Valuation of Zero Coupon Bonds Using Excel

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***Abstract* - The valuation of zero coupon bonds, or bonds which are paying no coupons or interests and are selling at a discount, is a straightforward discounting problem.**

To analyze zero coupon bonds, i.e. their present value, yield to maturity and duration, adequate models will be presented and a special spreadsheet developed using only plain vanilla Excel with its basic financial functions.

The reason for using only plain vanilla Excel is a better compatibility and portability of such spreadsheets, for students and other users, on different platforms or computers.

***Keywords* - valuation; bonds; zero coupon; Excel**

I. INTRODUCTION

Educational software is often used in finance courses for students, so they could better understand the investment process in securities, like bonds. It is also necessary to apply the methods of calculating prices, rates of return of fixed income securities in everyday practice. Because students at Universities especially at faculties of economics are trained to use Excel, it is possible to use such a spreadsheet program and its built in financial functions to calculate and evaluate interest rate risk in bond investments in general and especially in the case of zero coupon bonds. Besides that, very good books about Using Excel in Finance and Business do exist. [1] [11] [13]

For valuation of zero coupon bonds (zero bonds) in this article, the basics of ordinary or coupon bearing bonds will be presented first and after that the case of zero coupon bonds will be illustrated. After these theoretical considerations an Excel spreadsheet with examples is elaborated and the results are discussed like in two other articles presented at MIPRO Conferences a few years ago. [8] [9]

II. ORDINARY OR COUPON BEARING BONDS

Since bonds are securities and listed on secondary capital markets they can be traded like shares and it is therefore necessary to understand beside their nominal or face value, nominal interest rate and maturity, such categories as yield to maturity and present value or price. After that, more sophisticated aspects of bond investments like annual and semiannual compounding and duration will be discussed. [3]

A. Yield to maturity and present value of bonds

The yield to maturity is a measure of the rate of return that will be earned on a bond if it is bought now and held until maturity. To calculate the yield to maturity (y) it is necessary to solve the bond price equation for the interest rate given the price of the bond. In the case of annual compounding the formula or equation is: [2]

$$PV = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \ldots + \frac{C}{(1+y)^n} + \frac{FV}{(1+y)^n} \quad (1)$$

where:

PV - present value of the bond

C - coupon

n - number of years

y - yield to maturity/rate of return

FV - future value of the bond

For semiannual compounding of interests the adequate model is: [5]

$$PV = \frac{C/2}{1+0.5y} + \frac{C/2}{(1+0.5y)^2} + \ldots + \frac{C/2}{(1+0.5y)^{2n}} + \frac{FV}{(1+0.5y)^{2n}} \quad (2)$$

where:

PV - present value of the bond

C - coupon

n - number of years

y - yield to maturity/rate of return

FV - future value of the bond

According to the equations presented above it is possible to prepare an Excel spreadsheet using built in financial functions to determine the yield to maturity of a bond given the current purchase price. This is the true rate of return on a bond in the case that all coupons are reinvested at the rate of return when the bond was bought.

The present value of a bond is the sum of present values of coupons and the present value of the par or nominal value of a bond. It means that the equations presented above (1) and (2) are to be solved by calculating the present value or PV given the yield to maturity or rate of return in both cases, i.e. for paying interests annually or semiannually.

In an Excel spreadsheet presented later and developed especially for zero coupon bonds, the present value of coupon bonds can also be determined at the rate of return selected by the student. It is necessary to prepare basic input data, like the nominal interest rate and other necessary financial information on the bond. After that the present value can be calculated, but coupon bearing bonds are not in the focus of this article.

B. Duration of bonds

The duration is a measure of the average life of a bond, defined as the weighted average of the times until each payment is made, with weights proportional to the present value of the payment: [6]

$$D=\left[\sum_{t=1}^{N}\frac{t_C}{(1+y)^t}+\frac{NF}{(1+y)^N}\right]:\left[\sum_{t=1}^{N}\frac{C}{(1+y)^t}+\frac{F}{(1+y)^N}\right] \qquad (3)$$

where:

D - duration of a bond

N - maturity in years

C - coupon

F - face value

y - rate of return/yield to maturity

t - year when the payment is made

Using duration instead of maturity it is possible to minimize interest rate risk affecting investments in bond portfolios. In the Excel spreadsheet (ZBonds.xls) presented later, duration can be calculated not only for zero coupon bonds, but also for coupon bearing bonds, both for annual and semiannual payments of interests.

With semiannual payment, interests on bonds are paid out faster than with annual payments, so duration of the bond has to be a smaller number.

However, any change in the size of *y*, i.e. the rate of return or yield to maturity, in the denominator of equation (3), e.g. by dividing by 2 in order to calculate the interest payment on a semiannual basis, would mean essentially a reduction in the rate of return or yield to maturity, which would increase the duration of the bond, what is not true.

Therefore, quite the opposite has to be done. To get the correct solution for the duration of coupon bonds paying interests semiannually, Macaulay's duration formula has to be corrected, namely: [10]

$$D'=\frac{D}{1+0{,}5y} \qquad (4)$$

where:

D' - duration corrected for semiannual payment of coupons

D - duration according to equation (3) for annual payment of coupons

y - yield to maturity/rate of return

Besides that, the most important general characteristics of duration are: [10]

a) The duration of a bond with coupon payments will always be less than its term to maturity because duration takes into account these interim payments.

b) An inverse relationship exists between coupon and duration. A bond with a larger coupon will have a shorter duration because more of the total cash flows are paid sooner in the form of interests.

c) There is a positive relationship between term to maturity and duration, i.e. all other factors being the same, a bond with longer term to maturity will almost always have a higher duration.

d) The relationship between the yield to maturity and duration is an inverse one. A higher yield to maturity of a bond reduces its duration.

III. ZERO COUPON BONDS

Zero coupon bonds (zero bonds) are bonds which are not paying any coupons, they are selling at a discount and provide payment of the nominal or face value only at maturity.

The main advantages for investors are the reduced probability to call and the ability to lock in a return. There is also no reinvestment problem for received interests, because if an investor buys a zero coupon bond, e.g. with a yield to maturity of 10%, than he will actually secure that return if he holds this bond to maturity.

A. Yield to maturity and present value of zero coupon bonds

Since zero coupon bonds do not pay any interests, the yield to maturity and present value are calculated by discounting the future value or nominal value of this bond. In the case of annual compounding equation (1) has to be reduced to the following form: [3]

$$PV=\frac{FV}{(1+y)^n} \qquad (5)$$

where:

PV - present value of the bond

n - number of years

y - yield to maturity/rate of return

FV - future value of the bond

Equation (5) implies that the price of a zero coupon bond is simply the present value of the so called maturity value or future payment to be made at maturity.

For semiannual compounding equation (2) has also to be reduced and translated into a different form, namely: [12]

$$PV=\frac{FV}{(1+0,5y)^{2n}} \tag{6}$$

where:

PV - present value of the bond

n - number of years

y - yield to maturity/rate of return

FV - future value of the bond

According to equation (6) it is evident, that in the present value calculation the number of periods used for discounting is not the number of years to maturity of the bond, but rather double the number of years. The discount rate had also to be corrected and is now one half of the yearly yield to maturity or rate of return.

Examples for both cases, i.e. for zero coupon bonds with annual and semiannual compounding will be presented in the Excel spreadsheet section of this article.

B. Duration of zero coupon bonds

It was already stressed that duration is a measure of the average life of a bond or in different terms a time- and value- weighted measure of a bond`s maturity. By using duration instead of maturity it is possible to minimize interest rate risk affecting investments in bond portfolios.

In the case of zero coupon bonds with annual compounding, duration in equation (3) can be transformed and according to relevant literature reduced to the fact that duration for zero coupon bonds is always exactly equal to the maturity of zero coupon bonds or: [4] [6] [14]

$$D=N \tag{7}$$

where:

D - duration of a bond

N - maturity in years

Concerning zero coupon bonds with semiannual compounding illustrated in equation (4) above, it can easily be seen that by inserting 0 for variable *y* in this equation, the corrected duration *D'* is equal to duration with annual compounding or *D*:

$$D'=\frac{D}{1+0,5y}=\frac{D}{1}=D \tag{8}$$

Therefore, according to equation (7) corrected duration *D'* is also equal to *N* or the maturity in years of zero coupon bonds:

$$D'=N \tag{9}$$

where:

D' - corrected duration of a zero coupon bond

N - maturity in years

Duration *D* and corrected Duration *D'* will also be calculated in the following Excel spreadsheet and illustrated by adequate examples.

At the end two specific characteristics of zero coupon bonds should be noted.

First, compared with coupon bonds of similar maturity and yield, zero coupon bonds show much stronger percentage price changes when market interest rates or yield to maturities change. This is obvious since the duration of a zero coupon bond which is equal to the bond`s maturity is always larger than an otherwise similar coupon bearing bond. With the zero coupon bond nothing is paid or received until maturity, while in the case of a coupon bond the present value or the price is a function of both the periodic coupon interest payments and the bond`s nominal or par value.

Second, the longer or shorter the maturity of the zero coupon bond, the more or less the percentage price changes as market interest rates fluctuate. This is also obvious since the duration of shorter maturity zero coupon bonds is less than the duration of zero coupon bonds of longer maturity.

IV. EXCEL SPREADSHEET - ZERO COUPON BONDS

For determining the value and duration of zero coupon bonds, both for annual and semiannual compounding, an ordinary or plain vanilla Excel spreadsheet was developed applying custom made formulas by authors and standard financial functions built in Excel. [7] For explanation of the models two examples are presented below.

A. Zero coupon bonds with annual compounding

According to the following data about a zero coupon bond with annual compounding, the present value and duration of the bond are calculated.

Maturity - 5 years

Bond`s par or nominal value (FV) - 100 kn

Yield to maturity - 5.4%

Compounding - annually

After typing all necessary data in the “input data” field of the spreadsheet model (ZBonds.xls) the results for this bond, i.e. the present value or market value and duration are displayed in the “output data” field. (Fig. 1)

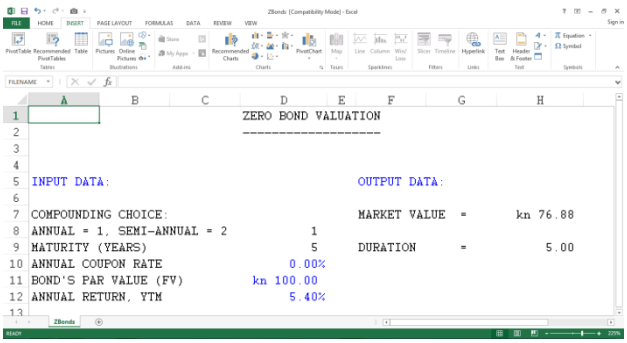

Figure 1. Spreadsheet screenshot of a zero coupon bond with annual compounding

The results show that the present value or market value of the zero coupon bond is kn 76.82 and the duration is 5 years. That means that the duration of a zero coupon bond with annual compounding equals the maturity of this bond in years.

B. Zero coupon bonds with semiannual compounding

Given the same basic information about a zero coupon bond, but this time with semiannual compounding, the present value or market value and duration of the bond are calculated again.

Maturity - 5 years

Bond`s par or nominal value (FV) - 100 kn

Yield to maturity - 5.4%

Compounding - semiannually

After typing all requested data in the "input data" field of the spreadsheet model (Zbonds.xls) the results for this zero coupon bond with the same general data, but with semiannual compounding are displayed in the "output data" field. (Fig. 2)

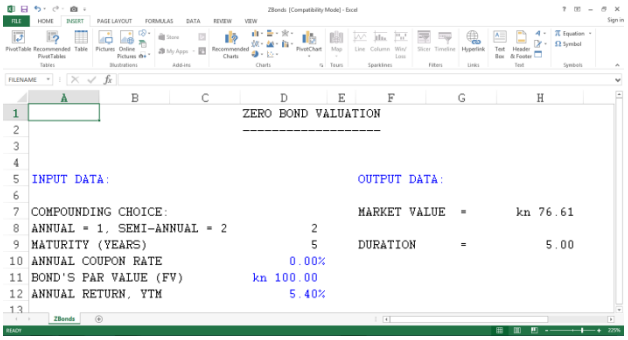

Figure 2. Spreadsheet screenshot of a zero coupon bond with semiannual compounding

Taking into account semiannual compounding of this zero coupon bond, the results show that the market value of the bond is lower and amounts to kn 76.61, but the duration is again 5 years. That means that the duration of zero coupon bonds, as a measure of interest rate risk, always matches the maturity of these bonds regardless of the compounding method, annually or semiannually.

On the other hand, the price or market value of a zero coupon bond compounded semiannually is always lower than the price of such a bond compounded annually, because it pays back sooner, what is consistent with bonds paying interests.

Comparing zero coupon bonds with coupon bearing bonds it can be concluded that the duration of zero coupon bonds which equals their maturity is always larger, or to be more precise, longer than the duration of coupon bearing bonds regardless of the compounding method. This implies that zero coupon bonds are more exposed to interest rate risk than coupon bearing bonds, or in other words, their price will fluctuate more up and down than the price of coupon bearing bonds when interest rates are falling or raising.

V. Delivery and Installation of the Excel Spreadsheet

Since all mentioned procedures and calculations are implemented in one Excel spreadsheet (ZBonds.xls), it is recommended to copy this file in the working directory of the PC where Microsoft Excel program can access it. Of course, Excel Analysis Toolpak Add-In can be installed to speed up eventually some specific financial functions or calculations, but is not necessary.

VI. Conclusion

In this article the valuation of zero coupon bonds (zero bonds) was elaborated both for annual and semiannual compounding. For this purpose, an Excel spreadsheet was developed using only plain vanilla Excel, i.e. by using only custom-made formulas developed by authors and usual financial functions already integrated in basic Excel. The theoretical models of present value or market price and duration of zero coupon bonds implemented in the Excel spreadsheet (ZBonds.xls) show that even more sophisticated discounting problems can be solved efficiently using Microsoft Excel program. In addition, it should be emphasized that by using only plain vanilla Excel, better compatibility and portability of such a spreadsheet, on different platforms or computers is also provided.

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