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Proceedings of IMECE2002
ASME International Mechanical Engineering Congress & Exposition
New Orleans, Louisiana, November 17-22, 2002

IMECE2002-32304

FATIGUE ASSESSMENT FOR COMBINED HCF/LCF LOADING

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ABSTRACT

The closed form expression for estimation of the crack initiation life at combined HCF/LCF loading is derived, and the way of reshaping the crack growth rate formulae in the form enabling their use in fatigue design at non-stationary loading is demonstrated. This new derived formula suggests an additional damage increase when crossing from one stress block to another. It is proposed to call this effect as "block crossing effect". Herein, the reshaped crack growth rate formula is applied for the fatigue design of structures and components made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading. For the stress history simplified in the way that it consists of one LCF stress block with number of cycles equal to number of start-up in-service operations, at load ratio $r = 0$, followed by one HCF stress block at load ratio $r > 0$ with summed-up all HCF cycles, the closed form expression is derived for estimating the crack propagation life at combined HCF/LCF loading.

Smith and Haigh diagrams as design tool for estimating the fatigue strengths for designed fatigue life, known load ratio and various number of HCF cycles per one combined stress block, are obtained for same material and same loading.

Keywords: Crack initiation; Reshaping crack growth rate formula; Haigh diagram.

INTRODUCTION

Any machine part subjected to substantial load due to start-stop operations has basically a similar stress history consisting of N_B stress blocks (one for each operation) with n_{HCF} high cycle fatigue (HCF) cycles and one low cycle (LCF) cycle (Fig. 1). LCF stresses are actually the "steady" stresses, which result in one cycle for every start-up and shutdown operation [1], and HCF stresses are caused by in-service vibrations. The integrity of the parts of high-speed engines, especially the turbine and compressor discs and blades is particularly critical, because the usually extremely high cyclic frequencies of in-service loading spectra cause that the fatigue life of e.g. 10^7 cycles can be reached in a few hours. It was one of the reasons that a number of fatigue failures has been detected e.g. in US fighter engines [1]. It is important therefore, to keep looking for a simple procedure enabling designer the reliable estimation of both crack initiation (CI) and crack propagation life for a given applied load, or to obtain the (boundary) load (or strain), at which the component would not experience the unpermissible damage during the designed life. The damage tolerant design normally refers to the design methodology in which Fracture Mechanics analyses predict remaining life and quantify inspection intervals. That philosophy allows the flows to remain in the structure, provided they are well below the critical size. Among the significant learned papers treating this matter, there is no one taking into account the additional damage when crossing from HCF stress block to LCF one, or reversely. That is the one more reason for this paper.

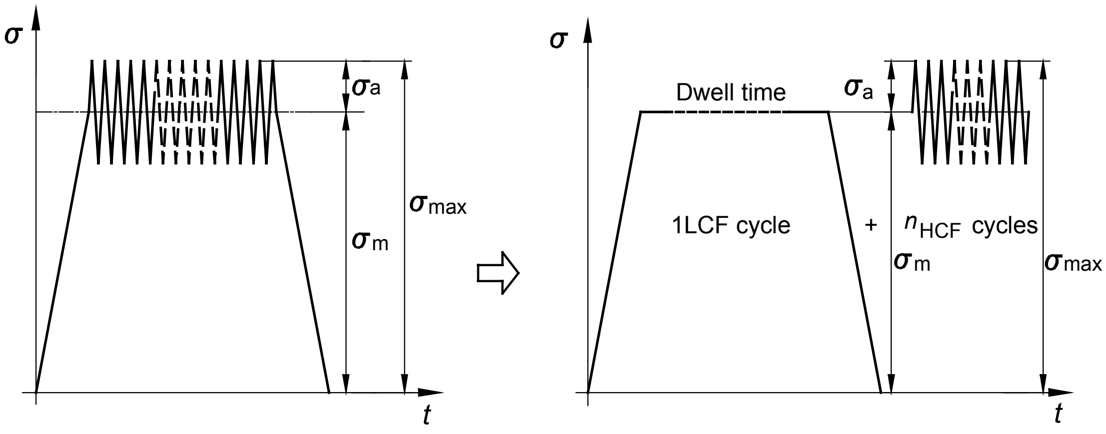

Figure 1: Common stress history of one combined stress block and its separation in one LCF stress cycle and one HCF stress block.

CRACK INITIATION ASSESSMENT

Crack initiation life at HCF loading

When French [2] established his well-known curve, and suggested distribution of σ - N diagram field in three regions - the region of damage between Wöhler curve and French curve, the region of failure on the right of the Wöhler curve, and the region of "overload" on the left of the French curve - he didn't know that he actually plotted the crack initiation curve, a years before the Fracture Mechanics was established. The original French testing procedure consists in cyclic loading that is stopped after a predetermined number of cycles, and after continuing at the endurance limit level, or slightly below it. If the specimen is fractured after sufficiently long number of cycles, it means that the specimen had been damaged (i.e. cracked) in the previous loading. Thus, the unfractured specimens had not been damaged. All the tests resulting in initial crack and all the tests resulting in uncracked specimen, represented by corresponding points, are separated by the French curve. In the strain approach to fatigue design, more suitable to LCF loading, those points are distributed by corresponding crack initiation curve (CIC) in log N - log ε diagram. Recently, the French procedure is simplified, because the crack initiation is perceived by modern devices, but the name of French is no more in use. In the region of the finite fatigue life, clasping the fatigue lives between the boundary of quasi-static failure N_q and the boundary of the infinite fatigue life region, this curve is well described by the Wöhler type equation [1,3].

$$N_i \sigma^{m_i} = C_i \qquad (1)$$

where N_i is the crack initiation life for a certain stress level σ, and m_i and C_i are the material constants.

At steady loading (N = 1/4), the CIC equals ultimate strength σ_U, and for the sufficiently long fatigue life, which can be taken as e.g. N_{gr}, it equals the endurance limit σ_0, which mean the entire fatigue life at the endurance limit level consists of the crack initiation life. On the basis of assumption that there is a unique CIC between these two points, its slope was approximated by the author [4] as

$$m_i = \frac{\log(4N_{gr})}{\log(\sigma_U / \sigma_0)} \qquad (2)$$

This expresion was found to be in good correllation with experimentaly obtained values. For example, the fatigue strength exponent b of steel 42 Cr Mo 4V (after DIN) for initiation life at r = -1 loading, was found to equal 0,0692 [5], thus m_i = 1/b = 14,5. Exactly the same value was obtained after Eq. (2) for $N_{gr} = 3 \cdot 10^7$. It is also in line with novel investigations of Singh [3].

Whereas at the endurance limit stress level the initiation life practically equals the total fatigue life, the constant C_i can be assessed as $C_i = N_{gr} \sigma_0^{m_i}$, where N_{gr} is the number of cycles at the knee of the S-N curve. For the purpose of this paper, the French curve at r = 0 is used, which enables determining the level of the pulsating stress at the CI boundary for certain N_i, by knowing the crack initiation life N_{gr} at the endurance limit level:

$$\sigma_{0N,i} = \sigma_0 \left(k_{gr} N_{gr} / N_i \right)^{1/m_i} \qquad (3)$$

Crack initiation life at combined HCF/LCF loading

For the stress history described in Fig. 1., the crack initiation life expressed in number of stress blocks $N_{B,i}$, is derived on the basis of Palmgren-Miner hypothesis of linear damage accumulation, where the level of damage is defined as

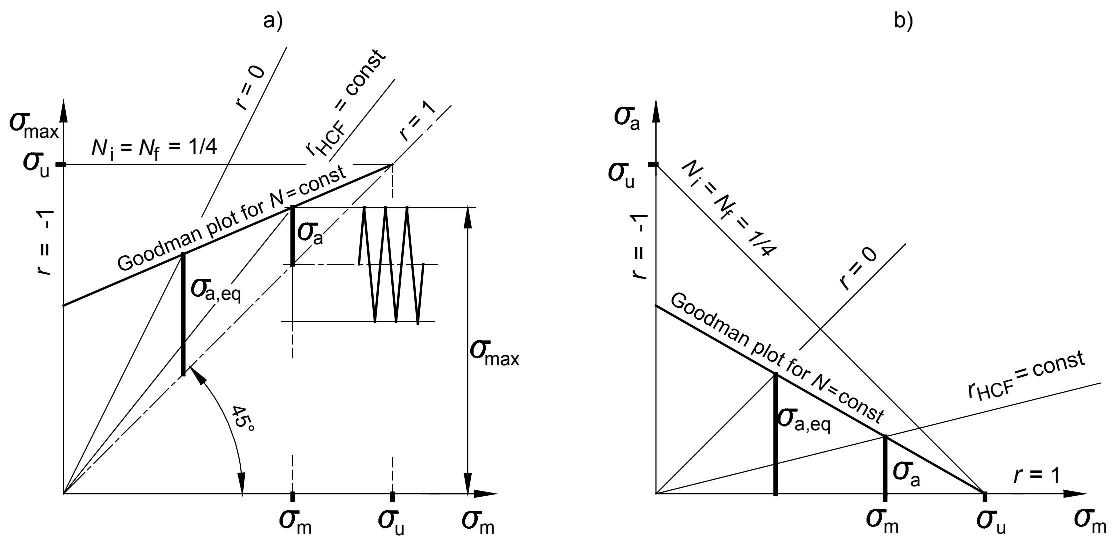

Figure 2: Reducing the HCF stress amplitude σ_a to an equivalent stress amplitude $\sigma_{a,eq}$ at r = 0 in a) Smith diagram, and b) Haigh diagram.

$$D_i = \sum_{j=1}^{n_B} \frac{n_j}{N_j} = \sum_{j=1}^{n_B}\left(\frac{n_{HCF,j}}{N_{HCF,j}} + \frac{1}{N_{LCF,j}}\right) = N_{B,i}\left(\frac{n_{HCF}}{N_{HCF,i}} + \frac{1}{N_{LCF,i}}\right) \quad (4)$$

The process of CI is finished for $D_i = 1$, when number of blocks n_B reaches the boundary value $N_{B,i}$. It is easy than to calculate the CI life expresed in stress blocks [1,4]:

$$N_{B,i} = \frac{1}{\frac{n_{HCF}}{N_{HCF,i}} + \frac{1}{N_{LCF,i}}} \quad (5)$$

It is easy now to obtain the total initiation life:

$$N_i = N_{B,i}(1 + n_{HCF}) \cong N_{B,i} \cdot n_{HCF} = \frac{1}{\frac{1}{N_{HCF,i}} + \frac{1}{N_{LCF,i} \cdot n_{HCF}}} \quad (6)$$

The initiation life $N_{LCF,i}$ is obtained after the CIC (3) at r = 0:

$$N_{LCF} = N_{gr}\left(\sigma_0 / \sigma_m\right)^{m_i} \quad (7)$$

Since the Palmgren-Miner hypothesis is valid for various stress blocks at the same stress ratio, this equation is also used for the calculation of the HCF initiation life, but by substituting in it an equivalent stress range obtained by reducing a HCF stress range (with stress ratio $r_{HCF} > 0$) to an equivalent stress range at r = 0 (Fig. 2.). This equivalent stress range is obtained as intersection point of Goodman plot N_i =const having the slope $(\sigma_U - \sigma_{max})/(\sigma_U - \sigma_m)$ and load line at r=0. It is obtained:

$$\Delta\sigma_{eq} = 2\sigma_{a,eq} = \frac{2\sigma_a \sigma_U}{\sigma_U + \sigma_a - \sigma_m} \quad (8)$$

Thus, by substituting Eq. (8) in Eq. (7) twice (for a LCF stress σ_m , and for a reduced HCF stress after Eq. (9)), the explicit formula is obtained for determining the crack initiation life at combined HCF/LCF loading:

$$N_i = \frac{N_{gr}\sigma_0^{m_i}}{\left(\frac{2\sigma_U \sigma_a}{\sigma_U + \sigma_a - \sigma_m}\right)^{m_i} + \frac{\sigma_m^{m_i}}{n_{HCF}}} \quad (9)$$

CRACK PROPAGATION ASSESSMENT FOR COMBINED HCF/LCF LOADING

As most appropriate for the purpose of this paper, the Ritchie fatigue crack growth rate formula [6]

$$\frac{da}{dN} = C\Delta K^m K_{max}^n \quad (10)$$

is applied for determining the damage size. In this formula $\Delta K = \Delta\sigma Y\sqrt{\pi a}$ is the stress intensity range, $K_{max} = \sigma_{max} Y \sqrt{\pi a}$ is the upper value of the stress intensity factor, m and n are material constants, $\Delta\sigma = 2\sigma_a$ is a stress range, σ_{max} is a maximum stress, Y is a crack form factor, and a is a crack size. For titanium alloy Ti-6Al-4V, the following values of material constants were obtained: $C = 5{,}2 \cdot 10^{-12}$, m = 2,5 and n = 0,67.

By introducing into the Eq. (10) the damage ratio $D = a/a_c$, where a_c is a critical crack size and fracture toughness $K_c = \sigma_{max} Y \sqrt{\pi a_c}$, it can be reshaped in the form

$$\frac{dD}{dN} = \frac{B}{a_c}(1-r)^m D^{\frac{m+n}{2}} \tag{11}$$

where $B = 2^m C K_c^{m+n}$ is a material constant. By integrating this equation, it is easy to determine the damage ratio after N propagating cycles:

$$D = \frac{D_0}{\left[1 - D_0^{\frac{m+n}{2}-1} \frac{B}{2a_c}(1-r)^m (m+n-2)N\right]^{\frac{2}{m+n-2}}} \tag{12}$$

where $D_0 = \pi a_0 \left(Y\sigma_m / K_c\right)^2$ is initial damage ratio, a_0 is an initial crack size, $r = \sigma_{min}/\sigma_{max} = K_{min}/K_{max}$ is a load (stress intensity) ratio, and the form factor is approximated after Raju and Newman [7] as $Y = 0{,}78(1 + a/d)$, where d is a bar diameter. By substituting in this formula $D = 1$, the crack propagation life at constant amplitude loading can be determined.

Equation (11) can be used also in fatigue assessments at variable amplitude loading [8], but in such a case a_c changes, if σ_{max} changes, and Eq. (11) must be reshaped:

$$\frac{dD}{dN} = \frac{1}{a_c}\frac{da}{dN} - \frac{a}{a_c^2}\frac{da_c}{dN} = \frac{B}{a_c}(1-r)^m D^{\frac{m+n}{2}} + \frac{D}{D_0}\frac{dD_0}{dN} \tag{13}$$

By the same procedure, the Paris or any other expression for the crack growth rate, can be transformed in the form similar to that in Eq. (13). In the case of block loading, or if the spectrum loading is approximated with block loading, the second term of this equation always equals zero, except when crossing from one stress block to another- just when the first term becomes zero. During the change of a_c, Eq. (13) can be written in the form

$$\frac{dD}{D} = -\frac{da_c}{a_c} \tag{14}$$

By integrating it, the increased value of damage ratio caused by the change of the critical crack size between two stress blocks, is obtained:

$$D_2 = D_1 \frac{a_{c1}}{a_{c2}} \tag{15}$$

The expression in Eq. (13) is appropriate for the crack propagation assessment at any loading conditions, including non-stationary one, where maximum stress, crack form factor and load ratio change.

Herein, the Eq. (13) is applied for the crack propagation life estimation in the gas turbine and compressor discs and blades made of the titanium alloy Ti-6Al-4V, at combined HCF/LCF loading. If the stress history is simplified in the way that it consists of one LCF stress block with $N_{LCF} = N_B$ cycles at maximum stress σ_m and load ratio $r = 0$, followed by one HCF stress block with $n_{HCF} \cdot N_B$ cycles at maximum stress σ_{max} and load ratio $r = (\sigma_{max} - 2\sigma_a)/\sigma_{max}$, then the damage ratio D_{LCF} after the LCF stress block is determined after Eq. (12). According to Eq. (13), at the beginning of the HCF stress block, the damage ratio is

$$D_{0,HCF} = D_{LCF}\frac{a_{cL}}{a_{cH}} \tag{16}$$

where a_{cL} and a_{cH} are the critical values of the srack size at LCF and HCF loading, respectively. Those values can be determined by solving their equations. E.g. a_{cH} is determined from the equation

$$a_{cH} = \frac{1}{\pi}\left[\frac{K_c}{Y(a_{cH})\sigma_{max}}\right]^2 \tag{17}$$

where K_c=50 MPa m$^{1/2}$ for Ti-6Al-4V alloy, after Ritchie [6]. The damage ratio D_{HCF} at the end of the HCF stress block, as the final damage ratio, is obtained again after Eq. (12) by substituting in it the corresponding values of initial damage ratio, stress ratio and number of cycles. The fatigue fracture occurs when this damage ratio reaches the value of one. Then, it is not difficult to solve the mentioned three equations for the N_B and consequently for the entire crack propagation life. It is obtained:

$$N_p = 2\frac{\left(a_0/a_{cH}\right)^{1-\frac{m+n}{2}} - 1}{B(m+n-2)\left[(1-r)^m n_{HCF}/a_{cH} + a_{cL}^{-1}\left(a_{cL}/a_{cH}\right)^{1-\frac{m+n}{2}}\right]} n_{HCF} \tag{18}$$

Thus, the explicit expression is derived, enabling the estimation of the crack propagation life at combined HCF/LCF loading, for certain values of the stress levels σ_{max} and σ_m, which are hidden in a_{cH} and a_{cL}. When no "block crossing" effect is applied, the expression for the crack propagation life becomes

$$N_p = 2\frac{D_0^{1-\frac{m+n}{2}} - 1}{B(m+n-2)\left[(1-r)^m n_{HCF}/a_{cH} + a_{cL}^{-1}\right]} n_{HCF} \tag{19}$$

Assumption that stress history consists of one LCF cycle followed by one HCF stress block consisting of n_{HCF} cycles, followed by one LCF cycle etc. is much closer to real operational conditions. Thus, for more precise calculations, damage ratio is calculated after one LCF cycle, its increase according to Eq. (13), after the HCF stress block, then damage decrease according to Eq. (13), etc. The fatigue fracture occurs and calculation procedure is stopped at the moment when damage ratio reaches the value of one.

FATIGUE LIMITS FOR COMBINED HCF/LCF LOADING

In fatigue design generally, and especcially in design of components subjected to combined HCF/LCF loading, the Haigh diagram is a very useful tool, presenting the areas, i.e. the stress levels at which the required fatigue life will not be reached. The corresponding curves obtained, enable damage tolerant design, i.e. they divide the diagram area into two zones: the zone of stress states resulting in allowable and unallowable fatigue lives, that is in allowable and unallowable damage level. The procedure is the same as described in previous chapter, but for the fatigue life as input data. Thus, for certain values of fatigue lives, the fatigue strength curves are obtained indicating the stress levels in Haigh diagram causing the fatigue failure after $N_f = C_f$ cycles. The calculations are carried out for various values of C_f, and for a number of HCF cycles per one stress block $n_{HCF} = 10^2 \ldots 10^5$. The fatigue limit curves obtained precisily exibit the reduction of the design area in Haigh diagram compared to HCF loading only, the more so as the share of LCF loading is greater.

As an example, the resulting N_f=10^7 and N_f=10^6 curves for titanium alloy Ti-6Al-4V, and for $n_{HCF} = 10^2 \ldots 10^5$, are exhibited in Haigh diagrams, Fig. 3. In view of these curves, which share the diagram space on the safe and the unsafe one, it is observed:

- At the region of lower mean stresses, they make one with Goodman line, then separate from it, and finaly turn down at constant mean stresses. Thus, presence of the LCF component restricts the safe design space compared to that in case of pure HCF, the more so as the share of the LCF component is greater.
- The block crossing effect does not influence significantly the curves of constant fatigue life.
- Between the curves of constant fatigue life based on the initial crack sizes of 0,1 mm and 0,05 mm was not observed a significant difference.
- The curves of constant fatigue life obtained on the basis of the derived closed form fatigue life formula, and those obtained on the basis of growth increments computed for one LCF cycle, n_{HCF} cycles, next LCF cycle, etc., does not differ significantly.

SUMMARY AND CONCLUSIONS

The closed form expression for estimation of the crack initiation life at combined HCF/LCF loading is derived, and the way of reshaping the crack growth rate formulae in the form enabling their use in fatigue design at non-stationary loading is demonstrated. This new derived formula suggests an additional damage increase when crossing from one stress block to another. So, fatigue design becomes more conservative, broaching the subject of reliability of recent fatigue assessment of the components under variable amplitude loading. Herein, the reshaped crack growth rate formula is applied for the fatigue design of aircraft components made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading. For the stress history simplified in the way that it consists of one LCF stress block with $N_{LCF} = N_B$ cycles at maximum stress σ_m and load ratio $r = 0$, followed by one HCF stress block with $n_{HCF} \cdot N_B$ cycles at maximum stress σ_{max} and load ratio $r = (\sigma_{max} - 2\sigma_a)/\sigma_{max}$, the closed form expression is derived for estimating the crack propagation life at combined HCF/LCF loading.

Haigh diagrams as design tools for estimating the fatigue strengths for designed fatigue life, known load ratio and various number of HCF cycles per one combined stress block, are obtained for the parts made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading.

The results of this research should be taken as a guide because

- The small crack behaviour has not been taken into account,
- The presence of other damage mechanisms like creep fatigue, oxidation and other environmental effects are ignored,
- The residual stresses have not been handled,
- The stress concentration has been ignored,
- Technology faults, material quality and operating conditions (like elevated temperature), have not been taken into account,
- Linear damage summation rule has been applied, although more precise techniques exist,
- The presence of inclusions and the service-induced damages could not be clasped,
- The reliability aspect of the design has been ignored.

At the same time, these imperfections are the sign-posts in the direction of building an expert system for the fatigue design of the aircraft components subjected to combined HCF/LCF loading.

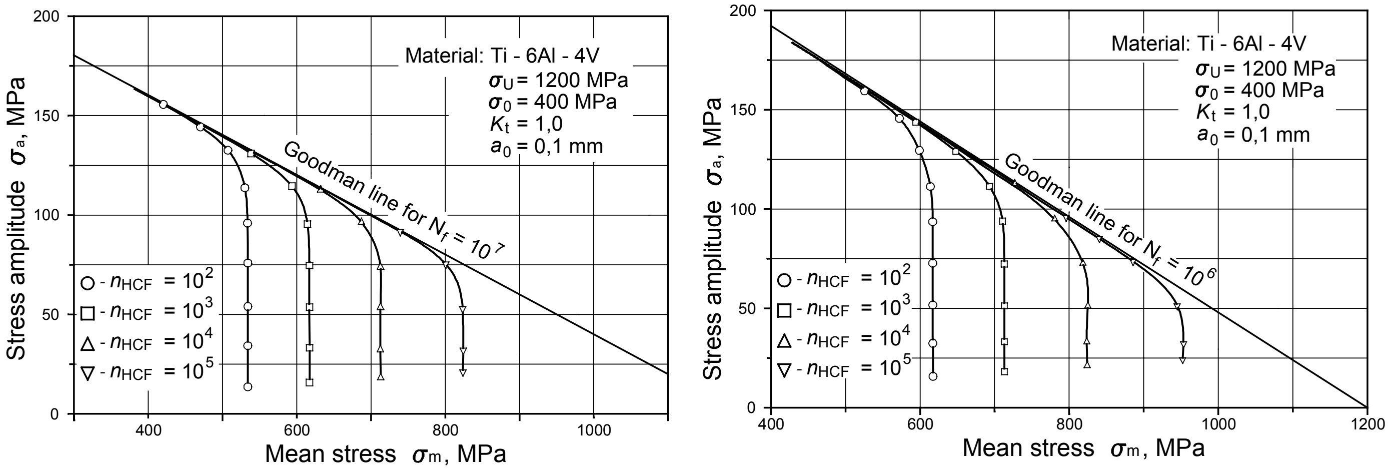

Figure 3: Fatigue limits at fatigue life *N* =10^7 and *N* =10^6 cycles in Haigh diagram for a combined HCF/LCF loading of a titanium alloy Ti-6Al-4V.

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