Generalized Parton Distributions (GPDs) through DVCS and DVMP

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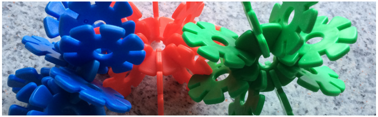

ACHT2015

"Strong Interactions in Quantum Field Theory"

Leibnitz, Oct, 8th, 2015.

Outline

1. Introduction
   - Resolving nucleon structure (form factors, PDFs, ...)
2. DVCS, DVMP, GPDs — theory
   - Deeply virtual Compton scattering (DVCS)
   - ..., deeply virtual meson electroproduction (DVMP)
   - Generalized parton distributions (GPDs)
3. DVCS, DVMP, GPDs — phenomenology
   - Experimental status
   - Towards unravelling GPDs
   - Modeling venues
   - One example approach...
4. Summary

Resolving nucleon structure

SCATTERING

$\rightarrow$ elastic	$(e^- p \rightarrow e^- p)$	}	exclusive
$\rightarrow$ inelastic	$(e^- p \rightarrow e^- \pi p)$	}	exclusive
	$(e^- p \rightarrow e^- X)$	}	inclusive

Resolving nucleon structure

SCATTERING

→ elastic	$(e^- p \to e^- p)$	}	exclusive
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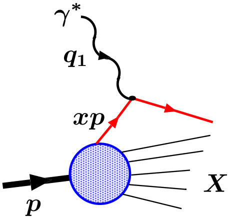
Parton distribution functions

- Deeply inelastic scattering $-q_1^2 \equiv Q^2 \to \infty,\ x_{BJ} \equiv \frac{-q_1^2}{2p\cdot q_1} \to \text{cte.}$ (Bjorken limit)



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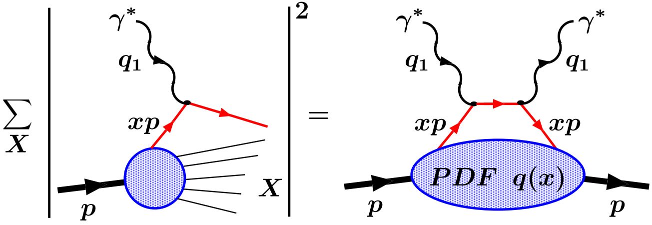

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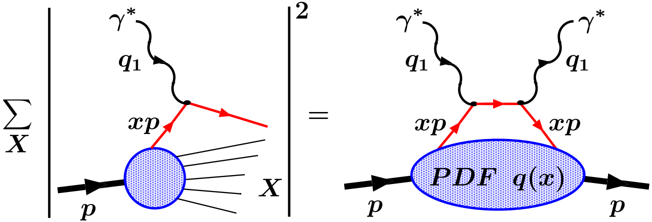
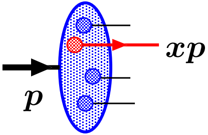
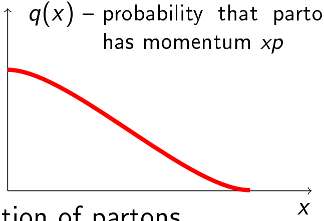
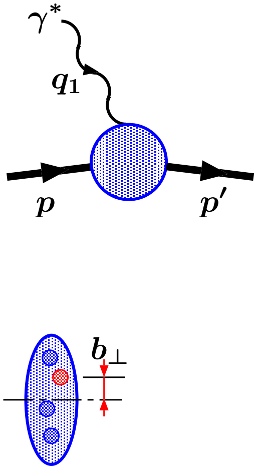






- no information on spatial distribution of partons

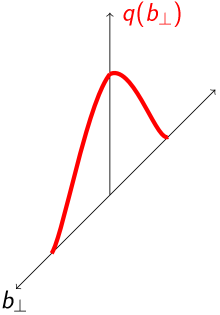
Electromagnetic form factors



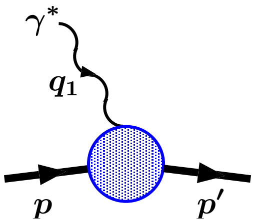
- Form factors → charge distribution

$$\Gamma^\mu(\gamma^* p \to p) = \gamma^\mu F_1(Q^2) + \frac{\kappa_p}{2M_p} i\,\sigma^\mu_\nu q_1^\nu F_2(Q^2)$$

$$q(b_\perp) \sim \int dq_1\, e^{i q_1 \cdot b_\perp} F_1(t = q_1^2)$$



Electromagnetic form factors



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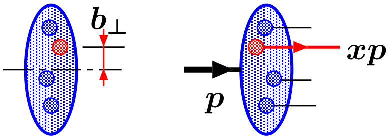

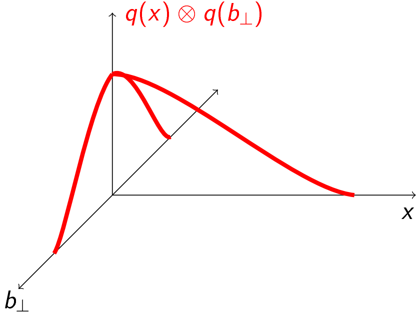

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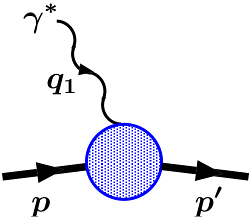

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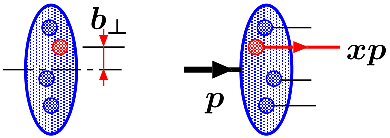

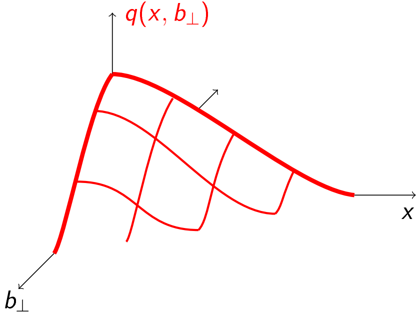

DIS and Compton scattering

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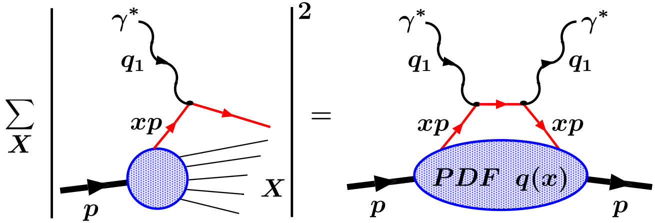
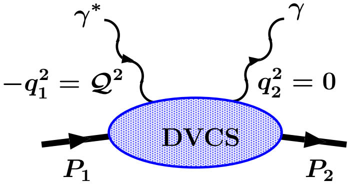


$$\sigma_{tot}(\gamma^* p \to X) \overset{\textit{optical theorem}}{\propto} Im\,\mathcal{A}(\gamma^* p \to \gamma^* p)$$

forward Compton scattering

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]



$$P = P_1 + P_2$$

$$q = (q_1 + q_2)/2$$

$$\Delta = P_2 - P_1$$

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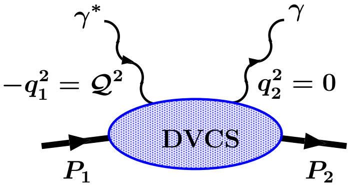


$$P = P_1 + P_2$$
$$q = (q_1 + q_2)/2$$
$$\Delta = P_2 - P_1$$

generalized Bjorken limit:

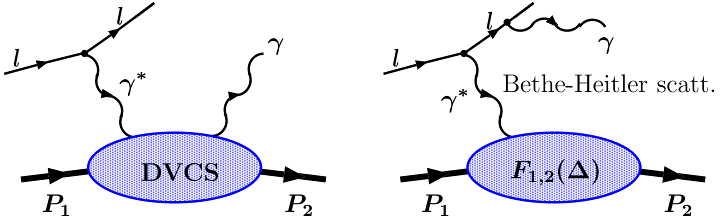
$$-q^2 (\overset{\mathrm{DVCS}}{\simeq} \mathcal{Q}^2/2) \to \infty \qquad \vartheta = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \approx \frac{\eta}{\xi} \overset{\mathrm{DVCS}}{=} 1$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const } (\text{as } x_B) \qquad t = (P_2 - P_1)^2 = \Delta^2$$

$$\sigma \propto |\mathcal{A}(\gamma^* p \to \gamma p)|^2$$

Deeply virtual Compton scattering

- Measured in $ep \to e\gamma p$



- There is a background process but it can be used to our advantage:

$$\sigma \propto |\mathcal{A}_{\rm DVCS}|^2 + |\mathcal{A}_{\rm BH}|^2 + \mathcal{A}^*_{\rm DVCS}\mathcal{A}_{\rm BH} + \mathcal{A}_{\rm DVCS}\mathcal{A}^*_{\rm BH}$$

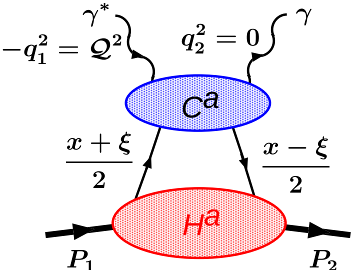
- Using $\mathcal{A}_{\rm BH}$ as a referent "source" enables measurement of the phase of $\mathcal{A}_{\rm DVCS}$

Factorization of DVCS $\longrightarrow$ GPDs

$\rightarrow$ cross-section can be expressed in terms of (the squares of)

Compton form factors: $\mathcal{H}(\xi, t, \mathcal{Q}^2)$, $\mathcal{E}(\xi, t, \mathcal{Q}^2)$, $\tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2)$, $\tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2)$, $\ldots$

[Collins and Freund '99]



$a = q, G$

- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2) \; H^a(x, \eta = \xi, t, \mu^2)$$

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)

Factorization of DVCS $\longrightarrow$ GPDs

$$ {}^{a}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \; H^a(x, \eta = \xi, t, \mu^2) $$

- $C^a(x, \xi, \mathcal{Q}^2/\mu^2)$ . . . hard scattering amplitude

  $\rightarrow$ pQCD

Factorization of DVCS $\longrightarrow$ GPDs

$$ {}^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x\; C^a(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu))\; H^a(x, \eta = \xi, t, \mu^2) $$

- $C^a(x, \xi, \mathcal{Q}^2/\mu^2)$ ... hard scattering amplitude

  $\rightarrow$ pQCD

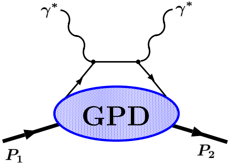
- $H^a(x, \eta = \xi, t, \mu^2)$ ... GPD

  $\rightarrow$ nonperturbative input

  $\rightarrow$ evolution $\Leftarrow$ pQCD (limiting cases DGLAP ($\eta = 0$) and ERBL ($\eta = 1$) evolution equations)

$$ \mu^2 \frac{d}{d\mu^2} \mathbf{F}(x, \eta, t, \mu^2) = \int_{-1}^{1} \frac{dy}{2\eta} \mathbf{V}\left(\frac{\eta + x}{2\eta}, \frac{\eta + y}{2\eta}; \eta \middle| \alpha_s(\mu) \right) \cdot \mathbf{F}(y, \eta, t, \mu^2) $$

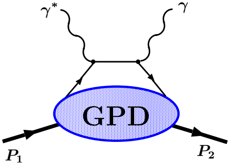
Complementary processes



(double) DVCS

$\boxed{\gamma^* p \to \gamma^* p}$

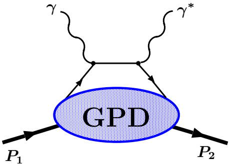
$(ep \to epl^+l^-)$



spacelike DVCS

$\boxed{\gamma^* p \to \gamma p}$

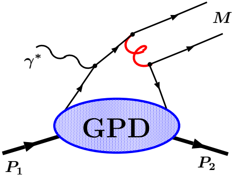
$(ep \to ep\gamma)$



timelike DVCS

$\boxed{\gamma p \to \gamma^* p}$

$(\gamma p \to pl^+l^-)$



Deeply virtual production of mesons (DVMP)

more difficult, but access to flavours

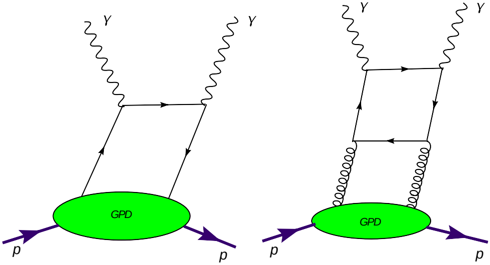
$\boxed{\gamma^* p \to Mp}$

factorization: [Collins, Frankfurt, Strikman '97]

Hard-scattering amplitudes

DVCS

$$\gamma^* q \to \gamma q, \; \gamma^* g \to \gamma g$$

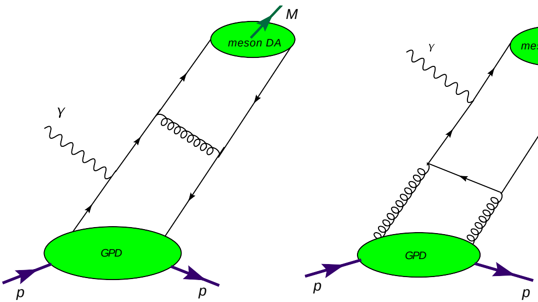


NLO: [Ji et al, Belitsky et al, Mankiewicz et al, '97]

Hard-scattering amplitudes

DVMP

$$\gamma^* q \to (q\bar{q})q, \; \gamma^* g \to (q\bar{q})g$$



NLO: [Belitsky and Müller '01, Ivanov et al '04]

Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$\widetilde{F}^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=}$$

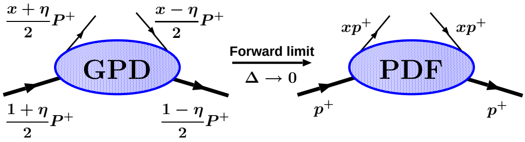
$$\widetilde{F}^g(x,\eta,t=\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)\widetilde{G}_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$

- Decomposing into helicity conserving and non-conserving part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

Properties of GPDs

- Forward limit ($\Delta \to 0$, $\eta \to 0$): $\Rightarrow$ $\widetilde{H}$-GPDs $\to$ PDFs



$P = P_1 + P_2$

$\Delta = P_2 - P_1$, $\Delta^2 = t$

$\eta = -\frac{\Delta^+}{P^+}$

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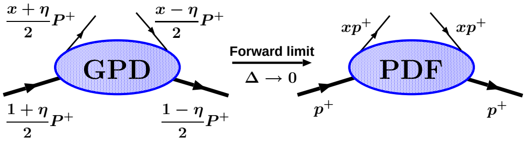

$$P = P_1 + P_2$$

$$\Delta = P_2 - P_1,\ \Delta^2 = t$$

$$\eta = -\frac{\Delta^+}{P^+}$$

- Sum rules: $\Rightarrow$ GPD $\to$ form factors

$$\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x,\eta,t) \\ E^q(x,\eta,t) \end{array} \right. = \left\{ \begin{array}{l} F_1(t) \\ F_2(t) \end{array} \right.$$

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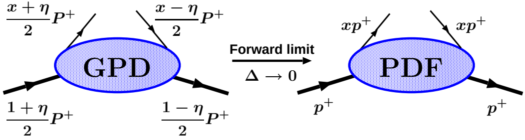

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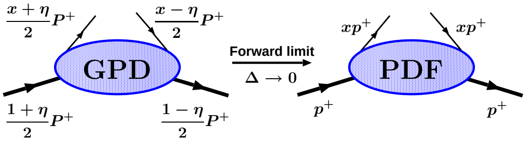
- Possibility of solution of proton spin problem

$$\frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\eta,t) + E^q(x,\eta,t) \right] = J^q(t) \qquad \text{[Ji '96]}$$

Properties of GPDs

- Forward limit ($\Delta \to 0,\ \eta \to 0$): $\Rightarrow$ $\widetilde{H}$-GPDs $\to$ PDFs



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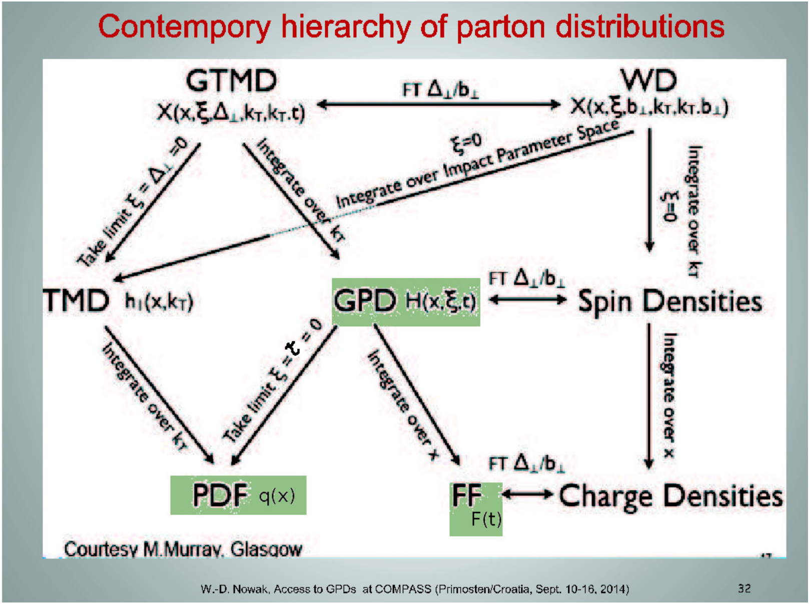
- polinomiality and positivity constraints


Contempory hierarchy of parton distributions
GTMD
X(x,ξ,Δ⊥,kT,kT,t)
FT Δ⊥/b⊥
WD
X(x,ξ,b⊥,kT,kT,b⊥)
Take limit ξ = Δ⊥ =0
Integrate over kT
ξ=0
Integrate over Impact Parameter Space
ξ=0
Integrate over kT
TMD h1(x,kT)
GPD H(x,ξ,t)
FT Δ⊥/b⊥
Spin Densities
Integrate over kT
Take limit ξ = t = 0
Integrate over x
Integrate over x
FT Δ⊥/b⊥
PDF q(x)
FF F(t)
Charge Densities
Courtesy M.Murray, Glasgow
W.-D. Nowak, Access to GPDs at COMPASS (Primosten/Croatia, Sept. 10-16, 2014)
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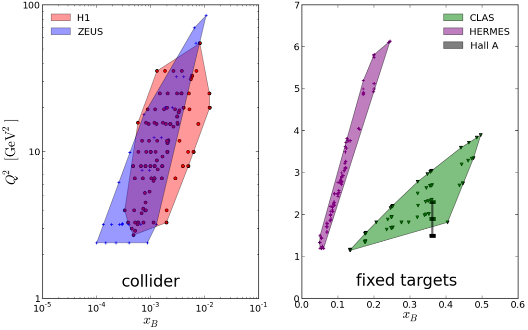
Experimental status

DVCS



[from Kumericki et al. 2015]

DVMP

- in the last decade: vector meson (ρ, J/Ψ, ϕ) production at H1 and ZEUS, COMPASS, pseudoscalar mesons (π, η) at CLAS . . .

→ new results from COMPASS, JLab12 (EIC)

Towards unravelling GPDs

DVCS: Compton form factors

$$ {}^{a}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2)) \; H^a(x, \xi, t, \mu^2) \Big|_{a=q,G} $$

or NS,S(Σ,G)

DVMP: transition form factors

$$ {}^{a}\mathcal{T}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \int \mathrm{d}y \; T^a(x, \xi, y, \mathcal{Q}^2/\mu^2)) \; H^a(x, \xi, t, \mu^2) \; \phi(y, \mu^2) $$

- **Complete deconvolution is impossible** and to extract GPDs from the experiment we need to **model** their functional dependence, or alternatively model form factors for start.
- *"Curse of the dimensionality"*
  When the dimensionality increases, the volume of the space increases so fast that the available **available data become sparse**.
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.

Modeling venues

- double distribution amplitude (DDA) satisfy automatically the polinomiality constraint so many models based on it, or specificaly Radyushkin's DDA (RDDA) (VGG code, [Goeke et al. 01], BMK model [Belitsky, Muller, Kirchner 01], GK model [Goloskokov, Kroll 05]))
- 'aligned jet' model [Freund, McDermott, Strikman 02], polynomials [Belitsky et al. '98], [Liuti et al. '07], [Moutarde '09]
- 'dual model' [Polyakov, Shuvaev 02], [Guzey, Teckentrup 06], [Polyakov 07]
- various models in Mellin-Barnes integral representation [Kumericki, Muller, Passek-K 08, . . .]
- fitting Compton form factors with neural networks [Kumericki, Muller, Schaefer 11]

- factorization formula for singlet DVCS CFFs:

$$ {}^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \; \mathbf{H}(x, \xi, t, \mu^2) $$

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- ...in terms of conformal moments

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \, \mathbf{H}_j(\xi = \eta, t, \mu^2) $$

$$ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) $$

$$ \ldots $$

H_j^a even polynomials in η with maximal power η^{j+1}

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$$ {}^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x\, \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}(x, \xi, t, \mu^2) $$

- …in terms of conformal moments

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$ = 2\sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, t, \mu^2) $$

$$ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) $$

…

H_j^a even polynomials in η with maximal power η^{j+1}

- series summed using Mellin-Barnes integral over complex j:

$$ = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right)\right] \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, t, \mu^2) $$

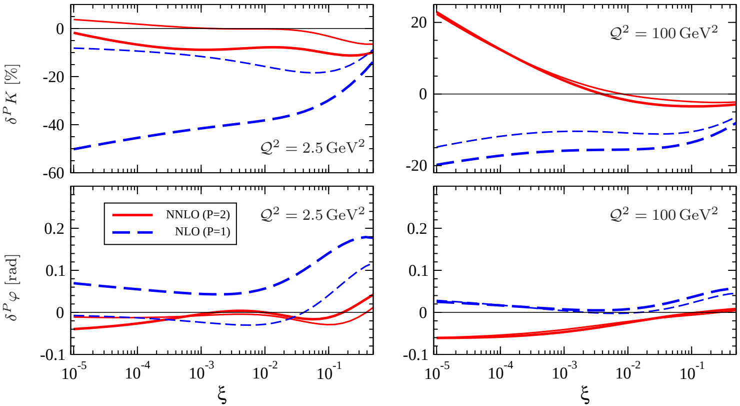
Advantages of conformal moments and Mellin-Barnes representation

- enables simpler inclusion of **evolution** effects
- powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
- opens the door for interesting **modelling of GPDs**
- possible efficient and stable numerical treatment $\Rightarrow$ stable and fast **computer code** for evolution and fitting
- moments are equal to matrix elements of local operators and are thus directly accessible on the **lattice**

- **NNLO corrections for DVCS** accessible by making use of conformal OPE and known NNLO DIS results

NLO and NNLO corrections

for generic parameters

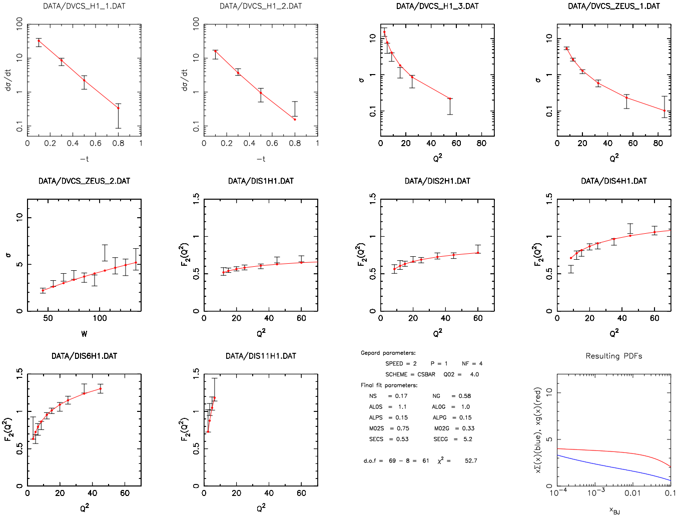


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0) = \alpha_\Sigma(0) + 0.05$

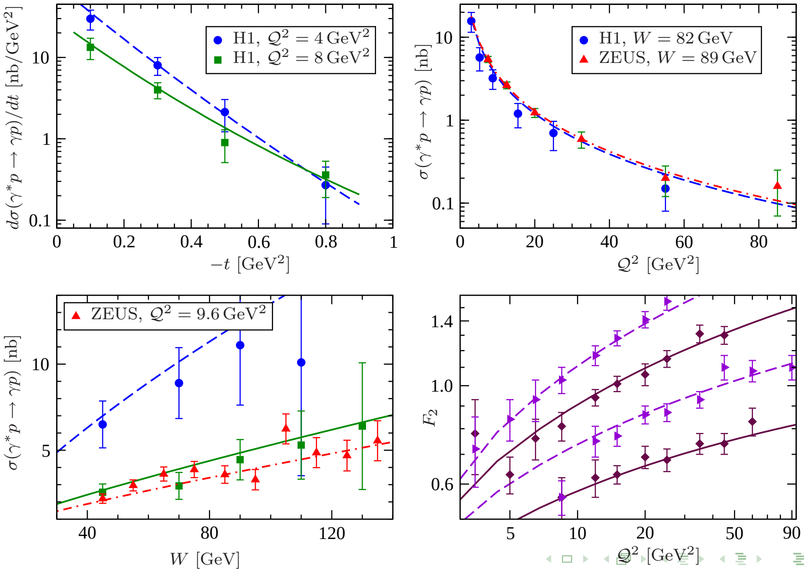
Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0) = \alpha_\Sigma(0) - 0.02$

$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

Fits (GeParD output)



NNLO fit to HERA DVCS+DIS data



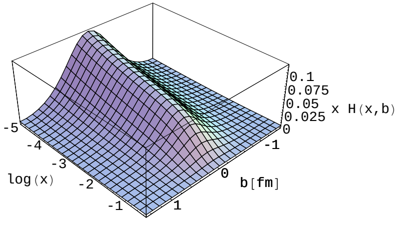
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b [Burkardt '00, '02]

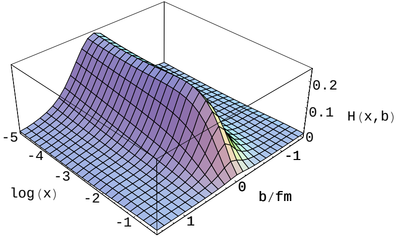
$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2) ,$$

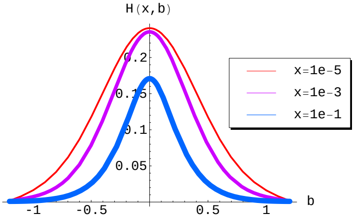
Three-dimensional image of a proton

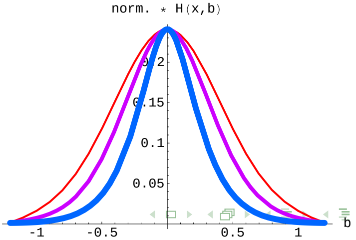
Quarks:



Gluons:







Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP ... different processes offer different insight and should provide more complete picture.

Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP . . . different processes offer different insight and should provide more complete picture.
- Extraction of GPDs is extremely chalenging but efforts for global fits are being made.
- New data are expected from COMPASS and JLab12. DVCS and related processes have a large role in EIC proposal.

The End