Thought Experiments between Nature and Society

Thought Experiments between Nature and Society:

A Festschrift for Nenad Miščević

Edited by

Bojan Borstner and Smiljana Gartner

Cambridge
Scholars
Publishing

Thought Experiments between Nature and Society:
A Festschrift for Nenad Miščević

Edited by Bojan Borstner and Smiljana Gartner

This book first published 2017

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-4438-8643-2
ISBN (13): 978-1-4438-8643-7

TABLE OF CONTENTS

PREFACE

> "*Suppose now that there were two such magic rings, and the just put on one of them and the unjust the other; no man can be imagined (hos doxeien) to be of such an iron nature that he would stand fast in justice...*" (Plato)

> "*Indeed, the possibility of thought experiments rests upon our ideas as being the more or less exact copy of facts.*" (Ernst Mach)

> "*I was sitting on a chair in my patent office in Bern. Suddenly a thought struck me: If a man falls freely, he would not feel his weight. I was taken aback. This simple thought experiment made a deep impression on me. This led me to the theory of gravity.*" (Albert Einstein)

The idea for this book has its roots in a discussion that happened in the middle of May in 2014, when the researchers of the Department of Philosophy, Faculty of Arts, University of Maribor, were completing the work on the research program *The Concept of Virtue in Theoretical and Practical Philosophy*. We asked ourselves: "What should be the main subject of our future research program?" We were operating with the idea that contemporary philosophy is often very fragmented, with extremely narrow specialization. Having this in mind, we came, after a long discussion, to a proposal which should be able to rectify this narrowness – thought experiment as a common place or starting point that can serve as a common frame for our research in different fields of philosophy. Our proposal for the research framework has been approved, and starting in 2015 we began our research work in the research program *Laboratorij duha: Miselni eksperimenti med naravo in družbo* (*Laboratory of Mind: Thought Experiments from Nature and Society*), with Professor Miščević being the leading researcher. There is another additional point worth mentioning: in 2015, Professor Miščević celebrated his sixty-fifth birthday, so combining the research task with his celebration seemed like the obvious thing to do. Thus we also organized the conference in November 2015 with the same title as this book has.

What are thought experiments (TE), and why are TEs something that can help us in the defragmentation of, at least in a methodological sense, contemporary philosophy? There are many possible ways to answer these

questions. We will highlight only one: just start with some paradigm cases and, through analysis, step by step come to the most important properties, features of TEs. We have been told that in philosophy and science one encounters a lot of TEs; for instance, the story of the *Ring of Gyges* in the second book of Plato's *Republic*. Indeed, the entire *Republic* might be viewed as an extended TE, in which the picture of an ideal state is built "in the logos", as Plato puts it. Is this really a TE? There is an ongoing debate between those who try to justify the idea that even some of the Presocratic philosophers of nature provide us with a bunch of good examples of TEs and others who claim just the opposite and seriously doubt the possibility of using the term as referring to the ancient practice of appealing to intuition. However, no one doubts that in TEs, if there are such cases, there is a special kind of reasoning at work which is often described as 'armchair' reasoning, in contrast to laboratory or outdoor research. It is not only used in philosophy but also in science (e.g. physics, see famous TEs by Einstein and Galileo, economics, etc.). In philosophy, it is used in political philosophy, ethics, ontology, philosophy of mind, and epistemology. Philosophers reflect on such knowledge, and in the last twenty-five years, they have gained support from cognitive scientists in this effort. The cooperation between cognitive scientists and philosophers offers a chance for a unified, respectably scientific account of human knowledge.

When Ernest Mach starts to think about TEs (Gedankenexperimente), he realizes that, in a certain sense, any sensibly designed real experiment should be preceded by a thought experiment which anticipates at any rate the possibility of its outcome. We can add that the purpose of a TE is to strip away the things that complicate matters in reality in order to focus more clearly on the most important properties of the problem that we are trying to solve.

Typical stages in thought experimenting and reflection are understanding, building a model, reaching a particular judgment-intuition, 'intuitive induction', and then a leap to a more general judgment. However, there are at least three points which are closely connected with our understanding of TEs.

Firstly, explanatory matters. Are TEs reliable sources of data for philosophers and scientists? If no, how should philosophy proceed? If yes, how is their reliability to be accounted for? The mysterious Platonic 'perception' does not sound like a good explanation, but the opposite stance, that judgments-intuitions from TEs are just ordinary judgments,

seems an oversimplification. We need an explanatory proposal, and a TE tries to offer one.

Secondly, normative problem – a priori and a posteriori. In a typical TE one can separate the contribution of general empirical knowledge from topic-specific (mathematical, moral, and probably normative epistemological, etc.) contribution, which might be deeply or superficially a priori. Therefore, their typical products, intuitional judgments, have a structured justification encompassing a priori and a posteriori elements. Most of them are *prima facie* justified without appeal to experience, but only *prima facie*. Full reflective justification of a given intuition is a posteriori. For instance, in the case of kind-intuitions, a posteriori justification derives from the general stability and success of thinking in terms of kinds. In the case of knowledge, from the general stability and success of appealing to knowledge, and distinguishing it from its lesser cousins.

Thirdly, application issues. How could TE-intuitions be made better? How could people, in principle, come to ameliorate their intuitions? How could enlightened individuals (experts) guide novices to avoid typical mistakes? TEs have stages, including reflection on them. Therefore, look at the stages; divide labour among stages – it might help. Dangers and promises are unequally distributed in the biography of a typical philosophical intuition – dangers abound in early stages, promises come in later. Collating TEs and performing intuitive inductions might help (specially to counter the opacity of each intuition taken in isolation); individual reflective equilibrium as well. It caters for coherence, in good cases. One then recognizes the margins of one's own reliability of intuiting. Finally, the collective wide reflexive equilibrium brings in factual knowledge; it should help. So, in principle, we can (hopefully) avoid the problem of regress or circularity, and show how expertize is possible.

To summarize, the basic goal of this book is to deepen the understanding of TEs. The additional goal is defending their reliability against sceptics. The practical goal is proposing new applications of TEs, both in theoretical philosophy and in ethical and political thinking, with important practical consequences.

And the last but not least important goal of the book is to honour Professor Miščević at his 65th birthday.

We received invaluable research assistance from Tadej Todorović in the final stages of the project.

Financial support for the project as a whole was provided by the Slovenian Research Agency (research core funding No. P6-0144).

Bojan Borstner, Smiljana Gartner

INTRODUCTION: THOUGHT EXPERIMENTS AS AN IMPORTANT PART OF PHILOSOPHICAL METHODOLOGY

BOJAN BORSTNER, SMILJANA GARTNER
UNIVERSITY OF MARIBOR

Thought experiments (TEs for short) have become a hot topic in the methodology of philosophy as well as in philosophy of science.

The reason for that is that they have been present throughout the whole history of the discipline, and have become indispensable for analytic philosophy. Collections of papers dedicated to particular philosophical topics or arguments are nowadays often organized around famous TEs, say Brain-in-a-Vat, or Original Position, or Twin Earth and its cognates. The indispensability is at least practical and valid for normative disciplines, from epistemology, through philosophy of language, to ethics, and political philosophy. In descriptive-explanatory disciplines like metaphysics they are required for arriving at a good pattern of our everyday concepts (for instance, identity of a material object through time) to be compared with the counterpart scientific one.

Let us start with a few examples of famous thought experiments. For historical examples think of Plato's Glaucon (*The Republic*: 359-360):

> Glaucon: But as for the second point, that those who practice it do so unwillingly and from want of power to commit injustice—we shall be most likely to apprehend that if we entertain some such supposition as this in thought: tei dianoiai if we grant to each, the just and the unjust, licence and power to do whatever he pleases, and then accompany them in imagination and see whither his desire will conduct each.
>
> Suppose now that there were two such magic rings, and the just put on one of them and the unjust the other; no man can be imagined (hos doxeien) to be of such an iron nature that he would stand fast in justice...

"How would you react?" the interlocutor is asked, and the thought experiment is under way.

Staying in ancient Greece, we have The Ship of Theseus thought experiment and its leading question about which of the two ships is identical with the original, the rebuilt ship or the one still sailing; its applications leading to the topic of personal identity, already present in early modern times with Locke and Hume.

On contemporary scene one of the most famous thought experiments is the Gettier one. Smith goes through various exercises, forms a belief, and the reader is confronted with the question "Does Smith know?" The thought experiment has revolutionized epistemology in the sixties.

Some years later Hilary Putnam (1973, 1975) has proposed a famous thought experiment: "Imagine a Twin Earth with water-like stuff of different chemical composition, say XYZ. Is Twin water water?" "No, since it is not H_2O." The negative verdict about Twin Earth stuff is interpreted as revealing that, when it comes to our understanding of essential property and of stuff, composition is essential for stuff.

In philosophy of mind think of Searle's Chinese room, with a human being, ignorant of Chinese language "locked in a room full of boxes of Chinese symbols (a data base) together with a book of instructions for manipulating the symbols (the program)" (Searle, 1999: 115). He gets incoming texts in Chinese, and, with the help of instructions, translates them, and then presents the answer in Chinese again. The whole, the-room-plus-person, would thus pass the Turing test: but does anybody (the person) or anything know Chinese here? No. But if our brain is like a computer, then it is like a Chinese room, and does not understand anything. But we do understand things, for instance, languages. So, our brain is not just a computer. "I demonstrated years ago with the so-called Chinese Room Argument that the implementation of the computer program is not by itself sufficient for consciousness or intentionality", writes Searle (2010: 17).

On the side of ethics, we have to mention the Trolley problem, without retelling it, and on the side of political philosophy we have the Original Position as a "thought experiment for the purpose of public- and self-clarification", as Rawls puts it in *Justice as Fairness: A Restatement* (2001: 17). The veil in the Original Position is "a guide to intuitions" (Rawls, 1971: 139). "The idea here is simply to make vivid to ourselves

the restrictions that it seems reasonable to impose on arguments for principles of justice, and therefore on these principles themselves" (Rawls, 2001: 16).

So much for examples. In the rest of the text we will briefly mention various topics connected to thought experiments. All of them are being discussed in the papers that follow, but not all to the same extent.

Firstly, what is a thought experiment? Einstein and Infeld wrote: "we recognized the importance of the idealized experiment created by thought" (1938: 226). The idealized experiment can never be actually performed, although it leads to a profound understanding of real experiments. They present the following characteristics of TEs:

- they are 'fantastic', they can neither be derived from experiment nor actually performed;
- they are consequently created 'by thought' and even by 'speculative thinking';
- they are however 'consistent with observation' and lead to 'a profound understanding of real experiments'. (ibid.)

More recently, John Norton offered the following characterization of TEs:

> "Thought experiments are arguments which: (i) posit hypothetical or counterfactual states of affairs, and (ii) invoke particulars irrelevant to the generality of the conclusion" (1991: 129).

Sören Häggqvist talks about "hypothetical cases intended to function as experiments, in the following sense: they aspire to test hypotheses or theories" (2009: 57).

A related question concerns the intellectual 'relatives' of TEs. J. R. Brown (1991, 1999, 2004) has proposed that some mathematical ways of reasoning are very close to TEs, people working in philosophy of literature (like Catherine Elgin, 2006, 2014) talk about some famous stories, like Orwell's *1984*, in terms of TEs, and Descartes, in his discussion of *Meditations*, draws some serious analogies between thought experiments and religious meditations. Can this bewildering variety be somehow organized, and what is the wider genus to which TEs belong (the question is tackled briefly in the book)?

Once these questions are answered, one can tackle the most difficult part, the accounting and explanation of TEs and the presumed insights that they

enable us. J. R. Brown has, in *Laboratory of the Mind: Thought Experiments in the Natural Sciences* (1991), proposed a list of explananda, i.e. of challenges that TEs pose to the theoretician and philosopher of science. The first challenge is posed by the fact that in TE "there has been no new observational data" (Brown, 1991: 11). Other puzzles range from quickness to universality.

There are Platonist, empiricist argumentativist, Kantian, and naturalist-cognitivist proposals available for explanation. J. R. Brown has, on the contemporary scene, pioneered a Platonic proposal that imaginative intuitions are windows upon Platonic heaven. In the present volume, Majda Trobok comes closest to this view, in application to mathematical intuitions.

John Norton (2004) has famously been offering a more empiricist proposal: materials of a TE are empirical, and the method is just logical argumentation:

(i) "My goal in this chapter is to state and defend an *account of thought experiments as ordinary argumentation* that is disguised in a vivid pictorial or narrative form.
(ii) TEs invoke particulars irrelevant to the generality of the conclusion" (2004: 45).

Timothy Williamson agrees on the importance of logic, but disagrees on the role of particular cases: for him they are crucial in mobilizing our imagination, which then, together with abstract thinking, provide an answer.

The Kantian Buzzoni (2008) talks about "indeterminate multiplicity of categories individually constructed by the mind and tested *operationally* to solve specific problems" (2008: 15), ascribing to the categories a relative apriority.

On the naturalist-cognitivist side, the most popular proposal engages mental models (Nersessian 1992, 2007; Miščević, 2007, 2017). Models are being discussed a lot in the literature. A recent account due to Chris Frith (2007) claims that in general, perception and action involve the generation of mental models. These models are continuously tested against reality and adapted using sensory signals and prediction errors. Although the basic mechanism enabling the construction and manipulation of the representations might be computational, the most important feature of representations and

operations having to do with mental models is precisely their concrete and quasi-spatial character. This has earned them the name of imaginative 'mental models'. Mental models are typically involved in understanding stories or in ordinary planning of activities. When a reader encounters a description of a situation, she builds a model, a quasi-spatial 'picture' of it. As new details are supplied by the story-teller the model gets updated. The background conditions are dictated by the thought experimenter's general knowledge about the world. Miščević claims: "Much more important is the possibility of producing new data by manipulating the old ones. So, the manipulability of elements in the model and the mobilizing force of particular instances, more friendly to our ordinary reasoning process than the abstract general arguments, should account for the heuristic value of TEs. The theory of mental models makes the correct prediction that a visualizable TE will have more heuristic value than a corresponding propositional alternative, and explains in principle why. It is plausible that manipulating quasi-spatial models can in heuristic value come very close to actually seeing the movement of pieces manipulated" (Miščević, 2007).

The next issue to be addressed in the explanation is the nature of abilities or capacities involved in thought-experimenting. A Platonist has to propose such special capacities; others might opt for a more naturalistic approach. One line of thought is to just claim that all our cognitive abilities contribute, and that in principle it is impossible to separate the contribution of the more specialized ones from the rest. This leads into 'ordinarism', the holistic view that would, on the justification side, treat the resulting insight as all a posteriori, or would question the very distinction between *a priori* and *a posteriori* justification (Williamson, 2013). On the opposite side is the view stressing particular competences, and thus allowing for some, say the mathematical ones, to yield *a priori* insights. M. Devitt (2013), himself more of an ordinarist, has ironically called it 'Voice of Competence View'. This position is briefly defended by Miščević in the present volume.

In recent times the debates just mentioned have been overshadowed by a new, big disagreement. A group of philosophers, most of them initially inspired by Stephen Stich, have decided to test the reliability of thought experiments, including the homogeneity of answers, dependence on irrelevant factors, and the like, in the way which is normally central in psychology. The new school has called itself experimental philosophy, X-phi for short. The experiments have produced some worrisome results that have inspired the creation of a more negative, critical stream within the school (E. Machery, R. Mallon, S. Nichols, S. Stich). Several lines of

defence of thought experiments against the accusations of negative experimental philosophers have been developed, a prominent one being the so-called expertize defence (T. Williamson, 2007; R. Sorensen, 1992). Miščević (2017) proposed an alternative line, building upon the distinction between stages of thought experiments and pointing to ways to avoid traps typical for each stage.

A final topic to be mentioned is the historical one. Thought experiments have resulted in traditions, some of them very long, for instance the Social Contract tradition (with the Original Position thought experiments starting as a sub-tradition within it), and the Cartesian sceptical tradition. Others, like the Twin Earth thought experiment or Gettier cases, have prompted at least half a century long trails. One cannot understand contemporary analytic philosophy without understanding these trails and traditions. It is an interesting question how the structure of a typical TE relates to the trail and what is the relation between its synchronic pattern and its diachronic posterity.

The book *Thought Experiments between Nature and Society*, a Festschrift for Nenad Miščević, is divided into three parts: Thought Experiment – General; Thought Experiments and Nature; and Thought Experiments and Society. Some of the texts are more explicitly about thought experiments, and some only glance at the basic problem, as they are more focused on other areas of Miščević's philosophical contributions. Each text is accompanied by Miščević's reply on the work of his colleagues.

The first part of the book (Thought Experiment – General) starts with **Timothy Williamson's** text, **From Anti-Metaphysics to Metaphysics,** where he emphasizes that there is a more general moral about the famous 'linguistic turn', a phrase which has looked less and less appropriate as a description of mainstream analytic philosophy over recent decades. Nevertheless, although analytic philosophers are ceasing to regard their central questions as linguistic or even conceptual in any sense that would distinguish them from questions asked in other disciplines, the traces of the linguistic turn are not simply being erased. However, we learned from history that misconceptions in semantics often induce misconceptions in metaphysics. By causing fallacious metaphysical arguments to be treated as valid, coherent metaphysical views are incorrectly dismissed as confused or inconsistent. Therefore, a rich legacy of methodological sophistication with traces of the linguistic turn is useful in testing the soundness of arguments about non-linguistic matters, and analytic

philosophers regularly and justifiably draw on this work in both semantics and pragmatics.

Howard Robinson, in **Intuitions and Thought Experiments**, says that thought experiments are not at all in the speculative hunch category, like Dennett's imagined case. The issue with Mary is whether one can tell directly that the qualitative nature delivered in experience is a different property from, say, a neural firing structure, as represented in science. The issue relevant here is whether the kind of appeal to intuition involved in claiming to be able to identify the nature of a property by introspective awareness of its presence is, *prima facie*, question begging or illicit. For Robinson, it is clear that one cannot block the argument simply by saying it relies on an appeal to intuition, as if all such were illegitimate. In the classic 'mind' cases, the appeal is always to some version of the intuitively obvious, in a way that invites arguments challenging whether they are obvious in the way they seem.

Maja Malec and Olga Markič, in their text **Miščević on Intuitions and Thought Experiments**, begin with the assumption that, in the case of philosophical thought experiments concerning metaphysical modality, imagination has a more difficult task. We are often required to imagine situations that are very different from actuality, and what we end up with cannot be simply verified in actuality. Take, for example, Chalmers' thought experiment purporting to show that consciousness cannot be explained in physicalist's terms. We are invited to imagine our zombie twin, who is our physical duplicate, psychologically and functionally identical to us, behaving exactly like us, and being situated in the same environment like us, with the only difference being that our zombie twin lacks conscious experience. In other words, there is nothing it is like to be a zombie. If such a situation is imaginable and consequently possible, then physicalism is false. Obviously, if spatial perceptual constraints are not relevant here, what is? It is very simple to imagine my zombie twin, but is this imagined situation really possible? Clearly, I cannot actually create such a creature and thus confirm its possibility, but this does not mean that Chalmers' hypothesis is false. The matter stays undecided. Therefore, it is not enough that imagination is a good guide to actuality; it must be a good guide to possibility.

Nenad Smorkrović, in **Curiosity and the Argumentative Process**, begins with the idea that the issue concerning argumentative process has its source in early theoretical differentiation, going back to Plato and Aristotle, distinguishes between *rhetoric* and *dialectic*. Aristotle famously

made a difference between *rhetoric* and *dialectic*. He was the first who stressed the dialectical character of argumentation conceived as a critical discussion, in contrast to rhetoric, which is primarily motivated by finding the available means of persuasion. The intrinsic curiosity directs the dialogue towards the truth or towards the extension of knowledge. Being sincerely curious and acting in a joint effort to extend their knowledge concerning the truth of a proposition, the participants are motivated to do their best to achieve it. This means that in the AP, more than in any other format of reasoning, they are prone to de-bias their reasoning capacities.

Peter Gärdenfors - Semantic Transformations tackles the differences between metaphors and metonymies. Metaphors refer to mappings *between* domains and metonymies refer to meronomic and other relations *within* domains. For Gärdenfors, the role of metonymy is primarily referential. Metonymic concepts make it possible to conceptualize something by its relation to something else to which it is connected. By picking out one specific aspect of an entity, metonymy focuses attention on something salient to the situation, thereby helping one's understanding of it. The relevant parts and wholes in *pars pro toto* and *totum pro parte* metonymies need not be just spatial, but can be temporal as well. More abstract meronomic relations of a functional or causal nature also generate metonymies. He says that in the sentence "Proust is tough to read," the author stands for the book he wrote; while in "Napoleon attacked Russia," the highlight is Napoleon's function as the leader of the army. At the end, Gärdenfors' position is that by narrowing the use of the notion of a domain, one obtains a sharper division between metaphors and metonymies.

Danilo Šuster, in **Lucky Math: Anti-luck Epistemology and Necessary Truth**, has lumped several approaches in virtue epistemology together, since, according to his analysis, it is not clear how to augment the safety account with virtue-theoretic conditions. With the resources of virtue epistemology, why still stick with safety with all of its baggage (methods, ways of forming beliefs)? Why not virtues all the way? There are proposals for a virtue-oriented theory of luck on the market: one's belief is epistemically lucky when one fails to deserve credit for reaching the truth (Greco, 2003). Or, whether an agent is lucky depends on whether the agent is in control of her action, so one knows that p when one comes to have a true belief that p in a way that is sufficiently under one's control (Riggs, 2009). It seems that the approach of virtue epistemology will undermine the project that Miščević started with: formulate an anti-luck condition in the frame of a modal minimal change theory of luck.

Guido Melchior - Epistemic Luck and Logical Necessities: Armchair Luck Revisited, starts with the claim that the particular problems that Miščević and Pritchard encounter point towards a more general problem. In case of necessarily true propositions, we cannot consider possible worlds, which vary with respect to the truth value of the target proposition, but we can consider other variations, e.g. different cognitive apparatus (Miščević) or different propositions (Pritchard). However, for such cases, we can modify the setting in a way that any possible world where the variation takes place is modally far off – and in this case, the necessary conditions for knowledge are fulfilled. Thus, modal variation does not offer a way out of the problem.

The last chapter in the first part is **Smiljana Gartner's Did a Particularist Kill the Thought Experiments?** A further (dis)advantage, especially of ETEs serving as intuition pumps, is that we can easily and quickly change them, and that some of them form very distant possible worlds. But where is then stability, one of Miščević's desiderata? With changing ETEs, we are directly confronted with the complexity of our intuitions, situations, and ethical dilemmas. Let us take Foot's Tram Case. We can let the tram go its own course or we can redirect it. We can save five lives or one life. The intuition of the students says that we should divert the train and save five lives. But then we add the information that the one person is a baby and the five are 50-year-olds. Now they would choose the baby. Or we add the information that the baby has severe mental problems and that the five have small children. Now they would choose the five people again. By adding properties (perhaps to avoid the objection to under-description), the relevance of the features could change and, consequently, the conclusion could change as well, thus throwing stability out of the window.

The second part (Thought Experiments and Nature) opens with **Marian David's** paper, **Experimental Philosophy, Gettier-Cases, and Pragmatic Projection**. David presents an idea that there is a problem of principle in the approach of experimental philosophy. On the one hand, the experimenters want their 'intuition probes' to resemble the originals rather closely, otherwise they run the risk of missing their target. On the other hand, the originals are often perceived as rather odd, especially when taken out of the context of philosophy classrooms or textbooks, and they are usually rather sketchy. The situations they present are under-described in various ways (the philosophers who designed them put in only what they needed, which increases the risk that the subjects will fill in additional information when they encounter the cases in a survey setting –

consequentially they could arrive to unintended interpretations). However, if experimenters take measures to control against filling in, they run the risk that their 'intuition probes' will not probe untutored intuitions: the line between controlling for unintended interpretations and tutoring intuitions is fuzzy – and probing tutored intuitions is exactly what the experimenters do not want to do.

Peter Simons, in **Concepts in a World of Particulars**, starts his analysis with the following dilemma: If there is a discernible, identifiable abstractive basis to concepts, its *concreta* are either very complex, or very heterogeneous, or both. This is probably why it is so tempting to identify concepts with meanings. The problem is: although meanings themselves are less than straightforward compared with simple *abstracta* like numbers, masses, and shapes, their abstractive basis in the uses of words is fairly easy to identify. On the other side, concepts are involved with their sustaining acts, users, bits of language, activities, and compliants, and so on in a much messier and tangled way. This suggests that, despite the usefulness of talking about concepts, their status as putative *abstracta* is even shakier than that of 'better behaved' *abstracta*. Luckily for a nominalist, this causes no ontological anxiety: the sustaining individuals, within and outside the mind, are what matter and that do all the work.

Ilhan Inan, in **Is the Speed of Light Knowable A Priori?**, starts with the well-known idea that regarding any declarative sentence that expresses a proposition in a given context, one can know two things: the first is whether the sentence refers to a fact, and the other is the fact to which the sentence refers to (assuming that it is true). This requires us to distinguish between *knowing that a proposition is true* on the one hand, and *knowing the fact that makes a proposition is true* on the other. Knowing that, a proposition could then appear in two different ways. In the one case, the subject knows both that the proposition is true, and the fact that makes it true. This is ostensible knowledge. In the other case, the subject knows that the proposition is true, but does not know the fact that makes it true. This is inostensible knowledge. Illustrating this with a simple example: suppose a friend of yours says that his lover's eyes are his favourite colour. If you take his word for it, you now know a proposition expressed by the sentence (in the appropriate context): "His lover's eyes are his favourite colour." Even though you know that the proposition is true, you may still not know what makes it true, that is, you may not know the fact to which the sentence refers. This would happen if you do not know your friend's lover, if you do not know what is your friend's favourite colour, or both. The sentence would then be inostensible for you, not because you

do not know whether it expresses a truth – you already know that it does – but rather because you do not know the fact to which it refers.

Andrej Ule, in **Mental Models in Scientific Work**, explains us that mental modelling in science seems to be an integral part of public practices of the usage of non-linguistic or para-linguistic representations of empirical structures in finding their full theoretical explanation. It is only secondarily a part of individual mental practices. The latter may be the result of internalization of the public practices by well-trained scientists. However, he believes that the same holds true for many types of intuitive insight. We have to ask ourselves in what case, in what circumstances, do we say: "Now I see what you mean.", "Now I see how I can do this or that.", etc.; "when *has* a formula, a graph, or a mental picture occurred to me?" Sometimes, when we hit upon the seemingly indubitable statements about intuitive reasoning, we actually stumble across indubitable, but *public criteria* for meaningful use of statements on our basic intuitions, and not on some *a priori* true propositions. Therefore, some innate (cognitive) competences and abilities may help in this development, but it will be a problem for further research if they are their neural or genetic foundations.

Ferenc Huoranski, in **Natural Kinds and Conceptual Truth**, presents, at the very beginning of the paper, a fictional example, which is, in fact, a real life example about cases when the content of a natural kind term is determined by its microphysical constitution that exists in abundance. Even if it is true of the concept of water or gold that their content was originally determined by water's and gold's sensible qualities and that we discovered their constitution only later, this is pretty much a contingent fact *not* about gold or water, but about how we actually introduce a term into our language. A procedure in the opposite direction is equally possible. Some chemical elements' and compounds' names have been first introduced by identifying their constitution, and hence it seems an *a priori* matter that their concept refers to some stuff with that type of constitution. Consequently, by what properties we fix a natural kind term's reference – or how we provide cognitive content – should be irrelevant regarding the modal status of propositions involving the terms. For any moderately realist account of the nature of modal truth, it must be unacceptable that a truth's modal status is merely determined by such historical accidents.

Majda Trobok, in **Grasping the Basic Arithmetical Concepts - The Role of Imaginative Intuitions**, tries to explain and criticize Miščević's position about how to come to the notion of natural number. For him,

Hume's principle – taken informally – does play a crucial role. However, any attempt to show that such a principle is, in fact, intuitively clear, or that the grasping process of it is based on quasi-perceptual (or imaginative) intuition, is, Trobok suggests, highly controversial, since there is no practical-empirical or historical-mathematical support based on evidence for it. She says that even though Miščević does not ignore the mathematical practice, he does not look at the historical-mathematical side of it, deciding to concentrate rather on what developmental psychology has to say on how we grasp the notion of (natural) number. She tries to go one step further and claims that we do not have sufficient practical-empirical or historical-mathematical evidence in favour of the view that grasping such concepts is based on imaginative intuitions, neither for the concept of equinumerosity (Frege), nor for that of (natural) numbers.

Andraž Stožer and Janez Bregant, in **The Colour Dilemma: A Subjectivist Answer**, at the beginning delineate the neurophysiological mechanisms underlying the visual experience of colour, and then with the help of concrete results of practical neurophysiological experiments show that the physicalist and dispositionalist theories of colour, in the light of new, neurophysiological objections, do not present credible views on the nature of colour. These findings demonstrate that a direct electrical activation of neurons that usually respond to blue-purple stimuli suffices to evoke a perception of blue-purple colour by the subject. Therefore, answering the question of what colour really is, physicalism and dispositionalism should be, in the light of neurophysiological findings that speak in favour of colour not being out there in the world, replaced by subjectivism as the theory of colour that addresses the problem of colour realism based on scientific grounds.

Matjaž Potrč - Dasein in a Vat says that it would be wrong to start from a sceptical conclusion about the external world. There is a tendency (e.g. Searle) to conclude that the assertion of mine and my BIV counterpart's experiential world identity leads to external world scepticism, and accordingly to the denial of external world existence. For Potrč, this is a wrong conclusion. Notice that the author, hopefully and by presupposition not a BIV, has an experiential world. If he perceives a cat in his experiential world, his thought about the cat will be true. Whereas his BIV counterpart's thought will be false, for the counterpart has no real world similar to the one that the author is operating in. Identity of our experiential world (based upon sharing of the zero point essential indexical) does not imply identity of our external worlds. It is simple: author's BIV, in counterpart to himself, has no such external world at all,

he just shares the experiential world together with the zero point perspective with the author.

The third part of the book (Thought Experiments and Society) opens with **Pierre Jacob's** paper, **Knowing One's Own Mind.** His starting point is that even if preverbal infants can spontaneously represent the contents of others' false beliefs, most 3-year-olds may fail explicit change-of-location false-belief tasks and point to the toy's actual location for at least three complementary reasons. First of all, they might fail to compute the relevance of the agent's false belief for answering the experimenter's prediction question. Secondly, even if they do compute its relevance, they might lack the executive resources required for inhibiting their own *knowledge* about the marble's *actual* location from intruding into their answer to the experimenter's prediction question. Finally, they might misinterpret the experimenter's prediction question as a normative question about where the mistaken agent ought to look for her toy. Note further that in order to predict where the mistaken agent will look for her toy, it is sufficient to know where she last placed it. Whatever happens to the toy afterwards is irrelevant. So in the classical explicit task, children are provided, by the experimenter, with much information about the whereabouts of the agent's toy that is irrelevant to predicting the agent's action. In a nutshell, having the ability to track the contents of others' false beliefs is not sufficient for success at elicited-response false belief tasks. Therefore, failure at these tasks by 3-year-olds is not a demonstration that they lack a theory-of-mind. Nor does it show that 3-year-olds cannot think that they and others think.

Friderik Klampfer, in **The False Promise of Thought-Experimentation in Moral and Political Philosophy**, shows that one important reason for distrusting intuitions generated by thought experiments is their malleability, instability, and vulnerability to manipulation. Our intuitions are easily swayed one way or the other by simple rephrasing of the story, a change in the order of presentation, emotional and social priming, or simply by tampering with our physiological needs. What psychologists tell us about the mechanisms that produce them and what we know influences them doesn't exactly build confidence. Intuitions are quick, snap, unreflective, spontaneous, almost automatic judgments. If they are preceded by any reasoning at all, it must be subconscious. They rely, for their formation, on similar cognitive shortcuts, heuristics, which people use in their judgments in other domains (availability, representativeness). They are subject to the framing effects ('lives not saved' vs. 'lives lost') and moralizing spill-over effects and in many cases they are shaped by

mood, affection, emotion, fatigue, affected by the level of abstraction and sensitive to the order of presentation. It is important to stress that despite their contingent origin, they are mostly dogmatic, i.e. resistant to contrary evidence. When our intuitive judgments are challenged or questioned, we are seldom able to provide good reasons or compelling evidence in their support (or if we are, the reasons we adduce are often not those that were operative in the production of our judgment). Furthermore, Klampfer states that: we fail to see any need for that and, consequently, don't consider this to be a problem (what is called 'moral dumbfounding').

Miomir Matulović, in **Miščević, Mental Models, and Thought Experiments in Political Philosophy**, shows in his analysis that Miščević's proposal of the desiderata for good or successful political thought experiments sets very high standards that political thought experiments have to satisfy. It seems that in this way Miščević is trying to answer the critics of the method of thought experiments in political philosophy. But he is aware that these standards (fulfilment of the desiderata) are never fully satisfied by the actual, even most famous, thought experiments. Did the critic thus win? Or is Miščević thinking of a so far unformulated alternative? For the time being, it seems that Miščević is confronted with a dilemma: either raise the standards, in order to satisfy the critics of thought experiments, or admit that standards are never satisfied by the extant proposals. On the first horn, the critics might be satisfied with the demands but will point to the fact that these are never fulfilled, and on the second horn Miščević has to agree with the critics of thought experiments.

Boran Berčić, in **Are Nations Social Constructs? Nenad on Nations**, starts with the proposal that the idea that nations are social constructs can be spelled out in the following way: Individuals *a*, *b*, *c*, *etc*. form a nation *A,* iff they believe that they form a nation *A*. The relevant question here is what does it mean that *they* believe? What is a collective belief? It seems that it is not sufficient that each individual believes that he or she belongs to the nation. One has to believe that others have the same belief as well. Therefore, it might be said that individuals *a*, *b*, *c*, *etc.* believe that they form a nation iff: (1) each one of them believes that they form a nation, and (2) each one of them believes that every one of them believes that they form a nation. The next question that arises is what does it mean that they *believe*? One might say that it is sufficient that they behave *as if* they believe (1) and (2), for in that case we might ascribe corresponding beliefs to them. However, in this case, large herds or flocks of animals could satisfy the requirement. Therefore, it seems that one has to demand that

they must have some degree of rational reflection about their belonging to a nation. Berčić says that they must have a disposition to give an affirmative answer if asked whether (1) and (2).

Rudi Kotnik, in **Thought Experiments in Teaching: TE as a Suppositional Real Story**, thinks that there is a contrast between two 'sorts' of TEs: a) TE as an imagined story; and b) TE as a suppositional real story. They are both TEs, but with one single, tiny, but crucial difference: in the case of b) students believe (or are supposed to believe) that the story is real. It seems that we are justified to assume that their intuitions will be real. If (when) they are real, we can expect 'real' motivations, engagements, passions – and corresponding positions that they defend (in accordance to their intuitions). He explains that we do not ask students: "Imagine that refugees are on their way to our city, neighbourhood, and even to our school, etc.," but we say: "They are already coming etc." For sure, it is a fake information, perhaps morally questionable, but still an experiment and an interesting learning experience. If we would ask them: "Imagine that they come here etc.," they would, probably, respond in a politically correct manner, and the discussion about this issue and these kinds of issues of politics and morality would be quite (or perhaps even significantly) different. Therefore, we did not say: "Imagine that they come," but we said: "They are already coming." And the responses are expectable: "Yes, we understand the situation, but we cannot accept them."; "Why don't they send them to some other place?" Their real intuitions, revealed through the exercises, become a possibility to reflect upon them in contrast to the usual political correctness.

Boris Vezjak, in **The Ring of Gyges and the Philosophical Imagination**, asks this question: Is Plato's story of Gyges only a paradigmatic example of philosophical imagination and nothing else? What does it mean to say that? Imagining things is a part of philosophical activity and in itself has always been a tool for philosophizing. The dilemma is whether such imagination is only supplementary, or does it have a more fundamental part to play in constituting and developing philosophical arguments. Was Plato's idea about picturing the consequences of invisibility really meant to be something stronger than mythically inspired exercise in fictionalising? TEs have to be much more than simple exercises in fictional activity. It appears the interpretation about ethical TEs went beyond the bounds of Plato's possibility to conceptualize TE.

Reference list

Brown, J. R. (1991). *Laboratory of the Mind: Thought Experiments in the Natural Sciences.* London: Routledge.

—. (1999). *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*. London: Routledge.

—. (2004). "Peeking into Plato's Heaven". *Philosophy of Science*, 71, pp. 1126–1138.

Buzzoni, M. (2008). *Thought Experiments in Natural Science: An Operational and Reflexive Transcendental Conception*. Würzburg: Königshausen & Neumann.

Devitt, M. (2013). "Linguistic Intuitions are not 'the Voice of Competence'". In Haug, M. C. (Ed.), *Philosophical Methodology: The Armchair or the Laboratory?*. London: Routledge. p. 268.

Einstein, A., Infeld L. (1938). *The Evolution of Physics*. Originally from Cambridge University Press. Reprinted by Simon and Schuster. New York, 1966.

Elgin, C. (2006). "The Laboratory of the Mind". In Abel, G. (Ed.), *Kreativität: XX Deutscher Kongress für Philosophie, Kolloquiumsbeiträge*. Hamburg: Felix Meiner Verlag, pp. 772–784.

—. (2014). "Fiction as Thought Experiment". *Perspectives on Science*, 22, pp. 221–241.

Frith, C. (2007). *Making up the Mind: How the Brain Creates Our Mental World*. London: Wiley-Blackwell Publishing.

Greco, J. (2003). "Knowledge as Credit for True Belief". In DePaul, M., Zagzebski, L. (Eds.), *Intellectual Virtue: Perspectives from Ethics and Epistemology.* Oxford: Oxford University Press.

Häggqvist, S. (2009). "A Model for Thought Experiments". *Canadian Journal of Philosophy,* 39 (1), pp. 55–76.

Miščević, N. (2007). "Modelling Intuitions and Thought Experiments". Croatian Journal of Philosophy, 7 (2), pp. 181–214.

—. (2017). "Accounting for Thought Experiments – 25 Years Later". This book.

Nersessian, N. J. (1992). "In the Theoretician's Laboratory: Thought Experimenting as Mental Modelling" In Okruhlik, K., Hull, D., Forbes, M. (Eds.), *PSA 1992: Proceedings of the Biennial Meeting of the Philosophy of Science Association, Vol. 2*. Chicago: The University of Chicago Press, pp. 291–301.

—. (2007). "Thought Experiments as Mental Modelling: Empiricism without Logic". *Croatian Journal of Philosophy*, 7, pp. 125–161.

Norton, J. (1991). “Thought Experiments in Einstein's Work”. In Horowitz, T. Massey, G. (Eds.), *Thought Experiments in Science and Philosophy*. Lanham: Rowman & Littlefield, pp. 129–148.

—. (2004). “On Thought Experiments: Is There More to the Argument?”. *Philosophy of Science*, 71, pp. 1139–1151.

Plato. (1991). *The Republic*. Translated by B. Jowett, B. Vintage Classics edition.

Putnam, H. (1973). “Meaning and Reference”. *Journal of Philosophy*, 70 (19), pp. 699–711.

—. (1975). “The Meaning of ‘Meaning’”. In *Mind, Language, and Reality: Philosophical Papers* (Volume 2). Cambridge: Cambridge University Press, pp. 215–271.

Rawls, J. (1971). *A Theory of Justice*. Cambridge, Massachusetts: Harvard University Press.

—. (2001). *Justice as Fairness: A Restatement*, Kelly, E. (Ed.). Cambridge, Massachusetts: Harvard University Press.

Riggs, W. (2009). “Luck, Knowledge, and Control”. In Haddock, M., Pritchard, D. (Eds.), *Epistemic Value*. Oxford: Oxford University Press.

Searle, J. (1999). “The Chinese Room”. In Wilson, R. A., Keil, F. (Eds.), *The MIT Encyclopedia of the Cognitive Sciences*. Cambridge, Massachusetts: MIT Press, pp. 115–116.

—. (2010). “Why Dualism (and Materialism) Fail to Account for Consciousness”. In Lee, R. E. (Ed.), *Questioning Nineteenth Century Assumptions about Knowledge, III: Dualism.* New York: SUNY Press.

Sorensen, R. A. (1992). *Thought Experiments*. Oxford: Oxford University Press.

Williamson, T. (2007). *The Philosophy of Philosophy*. London: Blackwell.

—. (2013). “How Deep is the Distinction between A Priori and A Posteriori Knowledge?”. In Casullo A., Thurow, J. C. (Eds.), *The A Priori in Philosophy*. Oxford: Oxford University Press, pp. 291–312.

CONTRIBUTORS

❖ **Boran Berčić, University of Rijeka**

Boran Berčić earned his doctorate from the University of Zagreb. He is now Professor of Philosophy at the University of Rijeka, Croatia, where he teaches the courses Introduction to Philosophy, Metaphysics, and several electives. He has written a number of articles and five books: *Znanost i istina* (*Science and Truth*), HKD, Rijeka, 1995; *Realizam, relativizam, tolerancija* (*Realism, Relativism, Tolerance*), HKD, Rijeka, 1995; *Filozofija Bečkog kruga* (*Philosophy of the Vienna Circle*), KruZak, Zagreb, 2002; *Filozofija I* (*Philosophy I*), Ibis grafika, Zagreb, 2012; *Filozofija II* (*Philosophy I*), Ibis grafika, Zagreb, 2012.

❖ **Bojan Borstner, University of Maribor**

Bojan Borstner is a Slovenian philosopher. He received his doctorate at the University of Ljubljana, and is now Professor of Philosophy and Head of the Department of Philosophy at the Faculty of Arts, University of Maribor, Slovenia. His main research areas are ontology, philosophy of science, and the philosophy of nursing. He has published two books, *Problems of Realism* and *Virtual Teacher* (with B. Aberšek and J. Bregant), more than a hundred articles and shorter pieces, and edited three books – *Handbook of Phenomenology and Cognitive Science*, *Consciousness at the Crossroads of Philosophy and Cognitive Science*, and *Historicizing Religion: Critical Approaches to Contemporary Concerns*.

❖ **Janez Bregant, University of Maribor**

Janez Bregant is a Slovenian philosopher, with a Ph.D. from the University of Maribor, Slovenia, where he is now Associate Professor. His areas of interest are philosophy of mind and philosophy of artificial intelligence; his areas of competence are philosophy of art and critical thinking.

❖ **Marian David, University of Graz**

Marian David obtained his Ph.D. in 1990 from the University of Arizona, Tucson. After 20 years as Professor of Philosophy at the University of Notre Dame, Indiana, he now holds the Chair for Theoretical Philosophy at Karl-Franzens-University of Graz, Austria. He is co-editor of the journal

Grazer Philosophische Studien, and author of a number of works, including *Correspondence and Disquotation: An Essay on the Nature of Truth* (Oxford: New York, 1994), and "The Correspondence Theory of Truth" in Zalta, E. N. (Ed.): *The Stanford Encyclopedia of Philosophy*.

❖ **Peter Gärdenfors, Lund University Cognitive Science**

Peter Gärdenfors is Senior Professor of Cognitive Science at Lund University, Sweden. His main research interests concern models of concept formation, semantics of natural language, and the evolution of cognition. Some of his main publications are *Conceptual Spaces* (MIT Press, 2000), *How Homo Became Sapiens* (Oxford University Press, 2003) and *Geometry of Meaning* (MIT Press, 2014).

❖ **Smiljana Gartner, University of Maribor**

Smiljana Gartner is a Slovenian philosopher, currently Assistant Professor at the Faculty of Arts, University of Maribor, where she also obtained her doctorate. Her main areas of research are ethics (with a special interest in professional ethics), logic, and critical thinking. She has published more than twenty articles. She is also the president of the Slovenian Society for Analytic Philosophy and Philosophy of Science, and co-editor of *Historicizing Religion: Critical Approaches to Contemporary Concerns*.

❖ **Ferenc Huoranszki, Central European University**

Ferenc Huoranszki is Professor of Philosophy at the Central European University, Budapest. He works in philosophy of action and metaphysics. His current research interests include the problem of free will, abilities, dispositions, causation, and natural kinds. His last book is *Freedom of the Will: A Conditional Analysi*s (New York: Routledge, 2011).

❖ **Ilhan Inan, Bogazici University**

Professor Ilhan Inan received his doctorate from the University of California, Santa Barbara, in 1997 and has been lecturing at Bogazici University in Istanbul since then. Most of his research is within the philosophy of language, specialising in the semantic and epistemic aspects of our ability to conceptualise and refer to the unknown. He has published extensively both in English and in Turkish, and is the author of *The Philosophy of Curiosity* (Routledge, 2012), which is the first published philosophy book on curiosity.

❖ **Pierre Jacob, Institut Jean Nicod**

Pierre Jacob is a philosopher of mind and cognitive sciences. His earlier work was devoted to the metaphysics of intentionality from a naturalistic standpoint: can mental representations of the world be part of the represented world? In later work, in collaboration with the cognitive neuroscientist Marc Jeannerod, he has examined experimental findings providing evidence for sharp dissociations between visual perception and visually guided action. Much of his more recent work has been devoted to the study of human mindreading or theory of mind, i.e. the ability to ascribe psychological states to self and others. In particular, he has addressed the significance of the discovery of mirror neurons and mirroring processes in the brains of non-human primates for human mindreading. More recently, he has advocated a pragmatic resolution of the puzzle generated by discrepant developmental findings about false-belief understanding in human children and preverbal infants, according to whether this ability is being probed with explicit or implicit tests.

❖ **Friderik Klampfer, University of Maribor**

Friderik Klampfer, Associate Professor, lectures on ethics and political philosophy at the University of Maribor. He has published three books, an introduction to ethics, a collection of essays in bioethics, and a textbook in critical thinking (the last one in co-authorship) and several articles, both at home and abroad, spanning a wide range of topics, from moral responsibility, war, euthanasia, same-sex marriage, and capital punishment to issues in moral epistemology and philosophical methodology.

❖ **Rudi Kotnik, University of Maribor**

Rudi Kotnik, Associate Professor, lectures at the Faculty of Arts, University of Maribor, where he is involved in the domain of teacher education in general, and at the Department of Philosophy, where he is involved in philosophy education (didactics of philosophy) in particular.

❖ **Maja Malec, University of Ljubljana**

Maja Malec is Adjunct Assistant at the Faculty of Arts, University of Ljubljana. She obtained her Ph.D. at the Central European University in Budapest in 2010. The title of her doctoral dissertation was “The Essential/Accidental Distinction in Contemporary Metaphysics – The Modal and Definitional Characterizations”. She is the editor-in-chief of the journal *Analiza*, published by the Slovenian Society for Analytic Philosophy and Philosophy of Science.

❖ **Olga Markič, University of Ljubljana**

Olga Markič is Professor of Philosophy at the Faculty of Arts, University of Ljubljana and in the Middle European interdisciplinary Master's programme in Cognitive Science (Mei:CogSci). She is currently the president of the Slovenian Cognitive Science Society. Her main areas of research are philosophy of mind, philosophy of cognitive science, and neuroethics. Her recent publications include two monographs, *Kognitivna znanost: filozofska vprašanja* (2011) and *Mind in Nature: From Science to Philosophy* (together with M. Uršič and A. Ule, 2012).

❖ **Miomir Matulović, University of Rijeka**

Dr. sc. Miomir Matulović is Professor of Theory of Law and State and Philosophy of Law at the University of Rijeka, Faculty of Law, Croatia. He is the author of numerous books, collections of essays, and articles, including *Jezik, pravo i moral: Filozofija prava Herberta Harta* (*Language, Law, and Morality: H. L. A. Hart's Philosophy of Law*), 1986; *Ljudska prava: Uvod u teoriju ljudskih prava* (*Human Rights: Introduction to the Theory of Human Rights*), 1996; *Cleansing the Law of Legal Theory: A View from Croatia* (with Ivan Padjen), 1996; *International Law and the Use of Force at the Turn of Centuries. Essays in Honor of V. Đ. Degan* (with Vesna Crnić-Grotić), 2005; and *Kazneno pravo, Kazneno postupovno pravo i kriminalistika. Zbornik radova povodom 70. godine života Berislava Pavišića* (*Criminal Law, Criminal Procedure Law and Criminalistics. A Collection of Essays on the Occasion of Berislav Pavišić's 70th Birthday*) (with Eduard Kunštek).

❖ **Guido Melchior, University of Graz**

Guido Melchior is currently leading an FWF-project at the University of Graz. After receiving his doctorate there, he was a visiting scholar at the University of Maribor and at Rutgers University. His primary research interest is epistemology, including modal knowledge accounts and scepticism.

❖ **Nenad Miščević, University of Maribor**

Nenad Miščević wrote his Ph.D. thesis on "Theories of Communication Intention – Austin, Grice, Strawson" in 1983 at the University of Ljubljana. He is currently Professor of Philosophy at the Faculty of Arts, University of Maribor, Slovenia, and visiting professor at the Central European University, Budapest. He has been the president of the Croatian Philosophical Society, the president of the Union of Philosophical Societies of Yugoslavia, and the president of European Society for

Analytic Philosophy. He is editor of the *Croatian Journal of Philosophy*, co-editor of *Acta Analytica* and member of the editorial board of *International Studies in the Philosophy of Science* (Oxford), *Analiza* (Ljubljana, Slovenia), and *Agora* (Rijeka, Croatia). He is the author of fourteen books and more than a hundred and fifty articles and book chapters. He has also edited and co-edited ten books.

❖ **Matjaž Potrč, University of Ljubljana**

Matjaz Potrc is Professor of Philosophy at the University of Ljubljana, Slovenia. He has published several books, including *Austere Realism* (MIT Press), *Challenging Moral Particularism* (Routledge, Ed.), as well as many articles, and book chapters. Furthermore, he has been awarded several grants, from the French government (Kristeva, Lacan), AvH (Stegmueller) and Fulbright (Horgan). Moreover, he established the international Bled conferences and the journal *Acta Analytica* (Springer). Among many other functions, he was also the president of the Union of Yugoslav Philosophical Societies.

❖ **Howard Robinson, Central European University**

Howard Robinson is a professor and former provost of Central European University, Budapest. He is also a Senior Fellow of the Rutgers Center for Philosophy of Religion, New Brunswick, and a recurrent visiting scholar at Fordham University, New York. He has written on the philosophies of mind, perception, and religion, and the history of philosophy. His most recent work is *From the Knowledge Argument to Mental Substance: Resurrecting the Mind*, Cambridge University Press, 2016.

❖ **Peter Simons, Trinity College Dublin**

Peter Simons is Emeritus Professor of Philosophy at Trinity College Dublin, having previously taught at Bolton, Salzburg and Leeds. His main interests are in metaphysics and its applications, philosophy of language and logic, and the history of Central European and early analytic philosophy. A founder-member of the European Society for Analytic Philosophy, he is a Fellow of the British Academy, Member of Academia Europaea, and Member of the Royal Irish Academy. His output comprises two major books and over three hundred articles and shorter pieces.

❖ **Nenad Smokrović, University of Rijeka**

Nenad Smokrović is Professor of Philosophy at the Faculty of Humanities and Social Sciences, University of Rijeka. His main areas of interest are: philosophy of logic, theory of rationality, and philosophy of mind. He is

the author of two books and many articles. He is also the principal researcher in the international project: Rationality: Between Logical ideal and Everyday Reasoning.

❖ **Andraž Stožer, University of Maribor**

Andraž Stožer, MD Ph.D., is Assistant Professor of Physiology at the Faculty of Medicine, University of Maribor, where he is Head of the Institute of Physiology. Besides lecturing general human physiology to students of medicine in Maribor, he is a visiting lecturer at the Medical University of Vienna and also works regularly with students of biology and psychology. Scientifically, his main interest is insulin secreting beta cells of islets of Langerhans, the function of which his group is studying by means of advanced electro- and optophysiological methods. Philosophy has always been his passion and he has a decade-long tradition of research collaboration with the Department of Philosophy at the Faculty of Arts in Maribor.

❖ **Danilo Šuster, University of Maribor**

Danilo Šuster is Professor of Philosophy at the University of Maribor. He works mainly in epistemology, metaphysics, and philosophy of logic. He has edited a collection on free will, and is also the author of a textbook on logic, a monograph on informal logic, and many articles in the general area of analytic philosophy. He is also the editor-in-chief of the *Acta Analytica* journal.

❖ **Tadej Todorović, University of Maribor**

Tadej Todorović is a student of philosophy and English language and literature at the Faculty of Arts, University of Maribor. He is currently in the final year of his M.A. studies, writing his philosophy thesis on the problem of multiple realizability, and his English language and literature thesis on the connection between thought experiments and dystopias.

❖ **Majda Trobok, University of Rijeka**

Majda Trobok is Professor of Logic at the University of Rijeka, Croatia. She works mainly on the philosophy of logic and philosophy of mathematics. She is currently taking part in two international projects: “Logic, Concepts and Communication” and "Rationality: Between Ideal and Everyday Reasoning”.

❖ **Andrej Ule, University of Ljubljana**

Andrej Ule is Professor Emeritus of Analytic Philosophy and Philosophy of Science at the Faculty of Arts, University of Ljubljana, where he obtained his doctorate in 1981. His main fields of interest are analytic philosophy, Wittgenstein's philosophy, epistemology, philosophy of science, and comparative philosophy.

❖ **Boris Vezjak, University of Maribor**

Boris Vezjak is Associate Professor at the Department of Philosophy at the Faculty of Arts in Maribor, Slovenia. His fields of interest cover various topics, such as the history of philosophy, theory of discourse, media analysis, and theory of argumentation.

❖ **Timothy Williamson, University of Oxford**

Timothy Williamson is the Wykeham Professor of Logic at Oxford University and visiting professor at Yale. He has also taught at Edinburgh, Dublin, MIT, Princeton, the Chinese University of Hong Kong, and elsewhere. He is the author of *Identity and Discrimination*, *Vagueness*, *Knowledge and its Limits*, *The Philosophy of Philosophy*, *Modal Logic as Metaphysics*, *Tetralogue*, and many articles. He is a Fellow of the British Academy, member of the Academia Europaea, and foreign honorary member of the American Academy of Arts and Sciences.

Ivo Petricioli* (1925–2009). *Caricature of Nenad Miščević*, mid-1980s
*Professor Ivo Petricioli, PhD, art historian and archaeologist, member of Croatian Academy of Sciences and Arts, was Nenad's friend and colleague at the Faculty of Arts in Zadar.

GRASPING THE BASIC ARITHMETICAL CONCEPTS: THE ROLE OF IMAGINATIVE INTUITIONS[1]

MAJDA TROBOK
UNIVERSITY OF RIJEKA

Before starting the discussion about the role of intuitions in the perception of mathematical truths, it is necessary to say a few words about what intuitions are in the first place. In addition, when talking about intuitions, we have to keep in mind that different philosophers characterise such notions differently and that – as in most philosophical discourses – there is no agreement on what intuitions are and what *kind* of a role intuitions play, if at all, in the descriptive epistemic context.

Williamson, to mention just few of the most appealing theories on intuitions, proposes the view that:

> "So-called intuitions are simply judgments (or dispositions to judgment); neither their content nor the cognitive basis on which they are made need be distinctively philosophical" (Williamson, 2007: 3).

Bealer, on the other hand, describes intuitions as an intellectual capacity of seeming:

> "For you to have an intuition that A is just for it to *seem* to you that A. Here 'seems' is understood, not as a cautionary or 'hedging' term, but in its use as a term for a genuine kind of conscious episode. /…/ this kind of

[1] Research for this paper was carried out under the project "Logic, Concepts, and Communication". The project is funded by the Croatian Science Foundation. IP-2014-09-9378.

> seeing is *intellectual*, not sensory or introspective (or imaginative). The subject here is *a priori* (or rational) intuition"[2] (Bealer, 1996: 123).

Other authors (e.g. Parsons, Resnik, Miščević) – without denying the existence of intellectual intuitions – in the context of arithmetical knowledge preferred to concentrate on the quasi-perceptual, i.e. imaginative intuitions instead. According to them, *these* intuitions are basic not just for grasping geometrical truths, but also for the epistemic process of grasping the basic arithmetical concepts such as the concepts of equinumerosity and (natural) number.

In this paper, I will direct my criticism at Nenad Miščević's theory and concentrate on the arithmetical case.

I am interested in Miščević's view not just because it is appealing *per se*, as an imaginative-intuition-basic-role view, but because it also contains the epistemic structuralists' tenets in the context of mathematical knowledge. Mainstream structuralism, even though not homogeneous as a theory, epistemically amounts to the view that we grasp (mathematical) structures *via* grasping the systems that exemplify such structures. And such a process, at least in some segments, is based on quasi-perceptive intuition and imagination.

Last but not least, quasi-perceptual intuitions supposedly offer an alternative to the platonic intuition since:

> "/.../ epistemological Platonism with its postulation of a mysterious faculty of grasp of abstract objects does not offer any real explanation of numerical intuitions" (Miščević, 2016: 25).

To reiterate, I will concentrate on the epistemic view, according to which quasi-perceptual or imaginative intuitions play a central role in the perception of mathematical truths, and according to which intuitions represent the tool that we use in order to grasp the basic mathematical concepts, such as equinumerosity and the concept of (natural) number, the latter being to my mind far more controversial than the former.

---

[2] Interestingly, Bealer's intuitions have to be distinguished from beliefs. In fact, we might believe, when confronted with the proof of it (Cantor's diagonal procedure), that there are more real than natural numbers, despite the (intellectual) intuition, i.e. the fact that it still does not *seem* to be the case to us. On the other hand, it is possible to have the intuition that, for example, sums with an infinite number of addends *has* to be infinite, despite the fact that we do not believe it, since we know it is not (always) the case.

The idea is that quasi-perceptual or imaginative intuition represents the basis not just for geometrical, but also arithmetical knowledge, and that it plays a major epistemic role in the process of gaining mathematical knowledge.

Let us start with Miščević's characterisation of the nature of quasi-perceptual, i.e. imaginative (meaning image-based) intuition.

In Miščević's terms such an intuition amounts to

> "a belief-producing response that is *phenomenologically immediate* and accompanied by a feeling of *obviousness and certainty*" (Miščević, 2016: 7, my emphasis).

Image-based intuitions are hence non-inferentially formed beliefs that 'force themselves upon us' as being obviously true. Most primitive intuitions – and imaginative intuitions are such – are those connected to and forming the core of arithmetical knowledge.

According to Miščević, quasi-perceptual or imaginative intuitions represent the initial stage and the basis for the grasp of arithmetical truths.

Let us have a look at the way in which we grasp most of fundamental arithmetical notions: the concept of one-to-one correspondence and that of equinumerosity.

I shall start by taking a very simple example of someone inviting friends for dinner and waiting to offer some delicious baked apples for dessert. The dinner host prepares the apples and wants to be sure that there is exactly one apple for each guest and, of course, that each guest gets their dessert. How does the dinner host, prior to baking the apples, check if there are enough apples for the guests without having any leftovers? According to Miščević, the dinner host pairs each of the apples in front of him with the mental image of each of the invited guests. Such a process is what visual perception combined with imagination allows him to do. At this point, the quasi-perceptual (or imaginative) intuition enters the picture and the dinner host can 'see' that there are as many apples as there are guests. In technical, mathematical terms, he comes to know that there is a one-to-one correspondence (a bijection) between the group of apples and the list of images of guests. According to Miščević, the dinner host comes to grasp the notion of equinumerosity at the same time. He therefore grasps those concepts (one-to-one correspondence and equinumerosity) *via* imaginative intuition.

The whole process is basically non-inferential, based only on intuition. In defending his view, Miščević focuses on what developmental psychology has to say on how we grasp concepts and appeals to the experimental results of cognitive psychologists, such as Susan Carey.[3] Carey's theory, based on thousands of experiments, represents the contemporary mainstream theory of how children grasp the concepts of one-to-one correspondence, equinumerosity, and (natural) number.

I shall firstly analyse Miščević's views in connection with the concept of one-to-one correspondence and equinumerosity, and secondly in connection with the concept of (natural) number.

Let us hence first examine Miščević's idea of the notion of one-to-one correspondence and equinumerosity being intuitively given to us.

Why is such a view contentious?

I suggest two arguments that show Miščević's theory is problematic: (a) the argument from the history of mathematics, i.e. from the historical point of view and (b) the argument from psychology, i.e. from the psychological point of view.

(a) The argument from the historical point of view.

Such an argument is based on the methodological route according to which 'history is the teacher of epistemology'. Namely, I take the accepted methodology for epistemology of mathematics to be (Kitcher's) pragmatic naturalism, and the view that the epistemological route follows the historical one.[4] As Kitcher would put it, "the epistemological order of mathematics broadly recapitulates the historical order" (Kitcher, 2011: 518).[5] Since someone might object to the acceptance of such methodology, I will first try to justify the use and relevance of Kitchers' motto, and

[3] Carey, 2009.

[4] My endorsement of Kitcher's pragmatic naturalism is confined to his view that history is the teacher of epistemology. In ontology, I endorse Platonism in the philosophy of mathematics, and even though Kitcher's aim is to deny a priori knowledge, I suggest his history-is-the-teacher-of epistemology motto turns out to be self-defeating with respect to his denial of the platonic perception and a priori knowledge in general. For more details, see Smokrović and Trobok, "Mathematics and Pragmatic Naturalism".

[5] See his "Epistemology without History is Blind".

second, show why the history-is-the-teacher-of-epistemology perspective makes Miščević's theory less plausible.

Three are three aspects that, taken together, give us reason to endorse the view that 'epistemology without history is blind' and that history should be the teacher of epistemology: the acceptance of Williamson's expertise defence, the underlying ontology taken to be Platonism, and, finally, the distinction between the context of acquisition and the context of discovery.

Williamson's expertise defence concerns the genuine expertise in thought experiment.

As he puts it:

> "Yet philosophy students have to learn how to apply general concepts to specific examples with careful attention to the relevant subtleties /.../ We should not regard philosophical training as an illegitimate contamination of the data, any more than training natural scientists how to perform experiments properly is a contamination of their data. Although the philosophically innocent may be free of various forms of theoretical bias, just as the scientifically innocent are, that is not enough to confer special authority on innocent judgment, given its characteristic sloppiness" (Williamson, 2007: 191).

The expertise defend can, however, *mutatis mutandis* be applied in the epistemic context of acquiring arithmetical knowledge too. It suggests that we ought to look at the practice of working mathematicians in order to know how we grasp mathematical concepts. Looking elsewhere, to the experience of non-mathematicians, is misleading and uninformative. Looking elsewhere would also mean to ignore the distinction between the context of acquisition and the context of discovery. The former being about the process of *learning* the basic arithmetical notions, while the latter is about the way of *grasping* and subsequently introducing new mathematical concepts/theories in the mathematical practice. To focus, as Miščević suggests, on the results of cognitive psychologists, such as Carey[6] and the way in which children come to know about mathematics, is to shift the discussion from the context of discovery to that of acquisition. Moreover, to insist on Carey's results, firstly takes misleadingly the focus of attention to be the process of acquiring and secondly, it turns out to be a 'turn the tables' for Miščević's proposal. And the latter is due to the fact that Carey's results do not imply that imaginative intuitions are basic for

[6] Carey, 2009.

the process of acquisition of basic arithmetical notions. Or at least this is what I shall hopefully show is the case.

The last but not the least, the underlying ontology is (a version of) realism. Presumably, what mathematicians assert as being true about mathematics is, by and large, true. To look at singular/personal representations would be – in Frege's terms – to look at "'ideas' used in the psychological sense" or at "mental pictures or acts of an individual mind" (Frege, 1884/1960: Introduction) and that is *not* Miščević's aim.

Now that I have, hopefully, offered reasons for endorsing the history-is-the-teacher-of-epistemology motto, let us have a look at what the history of mathematics has to say with respect to the importance of image-based intuitions in the epistemic domain. I will take into consideration the concepts that Miščević's theory focuses on: firstly the concepts of one-to-one correspondence and equinumerosity and, later on, the concept of (natural) number.

Historically, the notion of one-to-one correspondence was not introduced in the mathematical practice without some turbulences. There were two candidates for the equinumerosity criteria: the rule that 'the whole is greater than the part' and the one-to-one correspondence. The former was introduced by Euclid in his *Elements* as Axiom 5, and was taken as obvious by mathematicians through centuries. On the other hand, equinumerosity, i.e. the existence of a one-to-one correspondence being the criterion for the sameness of cardinally, is a rather recent result. There were different proposals for the criteria for sameness of cardinality from Galileo to Bolzano. Intuitively, if A is a proper part of B, it follows that the cardinality of A is smaller than that of B. That seemed self-evident to mathematicians from Euclid onwards.[7] Of course, the controversy here was due to the infinite domain, contrary to the case of (small) finite sets of objects. On the other hand, the concept of infinity is not novel in mathematics or philosophy – the Greek word *to apeiron* (the infinite) appeared for the first time in early Greek thought with Anaximander of Miletus in 6th century BC, while the 'first real glimpse into the mathematical infinite' is taken to be the Pythagoreans' discovery that $\sqrt{2}$ is

[7] Axioms at the time were taken to be self-evident truths that did not have to be proved.

not a rational number in roughly 450 BC.[8] The notion of equinumerosity is hence not as intuitively clear as it might appear at first sight.

(b) The argument from the psychological point of view.

My next step is to show that the acceptance of Carey's theory is compatible with the refusal of (at least a part of) Miščević's theory.

My aim is to show that even if we accept Carey's theory (as Miščević suggests) there is still place for questioning Miščević's tenets. According to him, even children are capable of grasping the concept of one-to-one correspondence as well as natural number via imaged-based intuition. The studies that allegedly support such a view are Carey's. So let us start by having a look at the way in which children grasp the concept of one-to-one correspondence. According to Carey:

> "Each proposal assumes that the child first learns 'one, two, three, four, five…' as a list of meaningless lexical items. This is the no numeral knower of stage documented above. There is no doubt that children have the capacity to learn meaningless ordered lists of words – they learn sequences such as 'eeny, meeny, miny, mo' the alphabet, the days of the week, and so on. Indeed, non-human primates have this capacity, and so it is likely part of the innate computational machinery (e.g. Terrace, Son & Brannon, 2003). This step in the learning process - learning the arbitrary ordered list ('one, two, three, four, five, six …') is a paradigmatic example of one aspect of Quinian bootstrapping: the meaning of the counted words are exhausted initially, by tier interrelations, their relative order in the list. At his point of the process, the verbal numerals are placeholders with respect to the numerical meaning they will come to have" (Carey, 2009: 308).

Gelman and Gallistel,[9] whose work Carey's is based on, point out that:

> "The pre-schooler's normal principle for determining whether two sets are numerically equal is 'Count them and see' /…/ [T]he child's procedure actually presupposes the establishment of a one-to-one correspondence. In counting, the child establishes a one-to-one correspondence between the elements in his count sequence and the elements in the set being counted. From a logical point of view, the child's procedure for deciding numerical equivalence depends upon the fact that the numerosity of both sets can be placed in a relation of one-to-one correspondence with the same set of counting tags. But the child does not ordinarily take cognizance of the

[8] For more details, see Moore, 1990.
[9] Gelman and Gallistel, 1978.

> transitivity of one-to-one correspondence. He ignores or is indifferent to the fact that the cardinal numerous representing two equally numerous sets are identical precisely because both sets have been placed in one-to-one correspondence with that cardinal numeron" (Gelman, Gallistel, 1987: 198–199).

The idea is that children might be able to count, which means being able to answer to questions like: "How many…?" without having any understanding of one-to-one correspondence, or the concept of cardinality. In his article, Heck nicely explains what 'counting' amounts to for children:

> "Think of the matter from the child's point of view. I have been taught how to 'count': I have, that is to say, been taught to point at objects and say certain words. I have to say the words in a certain order. I'm supposed to count each of the objects once and only once. And most importantly, when I get to the end, I'm supposed to say the last of the words loudly and proudly. But what all of this has to do with anything, who knows? The grownups seem to like it and they're forever facing over me when I do it right. That's more than enough reason for me to play" (Heck, 2000: 197).

What Heck is proposing is not just a mere theoretical possibility; his point is well known to psychologists whose views Miščević is endorsing as well.

The imaginative intuition, as meant by Miščević, is not just the basic tool for grasping the notion of equinumerosity, but arithmetical knowledge in general.

I shall now concentrate on the epistemic status of the other basic arithmetical concept: that of (natural) number.

Usually, when connecting imaginative intuitions and mathematics, the first association that comes to our mind is that of geometry. Of course, we might think of Frege and his drawing of the analogy that holds between geometry and arithmetic, but such an analogy is to be confined to a very precise and strict context that Frege introduces. What Miščević aims to do is to spread the analogy outside the limits in which Frege confines it. While Frege introduces the two abstraction principles[10] – that of Direction[11], and Hume's principle[12] – the geometry-arithmetic analogy is

[10] Frege, 1884/1960: paragraph 64.

[11] Principle of direction: for any two lines, *a* and *b*, the direction of the line *a* is identical to the direction of the line *b*, iff the two lines are parallel. In symbols: $\forall a \, \forall b \, (\mathrm{d}(a)=\mathrm{d}(b) \leftrightarrow a \parallel b)$.

limited to the way in which we grasp the two one-to-one correspondences involved: parallelism and equinumerosity. They are both intuitively clear to us, Frege would say. To get from parallelism to the concept of direction of lines or, *as in Hume's principle*, from equinumerosity to the concept of number, intuitions play no role. Miščević, contrary to Frege, boldly asserts that even the way we come to the concept of (natural) number is based on image-based intuitions.

I shall examine such a theory from the psychological point of view first, and secondly concentrate on the perspective of the history of mathematics (as I did in the case of one-to-one correspondence and equinumerosity).

In we go back to the cognitive psychologists' view, than the first step in a children's process of acquisition of the concept of (natural) numbers is taken to be the so called AMR – Analogous magnitude representation.

> "AMRs are primitive representations of spatial, temporal, numerical, and related magnitudes. They are primitive because they represent magnitudes without presupposing the ability to represent any units of measurement or mathematically defined system of numbers. /…/ They are present in six-month-old human infants, a wide variety of mammals, many birds, and at least some fish" (Beck, 2015: 1).

Experiments and extensive studies have shown that both human and non-human animals have a system of language-independent mental magnitudes.[13] AMR-s obey Weber's law, which is the ability to discriminate between two magnitudes determined by their ratio. As the ratio of two magnitudes approaches 1:1 they become harder to discriminate, and beyond a certain rate (threshold), determined by the subject's 'Weber constant', they cannot be discriminated at all. AMR-s, however, are still far from allowing the grasp of natural numbers. And the step that allows us to grasp the notion of natural number from that of equinumerosity is far from trivial, and even further from being based on (imaginative) intuition.

---

[12] Hume's principle: $\forall F \forall G (n(F)=n(G) \leftrightarrow F \approx G)$; n(F) is the number that belong to the concept F, while '≈' is the (equivalence) relation of equinumerosity.

[13] One of the most interesting experiments is the one performed on ducks by Harper (see his 'Competitive Foraging in Mallards: Ideal Free Duck'). In the experiment, two persons are tossing morsels of bread to the ducks at two different rates. Within a minute, the ducks divide themselves in proportion to those rates. If the first person starts tossing morsels that are twice the size of the morsels tossed by the second person, the ducks alter their strategy, and within five minutes divide themselves in proportion to the (new) product of the morsel size and feeding rate.

Let us apply the history-is-the-teacher-of-epistemology motto in this discourse too. Historically, grasping the concept of natural number from that of equinumerosity was a process that had been going on for many centuries: from the Pythagorean theory of numbers to Hume's principle. Even though the Pythagoreans exhaustively developed the theory of natural numbers, it was not until Frege's theory that the notion of (natural) number was (properly) defined. Before that, we saw 'everything as through a fog, blurred and undifferentiated'. Let me call this argument the historical-mathematical argument. Someone might object that what is first in itself is last for us, but such an objection is a grist to my mill, since it stresses the distance between what is obvious to the naïve cognizer and what is the ultimate basis of the rest. The proper (implicit) definition of (natural) number is Hume's principle and it allows the transition from the notion of equinumerosity (which is a categorising relation) to the notion of (natural) number (which is an object).

According to Miščević, Hume's principle – taken informally – does play a crucial role in arriving at the notion of natural number.

However, any attempt to show that such a principle is in fact intuitively clear, or that the grasping process of it is based on quasi-perceptual (or imaginative) intuition is, I would suggest, highly controversial, since there is no practico-empirical or historico-mathematical support/evidence for it.

Even though Miščević does not ignore the mathematical practice, he does not look at the historical-mathematical side of it, deciding to concentrate rather on what developmental psychology has to say on how we grasp the notion of (natural) number.

In defending his view, Miščević appeals again to the experimental results of cognitive psychologists, such as Carey and her theory of how children grasp the concept of number, 'a' – as she calls it – 'quintessential abstract entity'. What she has in mind when talking about numbers is the standard realist view:

> "Number representations are conceptual; their content goes beyond spatiotemporal and sensory vocabulary" (Carey, 2009: 134).

She goes on specifying:

> "The domain-specific perceptual analysers that encode number, as well as the arithmetic computations defined over the resulting representations, are

> evolutionary ancient, most likely innate, and operate throughout the life span" (Carey, 2009: 134).

So how does that mechanism of acquiring the notion of number work, according to cognitive psychology?

During the practice of language acquisition, children learn linguistic quantifiers, as well as the singular/plural distinction. During this process, they *learn* to group objects (or representations of objects, document-files), e.g. *a*, *b*, *c*, into two-membered sets:

$\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and finally, the three-membered one $\{a, b, c\}$.

The model for number one is the content of the singleton, the atom – as Carey calls it. The child is correlating the atom (or the correspondent representation of the atom) with the word 'one' from the long-term memory, as the child might correlate any atom with the word 'one'. The process is analogous for the word 'two', which the child learns to apply just to those sets whose members can be put into one-to-one correspondence with the model $\{a, b\}$, and the word 'two' applies to the totality of pair-sets. According to Carey, this represents the way in which children grasp the number two. The story is analogous for all the other (natural) numbers.

Carey's theory leaves open the question about the intuitive notion of number that we have. What exactly does our intuition of number amounts to? And where does it enter the picture? Perhaps in the epistemic process of acquiring the notion?

Going back to Frege, Miščević asserted that Hume's principle – taken informally – does play a crucial role in arriving at the notion of natural number. However, any attempt to show that such a principle is in fact naively intuitively clear, or that the grasping process of it is based on quasi-perceptual (or imaginative) intuition is, I would suggest, highly controversial, since there is no practico-empirical or historico-mathematical support for it.

There is nothing intuitive, quasi perceptual in Hume's principle, in the same way in which there is nothing intuitive about the 'direction of the line', no matter how the relation of being parallel might be and is intuitively clear. Frege is right in insisting that the number-case is, in that sense, analogous: while equinumerosity might be intuitively accessible, it

is hard to see how the notion of number is given to us through our intuitions.

I would like to go one step further and claim that we do not have sufficient practico-empirical or historico-mathematical evidence in favour of the view that grasping such concepts is based on imaginative intuitions, neither for the concept of equinumerosity, nor for that of (natural) numbers.

Reference list

Bealer, G. (1996). "A Priori Knowledge and the Scope of Philosophy". *Philosophical studies,* 81 (2–3), pp. 121–142.

Beck, J. (2015). "Analogue Magnitude Representations: A Philosophical Introduction". *British Journal for the Philosophy of Science*, 66 (4), pp. 829–855.

Carey, S. (2009). *The Origin of Concepts*. Oxford: Oxford University Press.

Frege, G. (1884/1960). *The Foundations of Arithmetic* (*Die Grundlagen der Arithmetik*). Tr. Austin, J. L., second edition. New York: Harper.

Gelman, R., Gallistel, C. R. (1978). *The Child's Understanding of Number*. Cambridge: Harvard University Press.

Harper, D. G. C. (1982). "Competitive Foraging in Mallards: Ideal Free Ducks". *Animal Behaviour*, 30, pp. 575–84.

Heck, R. G. (2000), "Cardinality, Counting, And Equinumerosity". *Notre Dame of Formal Logic*, 41 (3), pp. 187–209.

Kitcher, P. (2011). "Epistemology without History is Blind". *Erkenntnis,* 75 (3): pp. 505–524.

Miščević, N. (2016). "Understanding Numerical Intuitions". In Arsenijević, M, Lazović, Ž. (Eds), *Svetlani Knjazevoj za 85. rođendan - Zbornik radova*. Beograd: Insitut za filozofiju i Univerzitet u Beogradu. In print.

Moore, A.W. (1990). *The Infinite*. New York: Routledge.

Smokrović, N., Trobok, M. (2014). "Mathematics and Pragmatic Naturalism". *Synthesis Philosophica*, 28 (1–2), pp. 263–270.

Williamson, T. (2007). *The Philosophy of Philosophy*. Oxford: Wiley-Blackwell.

NENAD'S REPLY

Majda, thanks for all the compliments and the nice history of our cooperation! Majda mentions that I taught her philosophy of math; she generously forgets to mention that she taught me the little math that I know. She did make a lot of effort, but I just was not a good enough pupil!

Let me just say a few words about my own proposal. Majda and I agree in contrasting quasi-perceptual to purely intellectual intuition; she prefers the latter, I do the former. My motivation is that starting from quasi-perceptual intuition we can appeal to patterns in the real world as possible sources of mathematical knowledge, we can integrate the results of structuralism and keep as close to naturalism as we like. Our concrete examples is Hume's principle, according to which the same number belongs to two collections iff they can be put into a 1-1 correspondence, i.e. iff they are equinumerous. We consider it in Frege's context, who first introduces his definition of the sameness of direction for lines, and then passes to Hume's principle and his theory of what a number is. My claim is that cognizers originally understand 1-1 correspondence in a quasi-perceptual, imaginative way, and that this understanding plays a (moderate) justificatory role for the rest; similarly for sameness of direction and parallelism.

There are three groups of claims that Majda uses against my quasi-perceptual proposal. The first concentrates upon the claim that there is "nothing intuitive, quasi perceptual in Hume's principle, in the same way in which there is nothing intuitive about the "direction of the line", no matter how the relation of being parallel might be and is intuitively clear". She writes: "while equinumerosity might be intuitively accessible, it is hard to see how the notion of number is given to us through our intuitions".

The second group of claims is much stronger. She claims that neither for the concept of equinumerosity nor for that of (natural) numbers do we have sufficient practico-empirical evidence in favour of the view that grasping such concepts is based on imaginative intuitions.

The third group introduces history of mathematics and of theorizing about it: there is no "historico-mathematical evidence in favour of the view that grasping such concepts is based on imaginative intuitions". But her other formulation is much more moderate:

> "There were two candidates for the equinumerosity criteria: the rule that "the whole is greater than the part" and the one-to-one correspondence. The former was introduced by Euclid in his Elements as Axiom 5, and was taken as obvious by mathematicians through centuries. On the other hand, equinumerosity, i.e. the existence of a one-to-one correspondence being the criterion for the sameness of cardinally, is a rather recent result. There were different proposals for the criteria for sameness of cardinality from Galileo to Bolzano. Intuitively, if A is a proper part of B, it follows that the cardinality of A is smaller than that of B. That seemed self-evident to mathematicians from Euclid onwards. Of course, the controversy here was due to the infinite domain, contrary to the case of (small) finite sets of objects. /…/ The notion of equinumerosity is hence not as intuitively clear as it might appear at first sight".

Why would this be important? Because history is the teacher of epistemology, she explains, and I agree. If there were really no historical evidence in favour of my view, that would be very bad for it.

Let me move on to the defence. First, the general point. I am happy to agree that "equinumerosity might be intuitively accessible"; I think that, in fact, it is. This might be enough for me, although I would be happy if I could prove that first steps to "sameness of number" are also performed in a quasi-perceptual setting.

Second, the psychological point of view. We both take Suzan Carrey[1] as our main source. Majda concentrates on her description of later stages of number acquisition, where she stresses the conceptual nature of mechanisms at work. However, when describing, the most elementary mechanism are not necessarily purely conceptual. Here is a typical example:

> "/…/ a long-term memory model of a set of two individuals could be created and mapped to the linguistic expression for a dual marker or "two", and so on for "three" and "four". These models could contain abstract symbols for individuals ({i}, {j k}, {m n o}, {w x y z}) or they could simply be long-term memory models of particular sets of individuals ({Mommy}, {Daddy Johnnie} …)" (Carey, 2009: 324).

[1] Carey, S. (2009). *The Origin of Concepts*. Oxford: Oxford University Press.

Take the couple Daddy-Johnnie; no need for abstract symbols, a concrete long term memory model will do, and the model, since it is not abstract, is for sure quasi-perceptual. Now, how does the child arrive to a number?

> "What makes these models represent "one" "two" and so forth is their computational role. They are deployed in assigning numerals to sets as follows: The child makes a working-memory model of a particular set he or she wants to quantify {e.g., cookie cookie}. He she then searches the models in long-term memory to find that which can be put in 1–1 correspondence with this working-memory model, retrieving the quantifier that has been mapped to that model" (Carey, 2009: 324).

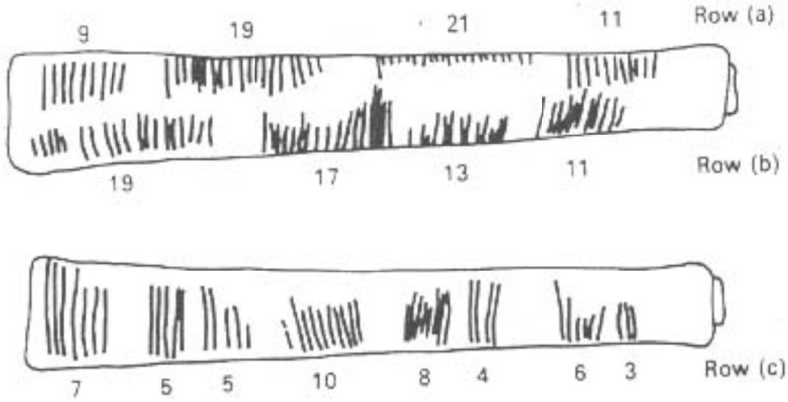
This is exactly what my view would predict: the numerosity is arrived at by putting in 1-1 correspondence elements from the working memory model, probably not abstract at all. And this is valid for pre-linguistic children:

> "All of the computational resources required for enriched parallel individuation are known to be available to prelinguistic infants. Prelinguistic infants create working-memory models of at least two separate sets and compare these on the basis of 1–1 correspondence" (Carey, 2009: 324).

Larger numbers may require more and more abstract representations. But my claim is that concrete, quasi-perceptual representation of 1-1 correspondence plays the crucial role. And Carey would agree with this.

Let me pass to the history-is-the-teacher-of-epistemology argument. Unfortunately, I have to stay brief. First, let me note that the ancient counting methods mostly rely on 1-1 correspondence. Take the method of tallies: the agent counts sheep by assigning one tally to each sheep, and not more than one. Five tallies, five sheep. Obviously, an awareness of 1-1 correspondence is there. Now the tallies and the sheep are given in perception; the correspondence will be perceptual or quasi-perceptual.

Next, cutting notches. Look at the reproduction of the Ishango Bone, discovered in 1960 at Ishango in Congo.



Supposedly, the agent counts items, say coconuts, by pairing each coconut with a notch. Seven notches, seven coconuts. Looks like a clear application of 1-1 correspondence in quasi-perceptual or perceptual setting. I think we may safely assume that this kind of counting was omnipresent in the time of the Greek civilization, and that theorists were very well aware of it.

I admit that it is almost impossible to find an explicit mention of 1-1 correspondence. However, some formulation comes close to it: Euclid's 'exact coincidence' in Axiom 4 of Book 1 of his *Elements*[2] is a good example. The Greek word is 'epharmozonta' εφαρμοζοντα, and the Axiom famously says that things which coincide with one another are equal to one another. Since number measures any multitude (plethos), set-like or geometrical, the Axiom should apply to collections, and in this case the exact coincidence would amount to 1-1correspondence.

Aristotle talks about abstracting number from a collection; take a collection of five sheep and of five deer (he wants collections to be similar, and this has been creating problems with generality).[3] What is common is, of course, the number of units, where each sheep is abstracted into one, and each deer as well. The two collections will then coincide, and

[2] Heath, T. L. (1908). *The Thirteen Books of Euclid's Elements*. Cambridge: Cambridge University Press.
[3] See Anas, J. (1976). *Aristotle's Metaphysics, Books M and N*. Oxford: Clarendon Press, p. 28.

the common number will be five. More material can be quoted, but I must stop here.

I agree with Majda that most formulations are far from being clear. My guess is that the pattern of 1-1 correspondence, in its simple, quasi-perceptual form, is in the background all the time; the reason why it is not made explicit is that mathematicians and philosophers are interested in more challenging matters, including the application of number to geometrical items (Euclid, books VII and VIII for sure), initial issues of infinity, plus, for philosophers, the issues of the ontology of number, its relation to Ideas and the like.

To conclude, let me mention an additional argument in favour of quasi-perceptual intuition. Majda rejects it, but if rejected, what are the alternatives open to us? If we don't want straightforward empiricism, we have a choice between logicism and intellectual intuitionism. Logicism is certainly not the way ordinary people have come to their number concept; neither from the historical, nor from the psychological point of view. The other alternative, pure intellectual intuitionism, postulates a faculty for which there is no explanation at all. So why not stay with the quasi-perceptual intuition?