004). Such a numerical scheme can be written as follows:

$$\begin{aligned} w_i^{n+1} = w_i^n &+ \frac{\Delta t}{\Delta x}\left(f_{i-1/2} - f_{i+1/2}\right) + \frac{\Delta t}{2\Delta x}\left[B_{i-1/2}\left(w_i^n - w_{i-1}^n\right) + B_{i+1/2}\left(w_{i+1}^n - w_i^n\right)\right] \\ &+ \frac{\Delta t}{2\Delta x}\left(v_{i-1/2} + v_{i+1/2}\right) + \frac{\Delta t}{\Delta x}\left(P_{i-1/2}^{+} s_{i-1/2} + P_{i+1/2}^{-} s_{i+1/2}\right) \end{aligned} , \tag{16}$$

where

$$f_{i+1/2} = \frac{1}{2}\left[f(\sigma_i, w_i) + f(\sigma_{i+1}, w_{i+1})\right] - \frac{1}{2}\left|\mathrm{A}_{i+1/2}\right|\left(w_{i+1} - w_i\right), \tag{17}$$

$$v_{i+1/2,[2]} = \frac{g}{2}\left(\frac{1}{\sigma_{i+1,1}} - \frac{1}{\sigma_{i+1/2,1}}\right)A_{i+1,1}^2 + \frac{g}{2}\left(\frac{1}{\sigma_{i+1/2,1}} - \frac{1}{\sigma_{i,1}}\right)A_{i,1}^2, \tag{18}$$

$$v_{i+1/2,[4]} = \frac{g}{2}\left(\frac{1}{\sigma_{i+1,2}} - \frac{1}{\sigma_{i+1/2,2}}\right)A_{i+1,2}^2 + \frac{g}{2}\left(\frac{1}{\sigma_{i+1/2,2}} - \frac{1}{\sigma_{i,2}}\right)A_{i,2}^2, \tag{19}$$

$$\begin{aligned} s_{i+1/2,[2]}^g = gA_{i+1/2,1}\Bigg[&\frac{1}{\sigma_{i+1/2,1}}\left(A_{i+1,1} + A_{i+1,2} - A_{i,1} - A_{i,2}\right) \\ &- \left(b_{i+1} + h_{i+1,1} + h_{i+1,2} - b_i - h_{i,1} - h_{i,2}\right)\Bigg] \end{aligned} , \tag{20}$$

$$s^{g}_{i+1/2,[4]} = gA_{i+1/2,2}\left[\frac{1}{\sigma_{i+1/2,2}}\left(A_{i+1,2} - A_{i,2}\right) + \frac{r}{\sigma_{i+1/2,1}}\left(A_{i+1,1} - A_{i,1}\right) \right. \\ \left. - \left(b_{i+1} + rh_{i+1,1} + h_{i+1,2} - b_i - rh_{i,1} - h_{i,2}\right)\right], \tag{21}$$

$$s^{f}_{i+1/2,[2]} = -C_d\left|u_{i+1/2,1} - u_{i+1/2,2}\right|\left(u_{i+1/2,1} - u_{i+1/2,2}\right)\sigma_{i+1/2,3}\Delta x - \\ - gA_{i+1/2,1}n^2\left|u_{i+1/2,1}\right|u_{i+1/2,1}R^{-4/3}_{i+1/2,1}\Delta x, \tag{22}$$

$$s^{f}_{i+1/2,[4]} = -C_d r\left|u_{i+1/2,2} - u_{i+1/2,1}\right|\left(u_{i+1/2,2} - u_{i+1/2,1}\right)\sigma_{i+1/2,3}\Delta x - \\ - gA_{i+1/2,2}n^2\left|u_{i+1/2,2}\right|u_{i+1/2,2}R^{-4/3}_{i+1/2,2}\Delta x, \tag{23}$$

$$P^{\pm}_{i+1/2} = \frac{1}{2}K_{i+1/2}\left[I \pm sgn\left(\Lambda_{i+1/2}\right)\right]K^{-1}_{i+1/2}, \tag{24}$$

$\Lambda_{i+1/2}$ is defined as a matrix whose diagonals are the eigenvalues of $A_{i+1/2}$, $K_{i+1/2}$ is a matrix whose columns are the eigenvectors corresponding to those eigenvalues, and I is an identity matrix.

The method used herein is explicit in time, thereby a CFL condition has to be satisfied. Castro et al. (2004) proposed the following condition:

$$max\left\{\left|\lambda_{i+1/2,l}\right|, 1 \le l \le 4, 1 \le i \le N\right\}\frac{\Delta t}{\Delta x} \le CFL, \tag{25}$$

where $\lambda_{i+1/2,l}$ are the eigenvalues of the matrix $A_{i+1/2}$.

NUMERICAL RESULTS

Simulation of the steady states and studying the response of the salt wedge in Rječina Estuary to changes in freshwater flow is the main aim of this numerical experiment. For this purpose, a detailed sampling campaign was carried out on June 3rd 2015, during a period from 10 am to 1 pm. Water stage at the mouth and freshwater discharges (approximated from the upstream water stage and a known rating curve) were obtained from the Croatian Waters real time Water Levels information center (http:\\vodostaji.voda.hr). Furthermore, during that 3 hour period, interface depths were measured at 10 different locations along the estuary every half hour. We tried to examine how well does the numerical model agree with the observations; before, during the discharge change, and after.

Channel geometry

In order to perform this numerical analysis, geometrical characteristics of the Rječina channel geometry was constructed. This is done by measuring 28 cross sections in a 1090 m long reach of the estuary. Each cross section was then interpolated by 100 points (red), and cross sections were connected by interpolated profile lines (gray) (Figure 4). Along the channel axis, every Δx = 1 m, interpolated lines were *cut* by a plane normal to the axis to obtain 1091 cross section. Each cross section was then individually analyzed to compute breadth, area and wetted perimeter for different water depths (Δz = 0,01 m spatial step).

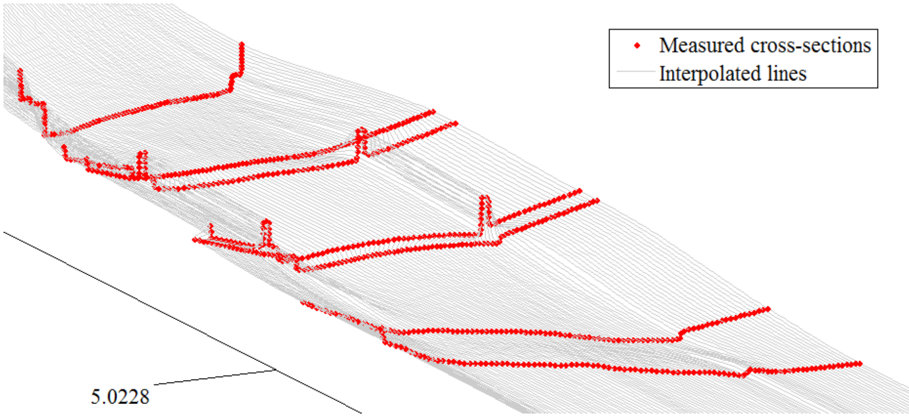


Fig. 4. Detail of Riječina channel geometry – cross section interpolation

Finally, calculated data for each cross section were combined into an array containing the geometrical characteristics for the whole channel under consideration. An example of such an array, in this case the breadth, is presented as an image in Figure 5. Color of each pixel represents the breadth in (m) at a specific distance along the channel and water depth. The bottom depth is approximated as the maximal depth at each cross section. Detailed description of this procedure can be found in (Travaš et al., 2015), with a note that in this case a cubic spline interpolation was used, instead of a Hermite interpolation.

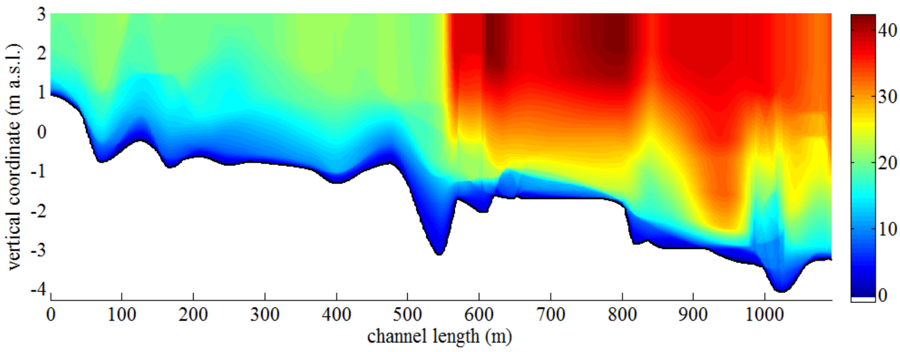


Fig. 5. Image of a breadth array – Riječina Estuary

Test1: Salt wedge dynamics in Riječina Estuary

A numerical simulation was carried out for comparison with the observed data. Figure 6 shows the initial condition, calculated by the two-layer steady state solver presented in (Krvavica et al, 2014), previously tested and compared to the observation at Riječina Estuary. The upper layer discharge is $Q_1 = 11{,}5$ m^3/s, lower layer is at rest $Q_2 = 0$ m^3/s. Water stage at the mouth is -0,04 m a.s.l., and the relative density ratio of two layers is $r = 0{,}97$. The spatial step is set to $\Delta x = 5$ m, and temporal to $\Delta t = 0{,}25$ s, which gives the CFL around 0,5. Total simulation time is $T = 180$ min. The interfacial drag coefficient is calibrated to fit the observed data, best fit is obtained when $C_d = 5\text{x}10^{-4}$, The Manning coefficient is taken from literature as $n = 0{,}035$ m$^{-1/3}$s for channels with a gravel bottom.

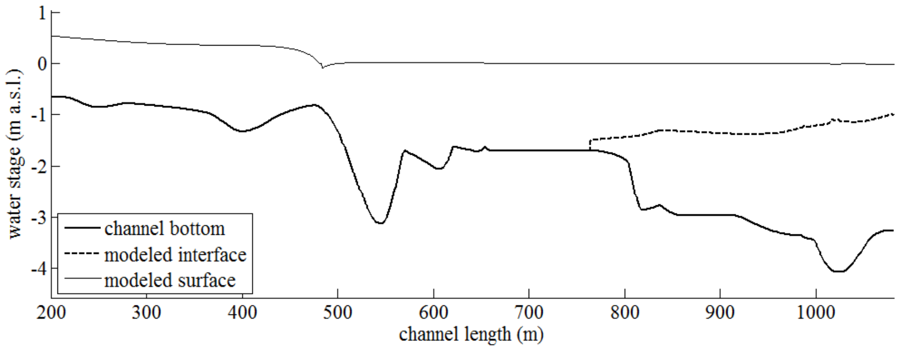
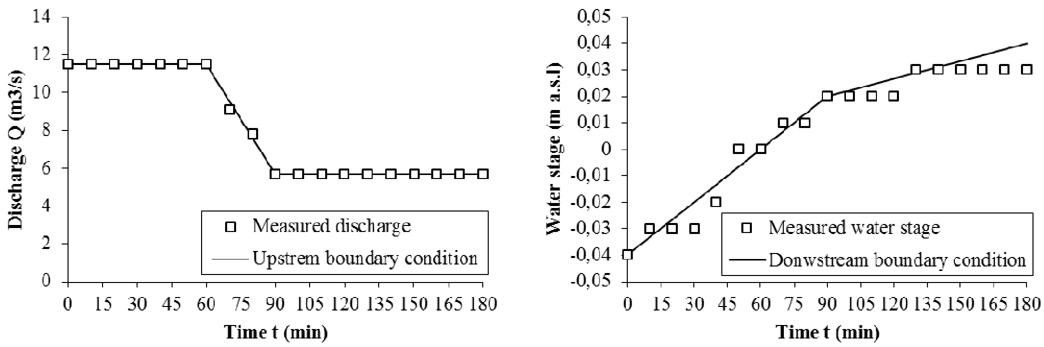


Fig. 6. Initial condition for the simulation – steady state solution for Q_1 = 11,5 m^3/s, Q_2 = 0 m^3/s, *SWL* = -0.04 m a.s.l.

The boundary conditions are set to match the discharges and water stages measured on 3rd June 2015 at Rječina Estuary (Figure 7). During that period there was a sudden change in the freshwater discharge due to the hydroelectric power plant operation – discharges decreased from 11,5 m^3/s to 5,7 m^3/s in 30 minutes. In that time sea water level at the mouth slightly rose from -0,04 to 0,04 m a.s.l.

At the upstream end a hydrograph for the upper layer (Figure 7a) and a zero flux for the lower layer are imposed as the boundary condition. At the downstream end the water stage (Figure 7b) and upper layer depth are imposed as the boundary conditions.



(a) **(b)**

Fig. 7. Boundary conditions:
(a) upstream - upper layer discharge, (b) downstream –water stage

Schijf and Schonfeld (1953) proposed that at the mouth of the river, when there is no appreciable influence of the tidal motions, the upper layer depth is critical. This condition was also confirmed in preliminary research of the salt wedge at Rječina Estuary by Krvavica et al. (2014). This means that one of the upper layer eigenvalues is equal to zero. As the analytical expressions for eigenvalues are unknown, a first order approximation given by Schijf and Schonfeld (1953), and adapted to irregular geometry by Castro et al. (2004), is used herein. It is said that the flow is critical when the composite Froude number is equal to one, $G^2 = 1$, where:

$$G^2 = F_1^2 + F_2^2 - (1-r)\frac{\sigma_2}{\sigma_3}F_1^2F_2^2 \ , \tag{26}$$

and F_1 and F_2 are the internal Froude numbers:

$$F_1^2 = \frac{u_1^2}{g(1-r)\frac{\sigma_2}{\sigma_3}\frac{A_1}{\sigma_1}}, \qquad F_2^2 = \frac{u_1^2}{g(1-r)\frac{A_2}{\sigma_3}} \ . \tag{27}$$

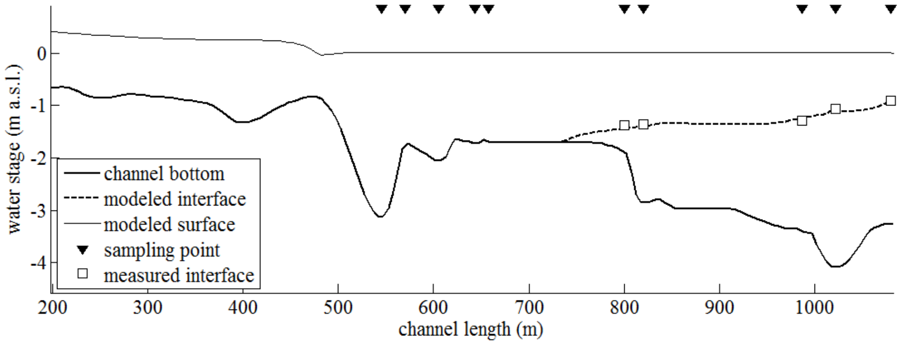
A problem of defining a suitable upper layer depth at the downstream boundary consist in the iterative calculation of h_1 at the mouth for which A_1 satisfies (Eq. 26 and 27) for a known water stage $h_2 + h_1$, density ratio r and discharges Q_1 and Q_2.

Figure 8 shows the evolution of salt wedge in time, i.e. changes in the water surface and interface during a 3 hour long simulation. As expected, total length of the salt wedge is smallest at the start, during a higher discharge (Q_1 = 11,5 m^3/s). As the discharge is decreasing, the salt wedge penetrates upstream by approximately 300 m. The upper layer depth h_1 at the mouth also decreases with time and agrees well with the boundary condition (Eq. 26). The agreement between the observed and predicted interface is excellent over a complete simulation time T. The tidal influence in comparison to the freshwater discharge is negligible in this case. It has to be emphasized that the excellent agreement is accomplished only with the calibration of the interfacial drag coefficient, which can have a strong influence on the result.

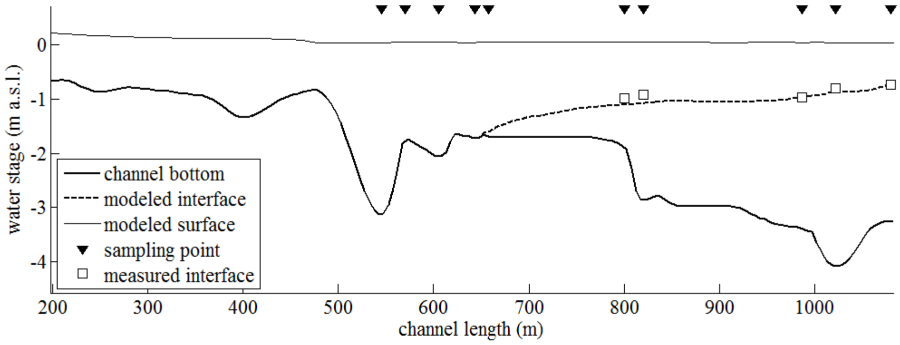
CONCLUSIONS

A formulation of the two layer shallow water equation for channels with irregular geometry, previously developed and used to simulate the exchange flow through Strait of Gibraltar, is used here for studying the salt wedge dynamics in Rječina Estuary. The coupled PDE system is numerically solved using a generalized Q-scheme (a first-order upwind finite volume method with an explicit time discretization). One experiment is presented to test the salt wedge response to changes in a freshwater flow. Numerical results agree well with the field observations, but only if the interfacial drag coefficient is chosen appropriately.

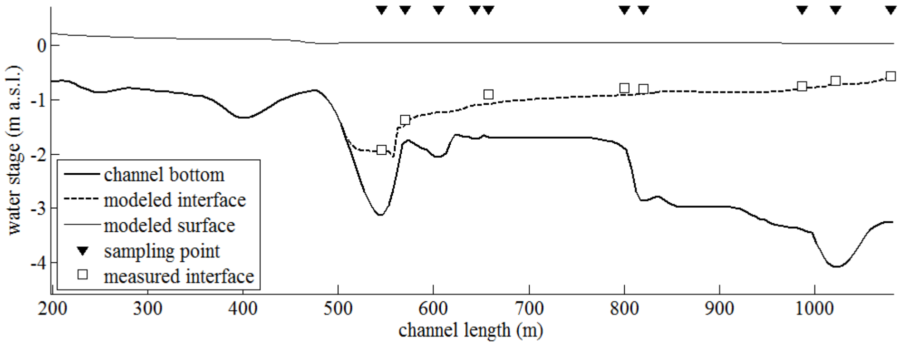
Although, these results seem very promising, further field studies and more comprehensive numerical experiments, under different hydrological conditions, are needed. Once this model undergoes a more thorough examination, it can be used to address a number of questions about the hydrodynamics processes in salt wedge estuaries, such as nature of the interfacial drag and mixing between layers, as well as the salt wedge response to the outside influences, such as freshwater inflow and tidal dynamics.



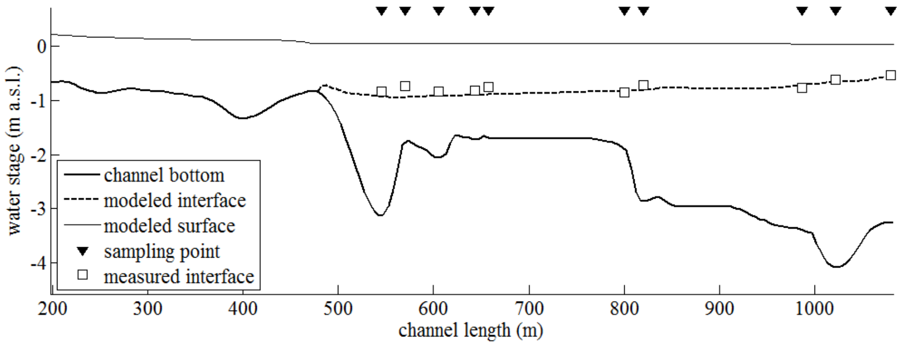
(a) simulation time *t* = 47 min, observed between 10:39 and 10:54 am



(b) simulation time *t* = 79 min, observed between 11:12 and 11:28 am



(c) simulation time *t* = 110 min, observed between 11:41 and 11.59 am



(d) simulation time *t* = 148 min, observed between 12:18 and 12:38 am

Fig. 8. Simulation of the free surface and interface in Rječina Estuary, with the observed interfaces on July 3rd 2015, from 10 am to 1 pm

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