The Comparison of the Reliability Margin Measure for the Different Concepts in the Slope Analysis

F. Dodigović, K. Ivandić, D. Štuhec, S. Strelec

***Abstract*—**The general difference analysis between the former and new design concepts in geotechnical engineering is carried out. The application of new regulations results in the need for real adaptation of the computation principles of limit states, i.e. by providing a uniform way of analyzing engineering tasks. Generally, it is not possible to unambiguously match the limit state verification procedure with those in the construction engineering. The reasons are the inability to fully consistency of the common probabilistic basis of the analysis. There are the fundamental effects of the material properties on the value of actions and the influence of the actions on resistance. Consequently, it is not possible to apply separate factorization with partial coefficients, as in construction engineering. For the slope stability analysis, design procedures problems in the light of the use of limit states in relation to the concept of allowable stresses is detailed in. The quantifications of the safety margins in the slope stability analysis for both approaches is done. When analyzing the stability of the slope, by the strict application of the adopted forms from the new regulations, and for significant external temporary and/or seismic actions, the equivalent margin of safety is increased. The consequence is the emergence of more conservative solutions.

***Keywords*—**Allowable pressure, Eurocode 7, Limit states, Slope stability

I. Introduction

FOLLOWING the adoption of the new technical regulations - the Eurocode [1] system at the general level, there was a need for the implementation of new forms of design in the field of geotechnical engineering. In general, the level of codification of everyday engineering practice in the Republic of Croatia (RH) was significantly lower in relation to the construction engineering area. There are relatively minor required changes. It can be say that the same principles with modified rules have to be applied. In geotechnical engineering, the application of new forms of uniform analysis is a significant change, i.e. there is a modification of the basic principles of the design. The reason for adopting a new concept of design – limit states, is the need to unify the analysis in terms of consistency in the application of design coefficients, i.e. by securing the given margin of safety, regardless of the load combinations, material characteristics and geometry of the specific task. Thus, the design task is viewed as a unique entity with the following components: structure, foundation, soil. The consequence of adopting the analysis towards the limit states is the use of so-called partial factors. These factors are generally applied to the actions - unfavorable, favorable, permanent and variable, and to resistance, as a material factor, or to a characteristic resistance. Values of partial factors are generally given over [2]. Each country has a duty to define its set of partial factors in National Annex, as in the RH [3]. It is shown that the quantity of different types of factors is well exceeded by 100, which is considerably more than the old design norms [4], same as in other European countries [5], [6].

Specifically, the current analysis of slope stability proofing procedures was based on the principles of permissible, or allowable stresses. In the new concept, the so-called limit state design is adopted. Three main design approaches (DA1, DA2 and DA3) are defined according to Eurocode 7 [2].

The paper deals with the mutual comparison of the concepts of permissible stresses and limit states. A simple slope stability analysis, with the design procedure *Assumed failure surface*, were performed. This method is most commonly used in everyday engineering practice. In most cases it is carried out using numerical procedures through analysis method of slices, or wedge-type method in rock or rock-like materials [7]. This is a simplified method which do not satisfy stress equilibrium and strain compatibility conditions simultaneously, but give priority to the failure, usually Mohr-Coulomb, criterion.

II. Concept of the Permissible Stress Design

Generally permissible stress design compares actual and permissible (allowable) stresses. There are two fundamental steps. The first is to determine actual, working stresses, which is gained by the appropriate equilibrium conditions. The second step is to find the value of the allowable stress. Now actual stress has to be less then maximum allowed stress to be applied on a particular structural element. The allowable stresses are defined by building codes, and for example for concrete is a fraction of their yield stress. In the case of slope stability design, the allowable stress is the maximum shear stress that can be applied on a particular failure surface. The values are defined as the fraction of the ultimate stress.

Actual safety margin (factor of safety – FS) is explicit expressed through the quotient of the ultimate stress and actual stress needed for equilibrium:

$$FS = \tau_u / \tau \tag{1}$$

The value of safety margin FS (factor of safety) has to be higher than demand value according to building code FS_p – prescribed factor of safety:

$$FS \geq FS_p \tag{2}$$

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Now it can be defined the ODF – Over Design factor – as a quotient of actual and prescribed safety margins:

$$ODF = FS / FS_p. \quad (3)$$

The optimal design is defined when ODF = 1. This means that the prescribed level of safety is reached, but not more that, which would lead to too conservative solutions, and the opposite less than unit is not allowed.

In Figure 1. it can be seen the schematic view of the forces and stresses for the plain (linear) sliding surface for the undrained soil condition and prescribed safety margin (MSS) for static loading case.

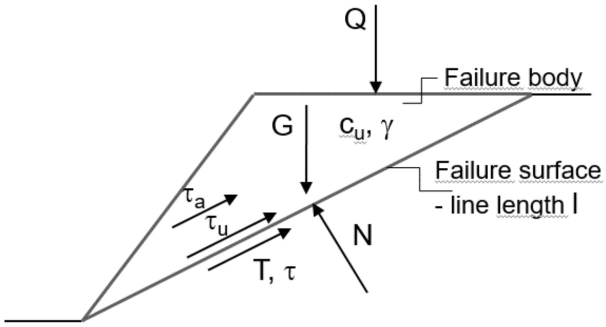

Fig. 1a View of the sliding (failure) body and plain sliding surface and corresponding action and stresses

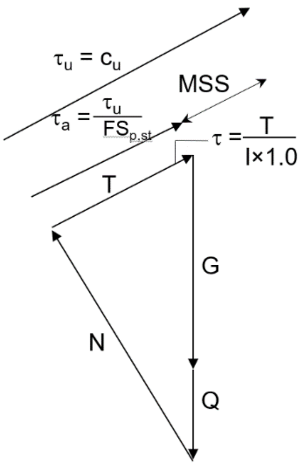

Fig 1b Prescribed margin of safety (MMS) - all for allowable stresses design, statical case and undrained soil condition for static load case

Where are:

G, Q, N, T – G – own weight - permanent action, Q – temporary action, N – normal, and T – tangential forces needed for equilibrium,

c_u, γ - undrained shear strength and volume weight respectively,

$\tau_u = c_u$ – ultimate shear stress at the failure surface, determined based on shear stress parameter - in this case undrained shear strength,

τ – shear stress from the equilibrium condition at the failure surface,

$FS_{p,st}$ –factor of safety from the building code for static case,

MSS – prescribed margin of safety from the building code.

Generally, the verification can be found in the form:

$$\tau \leq \tau_a = \tau_u / FS_p \quad (4)$$

The actual shear stress τ must be lower or equal compared with the allowable τ_a.

Demand (prescribed) factor of safety depends on the type of the unfavorable action – static or seismic, and durability of the construction – slope.

For example in RH from the [4], for the static cases and permanent structure $FS_{p,st}$ = 1.5 – 1.8, while for seismic action $FS_{p,se}$ = 1.0 – 1.1.

The form of the analysis verification is conducted through the unique application of the factor of safety. It acts at the position of the resultant actual shear stresses caused by the external actions, or at the position of the ultimate shear stress.

Generally, analyzes were performed in such a way that the actual, working factor of safety was defined as a global size, which was calculated as a result of all adverse effects and corresponding resistances. The analysis, or their final result, did not depend on the time and position of the safety factor application.

In this way, the default and realized safety margins are explicitly expressed, visible and mutually comparable, regardless of the project situation. Selected safety margins for different types of action are schematically shown in Figure 2. There are pair of values [τ, (τ_u or/and τ_a)] which defines the position for the acchieved safety margin evaluation. For some defined value of allowable stress τ_a, there is a shear stress τ needed from equilibrium conditions. If they are equal, or τ is a little bit lower than τ_a it presents a optimum design solution. If the τ values are significant lower than τ_a this is too conservative solutions.

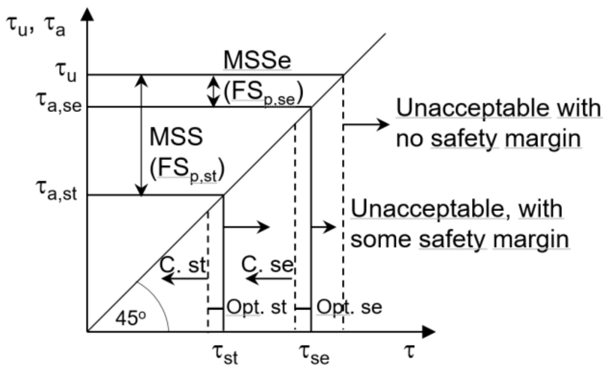

Fig. 2 Schematical view of the safety margin for statical and seismic loading cases

Where are:

MSS ($FS_{p,st}$), MSSe ($FS_{p,se}$) – prescribed margin of safety from the building code for static and seismic case respectively,

C. st, C. se – areas for too conservative for static and seismic solutions,

Opt. st, Opt. se – narrow areas of the optimal desgin for static and seismic solutions.

Allowable values of shear stresses for static and seismic case are not equal: $\tau_{a,st} \neq \tau_{a,se}$, precisely $\tau_{a,st} < \tau_{a,se}$. In this way, different safety margins are defined depending on the specific design situation.

III. Limit State Analysis

Generally, the new concept of limit states analyzes the unwanted state of the resultant forces. It leads to unacceptable limits of behavior. It is necessary to ensure that the occurrence of such a described, marginal state is unrealistic, respectively that the agreed margin of safety is pre-prescribed and accepted [8], [9].

In contrast to the concept of allowable stresses, where the actual stress condition has been observed, the concept of the limit states shows the possible resultant state of action and resistance. Such a state of the resultant force of action and resistance represents a potentially, or limit state. It follows that it is not realistic, but exists only as the possibility, which could never be realized.

TABLE I
PARTIAL FACTORS

DA	**DA1_1**	**DA1_2**	**DA2**	**DA3**
Co.	<u>A1</u>+M1+R1	<u>A1</u>+<u>M2</u>+R1	<u>A1</u>+M1+<u>R2</u>	<u>A1(A2)</u>+<u>M2</u>+R1

The main request for limit states (STR and GEO) in geotechnical engineering [1], [2] is expressed:

$$E_d \le R_d \quad (5)$$

Where E_d – design effects of action, R_d – design resistance.

It can be written for all partial factors in general form:

$$\gamma_E E\{\gamma_F F_{rep}; X_k/\gamma_M; a_d\} = E_d \le R_d = R\{\gamma_F F_{rep}; X_k/\gamma_M; a_d\}/\gamma_R \quad (6)$$

Partial factors may be applied either to the actions (F_{rep}) (F_k), ground properties (X), their effects (E) and resistance (R).

Where are:

E – generally effects of action,

F_{rep} (F_k) – representative (characteristic) value of action

R – resistance generally,

X_k – characteristic material property,

a_d – design value of geometrical data,

γ_E – partial factor for the effect of an action,

γ_F – partial factor for action,

γ_M – partial factor for a soil parameter,

γ_R – partial factor for resistance.

As it was for the allowable stress concepts, there is a change in safety margins between static and seismic case of loading. Partial factors for soil parameters remain same for both loading cases, but there is a unit values for partial factor for seismic actions. This approach is valid for the cases where there the partial factors for static cases are not unit values, or there are not such actions with no unit values. Consequently, there are just formal different safety margin between static and seismic load of cases, meaning that the seismic load becomes relevant in mostly cases.

Within geotechnical engineering, according to new code, there are three different Design approaches (DA): DA1, DA2 and DA3. The proposed and adopted standard Eurocode 7 (Eurocode7 EC7) by [2] does not use the explicitly expressed value of global safety factor. The impact of each element of the analysis is determined by multiplying by the appropriate partial factors. Certain combinations of factorization of permanent and variable unfavorable and favorable actions, material characteristics and resistances (A - Action - action, M - Material - Material Properties, R - Resistance - Resistance) have been defined.

In Table 1. a schematic view of the combination of partial factors different (larger - underlined) of the unit value (≠ 1) is shown.

where are:

A1 - structural actions,

A2 - geotechnical actions.

In RH for the slope stability limit state analysis regarding ultimate state GEO, DA3 is valid. Neverthenless this approach is valid in most countries for verification of the slope stability. It shows that there is some inconsistency with the use of DA2 in mentioned type of analysis. The relevant combination for the DA3 for slope and overall stability is <u>A2</u> + <u>M2</u> + R1. Partial factors in this case in RH according to National annex [3] for actions are: $\gamma_{G;sup} = \gamma_{G;inf} = \gamma_{Q;fav} = 1.0$ and $\gamma_{Q;unf} = 1.3$, and material properties: $\gamma_{\varphi'} = 1.25$, $\gamma_{c'} = 1.25$, $\gamma_{cu} = 1.4$, $\gamma_{qu} = 1.4$, $\gamma_{\gamma} = 1.0$.

Since the set of values under A2 are mostly unit (except for variable unfavorable action), the dominant position of the partial factor is on the properties of the material. For that reason, DA3 is also called *Material approach*.

The adopted approach is partly due to the fact that it is not possible to explicitly separate the unfavorable and favorable actions on the given sliding surface as well as the inability to define isolated zones of resistance and external activity itself. Reducing the characteristic values of soil strength parameters simultaneously acts to increase actions and reduce resistance, regardless of the specific position of a particular component of the design analysis.

In Figure 3. the schematic view of the forces and stresses for the flat sliding surface for the undrained soil condition and fragmented safety margin (MSS_G, MSS_Q, MSS_{cu},) for static loading case is shown.

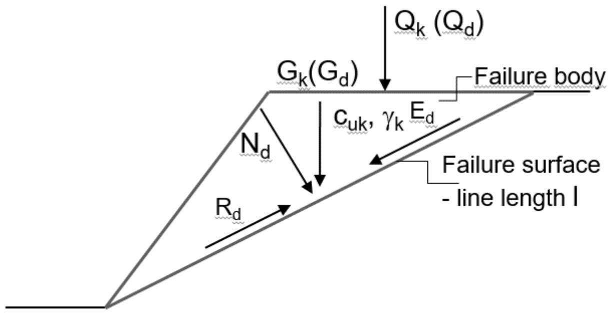

Fig. 3a View of the sliding (failure) body and plain sliding surface and corresponding resultant action and resistance.

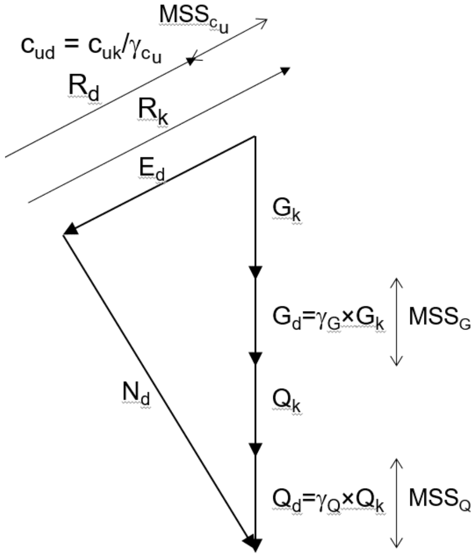

Fig. 3b Fragmentes safety margins (MSSG, MSSQ, MSScu) - all for limit state design, statical case and undrained soil condition

In a new concept tha margins of safety are fragmented. There is no unique position of the safety margin, as it was in concept of allowable stresses. There are two general positions for the partial factor acting: at the action and at the resistance. In the Figure 4, there is a schematic view of the relation between the design action and design resistance.

Optimum design relation is when E_d is equal R_d which means that the demanded margin of safety is reached. The case where $E_d > R_d$ is unacceptable. There is small area where $R_d > E_d$ which can be defined as the position of the optimum dessign. Above that, there is location of too conservative solutions.

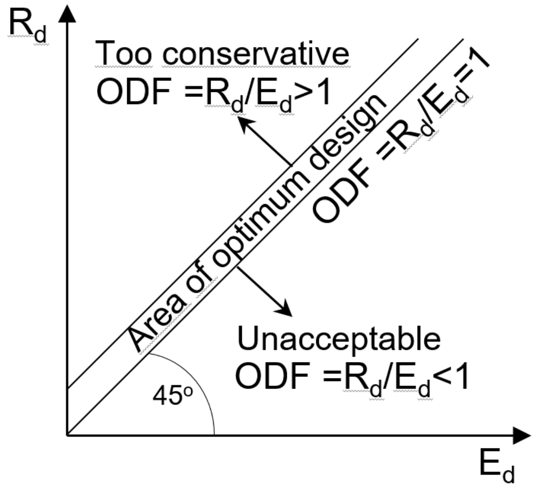

Fig. 4 Schematic view of the resultant safety margins for the limit state design concept

In Figure 5. there is a schematic view of the position of the safety margins related with the actions and resistance for static (st) and seismic (se) case of loading.

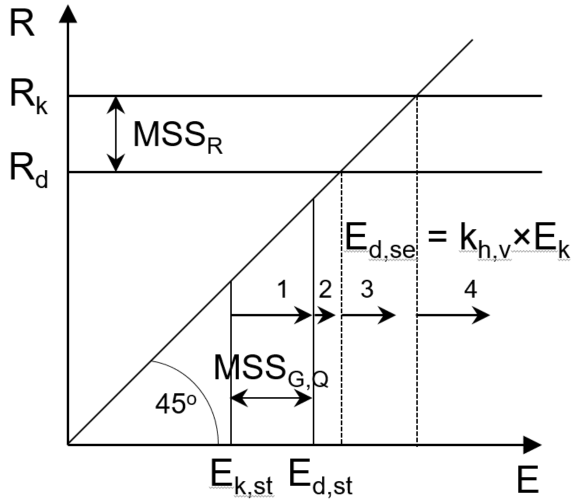

Fig. 5 Fragmented safety margins for the limit state design concept for static i seismic cases

On the side of the actions, for static load case, there is a prescribed safety margin $MSS_{G,Q}$, influenced generally by the partial factors γ_E, γ_F and γ_M which is a significantly different situation compared with structural engineering. Now material factor γ_M affects to the value of unfavorable and favorable actions. On the side of the resistance, for static case also, is safety margin MSS_R, influenced by the partial factors γ_F, γ_M and γ_R. Now the resistance is affected with partial factor for action γ_F, which is also not relevant in structural engineering.

For the seismic loading case, the computation must be performed by characteristic values of the actions E_k. Seismic force can be expressed as the fraction of the characteristic value of action $E_{d,se} = k_{h,v}\times E_k$. The sesismic force can be placed in three different areas:

1 $E_{d,se} < E_{d,st}$
2 $E_{d,st} < E_{d,se} < R_d$ - for the optimal designed statical solution
3 $R_d < E_{d,se} < R_k$
4 $E_{d,se} > R_k$

In the area 1 statical case is relevant. For the area 2 seismic force is greater than for the statical case, but, in the area of optimal desig smaller than R_d. The next one, area 3 seismic action is greater than design resistance R_d and the form of verification is not satisfied. Still there is a certain margin of safety in regard to R_k. The area 4 shows that there is no safety margin, compared with characteristic resistance R_k.

For the slope stability analysis according DA3 there is a situtaion which shows the relevance between compared static and seismic cases. In Figure 6. schematic diagram of possible solutions for slope stability analysis with DA3 for static and seismic case of loading without variable unfavourable actions is shown.

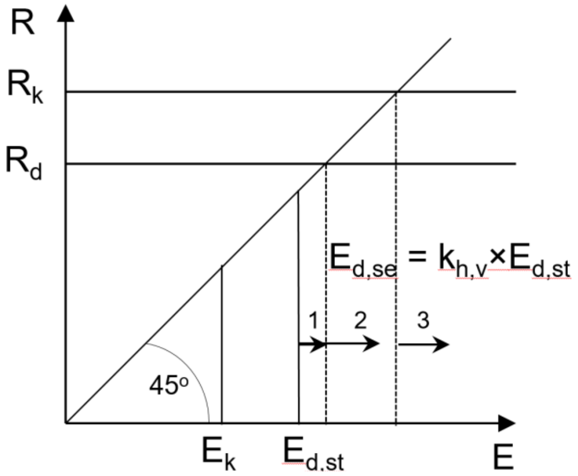

Fig. 6 Mutual relationship for design actions and design resistance

For the static case, the characteristic value of the permanent unfavorable action E_k is increased due to a decrease in the value of the soil strength strength parameters and is converted to the $E_{d,st}$ value. There is no increase in external action for the seismic load (unit values of partial factors). The design resistance is calculated with the design values of soil shear strength parameters. This means that the additional design seismic action $E_{d,se}$ counts as an increase from the value of the design static action $E_{d,st}$.

Consequently four zones of the fragmented safety margin are reduced to three.

1 $E_{d,st} < E_{d,se} < R_d$ - the design seismic action is in the narrow zone where necessarily higher than the design action for static case, but still within the prescribed margin of safety.
2 $R_d < E_{d,se} < R_k$ - still with some degree of safety related with R_k
3 $E_{d,se} > R_k$ – no margin safety

IV. Comparison of slope stability analysis

Figure 7 shows the geotechnical model of the design situation of the natural slope inclined α'. It is necessary to determine the inclination β of a permanently stable slope of homogeneous material for plain sliding surface at angle α for the following design conditions: h = 6 m, $\alpha = 30^o$, $\beta = 60^o$, $\gamma = 20$ kN/m^3, $c_u = 27$ kN/m^2, $k_h = 0.25$, $k_v = 0$ – seismic coefficients for pseudo-static force in horizontal and vertical direction respectively according allowable stress design - $FS_{p,st} = 1.5$ and $FS_{p,se} = 1.05$, and according DA3 - $\gamma_{cu} = 1.4$, $\gamma_G = 1.0$.

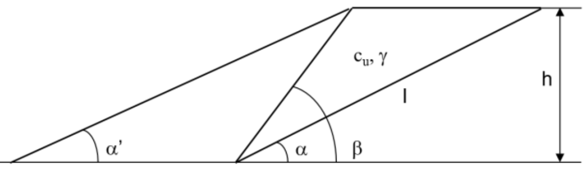

Fig. 7 Geotechnical design model for analysis of the permanent slope inclination

G = 414 kN/m
T_{st} = 414×sin30 = 414×0.5 = 207 kN/m
S = 0.25×414 = 104 kN/m'
T_{se} = 104×cos30 = 90 kN/m'

For the concept of the allowable stresses static ($FS_{p,st}$ = 1.5)
FS = $\tau_u/\tau = c_u$ / (T/l) = 27 / (207/12) = 1.56 - satisfies
and sesismic ($FS_{p,se}$ = 1.05) case
FS = 27 / [(207 + 90) /12] = 1.09 - satisfies
For the DA3
G_k = 414 kN/m'
$E_{d,st}$ = 1.0×414×sin30 = 207 kN/m'
$E_{d,se}$ = 1.0×(207 +90) = 297 kN/m'
$R_k = c_{uk}$×l = 27×12 = 324 kN/m'
$R_d = c_{uk}/\gamma_{cu}$×l = 27/1.4×12 = 231 kN/m'
Verification for static
$R_d / E_{d,st}$ = 231/207 = 1.11 - satisfies
and seismic case
$R_d / E_{d,se}$ = 231/297 = 0.77 - don't satisfies

The permanent slope inclination from the condition of $ODF_{se} = 1$ is $\beta = 48^o$. For natural slope inclination $\alpha' = 25^o$ there is increase in the excavation compared to the old procedure about 20%.

In Figure 8. It can be seen the positions of the relevant stresses and design forces for the numerical example.

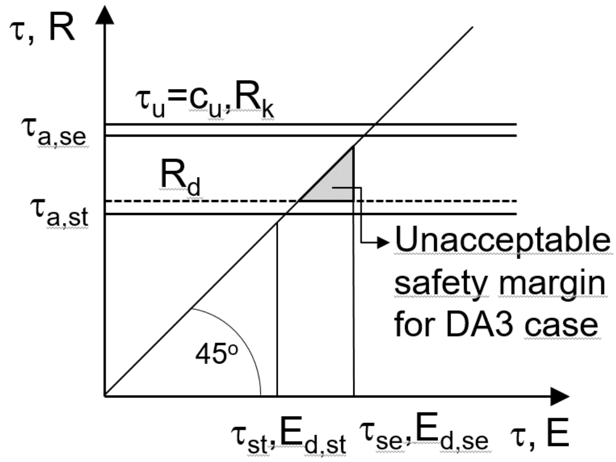

Fig 8 Stresses and design actions and resistance for slope stabilty example

At the Figure 8. the area of unsatisfactory safety margin for simple geotechnical model for plain surface and undrained condition can be seen. For the old fashion way of the analysis there are satisfactory results which ensure prescribed safety margins for both cases. For DA3 and seismic case there is a design resistance force exceeded.

V. Conclusion

In the new design approaches of slope limit analysis according to Eurocode 7, there are situations when the formal verification condition is not satisfied, although there is a certain margin of safety. The mentioned margin actual value should be compared with the equivalent margins obtained by applying the procedures of the allowable stresses. The introduction of new design patterns should have the effects like establishing a more uniform way of work, a higher degree organization and more cost-effective solutions. This means that for the same degree of safety, it would be possible to build with less material, or to dig slopes with larger inclinations.

On a theoretical basis and in a specific example, it is shown that the formal use of new design forms has provided a solution, which is more expensive up to 20% compared to the classic design approach.

Possible solution to overcome this challenge is to reduce the prescribed formal margin of safety for seismic design loads. There is a task to perform the analysis for the different design situations related to soil conditions (pore pressures, layered soil, drained and undrained situations), unfavorable actions (variable and seismic), computation models etc. The results should be calibrated in relation to previous calculation patterns and measured values for successfully executed geotechnical tasks.

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