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Bremsstrahlung emission during α -decay of ^{226}Ra

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We obtained the spectrum of probability of the bremsstrahlung emission accompanying the α -decay of ^{226}Ra ($E_\alpha=4.8$ MeV) by measuring the α - γ coincidences and using the model presented in our previous study on the α -decay of ^{214}Po ($E_\alpha=7.7$ MeV). We compare the experimental data with the quantum mechanical calculation and find a good agreement between theory and experiment. We discuss the differences between the photon spectra connected with the α -decay of the ^{226}Ra and ^{214}Po nuclei. For the two mentioned nuclei we analyze the bremsstrahlung emission contributions from the tunneling and external regions of the nucleus barrier into the total spectrum, and we find the destructive interference between these contributions. We also find that the emission of photons during tunneling of the α -particle gives an important contribution to the bremsstrahlung spectrum in the whole E_γ energy range of the studied ^{226}Ra nucleus.

Keywords: Alpha-decay; photon bremsstrahlung; spectrum for ^{226}Ra ; sub-barrier and external contributions; interference; tunneling.

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1. Introduction

In recent years many experimental and theoretical efforts^{1–12} have been made to investigate on the nature of the bremsstrahlung emission in the α -decay of heavy nuclei, because the behavior of the energy spectrum of photons is strongly related to the dynamics of the α -decay and alpha-nucleus particle potential. In some case the

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energy spectrum of bremsstrahlung shows some slight oscillations¹, in other case authors observed a minimum^{4,5}, in some experiment authors² have not observed evidence of any structure. Therefore the main problem to investigate the tunneling of the α -particle through the Coulomb barrier of nucleus is to enlarge the area of the study on other nuclei and to compare the characteristics of the γ -spectra. In this paper we present the results of the last experiment on the study of the bremsstrahlung emission in α -decay of ^{226}Ra . We also analyze and discuss the comparison between the experimental and theoretical results of the photon emission related to the ^{226}Ra and ^{214}Po nuclei. In Sec. 2 we shortly describe the characteristics of the experiment and the results of the bremsstrahlung emission probability. In Sec. 3 we present the basis of the model, the analysis of our calculation about the various contributions of the bremsstrahlung emission, and we discuss the results of the γ -emission for the α -decay of ^{226}Ra , in comparison with the ones previously obtained for ^{214}Po . Sec. 4 is devoted to our conclusion.

2. Experiment and results

The experimental set up was the same as described in our previous paper¹. The source of ^{226}Ra with a activity of about 10^4 α -particles/s was used. Along the decay chain of this nucleus the α -particles were recorded. The diameter of the radioactive spot on the source surface was about 8 mm and α -particles were detected by a silicon surface-barrier detector with energy resolution of about 20 keV at the α -particle energy of 4.8 MeV. The α -detector was 200 mm² in area and was placed at the distance of about 1 cm from the source. The time resolution of the α - γ coincidence technique was $\tau = 10$ ns.

The γ -rays were detected by the NaI(Tl)-detector with diameter of 3 cm and thickness of 3 cm, and the distance between source and γ -detector has been about 1.6 cm. The angles between the two detectors and the normal axis to the surface of the source were 45°. So the total angle between the α and γ detectors was about 90°. This angle value was chosen to increase the yield of E1 dipole bremsstrahlung photons and to reduce the influence of γ -rays from E2-quadrupole transitions of excited states of daughter nuclei.

The absolute values of the NaI(Tl)-detector efficiency was determined by measuring the intensities of lines of the standard γ -sources ^{241}Am ($E_\gamma = 59.6$ keV), ^{57}Co (122 and 136 keV), ^{226}Ra (186, 295, 532 and 609.4 keV), ^{137}Cs (662 keV) and ^{60}Co (1.17 and 1.33 MeV), by replacing each of them in the position of the ^{226}Ra -source of α -particles. The measurements of bremsstrahlung photons in coincidence with α -particles for five α -groups¹³ from decays of ^{226}Ra ($E_\alpha = 4.784$ MeV), ^{210}Po (5.304 MeV), ^{222}Rn (5.490 MeV), ^{218}Po (6.002 MeV) and ^{214}Po (7.687 MeV) were performed during about 1500 hours and the total number of $4.4 \cdot 10^6$ events was registered by the α - γ coincidence system. The analysis of these events in the (E_γ vs E_α)-plane located in the region of the total energy conservation line $E_\gamma + E_\alpha = \text{constant}$, taking into account the detectors energy resolutions (20 keV for α -particles

and 32 keV for photons), gives us the possibility to determine the yield of photons at energies up to 803 keV (1-st excited state of the daughter ^{206}Pb nucleus) for the α -decay of ^{210}Po . The range of $(E_\gamma \text{ vs } E_\alpha)$ -coincidences was collected for one of the experimental run with the measurement time of about 150 hours.

For other details on the recorded events and analysis, see our previous paper ¹ and references therein.

To take into account the defined angular dimensions of detectors we use the same procedure to obtain the angular averaged probability of the photon emission dP/dE_γ as described in Refs. 4, 5, where the $\alpha - \gamma$ angular correlation function $W(\theta)$, for the case of E1-dipole photon emission and point-like $\alpha - \gamma$ source is

$$W(\theta) = 1 + A_2 \cdot Q_2 \cdot P_2(\cos\theta) \quad (1)$$

where $A_2 = -1$ for the dipole E1 transitions, $P_2(\cos\theta)$ is the second order polynome of Legendre, Q_2 is the total geometrical attenuation coefficient of the second order: $Q_2 = Q_2^\alpha \cdot Q_2^\gamma$, $Q_2^{\alpha,\gamma}$ being the geometrical attenuation coefficients for α -particle and photon detectors.

The exact calculations in the framework of the density matrix theory^{14,15} give us the real experimental geometry values of the attenuation coefficients: for the α -particle detector it is $Q_2^\alpha = 0.90$, and for the photon detector the value of Q_2^γ -coefficient is varied from $Q_2^\gamma = 0.66$ for $E_\gamma = 100$ keV up to $Q_2^\gamma = 0.76$ for $E_\gamma = 800$ keV.

The probability of the photon emission at angle θ can be written as^{3,6}

$$\frac{d^2 P}{dE_\gamma d\Omega_\gamma} = N_{\alpha-\gamma}(\theta, E_\gamma) / (\Delta T_{meas} \cdot n_\alpha \varepsilon_\gamma(E_\gamma) \cdot W(\theta)) \quad (2)$$

where $N_{\alpha-\gamma}(\theta, E_\gamma)$ is the total number of $\alpha - \gamma$ coincidences during the measurement time ΔT_{meas} , in the intervals of photon energies $E_\gamma \pm \Delta E_\gamma / 2$ and angles $\Omega_\gamma \pm \Delta \Omega_\gamma / 2$, n_α is the intensity of particles in the α -detector and $\varepsilon_\gamma(E_\gamma)$ is the absolute efficiency of the γ -detector. Therefore, the total probability of the bremsstrahlung emission is

$$dP/dE_\gamma = 4\pi \cdot d^2 P / dE_\gamma d\Omega_\gamma. \quad (3)$$

The check on the experimental data has been made by measuring the coincidences between the α -particles emitted to the first excited level of the daughter nuclei ^{222}Rn , ^{218}Po and ^{210}Pb , corresponding to γ -rays with energies 186, 510 and 800 keV¹³. The angular correlation function $W(\theta)$, in the case of γ -ray E2-transitions, can be presented as^{14,15}

$$W(\theta) = 1 + A_2 \cdot Q_2 \cdot P_2(\cos\theta) + A_4 \cdot Q_4 \cdot P_4(\cos\theta), \quad (4)$$

where $A_2 = 5/7$, $A_4 = -12/7$, $Q_{2,4} = Q_{2,4}^\alpha \cdot Q_{2,4}^\gamma$, $Q_4^\alpha = 0.68$, $Q_4^\gamma = 0.16$ for $E_\gamma = 100$ keV and $Q_4^\gamma = 0.37$ for $E_\gamma = 800$ keV.

The bremsstrahlung spectra have been averaged over a photon energy interval of 25 keV. The measured values of the photon emission probability dP/dE_γ due to the bremsstrahlung process accompanying the α -decay of ^{226}Ra are shown by solid squares in Fig. 1.

In the following Sec.3 we calculate the bremsstrahlung spectrum accompanying the α -decay of ^{226}Ra and present, for a comparison, the above results for ^{226}Ra (where is $E_\alpha=4.8$ MeV) with the ones presented in our previous paper¹ for ^{214}Po (where is $E_\alpha=7.7$ MeV). In the theoretical description of the bremsstrahlung emission during the α -decay there are some problems: one is the choice of the realistic wave function of the α -particle inside the nuclear potential¹⁶. For example, the shape of the nuclear potential (the values of the nuclear radius R_n and the deepness V_n for a rectangular potential) influences the slope of the wave function near the nuclear surface and therefore the conditions of tunneling through the Coulomb barrier. Other problems can be connected with the influence of the nuclear surface deformation and electron screening of the Coulomb barrier.

3. Model, calculation and discussion

3.1. Calculation method

We define the bremsstrahlung probability during the α -decay of a nucleus in terms of the transition matrix elements for the compound quantum system (α -particle and daughter nucleus) from its state before photon emission (we name such a state as the *initial i-state*) into its state after the photon emission (we name such a state as the *final f-state*). If it is possible to separate total wave function of α -particle (before and after photon emission) into radial and spherical symmetric components (as in the approximation of the spherically symmetric α -decay), then one can find the expression for the total bremsstrahlung probability with the separation on the radial and angular components explicitly by an analytical way. Here, the radius defines the position of the particle with reduced mass relatively to the center of mass. The angular components contain all the detailed information about the directions of this particle motion (with taking into account its tunneling) before and after the photon emission and on the direction of the photon emission.

According to Ref. 17 we define the bremsstrahlung probability as

$$\frac{dP(w, \vartheta)}{dE_\gamma} = N_0 k_f w |p(w, \vartheta)|^2, \quad (5)$$

where

$$N_0 = \frac{Z_{\text{eff}}^2 e^2}{(2\pi)^4 m}, \quad (6)$$

$$p(w, \vartheta) = -\sqrt{\frac{1}{3}} \cdot \sum_{l=0}^{+\infty} i^l (-1)^l (2l+1) P_l(\cos \vartheta) \cdot \sum_{\mu=-1,1} h_\mu J_{m_f}(l, w), \quad (7)$$

$$h_\pm = \mp \frac{1}{\sqrt{2}} (1 \pm i), \quad k_{i,f} = \sqrt{2mE_{i,f}}, \quad w = E_i - E_f. \quad (8)$$

In (7) $J_{m_f}(l, w)$ is the radial integral independent on the angle ϑ :

$$J_{m_f}(l, w) = \int_0^{+\infty} r^2 R_f^*(r, E_f) \frac{\partial R_i(r, E_i)}{\partial r} j_l(kr) dr. \quad (9)$$

In determination of the wave functions of the initial and final states we use the selection rules for the quantum numbers l and m :

$$\begin{aligned} i\text{-state before emission: } l_i = 0, m_i = 0; \\ f\text{-state after emission: } l_f = 1, m_f = -\mu = \pm 1. \end{aligned} \quad (10)$$

In (9) $j_l(kr)$ is the spherical Bessel function of the order l , $R_i(r)$ and $R_f(r)$ are the radial components of the total wave functions $\psi_i(\mathbf{r})$ and $\psi_f(\mathbf{r})$ of the system in the initial i - and final f -state, respectively. For other notations and details see Ref. ¹, in accordance with Refs. 17, 18.

To describe the interaction between the α -particle and daughter nucleus (A, Z) we use the following potential ¹⁹:

$$V(r, \theta, l, Q) = v_C(r, \theta) + v_N(r, \theta, Q) + v_l(r) \quad (11)$$

where Coulomb $v_C(r, \theta)$, nuclear $v_N(r, \theta, Q)$ and centrifugal $v_l(r)$ components have such form:

$$v_C(r, \theta) = \begin{cases} \frac{2Ze^2}{r} \left(1 + \frac{3R^2}{5r^2} \beta_2 Y_{20}(\theta) \right), & \text{for } r \geq r_m, \\ \frac{2Ze^2}{r_m} \left\{ \frac{3}{2} - \frac{r^2}{2r_m^2} + \frac{3R^2}{5r_m^2} \beta_2 Y_{20}(\theta) \left(2 - \frac{r^3}{r_m^3} \right) \right\}, & \text{for } r < r_m \end{cases} \quad (12)$$

and

$$v_N(r, \theta, Q) = \frac{V(A, Z, Q)}{1 + \exp \frac{r - r_m(\theta)}{d}}, \quad v_l(r) = \frac{l(l+1)}{2mr^2}. \quad (13)$$

Here, Q is the Q -value for the α -decay, R is the radius of the daughter nucleus, $V(A, Z, Q, \theta)$ is the strength of the nuclear component; r_m is the effective radius of the nuclear component, d is the parameter of the diffuseness; $Y_{20}(\theta)$ is the spherical harmonic function of the second order, θ is the angle between the direction of the leaving α -particle and the axis of the axial symmetry of the daughter nucleus; β_2 is the parameter of the quadruple deformation of the daughter nucleus. The parameters of the Coulomb and nuclear components are defined in Refs. 1, 19.

In order to obtain the spectrum, we have to know wave functions in the initial and final states. In the spherically symmetric approximation one can rewrite the total wave functions by separating the radial and angular components:

$$\begin{aligned} \varphi_i(r, \theta, \phi) &= R_i(r) Y_{l_i m_i}(\theta, \phi) = \frac{\chi_i(r)}{r} Y_{l_i m_i}(\theta, \phi), \\ \varphi_f(r, \theta, \phi) &= R_f(r) Y_{l_f m_f}(\theta, \phi) = \frac{\chi_f(r)}{r} Y_{l_f m_f}(\theta, \phi). \end{aligned} \quad (14)$$

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We find the radial components $\chi_{i,f}(r)$ numerically on the base of the given alpha-nucleus potential. Here, we use the following boundary conditions: the i -state of the system before the photon emission is a pure decaying state, and therefore for its description we use the wave function for the α -decay; after the photon emission the state of the system is changed and it is more convenient to use the wave function as the scattering of the α -particle by the daughter nucleus for the description of the f -state. So, we impose the following boundary conditions on the radial components $\chi_{i,f}(r)$:

$$\begin{aligned} \text{initial } i\text{-state: } \chi_i(r \rightarrow +\infty) &\rightarrow G(r) + iF(r), \\ \text{final } f\text{-state: } \chi_f(r = 0) &= 0, \end{aligned} \quad (15)$$

where F and G are the Coulomb functions as used in Refs. 17, 18.

3.2. *Bremsstrahlung components from different spacial regions, and interference term*

It is interesting to estimate how much is the emission of photons from the tunneling region and the ones from the internal and external regions concerning the barrier versus the distance r . Such a question was put for the first time by J. Kasagi *et al.* in Ref. 5 in the tentative to explain the difference between the experimental data and the calculated spectrum for the ^{210}Po nucleus. Some later, a constructive analysis was proposed by N. Takigawa *et al.* in Ref. 9 and, independently E. Tkalya in Ref. 10, both giving the theoretical basis to work with the contributions of the photon emission from different regions and interference term by different approaches. As we see and shall show further, the study of the photon emission from the different spacial regions allows one not only to analyze the difference between the experimental data and the calculated spectrum for ^{210}Po , but gives a more important information about the studied process. For this reason, we shall develop a formalism of the photon emission from the different spacial regions in the framework of our model.

Let us define the bremsstrahlung probabilities of photons emitted from different spacial regions, characterized by the points R_1 and R_2 in Fig. 2. According to Ref. 18, we shall assume that the emission of photons from the internal spacial region (defined from $r = 0$ up to $r = R_1$) is very small in a comparison with the total photon emission, and therefore we can neglect it. In practical calculations of the spectra, we find the integral (9) by a reasonable approximation taking an any finite value R_{\max} into account instead of $r = +\infty$. By such reasons, we shall analyze the integral (9) only inside the spacial intervals from the point R_1 up to R_{\max} .

We separate this integral into two items:

$$J_{m_f}(l, w) = J_{m_f}^{(\text{tun})}(l, w) + J_{m_f}^{(\text{ext})}(l, w) \quad (16)$$

where

$$\begin{aligned} J_{m_f}^{(\text{tun})}(l, w) &= \int_{R_1}^{R_2} r^2 R_f^*(r, E_f) \frac{\partial R_i(r, E_i)}{\partial r} j_l(kr) dr, \\ J_{m_f}^{(\text{ext})}(l, w) &= \int_{R_2}^{R_{\max}} r^2 R_f^*(r, E_f) \frac{\partial R_i(r, E_i)}{\partial r} j_l(kr) dr \end{aligned} \quad (17)$$

where (R_1, R_2) is the tunneling region, (R_2, R_{\max}) is the external one, and E_i is the energy of the system in the initial i -state. Then, the formula (5) of the bremsstrahlung probability is transformed into the following:

$$\frac{dP(w, \vartheta)}{dE_\gamma} = \frac{dP_{\text{tun}}(w, \vartheta)}{dE_\gamma} + \frac{dP_{\text{ext}}(w, \vartheta)}{dE_\gamma} + \frac{dP_{\text{interference}}(w, \vartheta)}{dE_\gamma}. \quad (18)$$

where the three components P_{tun} , P_{ext} and $P_{\text{interference}}$ are defined through the finite integrals $J_{m_f}^{(\text{tun})}$ and $J_{m_f}^{(\text{ext})}$.

In particular, at $l = 0$ (and assuming that the radial wave function $R_f(r)$ in the final f -state does not depend on quantum number m_f at $l_f = 1$ like as Coulomb functions) one can obtain:

$$\frac{dP_{\text{tun}}(w, \vartheta)}{dE_\gamma} = \frac{2}{3} N_0 k_f w \left| J^{(\text{tun})}(0, w) \right|^2, \quad (19)$$

$$\frac{dP_{\text{ext}}(w, \vartheta)}{dE_\gamma} = \frac{2}{3} N_0 k_f w \left| J^{(\text{ext})}(0, w) \right|^2, \quad (20)$$

$$\frac{dP_{\text{interference}}(w, \vartheta)}{dE_\gamma} = \frac{4}{3} N_0 k_f w \Re \left(J^{(\text{tun})}(0, w) \cdot J^{(\text{ext}),*}(0, w) \right). \quad (21)$$

Therefore, we affirm the following:

- (i) the total bremsstrahlung spectrum is not simply the summation of the direct (pure) probabilities from tunneling and external regions P_{tun} and P_{ext} , but it also includes the interference term $P_{\text{interference}}$ (see also Refs. 9, 10);
- (ii) the probabilities P_{tun} and P_{ext} from tunneling and external regions are only positive, the interference term $P_{\text{interference}}$ can be positive or negative.

3.3. Bremsstrahlung spectrum for ^{226}Ra

We have applied the above described method to calculate the bremsstrahlung spectrum emitted during the α -decay of the ^{226}Ra nucleus. The results are presented by the full line in Fig. 1 together with the experimental data (solid squares). In calculation we have used the approximation of the spherically symmetric α -decay $l = 0$ only in determination of $p(w, \vartheta)$ in (7), and the angle $\vartheta = 90^\circ$ between the directions of the α -particle motion (with possible tunneling) and the photon emission. The energy E_i of the α -particle in the initial i -state is 4.8 MeV, according to Ref. 21.

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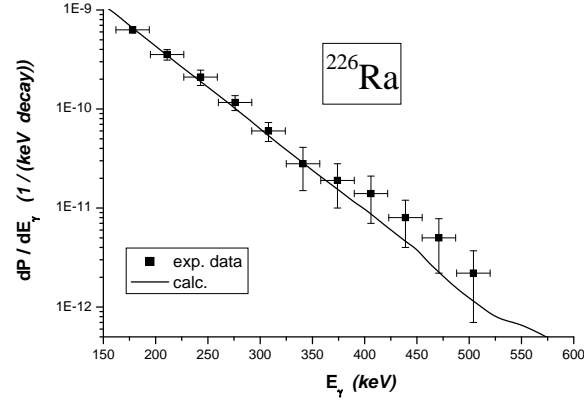


Fig. 1. The photon emission probability dP/dE_γ accompanying the α -decay of ^{226}Ra . Full square are the experimental data, full line is the calculation for this nucleus.

The approximation of the spherically symmetric α -decay is used in the present paper for the following reasons (see also the analysis discussed in Ref. 12). At the range of the nucleus surface and maximum value of the Coulomb barrier, the potential form changes more rapidly than for larger distances outside the nucleus surface and also inside the nucleus (see Fig. 2).

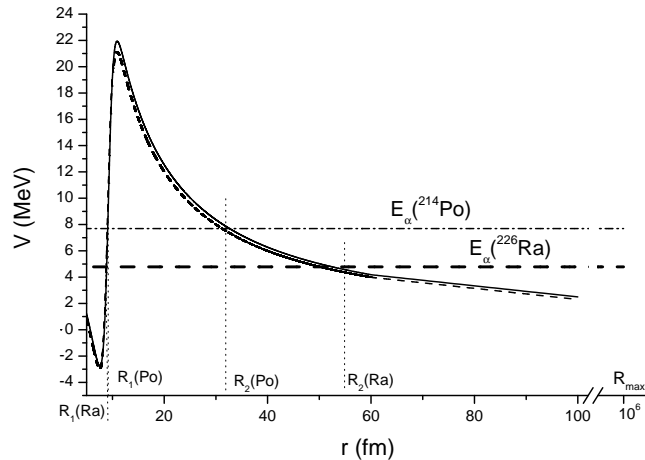


Fig. 2. Alpha-nucleus potential V versus r distance from the center of the decaying nucleus: ^{226}Ra (full line), ^{214}Po (dashed line). E_α represents the energy of the α -decay for the ^{226}Ra and ^{214}Po nuclei, R_1 and R_2 are the points delimiting the tunneling region; R_{max} is the maximum value of r considered in calculation of the radial integral (17) instead of $r=+\infty$.

Therefore at larger distances from the nucleus (and also inside nucleus), where the changing of the potential is much slower than in the surface region, the emission photon energy and influence of the deformation parameter of the nucleus both are smaller than near the surface region. Considering that the external re-

gion (R_2, R_{max}) included between several fm and more large distances (much \AA and further) is very wide, the photon emission probability from such wide space is bigger than the one from the narrow range of the nuclear surface region (R_1, R_2). Therefore, the description of the bremsstrahlung spectrum of Fig. 1 by the spherical-nucleus approximation is rather good for lower photon energies (near about 100–300 keV where the γ -emission probability dP/dE_γ is high). Of course, for photon energies larger than 400 keV it is useful to take into account the deformation parameter of the ^{226}Ra nucleus ($\beta_2=0.151$)²⁰. Such an improvement of the theory that takes into account the deformation parameter of the nucleus will be made in the next future because the upgrading of the model overcomes the aim of the present paper.

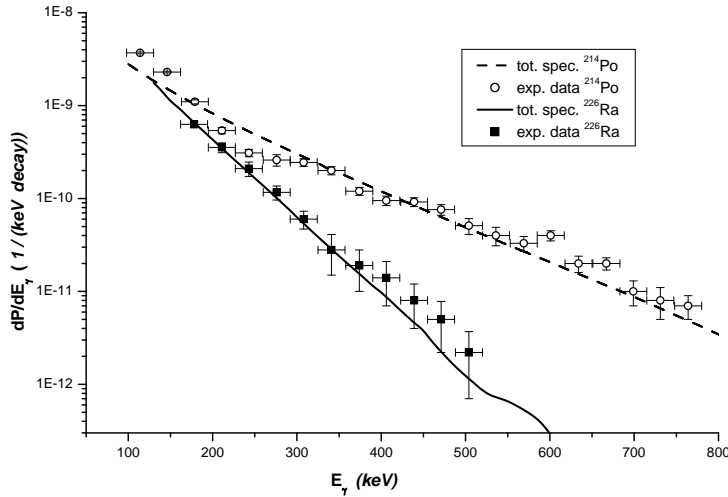


Fig. 3. Calculation (full line) of the photon emission probability dP/dE_γ accompanying the α -decay of ^{226}Ra ($E_\alpha = 4.8$ MeV), and experimental data (full squares) dP/dE_γ for ^{226}Ra already presented in Fig. 1. For a comparison, we also include the experimental data (open circles) and calculation (dashed line) obtained for the α -decay of ^{214}Po ($E_\alpha = 7.7$ MeV).

In Fig. 3 we report, for a comparison, the presented results for ^{226}Ra together with the ones found for the ^{214}Po nucleus¹. As Fig. 3 shows, both experimental and theoretical results of the photon emission probability obtained for the α -decay of ^{226}Ra are clearly lower than the ones obtained for ^{214}Po . The difference between the two sets of data can be attributed to the different structure of the two nuclei, which affects the motion of the α -particle inside the barrier. The ratio between the two sets of data of the photon emission probability dP/dE_γ is strongly characterized by the different α -decay energy for ^{214}Po ($E_\alpha = 7.7$ MeV) and ^{226}Ra ($E_\alpha = 4.8$ MeV) concerning the shapes of the alpha-nucleus barriers for these nuclei. In the α -decay of ^{214}Po , the α -particle with energy of 7.7 MeV passes under the barrier in the upper part where the potential form changes more strongly emitting photons of high energies, while in the case of ^{226}Ra the α -particle with energy of 4.8 MeV

passes under the barrier in the lower part where the potential changes partially more slowly emitting photons of lower energies.

To explain the difference between the slopes of the bremsstrahlung spectra for the two considered nuclei, we formulate the following consideration: the slope of the bremsstrahlung spectrum is defined directly by the principal difference between the emission of photons during tunneling of the α -particle and the emission of photons during its motion. To understand why two nuclei have such different slopes of the bremsstrahlung spectra, we estimate how much the emission of photons during tunneling of the α -particle differs from the emission of photons during further motion of this α -particle in the external region. In the Fig. 4 we report the tunneling and external contributions of the γ -emission accompanying the α -decay of ^{214}Po and ^{226}Ra nuclei and the interference terms which are calculated by (19)–(21). To make the analysis clearer, here we present the different contributions and interference terms for each nucleus separately. From calculation represented in this figure we establish that:

- (i) for both nuclei, the interference term has negative values in the whole energy region of the emitted photons and plays a destructive role in addition to the contributions from tunneling and external regions in the forming total spectrum;
- (ii) the calculated total emission probability (full line) for ^{226}Ra is always smaller than the one for ^{214}Po , and the external emission probability (dash-dotted line) for ^{226}Ra is smaller than the one for ^{214}Po , for photon energies about $E_\gamma > 120$ keV.

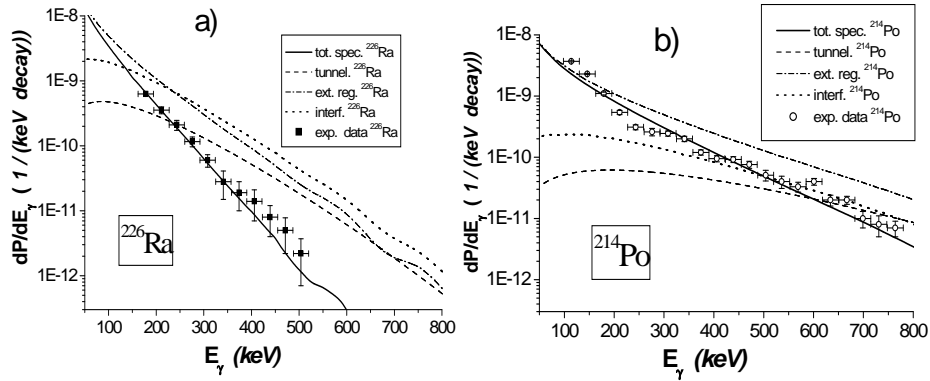


Fig. 4. Spectra of photons emitted from the tunneling region (dashed line), from the external region (dash-dotted line), and the absolute value of contribution of the interference term (dotted line). One can see for both nuclei the similar behavior of contributions from tunneling, external regions and interference term concerning the total spectrum.

Since the α -particle energies for the decaying ^{214}Po and ^{226}Ra nuclei are 7.7 MeV and 4.8 MeV, respectively, the tunneling region (R_1, R_2) for ^{226}Ra is longer

than for the case of ^{214}Po (see Fig. 2). Therefore, the bremsstrahlung photons are emitting through a greater distance under the barrier of the ^{226}Ra nucleus than for ^{214}Po . For this reason, the relative contribution of the photon emission from the tunneling region into the total spectrum for ^{226}Ra is larger than for ^{214}Po , at least up to about $E_\gamma=450$ keV (see the dashed lines in panels a) and b) of Fig. 4). In our results, the relative contribution of the photon emission from the external region (R_2, R_{max}) into the total spectrum for ^{226}Ra is smaller than for ^{214}Po (see dash-dotted lines in the cited panels a) and b)), because (R_2, R_{max}) is more large for ^{214}Po (see Fig. 2), but such a contribution from the external region overcomes the total bremsstrahlung spectrum and experimental data for both cases of the ^{226}Ra and ^{214}Po nuclei. This result clearly confirms that without the appreciable contribution of the photon emission during tunneling and the interference term (negative for these cases), it is impossible that the total bremsstrahlung emission can reach and agree with the experimental results. If the photon emission during tunneling of α -particle is smaller than the photon emission during the external motion of this α -particle, then we have to obtain smaller total bremsstrahlung spectrum for ^{226}Ra than the spectrum for ^{214}Po (at all photon energies). Figures 3 and 4 show this effect and confirm (theoretically and experimentally) our results and considerations.

The smaller values of the calculated total emission probability for ^{226}Ra than the one for ^{214}Po (and the smaller values of the external emission probability for ^{226}Ra than the one for ^{214}Po) can be explained by a consequence of the fact that outside the barrier the Coulomb field (and its derivative respect to r), that acts on the α -particle, in the case of ^{226}Ra is smaller than in the case of ^{214}Po because the external wide region results for the ^{214}Po nucleus larger than for ^{226}Ra and therefore the γ -emission probability for the ^{214}Po nucleus is bigger.

4. Conclusions

We have obtained a good agreement between theory and experiment for the bremsstrahlung spectrum of photons emitted during the α -decay of the ^{226}Ra nucleus, for E_γ energies up to about 500 keV. We think that for photon energies higher than 400 keV it is useful to take into account the deformation parameter of the decaying nucleus in the model, in order to reach a complete good agreement between data and calculation also at higher energies. We affirm that the photons with higher energies are emitted near the narrow region of the barrier where the potential form changes more rapidly, while the photons of lower energies are emitted at larger distances from the nucleus and in a very wide space where the changing of the potential is much slower. Therefore such a large wide space contributes with photons of low energies but with a higher emission probability. Moreover, we find that the slope of the bremsstrahlung spectrum accompanying the α -decay of the ^{226}Ra nucleus decreases more swiftly than the one registered for ^{214}Po because the α -particle energy is lower ($E_\alpha = 4.8$ MeV) for ^{226}Ra than for ^{214}Po ($E_\alpha = 7.7$ MeV), and this plays an important role both on the motion of the α -particle and in the

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determination of the photon emission probability in tunneling and external regions. In both cases we find a destructive interference between the photons emitted from the two above-mentioned regions, and, for the first time, we find an important contribution of the bremsstrahlung photons emission during tunneling of the α -particle in ^{226}Ra , taking into account the realistic α -nucleus potential in the model.

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References

1. G. Giardina, G. Fazio, G. Mandaglio, M. Manganaro, C. Saccá, N.V. Eremin, A.A. Paskhalov, D.A. Smirnov, S.P. Maydanyuk and V.S. Olkhovsky, in press in Eur. Phys. J. **A 36**, (2008)(published on-line DOI 10.1140/epja/i2007-10548-5)
2. H. Boie, H. Scheit, U. D. Jentschura, F. Köck, M. Lauer, A. I. Milstein, I. S. Terekhov, and D. Schwalm, Phys. Rev. Lett. **99**, 022505 (2007).
3. A. D'Arrigo, N. V. Eremin, G. Fazio, G. Giardina, M. G. Glotova, T. V. Klochko, M. Sacchi, and A. Taccone, Phys. Lett. **B332**, 25 (1994).
4. J. Kasagi, H. Yamazaki, N. Kasajima, T. Ohtsuki, and H. Yuki, Phys. Rev. Lett. **79**, 371 (1997).
5. J. Kasagi, H. Yamazaki, N. Kasajima, T. Ohtsuki, and H. Yuki, J. Phys. G. **23**, 1451 (1997).
6. N. V. Eremin, G. Fazio, and G. Giardina, Phys. Rev. Lett. **85**, 3061 (2000).
7. I. S. Batkin et al., Sov. J. Nucl. Phys. **44**, 946 (1986).
8. Th. Papenbrock and G. F. Bertsch, Phys. Rev. Lett. **80**, 4141 (1998).
9. N. Takigawa, Y. Nozawa, K. Hagino, A. Ono, and D. M. Brink, Phys. Rev. **C59**, 593 (1999).
10. E. V. Tkalya, Phys. Rev. **C60**, 054612 (1999); E. V. Tkalya, JETP **89** (2), 208–218 (1999).
11. M. I. Dyakonov, Phys. Rev. **C60**, 037602 (1999).
12. A. D'Arrigo, G. Giardina, V.S. Olkhovsky, V.S. Sergeyev, A. Taccone, and V.D. Tchesnokova, Phys. Atom. Nucl. **56**, 328 (1993).
13. R. B. Firestone, V. S. Shirley, *Tables of Isotopes* (Eight edition, Lawrence Berkley National Laboratory, 1996).
14. A. J. Ferguson, *Angular correlation methods in gamma-spectroscopy* (Russian edition, Atomizdat, Moscow, 1969).
15. D. C. Camp and A. L. Van Lehn, Nucl. Instr. Meth. **76**, 192 (1969).
16. S. D. Kurgalin, Yu M. Chuvilsky and T. A. Churakova, Journal "Izvestiya Akademii Nayk. Ser. Fizicheskaya" **65**, 666 (2001)(in Russian).
17. S. P. Maydanyuk and V. S. Olkhovsky, Eur. Phys. Journ. **A28**, 283 (2006).
18. S. P. Maydanyuk and V. S. Olkhovsky, Prog. Theor. Phys. **109**, (2003) 203.
19. V. Yu. Denisov, and H. Ikezoe, Phys. Rev. **C72**, (2005) 064613.
20. P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, Atomic Data Nucl. Data Tables **59**, 185-381 (1995).

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21. Y. A. Akovali, Nuclear Data Sheets **77**, 433 (1996).