24th International Conference & Exhibition on Electricity Distribution (CIRED)

12-15 June 2017

Session 5: Planning of power distribution systems

Improved three-phase power flow method for calculation of power losses in unbalanced radial distribution network

ISSN 2515-0855
doi: 10.1049/oap-cired.2017.0489
www.ietdl.org

Tomislav Alinjak[1] ✉, *Ivica Pavic*[2], *Kruno Trupinic*[1]

[1]*HEP ODS, Zagreb, Croatia*
[2]*FER, Zagreb, Croatia*
✉ *E-mail: tomislav.alinjak@hep.hr*

Abstract: Recently, the need for improving the efficiency of distribution network in terms of power losses is being emphasised. A large share of total power losses refers to low-voltage networks which are usually unbalanced. For the power flow analysis, it is necessary to use three-phase modelling. This study presents modelling of distribution network elements and their implementation in backward/forward sweep (BFS) power flow method. An improvement of the BFS method is developed by using the breadth-first search method for network renumbering and creation of modified incidence matrix. The improved method minimises the read elements of each iteration and results in a significant reduction in total calculation time without accuracy loss. This improvement makes this method more suitable for using in real-time calculations. The proposed method is used for calculation of power losses in unbalanced and symmetrical network which are compared. The purpose of this test is to show the advantage of a three-phase power flow analysis compared with a symmetrical model.

1 Introduction

Recently, power loss reduction and increase in distribution network efficiency of distribution networks are being emphasised. Distribution networks are mostly radial systems that are characterised with short branches with many laterals, large number of nodes, three-phase, and single-phase users that cause unbalanced loads. Owing to increasing penetration of distributed generation, power can flow in both directions of radial system that also impacts on electrical conditions and further complicates the management of distribution network.

To ensure distribution network reliability, increase its efficiency, and to enable connections of distributed generation, it is necessary to increase network automation and implement an active distribution management that also includes optimisation of power losses. For optimisation of power losses, it is necessary to make power flow analysis. Owing to high R/X ratio of distribution network, power flow methods that are commonly used for transmission network analysis, like Gauss–Seidel or Newton–Raphson, are not suitable because they do not always converge. For power flow analysis of distribution networks are rather used BFS method and ladder network theory method that are both described in [1]. Comparison of these methods is made in many researches and many of them more prefer BFS method like [2, 3]. Analysis of unbalanced networks by using a symmetrical model can cause inaccurate results which are particularly expressed in low-voltage networks. Hence, for better accuracy, it is necessary to use three-phase models.

The purpose of this paper is to present the way of modelling of three-phase distribution network elements, including lines, transformers, and loads. Special case is low-voltage network that is four-wired line with neutral wire. Hence, it has to be transformed in a three-phase model, so it can be used in the same calculation with three-phase models of middle-voltage networks.

After defining three-phase models, an iterative three-phase power flow BFS method algorithm will be described. That method will be used for calculation of total power losses in distribution networks. For large networks, calculation time can be significantly increased, so it makes some difficulties in implementation of this method in a real-time analysis. In this paper, authors made an improvement of the commonly used BFS method by renumbering of network nodes and branches by using breadth-first search graph theory method. The proposed method minimises the number of read elements of incidence matrix, which is a sparse matrix, and thus reduces the calculation time.

Comparison of calculation time of the commonly used BFS method and the proposed method will be made for networks with various numbers of nodes.

Developed algorithm will be used for comparison of the results of power losses calculation by using symmetrical model and the three-phase model. The analysis will be made on a real low-voltage network and the results will prove the advantages of a three-phase model.

2 Three-phase distribution network elements

2.1 Line model

Distribution middle-voltage lines consist of three-phase conductors, while low-voltage lines consist of three phase and on neutral conductor. For three-phase model of a line, it is necessary to define impedance and admittance matrix, both with dimensions 3×3. For four-wired low-voltage network, initial impedance and admittance matrix have dimension 4×4 and they need to be transformed to matrices with dimension 3×3. Fig. 1 presents four-wired line section model between nodes p and q. The conductor impedances between nodes p and q of the same phase are called self-coupling impedances and impedances between conductors of different phases are called mutual-coupled impedances.

For the utility frequency of 50 Hz, the formula for self-coupling impedances is (1) and for mutual-coupling impedances is (2) [4]:

$$Z_{ii}^{pq} = R_1 + 0.05 + \mathrm{j}0.0628\ \ln\frac{93\sqrt{\rho}}{D_\mathrm{S}}\ \Omega/\mathrm{km} \tag{1}$$

$$Z_{ij}^{pq} = 0.05 + \mathrm{j}0.0628\ \ln\frac{93\sqrt{\rho}}{D_{ij}}\ \Omega/\mathrm{km} \tag{2}$$

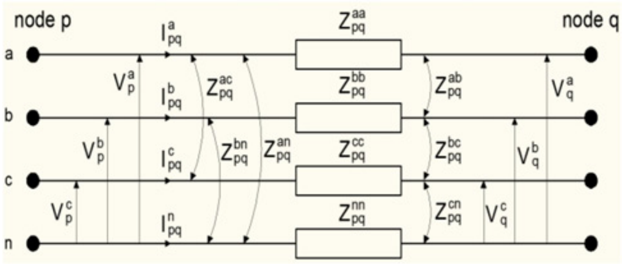

Fig. 1 *Line model of a four-wired network*

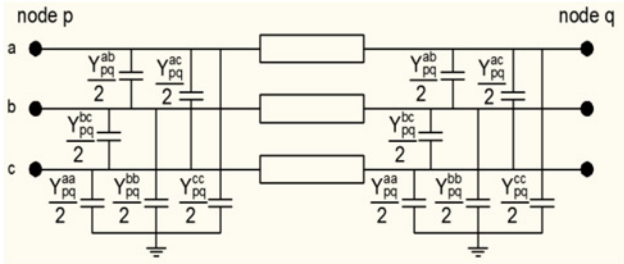

Fig. 2 *Line section model with shunt capacitances*

where, R_1 is the resistance of the conductor (Ω/km), ρ the earth resistivity (Ω m), D_S, geometric mean radius of the conductor (m), and D_{ij} the distance between phases i and j (m).

These impedances are used for creation of impedance matrix 4×4 (3)

$$\left[\boldsymbol{Z}^{abcn}_{pq}\right] = \begin{bmatrix} \boldsymbol{Z}_{aa} & \boldsymbol{Z}_{ab} & \boldsymbol{Z}_{ac} & \boldsymbol{Z}_{an} \\ \boldsymbol{Z}_{ba} & \boldsymbol{Z}_{bb} & \boldsymbol{Z}_{bc} & \boldsymbol{Z}_{bn} \\ \boldsymbol{Z}_{ca} & \boldsymbol{Z}_{cb} & \boldsymbol{Z}_{cc} & \boldsymbol{Z}_{cn} \\ \boldsymbol{Z}_{na} & \boldsymbol{Z}_{nb} & \boldsymbol{Z}_{nc} & \boldsymbol{Z}_{nn} \end{bmatrix} \tag{3}$$

Kron reduction is used for reduction of this matrix to dimensions 3×3. Voltage equation between nodes p and q is (4) [5]:

$$\left[\begin{array}{c} \boldsymbol{V}^a_p \\ \boldsymbol{V}^b_p \\ \boldsymbol{V}^c_p \\ \hline \boldsymbol{V}^n_p \end{array}\right] = \left[\begin{array}{c} \boldsymbol{V}^a_q \\ \boldsymbol{V}^b_q \\ \boldsymbol{V}^c_q \\ \hline \boldsymbol{V}^n_q \end{array}\right] + \left[\begin{array}{ccc|c} \boldsymbol{Z}_{aa} & \boldsymbol{Z}_{ab} & \boldsymbol{Z}_{ac} & \boldsymbol{Z}_{an} \\ \boldsymbol{Z}_{ba} & \boldsymbol{Z}_{bb} & \boldsymbol{Z}_{bc} & \boldsymbol{Z}_{bn} \\ \boldsymbol{Z}_{ca} & \boldsymbol{Z}_{cb} & \boldsymbol{Z}_{cc} & \boldsymbol{Z}_{cn} \\ \hline \boldsymbol{Z}_{na} & \boldsymbol{Z}_{nb} & \boldsymbol{Z}_{nc} & \boldsymbol{Z}_{nn} \end{array}\right] \cdot \left[\begin{array}{c} \boldsymbol{I}^a_{pq} \\ \boldsymbol{I}^b_{pq} \\ \boldsymbol{I}^c_{pq} \\ \hline \boldsymbol{I}^n_{pq} \end{array}\right] \tag{4}$$

Matrices are divided in blocks by lines, so it can be also written as:

$$\begin{bmatrix} \boldsymbol{V}^{abc}_p \\ \boldsymbol{V}^n_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}^{abc}_q \\ \boldsymbol{V}^n_q \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z}_{abc} & \boldsymbol{Z}_n \\ \boldsymbol{Z}^T_n & \boldsymbol{Z}_{nn} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}^{abc}_{pq} \\ \boldsymbol{I}^n_{pq} \end{bmatrix} \tag{5}$$

If the neutral wire is grounded, then $\boldsymbol{V}^n_p$ and $\boldsymbol{V}^n_q$ are equal and the result is:

$$I^n_{pq} = -Z^{-1}_{nn} \cdot Z^T_n \cdot I^{abc}_{pq} \tag{6}$$

Inserting (6) in (5) gives:

$$\begin{aligned} V^{abc}_p &= V^{abc}_q + Z_{abc} \cdot I^{abc}_{pq} + Z_n \cdot (-Z^{-1}_{nn} \cdot Z^T_n \cdot I^{abc}_{pq}) \\ &= V^{abc}_q + Z^{abc}_{pq} \cdot I^{abc}_{pq} \end{aligned} \tag{7}$$

where

$$Z^{abc}_{pq} = Z_{abc} - Z_n \cdot Z^{-1}_{nn} \cdot Z^{\mathrm{T}}_n \tag{8}$$

Finally, impedance matrix can be reduced to dimensions 3×3 (9) [5]:

$$\left[\boldsymbol{Z}^{abc}_{pq}\right] = \begin{bmatrix} Z'_{aa} & Z'_{ab} & Z'_{ac} \\ Z'_{ba} & Z'_{bb} & Z'_{bc} \\ Z'_{ca} & Z'_{cb} & Z'_{cc} \end{bmatrix} \tag{9}$$

Fig. 2 presents the line section model with phase-to-phase and phase-to-ground shunt capacitances.

Self and mutual potential coefficients are defined as (10) and (11) [1]

$$P^p_{ii} = 18 \times 10^6 \ln \frac{D_{i'i}}{D_{ii}} \text{ km/F} \tag{10}$$

$$P^p_{ij} = 18 \times 10^6 \ln \frac{D_{i'j}}{D_{ij}} \text{ km/F} \tag{11}$$

where $D_{i'i}$ is the distance from the conductor i to its image i' (m), D_{ii} is the radius of the conductor i (m), $D_{i'j}$ is the distance from the conductor i to the image of conductor j (m), and D_{ij} is the distance from the conductor i to conductor j (m).

These potential coefficients are used for creation of admittance matrix of node p (12):

$$\left[\boldsymbol{Y}^{abc}_p\right] = \mathrm{j}\omega \cdot \begin{bmatrix} \boldsymbol{P}^p_{aa} & \boldsymbol{P}^p_{ab} & \boldsymbol{P}^p_{ac} \\ \boldsymbol{P}^p_{ba} & \boldsymbol{P}^p_{bb} & \boldsymbol{P}^p_{bc} \\ \boldsymbol{P}^p_{ca} & \boldsymbol{P}^p_{cb} & \boldsymbol{P}^p_{cc} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{B}_{aa} & \boldsymbol{B}_{ab} & \boldsymbol{B}_{ac} \\ \boldsymbol{B}_{ba} & \boldsymbol{B}_{bb} & \boldsymbol{B}_{bc} \\ \boldsymbol{B}_{ca} & \boldsymbol{B}_{cb} & \boldsymbol{B}_{cc} \end{bmatrix} \tag{12}$$

If the values of self and mutual admittances are known, admittance matrix is equal to (13):

$$\left[\boldsymbol{Y}^{abc}_p\right] = \frac{1}{2} \cdot \begin{bmatrix} -\sum_i Y^{ai}_{pq} & Y^{ab}_{pq} & Y^{ac}_{pq} \\ Y^{ba}_{pq} & -\sum_i Y^{bi}_{pq} & Y^{bc}_{pq} \\ Y^{ca}_{pq} & Y^{cb}_{pq} & -\sum_i Y^{ci}_{pq} \end{bmatrix}, \quad i = \{a, b, c\} \tag{13}$$

Shunt currents in node p are equal to (14):

$$\left[\boldsymbol{Ish}^{abc}_p\right] = \begin{bmatrix} \boldsymbol{Ish}^a_p \\ \boldsymbol{Ish}^b_p \\ \boldsymbol{Ish}^c_p \end{bmatrix} = \sum_i \left[\boldsymbol{Y}^{abc}_p\right]_i \cdot \begin{bmatrix} \boldsymbol{V}^a_p \\ \boldsymbol{V}^b_p \\ \boldsymbol{V}^c_p \end{bmatrix} \tag{14}$$

2.2 Spot load models

Three-phase model of a star-connected load with constant power at node p can be expressed as

$$\left[\boldsymbol{Iload}^{abc}_p\right] = \begin{bmatrix} \boldsymbol{Iload}^a_p \\ \boldsymbol{Iload}^b_p \\ \boldsymbol{Iload}^c_p \end{bmatrix} = \begin{bmatrix} \left(\dfrac{S^a_p}{V^a_p}\right)^* \\ \left(\dfrac{S^b_p}{V^b_p}\right)^* \\ \left(\dfrac{S^c_p}{V^c_p}\right)^* \end{bmatrix} \tag{15}$$

CIRED, Open Access Proc. J., 2017, Vol. 2017, Iss. 1, pp. 2361–2365

Single-phase load currents are calculated by using (15), but only in a phase where the load is connected, while the current in other two phases is equal to zero.

2.3 Current and voltage line equations

Total current that flows through line section pq is equal to sum of all shunt currents in node q, all load currents in node q and currents of all branches that exit from node q $I_{\text{exit}q}$:

$$\left[\boldsymbol{I}_{pq}^{abc} \right] = \begin{bmatrix} \boldsymbol{I}_{pq}^{a} \\ \boldsymbol{I}_{pq}^{b} \\ \boldsymbol{I}_{pq}^{c} \end{bmatrix} = \begin{bmatrix} \sum_i I\text{load}_{qi}^{a} + \sum_i \text{Ish}_{qi}^{a} + \sum_i I\text{exit}_{qi}^{a} \\ \sum_i I\text{load}_{qi}^{b} + \sum_i \text{Ish}_{qi}^{b} + \sum_i I\text{exit}_{qi}^{b} \\ \sum_i I\text{load}_{qi}^{c} + \sum_i \text{Ish}_{qi}^{c} + \sum_i I\text{exit}_{qi}^{c} \end{bmatrix} \tag{16}$$

Voltage equation is equal to:

$$\begin{bmatrix} \boldsymbol{V}_{q}^{a} \\ \boldsymbol{V}_{q}^{b} \\ \boldsymbol{V}_{q}^{c} \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_{p}^{a} \\ \boldsymbol{V}_{p}^{b} \\ \boldsymbol{V}_{p}^{c} \end{bmatrix} - \begin{bmatrix} Z'_{aa} & Z'_{ab} & Z'_{ac} \\ Z'_{ba} & Z'_{bb} & Z'_{bc} \\ Z'_{ca} & Z'_{cb} & Z'_{cc} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_{pq}^{a} \\ \boldsymbol{I}_{pq}^{b} \\ \boldsymbol{I}_{pq}^{c} \end{bmatrix} \tag{17}$$

2.4 Transformer model

A single-phase transformer is modelled as a four-pole model (Fig. 3).

Depending on the transformer connection vector group, the model is derived by connecting three single-phase models. One of the most commonly used transformers in distribution network is Dyn5. Its three-phase model is shown in Fig. 4 and its current–voltage equation is:

$$\begin{bmatrix} \boldsymbol{I}^{A} \\ \boldsymbol{I}^{B} \\ \boldsymbol{I}^{C} \\ \boldsymbol{I}^{a} \\ \boldsymbol{I}^{b} \\ \boldsymbol{I}^{c} \end{bmatrix} = \begin{bmatrix} 2Y_{\alpha} & -Y_{\alpha} & -Y_{\alpha} & Y_{\alpha\beta} & 0 & -Y_{\alpha\beta} \\ -Y_{\alpha} & 2Y_{\alpha} & -Y_{\alpha} & -Y_{\alpha\beta} & Y_{\alpha\beta} & 0 \\ -Y_{\alpha} & -Y_{\alpha} & 2Y_{\alpha} & 0 & -Y_{\alpha\beta} & Y_{\alpha\beta} \\ Y_{\alpha\beta} & -Y_{\alpha\beta} & 0 & Y_{\beta} & 0 & 0 \\ 0 & Y_{\alpha\beta} & -Y_{\alpha\beta} & 0 & Y_{\beta} & 0 \\ -Y_{\alpha\beta} & 0 & Y_{\alpha\beta} & 0 & 0 & Y_{\beta} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{V}^{A} \\ \boldsymbol{V}^{B} \\ \boldsymbol{V}^{C} \\ \boldsymbol{V}^{a} \\ \boldsymbol{V}^{b} \\ \boldsymbol{V}^{c} \end{bmatrix} \tag{18}$$

3 Solution algorithm

The proposed method is based on the BFS method. Input parameters are feeder voltage, loads in all nodes, line parameters and mismatch tolerance.

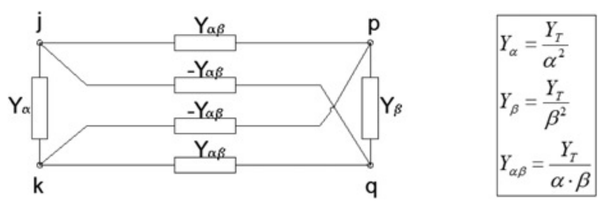

Fig. 3 *Single-phase transformer model*

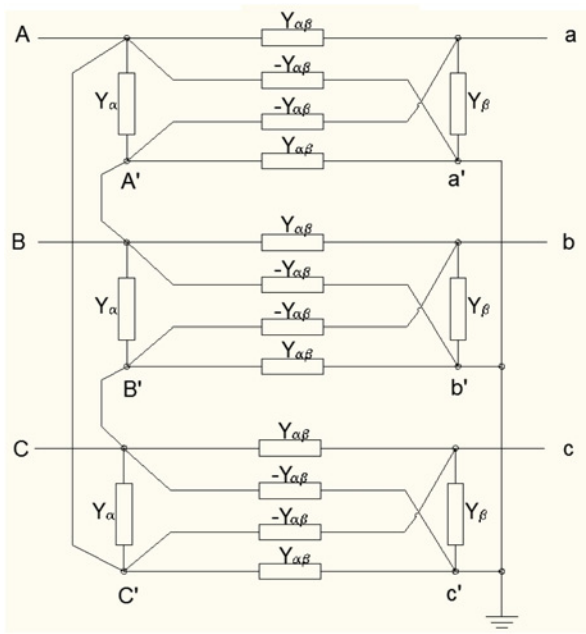

Fig. 4 *Equivalent model of a three-phase transformer Dyn5*

3.1 Modified incidence matrix

The first step after initialisation is to create an incidence matrix of the network. Distribution networks are usually radial with laterals. Their incidence matrix is a sparse matrix where zero elements mean 'no connection', −1 represents sending nodes which are located on a matrix diagonal, and 1 represents receiving nodes. Reading of all the elements significantly prolongs calculation time. The number of read elements NRE for every step of BFS is equal to the number of non-diagonal elements of upper-triangular matrix ($n \times n$):

$$\text{NRE} = \frac{n(n-1)}{2} \tag{19}$$

The idea of the proposed method is to create modified incidence matrix MIM by using breadth-first search method where all the non-diagonal and non-zero elements of each row will be sorted sequentially and thus minimise NRE. The method will be explained on a nine-node sample network in Fig. 5.

The numbers below the nodes are node labels. The first step of the method is to give the ordinal numbers to nodes and branches. The first node is the feeder node (usually busbar or substation) and the first branch is that which enters the feeder node. The nodes that are connected to the first node are successively assigned with next ordinal numbers, and the ordinal numbers of branches are equal to the ordinal number of the node that they enter. Then for each of these nodes, their neighbour nodes are searched and they get the next free ordinal number. Searching is finished when the end node of the network is reached. In Fig. 5, the node ordinal numbers are placed above the nodes and the branch numbers are placed in

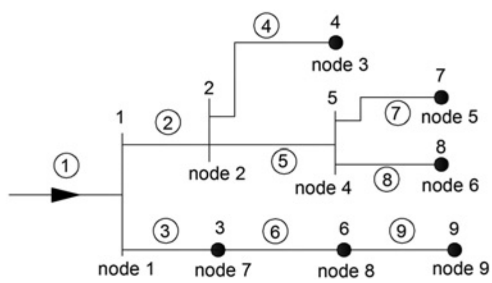

Fig. 5 *Nine-node sample network*

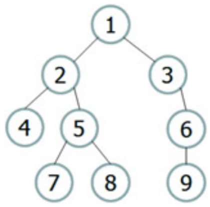

Fig. 6 *Renumbering scheme for a nine-node sample network*

$$[IM]=\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \Rightarrow [MIM]=\begin{bmatrix} 1 & 2 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$

Fig. 7 *Incidence matrix and its modification*

circles. A scheme of renumbering for the sample network is shown in Fig. 6.

New ordinal numbers are used to create MIM. The *i*th node is placed on the diagonal element of the *i*th row of the incidence matrix. The diagonal elements represent receiving nodes of branches. All nodes that are connected to the *i*th node are placed in the upper-triangular part of the matrix in the column that is equal to their ordinal number. Fig. 7 represents incidence matrix and its modification MIM and the line connects only elements that are read in every loop.

3.2 BFS method

After creation of MIM, an iterative process of BFS starts. In each iteration k, the first step is calculation of nodal currents in all nodes:

$$\left[I_p^{abc}\right]^{(k)}=\sum_i \begin{bmatrix} \left(\dfrac{S_p^a}{V_p^{a(k-1)}}\right)^* \\ \left(\dfrac{S_p^b}{V_p^{b(k-1)}}\right)^* \\ \left(\dfrac{S_p^b}{V_p^{b(k-1)}}\right)^* \end{bmatrix} - \sum_{\mathrm{i}} \left[Y_p^{abc}\right]\cdot\left[V_p^{abc}\right]^{(k-1)} \tag{20}$$

The next step is backward sweep which starts from the end node and successively moves to the feeder node. The branch currents are calculated as the sum of nodal currents in receiving node and currents of all branches that exit the receiving node:

$$\left[I_{pq}^{abc}\right]^{(k)}=\sum_j\left[I_{qj}^{abc}\right]^{(k)}+\sum_i\left[\mathrm{Iexit}_{qi}^{abc}\right]^{(k)} \tag{21}$$

The third step is forward sweep which starts from the feeder node and moves towards the end node. Nodal voltages are calculated by using:

$$\left[V_q^{abc}\right]^{(k)}=\left[V_p^{abc}\right]^{(k)}-\left[Z_{pq}^{abc}\right]\cdot\left[I_{pq}^{abc}\right]^{(k)} \tag{22}$$

The final step of each iteration is calculation of voltage mismatch for every node i:

$$\Delta\left[V_i^{abc}\right]^{(k)}=\left[V_i^{abc}\right]^{(k)}-\left[V_i^{abc}\right]^{(k-1)} \quad i=1,2,\ldots,n \tag{23}$$

If the voltage mismatch for every node in all phases is lower than tolerance limit, iteration process stops.

Total power losses are calculated as sum of power losses in all branches and losses in shunt capacitances:

$$S_{\mathrm{loss}}=\sum_{i,j} S\mathrm{loss}_{\mathrm{branch}ij}+\sum_i S\mathrm{loss}_{\mathrm{shunt}i} \tag{24}$$

4 Test results

4.1 Performance test

The program code of commonly used BFS and the proposed method is made in MATLAB in order to compare their execution times for networks with various numbers of nodes. All tests are made on computer with AMD Athlon 64 X2 dual-core processor 2.50 GHz and 6.00 GB RAM.

The performance tests are made on sample three-phase distribution grids with 9, 34, 100, 250, 500, and 1000 nodes. Each test is performed for convergence tolerance $\epsilon = 0.0001$. Calculation durations are compared in Table 1. Given results are average values of ten consecutive calculations. Total numbers of incidence matrix elements that are read during the iteration process (NRE) for both methods are compared in Table 2.

Based on test results, it can be concluded that the proposed method is significantly more efficient, and the efficiency is more emphasised in larger networks. For the network with 1000 nodes that converges in six iterations, the proposed method reduces the number of incidence matrix read elements for more than seven times which causes the three times shorter execution time in comparison with the commonly used BFS method.

4.2 Power losses analysis of unbalanced networks

One of the goals of this paper is to compare results of power losses calculation in unbalanced distribution networks with three-phase model and symmetrical model. Test was performed for five cases of daily diagram of low-voltage unbalanced network that is supplied from substation 10/0.4 kV Sv. Matije near Slavonski Brod, Croatia, with 64 three-phase and 18 single-phase customers (Fig. 8) and test results are shown in Table 3.

Table 1 Comparison of calculation durations ($\epsilon = 0.0001$)

Nodes	$\epsilon = 0.0001$			
	Iterations	BFS, s	Proposed method, s	Time reduction, %
9	5	0.0192	0.0163	15.10
34	8	0.0915	0.0507	44.59
100	2	0.2850	0.1958	31.30
250	6	2.9540	1.0531	64.35
500	6	11.6423	3.9384	66.17
1000	6	46.0294	15.2329	66.91

Table 2 Comparison of total NRE ($\epsilon = 0.0001$)

Nodes	$\epsilon = 0.0001$		
	BFS	Proposed method	Number reduction, %
9	1575	415	73.65
34	36,720	6064	83.49
100	79,800	11,086	86.11
250	1,498,500	195,708	86.94
500	5,997,000	766,458	87.22
1000	23,994,000	3,032,958	87.36

CIRED, Open Access Proc. J., 2017, Vol. 2017, Iss. 1, pp. 2361–2365

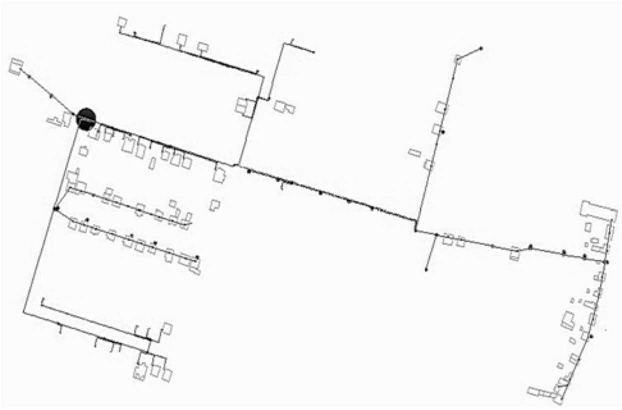

Fig. 8 *Low-voltage 0.4 kV network of substation 10/0.4 kV Sv. Matije*

Table 3 Comparison of power losses calculation results by three-phase model and symmetrical model

Time, h	Total power, kVA	Total power losses, kVA		Mismatch, %
		Three phase	Symmetric	
01:45	36.4832	0.1755	0.1681	4.22
07:00	63.0352	0.9364	0.8844	5.55
13:00	100.5001	2.5222	2.3400	7.22
19:00	142.7314	5.6432	5.1276	9.14
22:30	87.9337	1.4444	1.3480	6.68

For power losses analysis of unbalanced networks, three-phase model is more accurate because it distinguishes three-phase and single-phase loads as well as asymmetry of three-phase loads. Based on test results, it can be concluded that the more unbalanced load of the network causes a greater mismatch between the results. Thus, for power losses analysis of unbalanced distribution networks, especially of low-voltage networks, it is recommended to use three-phase model.

5 Conclusions

This paper presents how to use three-phase power flow calculation BFS method for unbalanced distribution networks. The authors developed an improvement of commonly used BFS by modification of incidence matrix by breadth-first search method which resulted in significant reduction in programme execution time. The proposed method is used for calculation of power losses in unbalanced networks and the conclusion is that it is more accurate than symmetrical model. Although three-phase calculation model is more complex than symmetrical model, and thus it lasts longer, an improvement presented in this paper, which significantly shortens programme execution time, makes the proposed method suitable for application in real-time analysis.

6 References

1 Kersting, W.H.: 'Distribution system modelling and analysis' (CRC Press, Boca Raton, USA, 2002)

2 Krushna Murthy, K., Jaya Ram Kumar, S.V.: 'Three phase unbalanced radial distribution load flow method', *Int. Refereed J. Eng. Sci.*, 2012, **1**, (1), pp. 039–042

3 Alsaadi, A., Gholami, B.: 'An effective approach for distribution system power flow solution', *Int. J. Electr. Comput. Energetic Elelctron. Commun. Eng.*, 2009, **3**, pp. 1–5

4 Pavic, I.: 'Trofazni proracun tokova snaga' (Udzbenici sveucilista u Zagrebu, Zagreb, Croatia, 2011)

5 Subrahmanyam, J.B.V.: 'Load flow solution of unbalanced radial distribution systems', *J. Theor. Appl. Inf. Technol.*, 2009, **6**, (1), pp. 040–051