INTRODUCTION

This special issue of the Journal of Energy is dedicated to the establishment of today’s Department for Energy and Power Systems (ZVNE), University of Zagreb Faculty of Electrical Engineering and Computing in 1934. For this reason, the history of the Department for Energy and Power Systems is presented in the introductory article, while the other articles are part of a broad scientific and professional work of the employees of the Department and some of the articles were created in wide cooperation with experts from the companies, that graduated from the Department.

Journal of Energy special issue: present 17 papers selected for publication in Journal of Energy after having undergone the peer review process. We would like to thank the authors for their contributions and the reviewers who dedicated their valuable time in selecting and reviewing these papers. We hope this special issue will provide you valuable information of some achievements at Department of Energy and Power Systems, Faculty of Electrical Engineering and Computing.

Short introduction of scientific and expert work of the Department for Energy and Power Systems (ZVNE):

Besides educational energy-related programmes for undergraduate, graduate and postgraduate students, Department Energy and Power Systems has been actively involved for many years in many scientific and expert studies. Studies on scientific projects include collaboration with industry, national institutions, electric utilities, and many foreign universities. The Department has developed valuable international cooperation with many research institutions around the world, either directly or through inter-university cooperation.

The Department is the leading institution in the field of electrical power engineering in the region, it has a long-lasting cooperation with the economic sector, and it is recognized for its scientific activity and a large number of published scientific papers in globally relevant journals, as well as numerous national and international scientific projects.

Main Department areas of activities are:

- Power Engineering and Power Technologies,
- Energy, Environment, Energy Management and
- Nuclear Power Engineering

In Power Systems Engineering the research is focused to development of both fundamental knowledge and applications of electrical power engineering. The research is generally directed to increasing the availability and the reliability of a power system with an emphasis on the adjustment to the open market environment. Specific goals include: improving models and methodologies for power system analysis, operation and control; development, production and application of models and methodologies for power systems planning, maintenance and development; application of soft-computing (artificial intelligence, meta-heuristics, etc.), information technologies (web-oriented technologies, geographic information systems, enterprise IT solutions, etc.) and operational research in improving processes of planning, development, exploitation and control of power systems; investigation on applications for coordinated control of power system devices and exploring the power system stability, security and economic operation; integration of intelligent devices and agents in energy management systems and distribution management systems equipment and software; advanced modeling of dynamics, disturbances and transient phenomena in transmission and distribution networks (in particular regarding distributed generation); advances in fault detection, restoration and outage management. The researches also cover high voltage engineering. At time of global changes in the energy sector, with emphasis on sustainable development, significant efforts are devoted to liberalization efforts, facilities revitalization, improved legislation and adoption of new standards.

In area of Power Technologies, Energy and Environment, Energy Management the main framework for the research are: sustainable electricity generation on a liberalized market, modeling energy market, renewable energy and climate change; power system optimization with emission trading; rational use of energy and energy savings; energy management in industry and buildings; energy conservation and energy auditing in industry and buildings. General objective of the research is to develop methodologies for quantitative assessment of the environmental impact of applicable energy technologies (electric power producing plants and their technology chains), as a base for estimating optimal long-term development strategy of the Croatian power system. Research work includes new strategies of energy sector and power system development; preparing medium and long-term electricity generation expansion plan for power system; comparison of energy, economic and environmental characteristics of different options for electric power generation; studies for rational use of energy and energy savings, assuming a centralized structure of the electricity market. Research work also includes renewable energy sources and its role in power sector, as well as electricity production considering cap on CO2 emissions. Research covers development of new models for power system generation optimization and planning under uncertainties on the open electricity market. The goal of that research is to create analytical tools and software tools which will enable a successful transition to the liberalized electricity market and ensure healthy and efficient power system operation in compliance with environmental requirements.

In the Nuclear Energy Field research cover nuclear physics reactor theory, nuclear power plants, fuel cycles and reactors materials and general objective of the research is to develop new methodologies for reliable assessment of the environmental impact of nuclear power plants operational safety. In the nuclear energy field the specific analysis cover calculations of transients and consequences of potential accidents in NPP Krsko. In the field of safety analyses of nuclear power plants the research activities are oriented to the mathematical modelling of nuclear power plant systems and components.

Guest Editors

prof. dr. sc. Zeljko Tomljen
prof. dr. sc. Igor Kuzle
Department of Energy and Power Systems
University of Zagreb Faculty of Electrical Engineering and Computing
MILJENKO BREZOVEC
miljenko.brezovec@hep.hr
HEP Generation Ltd
IGOR KUZLE          MATEJ KRPAN
igor.kuzle@fer.hr       matej.krpan@fer.hr
University of Zagreb Faculty of Electrical Engineering
and Computing

DETAILED MATHEMATICAL AND SIMULATION MODEL OF A SYNCHRONOUS GENERATOR

SUMMARY

Synchronous generator theory has been known since the beginning of its use, but the modelling and analysis of synchronous generators is still very existent in the present-day. Modern digital computers enable development of detailed simulation models, thus individual power system elements, including synchronous generators, are represented by the highest degree order models in power system simulation software packages. In this paper, first, a detailed mathematical model of a synchronous generator is described. Then, a simulation model of a synchronous generator developed based on the presented mathematical model. Finally, a transient stability after a short-circuit is simulated using real generator parameters.

Key words: block-diagram model, mathematical model, synchronous generator, short-circuit, transient stability
1. INTRODUCTION

In electric power system (EPS) simulation software packages today, individual elements can be represented by the models of the highest order of accuracy. By the virtue of state-of-the-art digital computers, in many cases it is not necessary to use simplified and reduced order models based on numerous assumptions anymore. In spite of this, using the most detailed mathematical models does not guarantee the quality and credibility of calculation results. The cause of unsatisfactory results is usually the lack of sufficiently accurate values of parameters on which a certain model is based on. Generally, the more detailed the mathematical model is, the more parameters it requires to be known. As many data for power system calculations (e.g. transient stability, short-circuit, power flow, etc.) are usually hard to obtain, it is clear that it isn’t always the best solution to use the most detailed mathematical models. Equipment manufacturers usually provide data about the most of needed parameters, e.g. of synchronous generators, but there are a lot of older generators in the operation today for which it is difficult to determine even the most basic parameters such as synchronous reactance or exciter forced voltage.

Different power system calculations have very different purposes so the demands on accuracy are different as well—from tuning of the protection relays or automatic regulators to analysis of assumed operational scenarios. The issue of detailed modelling, primarily of generators and turbines, and their control systems is especially accentuated in stability calculations. Detailed nonlinear models of generators are described in [1-4]. The most popular is simplified linearized third order model, used by Demello and Concordia [5]. This model is further developed in [6] for small-signal stability analysis. Automatic voltage regulator (AVR) with voltage control loop essentially changes the synchronous generator dynamics. In [7], extended state-space model including the effects of excitation system and generator amortisseurs is used. In this paper the influence of excitation system is not considered and focus is only on generator model. The impact of generator modelling complexity is the subject of many transient stability studies, such as [8-11].

When modelling the synchronous generator, the rest of the EPS is usually replaced with an infinite bus. When researching stability of a generator working in a multi-machine system where the total power is a lot larger than the power of the individual generator (along with a strong grid), only the impact of a short-circuit close to the generator terminals is analysed. As the length of a transient is relatively short (2 s do 5 s), physical properties of the analysed machine have the prevailing impact on the properties of machine swing response.

2. SYNCHRONOUS GENERATOR MODEL
Although the theory of synchronous generator has been known since the beginning of its application, the research of modelling and analysis of synchronous generators is still very much ongoing. Mathematical description of electromechanical systems operation such as synchronous generator generally leads to a system of differential equations which is regularly nonlinear due to the multiplication of state variables. With the increase of computing power, the capabilities for modelling and analysis are increased as well. This has resulted in a large number of models that differ depending on the type of research they are intended for and on the degree of desired accuracy.

There are different approaches when developing a mathematical model and the corresponding simulation model of a synchronous generator. The most common approach is based on general two-reaction theory upon which a three-phase winding of a generator is substituted by one equivalent, fictitious two-phase winding projected onto the direct ($d$) and quadrature ($q$) rotor axis. The field winding is represented as a $d$-axis winding and the reaction of damper winding caused by the eddy currents in the cylindrical rotor is substituted by fictitious windings in $d$-axis and $q$-axis.

### 3. PARK’S TRANSFORMATION

Mathematical description of a synchronous generator can be significantly simplified with proper variable transformation. One of the possible stator variables (currents, voltages, fluxes) transformation is known as Park’s or $d$-$q$ transformation. The number of variables after a transformation generally remains the same and in general case, substitution with new variables should be observed as a completely mathematical operation, thus no physical interpretation of fictitious is necessary. In this case, according to [1], the applied transformation can be physically interpreted because the new variables are obtained by projecting the real variables onto the three axes (direct, quadrature and stationary):

$$
i_{dq} = P i_{abc}
$$

$$
i_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}
$$

$$
P = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos \vartheta & \cos \left( \vartheta - \frac{2\pi}{3} \right) & \cos \left( \vartheta + \frac{2\pi}{3} \right) \\ \sin \vartheta & \sin \left( \vartheta - \frac{2\pi}{3} \right) & \sin \left( \vartheta + \frac{2\pi}{3} \right) \end{bmatrix}
$$
Current $i_d$ can be imagined as a current through a fictitious winding which rotates with the same speed as rotor windings and has such position that its axis always aligns with the field winding axis. The magnitude of current in this fictitious winding will be such that it will induce a magnetomotive force in the $d$-axis equal to the sum of magnetomotive forces in real phase windings. The current $i_q$ can be imagined in the same way, but the difference is that the axis of the fictitious winding aligns with the neutral axis of the rotor. Current $i_0$ is identical to the zero-sequence current component and it exists only when the sum of phase currents is different than zero. Zero-sequence is not considered in the generator analysis so the two-reaction representation is simplified which facilitates the setting of generator equations.

Park’s transformation is unique, thus an inverse transformation $P^{-1}$ exists as well, defined as:

$$i_{abc} = P^{-1} i_{dql}$$

$$P^{-1} = \begin{bmatrix}
\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{\cos \vartheta}{\sqrt{3}} & \sin \vartheta & \sin \left(\vartheta - \frac{2\pi}{3}\right) \\
\frac{\cos \left(\vartheta - \frac{2\pi}{3}\right)}{\sqrt{3}} & \cos \left(\vartheta + \frac{2\pi}{3}\right) & \sin \left(\vartheta + \frac{2\pi}{3}\right)
\end{bmatrix}\tag{5}$$

Coefficient $\sqrt{2}/3$ is chosen such that $P^{-1} = P'$ which means Park’s transformation is orthogonal.

4. VOLTAGE EQUATIONS

Figure 1 shows rotor and stator windings of a three-phase synchronous generator. The considered synchronous generator has three stator windings ($a, b, c$), a field winding ($F$) and two fictitious windings, one in $d$-axis ($D$) and one in $q$-axis ($Q$) which substitute the reaction of damper windings or dampening caused by eddy currents in a cylindrical rotor. These six windings are magnetically linked, and flux linkages are a function of the rotor position.

Voltage equations for these six linked circuits can be written in a matrix form:
where

\[ \mathbf{v}_n = -r_n \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - L_n \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \]  

(7)

By applying Park’s transformation, (6) becomes

\[
\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \psi_{a} \\ \psi_{b} \\ \psi_{c} \end{bmatrix} + \begin{bmatrix} v_n \\ 0 \end{bmatrix}
\]  

(6)
By substituting for flux linkages

\[
\begin{bmatrix}
\psi_0 \\
\psi_d \\
\psi_q \\
-\psi_F \\
0 \\
0
\end{bmatrix}
= -
\begin{bmatrix}
  r & 0 & 0 & 0 & 0 & 0 \\
  0 & r & 0 & 0 & 0 & 0 \\
  0 & 0 & r & 0 & 0 & 0 \\
  0 & 0 & 0 & r_F & 0 & 0 \\
  0 & 0 & 0 & 0 & r_D & 0 \\
  0 & 0 & 0 & 0 & 0 & r_Q
\end{bmatrix}
\begin{bmatrix}
i_0 \\
i_d \\
i_q \\
i_F \\
i_D \\
i_Q
\end{bmatrix}
- \begin{bmatrix}
-\omega \psi_q \\
\psi_d \\
\psi_q \\
-\omega \psi_F \\
\psi_D \\
\psi_Q
\end{bmatrix} + \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
\end{bmatrix}
\tag{8}
\]

(8) becomes:

\[
\begin{bmatrix}
\psi_0 \\
\psi_d \\
\psi_q \\
-\psi_F \\
0 \\
0
\end{bmatrix}
= -
\begin{bmatrix}
  r + 3r_n & 0 & 0 & 0 & 0 & 0 \\
  0 & r & 0 & 0 & 0 & 0 \\
  0 & 0 & r & 0 & 0 & 0 \\
  0 & 0 & 0 & r_F & 0 & 0 \\
  0 & 0 & 0 & 0 & r_D & 0 \\
  0 & 0 & 0 & 0 & 0 & r_Q
\end{bmatrix}
\begin{bmatrix}
i_0 \\
i_d \\
i_q \\
i_F \\
i_D \\
i_Q
\end{bmatrix}
- \begin{bmatrix}
  L_0 & 0 & 0 & 0 & 0 & 0 \\
  0 & L_d & 0 & kM_F & kM_D & 0 \\
  0 & kM_F & 0 & L_F & M_R & 0 \\
  0 & kM_D & 0 & M_R & L_D & 0 \\
  0 & 0 & kM_Q & 0 & 0 & L_Q
\end{bmatrix}
\begin{bmatrix}
i_0 \\
i_d \\
i_q \\
i_F \\
i_D \\
i_Q
\end{bmatrix}
\tag{9}
\]

As only balanced three-phase systems are usually analysed, the zero-sequence equations are usually omitted. By row-switching in order to group \(d\)-axis variables together and \(q\)-axis variables together, voltage equations (10) become
5. Rotor Swing Equation

Rotorswing equation is usually written in the following form:

$$ J \frac{d\omega_m}{dt} = M_m - M_e $$

(12)

where $J$ is the moment of inertia (kg·m²), $\omega_m$ is the mechanical angular velocity (rad/s), $M_m$ is the mechanical torque (Nm), $M_e$ the electrical torque (Nm). Difference between mechanical and electrical torque is called an accelerating torque. Equation (12) can be written in terms of power instead of torque:

$$ J \frac{d\omega_m}{dt} \omega_m = P_m - P_e $$

(13)

Electrical angular velocity is usually used instead of mechanical angular velocity. The relation between mechanical and electrical velocity is given by

$$ \omega = p\omega_m $$

(14)

where $p$ is the number of pole pairs. It can be shown [1] that by substituting mechanical angular velocity with electrical angular velocity and by introducing per-unit values instead of real values, (12) becomes

$$ \frac{2H}{\omega_R} \frac{d\omega}{dt} = M_m - M_e $$

(15)

where $H$ is an inertia constant (MWs/MVA), $\omega_R$ is the rated electrical speed (rad/s), $\omega$ is the electrical angular velocity (rad/s), while mechanical and electrical torque are in per-unit (p.u.). With the assumption that angular velocity $\omega$ is approximately
constant, the accelerating power is numerically approximately equal to the-accelerating torque (p.u.). Thus, the swing equation can be written as

\[
\frac{2H}{\omega_R} \frac{d\omega}{dt} \approx P_m - P
\]

(16)

Rated speed \( \omega_R \) is equal to

\[
\omega_R = 2\pi f_R
\]

(17)

where \( f_R \) is the nominal frequency (Hz), thus (16) can be written as

\[
\frac{d\omega}{dt} = \frac{\pi f_R}{H} (P_m - P_e)
\]

(18)

The generator swing equation is written in the form of (18). In the case of small disturbances the swing equation could be written as transfer function

\[
\frac{\Delta \omega}{\Delta m_m - \Delta m_e} = \frac{1}{2Hs}
\]

(19)

where \( s \) is the Laplace operator [12].

6. ELECTRICAL POWER AND ELECTRICAL TORQUE

Power at the three-phase synchronous generator’s terminals is generally calculated as

\[
P_e = v_d i_d + v_q i_q + v_{id_0} = v_{abc} i_{abc}
\]

(20)

By applying Park’s transformation on currents and voltages in (20), while keeping in mind that the transformation is orthogonal, the expression for generator power expressed in terms of new voltage and current variables is given as

\[
P_e = v_d i_d + v_q i_q
\]

(21)

As only balanced three-phase systems are usually observed, the expression (21) simplifies to

\[
P_e = v_d i_d + v_q i_q
\]

(22)

By substituting expressions for \( v_d \) and \( v_q \) from voltage equations the power equation becomes

\[
P_e = (i_d \psi_d + i_q \psi_q) + (i_d \psi_q - i_q \psi_d) \omega - r (i_d^2 + i_q^2)
\]

(23)
From this, by using certain assumptions, the simplified expression for an electric torque of a synchronous generator is obtained

\[ M_e = i_d \psi_d - i_q \psi_q \]  

which is usually used when modelling a synchronous machine.

7. EQUIVALENT CIRCUIT OF A SYNCHRONOUS GENERATOR

By expanding equation (9) for flux linkages, it can be shown that flux linkages of mutual inductances can be written as:

\[ \psi_{AD} = i_d (L_d - l_d) + kM_F i_F + kM_D i_D = L_{AD} (i_d + i_F + i_D) \]  

\[ \psi_{AQ} = i_q (L_q - l_q) + kM_Q i_Q = L_{AQ} (i_q + i_Q) \]

where \( L_{AD} \) and \( L_{AQ} \) are magnetizing inductances of windings in d and q axes.

\[ L_{AD} \equiv L_D - l_D = L_F - l_F = L_d - l_d = kM_F = kM_D = M_R \]  

\[ L_{AQ} \equiv L_Q - l_Q = L_q - l_q = kM_Q \]

Expressions (25) and (26) for flux linkages of mutual inductances can be represented by current injection in the magnetizing branch, Figure 2. In order to obtain a complete equivalent circuit, it is necessary to consider voltage equations. From (8), for \( d \)-axis windings, the following expressions are obtained:

\[ v_d = -r i_d - l i_d - L_{AD} (\dot{i}_d + i_F + \dot{i}_D) - \omega \psi_q \]  

\[ v_F = -r_F i_F - l_F i_F - L_{AD} (\dot{i}_d + i_F + \dot{i}_D) \]  

\[ v_D = -r_D i_D - l_D \dot{i}_D - L_{AD} (\dot{i}_d + i_F + \dot{i}_D) = 0 \]

Figure 2. Flux linkages inductances of a synchronous generator

These voltage equations are represented by an equivalent circuit shown in Figure 3. The three circuits (\( d \), \( F \) and \( D \)) in the \( d \)-axis are connected by the mutual inductance \( L_{AD} \) through which a sum of currents \( i_d, i_F \) and \( i_D \) is flowing. A voltage source \( \omega \psi_q \) is included in the \( d \)-axis stator winding circuit.
Voltage equations for $q$-axis windings are as follows:

\[ v_q = -r_i q - L_{Aq} (i_q + i_{Q}) + \omega \psi_d \]  
\[ v_Q = -r_{iQ} i_Q - L_{AQ} (i_q + i_{Q}) = 0 \]

and from these equations, the equivalent circuit of $q$-axis is constructed shown in Figure 4. Just like in $d$-axis, the sum of currents also flows through the magnetizing branch and a voltage source $\omega \psi_d$ exists in the $q$-axis winding circuit.

8. **FLUX LINKAGES STATE SPACE MODEL OF A GENERATOR**

It can be shown that the following relations between currents and flux linkages result from (9):

$d$-axis:

\[ i_d = \frac{1}{l_d} (\psi_d - \psi_{AD}) \]
\[ i_F = \frac{1}{l_F} (\psi_F - \psi_{AD}) \]  
\[ i_D = \frac{1}{l_D} (\psi_D - \psi_{AD}) \]  

where

\[ \psi_{AD} = \frac{L_{MD}}{l_d} \psi_d + \frac{L_{MD}}{l_F} \psi_F + \frac{L_{MD}}{l_D} \psi_D \]  

with equivalent \( d \)-axis inductance defined as:

\[ \frac{1}{L_{MD}} = \frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D} \]  

\( q \)-axis:

\[ i_q = \frac{1}{l_d} (\psi_q - \psi_{AQ}) \]  

\[ i_Q = \frac{1}{l_Q} (\psi_Q - \psi_{AQ}) \]  

where

\[ \psi_{AQ} = \frac{L_{MQ}}{l_q} \psi_q + \frac{L_{MQ}}{l_Q} \psi_Q \]  

with equivalent \( q \)-axis inductance defined as

\[ \frac{1}{L_{MQ}} = \frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} \]  

The expressions for flux linkages result from voltage equations (6):

\( d \)-axis:

\[ \dot{\psi}_d = -\frac{r}{l_d} \psi_d + \frac{r}{l_d} \psi_{AD} - \omega \psi_q - v_d \]  
\[ \dot{\psi}_F = -\frac{r}{l_F} \psi_F + \frac{r}{l_F} \psi_{AD} + v_F \]  
\[ \dot{\psi}_D = -\frac{r}{l_D} \psi_D + \frac{r}{l_D} \psi_{AD} \]  

\( q \)-axis:
\[ \dot{\psi}_q = -\frac{r}{L_q} \psi_q + \frac{r}{L_q} \psi_{Aq} + \omega \psi_d - v_q \]  
(46)

\[ \dot{\psi}_Q = -\frac{r_Q}{L_Q} \psi_Q + \frac{r_Q}{L_Q} \psi_{AQ} \]  
(47)

9. LOAD EQUATIONS

Equations (11), (15) and (24) represent a detailed model of a synchronous machine where the currents are state variables. With the assumption that \( v_F \) and \( M_m \) are known, the aforementioned system of equations does not completely describe the synchronous generator as long as the unknown variables \( v_d \) and \( v_q \) are not expressed in terms of state variables \( i_d \) and \( i_q \). The prerequisite for this is known conditions at the machine’s terminals, i.e. the load at the infinite bus must be taken into account as well as the value of impedance between the generator and the grid.

There are different ways to represent the load: constant impedance, constant power, constant current or any of the possible combinations of these three. For generator modelling, the load representation that will define relations between voltages, currents and angular velocity (load angle) obtained by solving the load flow is required. To simplify the generator model analysis, the rest of the electric power system is replaced by an infinite bus, thus the system influence is reduced to an impedance, and magnitude and angle of the voltage phasor at the infinite bus.

For a generator connected to an infinite bus via step-up transformer and a transmission line of equivalent resistance \( R_e \) and inductance \( L_e \), the terminal voltage of the generator is calculated as

\[
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c
\end{bmatrix}
=
\begin{bmatrix}
  v_{xa} \\
  v_{xb} \\
  v_{xc}
\end{bmatrix}
+
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\begin{bmatrix}
  R_e \\
  L_e
\end{bmatrix}
\]  
(48)

The infinite bus voltage is a balanced three-phase voltage

\[
v_{xabc} = \sqrt{2} V_{\infty} \begin{bmatrix}
  \cos(\omega_r t + \alpha) \\
  \cos(\omega_r t + \alpha - 2\pi / 3) \\
  \cos(\omega_r t + \alpha + 2\pi / 3)
\end{bmatrix}
\]  
(49)

where \( V_{\infty} \) is the RMS value of the grid voltage.

It can be shown that by using Park’s transformation and (50)

\[ \theta = \omega_r t + \delta + \pi / 2 \]  
(50)

expression (49) becomes
thus, the expression (48) in 0dq system is as follows:

\[
\begin{align*}
\mathbf{v}_{0dq} &= V_n \sqrt{3} \begin{bmatrix} 0 \\ -\sin(\delta - \alpha) \\ \cos(\delta - \alpha) \end{bmatrix} + R_i \mathbf{i}_{0dq} + L_e \mathbf{i}_{0dq} - \omega L_e \begin{bmatrix} 0 \\ -i_q \\ i_d \end{bmatrix} \\
\end{align*}
\]

(52)
Figure 6. Calculation of currents and flux linkages of mutual inductances

Figure 7. Calculation of flux linkages
11. SYNCHRONOUS GENERATOR PARAMETERS

Data acquisition necessary for calculations and parameter determination is an important step in the modelling process. Sometimes, acquiring even the most basic generator and corresponding control systems data can present a huge obstacle, especially when dealing with older machines that are still in operation. Thus, generator models with standard parameters are often used, i.e. reactances and time constants identified for the equivalent circuits in the $d$ and $q$ axis which are given by most generator manufacturers. Standard parameters are being used for the detailed generator model presented in this paper.

12. Standard generator parameters

During a disturbance in the rotor circuits, certain currents are induced under the terms of which some of them diminish more quickly than the others. Thus, the following generator parameters differ:

- subtransient – determine the quickly diminishing components,
- transient – determine the slowly diminishing components,
- synchronous – determine the constant (steady) components

Standard generator parameters are reactances as seen from generator terminals associated with fundamental frequency during steady-state, transient
and subtransient states along with corresponding time constants that determine the currents and voltages falloff gradient.

Besides reactances and time constants as standard generator parameters, it is also necessary to know the inertia constant \( H \) which determines the dynamic behaviour of the turbine-generator. The value of the inertia constant (MWs/MVA) can be determined using (53)

\[
H = \frac{1}{2} \frac{J \omega_m^2}{S_n}
\]  

(53)

where \( J \) is the moment of inertia of the turbine-generator (t·m²), \( \omega_m \) the (nominal) mechanical speed of the shaft (rad/s), \( S_n \) the volt-ampere base of the turbine-generator, usually the nominal apparent power (kVA). Moment of inertia describes the influence of the total rotating mass of the turbine-generator consisting of rotating mass of the turbine and rotating mass of the generator, while the contribution of the water mass must also be considered when dealing with hydroelectric turbines.

13. DETERMINING THE MODEL PARAMETERS FROM STANDARD GENERATOR PARAMETERS

Calculation of rotor mutual inductances is done according to the equivalent circuits (Figure 3 for \( d \)-axis, Figure 4 for \( q \)-axis) and by utilizing (27) and (28).

![Equivalent circuit for d-axis inductance: (a) transient, (b) subtransient](image)

\( d \)-axis transient inductance according to Figure 9a is given by

---

\[ L_d' = I_d + \frac{L_{AD}I_F}{L_{AD} + I_F} \] (54)

from which the field winding leakage inductance can be expressed as

\[ I_F = L_{AD} \frac{L_d' - I_d}{L_d - L_d'} \] (55)

Similarly, according to Figure 9b, \(d\)-axis subtransient inductance is given by

\[ L_d' = I_d + \frac{1}{1/L_{AD} + 1/I_D + 1/I_F} \] (56)

from which the \(d\)-axis damper winding leakage inductance can be expressed as

\[ I_D = L_{AD}I_F \frac{L_d' - I_d}{L_{AD}I_F - L_F(L_d' - I_d)} \] (57)

Finally, \(d\)-axis damper windings inductance and field winding inductance are given by

\[ L_D = L_{AD} + I_D \] (58)

\[ L_F = L_{AD} + I_F \] (59)

Analogously for the \(q\)-axis, from Figure 10 follows:

\[ L_q' = I_q + \frac{L_{AQ}I_Q}{L_{AQ} + I_Q} \] (60)

From which the \(q\)-axis damper winding leakage inductance is expressed as

\[ I_Q = L_{AQ} \frac{L_q' - I_q}{L_q - L_q'} \] (61)

and then the \(q\)-axis damper winding inductance is given by
\[ L_Q = L_{dQ} + l_Q \]  

It can be shown that the field winding resistance and the damper windings resistance can be determined from aforementioned reactances by using the following expressions:

\[ r_F = \frac{l_F + L_{dQ}}{\omega R T_{d0}^*} \]  
\[ r_D = \frac{l_D + L_d^* - l_q^*}{\omega R T_{d0}^*} \]  
\[ r_Q = \frac{l_Q + L_{dQ}}{\omega R T_{q0}^*} \]

where the time constants are in (s).

Time constants of short-circuited windings are given by

\[ T_d^* = T_{d0}^* \frac{L_d^*}{L_d} \]  
\[ T_q^* = T_{q0}^* \frac{L_q^*}{L_q} \]

where subscript 0 denotes open circuit time constants.

14. SYNCHRONOUS GENERATOR PARAMETERS

Synchronous generator model described in chapter 2 represents a system of time dependent differential equations. In steady-state, differential equations disappear because all magnitudes are constant. Stability analysis of some system generally begins from a steady state of that system. Then, a disturbance is applied and dynamic behavior is then observed.

Phasor diagrams are usually used to display steady-state relations as shown in Figure 11. Figure 11 displays the phasor diagram for the developed generator model connected to an infinite bus through impedance \( R_e + j X_e \).

Steady state can be defined in multiple ways. The most common way is defined by conditions at the generator terminals—voltage, active and reactive power. In this case, the power factor is calculated as
\[
\cos \varphi = \frac{P}{\sqrt{P^2 + Q^2}}
\]  
(69)

where \( P \) and \( Q \) are initial active and reactive power, respectively.

To calculate \( d \)-axis and \( q \)-axis components of currents and voltages of the generator and of the grid voltage, angles \( \delta, \beta \) and \( \varphi \) (see Figure 11) have to be known. \( \delta \) and \( \beta \) are determined from the phasor diagram and \( \varphi \) is determined from the power factor.

First, generator current is calculated:

\[
I = \frac{P}{V \cos \varphi}
\]  
(70)

Then, active and reactive component of generator current are calculated:

\[
I_r = I \cos \varphi \quad I_q = -I \sin \varphi
\]  
(71)

Angle between \( q \)-axis and terminal voltage vector is calculated by

\[
\delta - \beta = \arctan \frac{x_q I_r + r I_x}{V + r I_r - x_q I_x}
\]  
(72)

\( d \)-axis and \( q \)-axis components of generator currents and terminal voltage:

\[
I_d = -I \sin(\delta - \beta + \varphi) \quad I_q = I \cos(\delta - \beta + \varphi)
\]  
(73)

\[
V_d = -V \sin(\delta - \beta) \quad V_q = V \cos(\delta - \beta)
\]  
(74)

Induced EMF and excitation current:
\[ E = V_q + rI_q - x_d I_d \]  \hspace{1cm} (75)

\[ I_F = \frac{E}{L_{AD}} \]  \hspace{1cm} (76)

**Flux linkages:**

\[ \psi_d = L_d I_d + L_{AD} I_F \]  \hspace{1cm} (77)

\[ \psi_F = L_{AD} I_d + L_F I_F \]  \hspace{1cm} (78)

\[ \psi_D = (I_d + I_F) L_{AD} \]  \hspace{1cm} (79)

\[ \psi_q = L_q I_q \]  \hspace{1cm} (80)

\[ \psi_Q = L_{Dq} I_q \]  \hspace{1cm} (81)

**Figure 12. Phasor diagram of generator terminal voltage and grid voltage**

**Grid voltage vector equation:**

\[ \vec{V}_\infty = \vec{V} - \vec{Z}_p \vec{I} \]  \hspace{1cm} (82)

According to Figure 12, (82) can be expressed as follows:

\[ V_\alpha \angle (\alpha - \beta) = V - Z_p I (\cos(-\varphi) + j \sin(-\varphi)) \]  \hspace{1cm} (83)

From (83), grid voltage \( V_\alpha \) and angle difference \( \alpha - \beta \) can be determined. Load angle (angle between grid voltage vector and \( q \)-axis) is determined from:

\[ \delta - \alpha = (\delta - \beta) - (\alpha - \beta) \]  \hspace{1cm} (84)
15. SIMULATION RESULTS

Time-domain simulations have been conducted using the synchronous generator model developed in this paper. Parameters from a real hydroelectric power unit in HPP Dubrava (42 MVA) are used in the simulations. Parameters are shown in Table I.

Table I: Generator parameters of HPP Dubrava

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$-axis synchronous reactance $x_d$ (p.u.)</td>
<td>1.346</td>
</tr>
<tr>
<td>$q$-axis synchronous reactance $x_q$ (p.u.)</td>
<td>0.940</td>
</tr>
<tr>
<td>$d$-axis transient reactance $x_{d'}$ (p.u.)</td>
<td>0.446</td>
</tr>
<tr>
<td>$d$-axis subtransient reactance $x_{d''}$ (p.u.)</td>
<td>0.330</td>
</tr>
<tr>
<td>$q$-axis subtransient reactance $x_{q''}$ (p.u.)</td>
<td>0.370</td>
</tr>
<tr>
<td>Stator leakage reactance $x_l$ (p.u.)</td>
<td>0.243</td>
</tr>
<tr>
<td>$d$-axis open-circuit transient time constant $T_{d o'}$ (s)</td>
<td>1.660</td>
</tr>
<tr>
<td>$d$-axis open-circuit subtransient time constant $T_{d o''}$ (s)</td>
<td>0.118</td>
</tr>
<tr>
<td>$q$-axis open-circuit subtransient time constant $T_{q o''}$ (s)</td>
<td>0.035</td>
</tr>
<tr>
<td>Stator resistance $r$ (p.u.)</td>
<td>0.006</td>
</tr>
<tr>
<td>Inertia constant $H$ (s)</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Three-phase short-circuit fault at the infinite bus has been simulated as a typical example for different initial conditions.

Current and voltages responses in figure 13 are simulation results for following initial conditions:

- generator active power $P = 0.75$ (p.u.)
- generator reactive power $Q = 0.25$ (p.u.)
- generator terminal voltage $V = 1$ (p.u.)

Simulations have been also made for different fault durations. Figure 14 show load angle responses for fault durations of 0.1 s and for fault durations equal to and larger than critical clearing time which is 0.165 s for given scenario.

Model is verified comparing simulated and measured values. Responses of HPP Dubrava generator A on three-phase short circuit in the neighbouring grid (HPP Varaždin) are shown in Figure 15. Simulated and measured responses agree
very well, and small differences are probably caused by model parameters which could be calibrated.

Figure 13. Simulated results – currents and voltages responses (the short-circuit is applied at 0.1 s and removed at 0.2 s)

Figure 14. Load angle response for different fault duration (0.1, 0.165 and 0.17 s)
16. CONCLUSION

Thanks to the modern digital simulation systems even the most complex mathematical models can be translated into adequate simulation models. Therefore, high-order models that provide the highest degree of accuracy (with respect to the existing theory) are used more and more for power system elements modelling instead of low-order simplified models for simulating power system operation. The presented mathematical and simulation model of a synchronous generator allows the analysis of all electrical and mechanical units during faults and in different time scales. As an example, in this paper, the generator response to a three-phase short-circuit fault at the infinite bus (most commonly used type of short-circuit fault in stability analysis) have been shown. With minor adjustments, other types of faults can be simulated as well. The change of initial conditions and parameters is simple so different responses can be simulated and compared in order to analyze the impact of different initial conditions and parameters on the dynamic response of a generator. The block diagram model can be easily integrated with other models (foremost, the excitation and voltage control system and turbine with turbine governor systems).

Because of very high accuracy, the described model is used in Power System Laboratory at the Department of Energy and Power Systems, Faculty of Electrical Engineering and Computing, University of Zagreb to compare computer simulations with recorded dynamics of the generator after some switching operations [13].
17. ACKNOWLEDGEMENT

This work was supported by the Croatian Science Foundation under project "FENISG – Flexible Energy Nodes In Low Carbon Smart Grid" (grant no. IP-2013-11-7766).

18. NOMENCLATURE

\( E \) internal EMF induced by excitation current
\( f_R \) nominal frequency (Hz)
\( H \) inertia constant (s)
\( i_a, i_b, i_c \) generator armature current - phases \( a, b, c \)
\( i_0, i_d, i_q \) generator current - 0, \( d, q \) system
\( i_F, i_D, i_Q \) field winding current, \( d \) and \( q \) axis damper winding current
\( I \) generator RMS current
\( I_r, I_x \) active and reactive component of generator current
\( I_d, I_q \) \( d \) and \( q \) axis generator current
\( I_F \) excitation current
\( J \) moment of inertia (kg\( \cdot \)m\(^2\))
\( k \) mutual inductance coefficient
\( L_0, L_d, L_q \) stator winding inductance - 0, \( d, q \) system
\( L_F, L_D, L_Q \) field winding inductance, \( d \) and \( q \) axis damper winding inductance
\( L_n \) generator neutral point grounding inductance
\( L_{d}', L_{q}' \) \( d \) and \( q \) axis transient inductance
\( L_{q''} \) \( q \)-axis subtransient inductance
\( l_d, l_q \) stator winding leakage inductance – \( d, q \) components
\( l_F, l_D, l_Q \) field winding leakage inductance, \( d \) and \( q \) axis damper winding leakage inductance
\( L_{AD}, L_{AQ} \) \( d \) and \( q \) axis winding magnetizing inductance
\( L_{MD}, L_{MQ} \) \( d \) and \( q \) axis equivalent inductances
\( L_e \) equivalent inductance between the generator and the infinite bus
\( M_F, M_D, M_Q \) armature winding and field winding mutual inductance, \( d \) and \( q \) axis damper winding mutual inductance
\( M_R \) field winding and \( d \)-axis damper circuit mutual inductance
\( M_e \) electrical torque
\( M_m \) mechanical torque
\( p \) number of pole pairs
\( P_e \) electrical power
\( P_m \) mechanical power
\( P \) (initial) generator active power
\( Q \) (initial) generator reactive power
\( r \) armature winding resistance
\( r_F, r_D, r_Q \) field winding resistance, \( d \) and \( q \) axis damper winding resistance
\( r_n \) generator neutral point grounding resistance
\( R_e \) equivalent resistance between the generator and the infinite bus
\( S_n \) generator nominal apparent power (kVA)
\( t \) time (s)
\( T_{d0}, T_{q0} \) \( d \) and \( q \) axis open-circuit transient time constant (s)
\( T_{d0}'' \) \( d \)-axis open-circuit subtransient time constant (s)
\( T_d', T_q' \) \( d \) and \( q \) axis short-circuit transient time constant (s)
\( T_d'' \) \( d \)-axis short-circuit subtransient time constant (s)
\( v_a, v_b, v_c \) generator terminal voltage - phases \( a, b, c \)
\( v_0, v_d, v_q \) generator terminal voltage - 0, \( d, q \) system
\( v_F, v_D, v_Q \) field winding voltage, \( d \) and \( q \) axis damper winding voltage
\( v_n \) generator neutral point voltage
\( v_{ea}, v_{eb}, v_{ec} \) infinite bus voltage - phases \( a, b, c \)
\( V_\infty \) infinite bus RMS voltage
\( V \) generator RMS voltage
\( V_d, V_q \) \( d \) and \( q \) axis generator voltage
\( x_d, x_q \) \( d \) and \( q \) axis synchronous reactance
\( X_e \) equivalent reactance between the generator and the infinite bus
\( Z_e \) equivalent impedance between the generator and the infinite bus
\( \alpha \) infinite bus voltage phase shift (rad)
\( \beta \) infinite bus voltage and generator voltage phase shift (rad)
\( \delta \) \( q \)-axis phase shift with respect to the reference axis; load angle (rad)
\( \varphi \) Phase shift between generator voltage and generator current (rad)
\( \psi_a, \psi_b, \psi_c \) stator winding flux linkages - phases \( a, b, c \)

\(\psi_0, \psi_d, \psi_q\) stator winding flux linkages - 0, d, q system

\(\psi_F, \psi_D, \psi_Q\) field winding flux linkage, d and q axis damper winding flux linkage

\(\psi_{AD}, \psi_{AQ}\) d and q axis mutual inductance flux linkages

\(\vartheta\) instantaneous generator voltage angle (rad)

\(\omega\) angular frequency (rad/s)

\(\omega_m\) mechanical angular frequency (rad/s)

\(\omega_R\) nominal (synchronous) angular frequency (rad/s)

All magnitudes for which no units have been specified are expressed in per-unit unless specified otherwise in the text.

19. REFERENCES


